Normal-mode models OGOPOGO and NOGRP applied to the 2006 ONR Reverberation Modeling Workshop problems

Dale D. Ellis
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Dale D. Ellis
Abstract

A Reverberation Modeling Workshop was held in Austin, Texas, 7–9 November 2006. Participants were asked to describe their models, and submit solutions to a number of problems. The problems were described in a number of documents accessible over the internet. Twenty problems were specified. All were shallow water environments, with various geometries, and scattering functions. Each problem typically had three frequencies, three receiver depths, and several pulse bandwidths; a towed array was specified for 5 problems. All problems used a homogeneous infinite seabed. Doing all the problems would require hundreds or even thousands of runs, so people were selective. There were essentially 3 classes of problems: 2D computationally intensive physics-based models with realizations of rough bottoms, 3D flat-bottom (pseudo-monostatic) problems with scattering functions (Lambert’s rule, and perturbation theory), and 3D bistatic range-dependent problems (with a towed array). This paper presents an overview of the DRDC Atlantic normal-mode reverberation models OGOPOGO and NOGRP. Comparative results are presented for a number of the 3D range-independent problems, including bistatic geometry, and towed array beam patterns.

Résumé

Un atelier sur la modélisation de la réverbération a eu lieu à Austin, au Texas, du 7 au 9 novembre 2006. On a demandé aux participants de décrire leurs modèles et de soumettre des solutions pour un certain nombre de problèmes. Les problèmes étaient décrits dans un certain nombre de documents accessibles sur Internet. Vingt problèmes étaient spécifiés. Tous portaient sur des environnements en eaux peu profondes, avec diverses géométries et fonctions de diffusion. Chaque problème avait typiquement trois fréquences, trois profondeurs de récepteur et plusieurs largeurs de bande des impulsions; un réseau remorqué était spécifié pour 5 problèmes. Tous les problèmes présumaient un fond marin infini homogène. La résolution de tous les problèmes exigerait des milliers d’essais, de sorte que les participants étaient sélectifs. Il y avait essentiellement 3 classes de problèmes : modèles 2D basés sur la physique et exigeant un grand nombre de calculs avec des réalisations de fonds rugueux, des problèmes de modèles 3D (pseudo monostatiques) à fond uni avec des fonctions de diffusion (loi de Lambert et théorie des perturbations) et des problèmes de modèles 3D bistatiques dépendant de la distance (avec réseau remorqué). Le présent rapport donne un aperçu des modèles de réverbération en mode normal OGOPOGO et NOGRP de RDCC Atlantique. Des résultats comparatifs sont présentés pour un certain nombre des problèmes de modèles 3D indépendants de la distance, y compris la géométrie bistatique et des diagrammes des faisceaux de réseau remorqué.
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Executive summary

Normal-mode models OGOPOGO and NOGRP applied to the 2006 ONR Reverberation Modeling Workshop problems


Background: A Reverberation Modeling Workshop was held in Austin, Texas, 7–9 November 2006. Participants were asked to describe their models, and submit solutions to a number of problems. The problems were described in a number of documents accessible over the internet. Twenty problems were specified. All were shallow water environments, with various geometries, and scattering functions. Each problem typically had three frequencies, three receiver depths, and several pulse bandwidths; a towed array was specified for 5 problems. All problems used a homogeneous infinite seabed. Doing all the problems would require thousands of runs, so people were selective. There were essentially 3 classes of problems: 2D computationally intensive physics-based models with realizations of rough bottoms, 3D flat-bottom (pseudomonostatic) problems with scattering functions (Lambert’s rule, and perturbation theory), and 3D bistatic range-dependent problems (with a towed array).

Principal results: This paper presents an overview of the DRDC Atlantic normal-mode reverberation models OGOPOGO and NOGRP. Comparative results are presented for a number of the 3D range-independent problems, including bistatic geometry, and towed array beam patterns.

Significance of results: Time series for the full set of solutions were submitted to the organizers, for quantitative comparison with other models. Initial comparisons at the Workshop indicated that the normal-mode predictions were in good agreement with results from other models. Refinement of the problems will lead to a set of reverberation benchmarks, similar to those already available for propagation models.

Future work: The problems were a good test of the reverberation codes. The NOGRP model was fast to run and easy to modify. It would be useful to incorporate some of the features into the bistatic model OGOPOGO, though it is more difficult to modify. The normal-mode framework is amenable to moderate range-dependence using the adiabatic approximation, and a joint effort is being considered with Penn State University to develop a range-dependent bistatic model using Matlab. A second
Reverberation Modeling Workshop is planned for May 2008, emphasizing other aspects including target echo, clutter, scattering from bubbles and fish, and sub-bottom scattering.
Sommaire

Normal-mode models OGOPOGO and NOGRP applied to the 2006 ONR Reverberation Modeling Workshop problems

Dalè D. Ellis; DRDC Atlantic TM 2006-289; R & D pour la défense Canada – Atlantique; juin 2008.

Contexte : Un atelier sur la modélisation de la réverbération a eu lieu à Austin, au Texas, du 7 au 9 novembre 2006. On a demandé aux participants de décrire leurs modèles et de soumettre des solutions pour un certain nombre de problèmes. Les problèmes étaient décrits dans un certain nombre de documents accessibles sur Internet. Vingt problèmes étaient spécifiés. Tous portaient sur des environnements en eaux peu profondes, avec diverses géométries et fonctions de diffusion. Chaque problème avait typiquement trois fréquences, trois profondeurs de récepteur et plusieurs largeurs de bande des impulsions ; un réseau remorqué était spécifié pour 5 problèmes. Tous les problèmes présumaient un fond marin infini homogène. La résolution de tous les problèmes exigerait des milliers d’essais, de sorte que les participants étaient sélectifs. Il y avait essentiellement 3 classes de problèmes : modèles 2D basés sur la physique et exigeant un grand nombre de calculs avec des réalisations de fonds rugueux, des problèmes de modèles 3D (pseudo-monostatiques) à fond uni avec des fonctions de diffusion (loi de Lambert et théorie des perturbations) et des problèmes de modèles 3D bistatiques dépendant de la distance (avec réseau remorqué).

Résultats : Le présent rapport donne un aperçu des modèles de réverbération en mode normal OGOPOGO et NOGRP de RDDC Atlantique. Des résultats comparatifs sont présentés pour un certain nombre des problèmes de modèles 3D indépendants de la distance, y compris la géométrie bistatique et des diagrammes des faisceaux de réseau remorqué.

Portée : Une série chronologique de l’ensemble des solutions a été soumise aux organisateurs aux fins de comparaison quantitative avec d’autres modèles. Des comparaisons initiales effectuées lors de l’atelier indiquaient que les prédictions pour le mode normal concordaient bien avec les résultats d’autres modèles. Le raffinement des problèmes aboutira à un ensemble de repères de réverbération, semblables à ceux qui sont déjà disponibles pour les modèles de propagation.

Recherches futures : Les problèmes étaient un bon test pour les codes de réverbération. Le modèle NOGRP était rapide à exécuter et facile à modifier. Il serait utile d’intégrer
certaines de ses caractéristiques au modèle bistatique OGOPOGO, bien que ce der-
nier soit plus difficile à modifier. Le cadre de mode normal est susceptible d’une
dépendance modérée de la distance faisant appel au développement adiabatique, et
nous envisageons un effort conjoint avec l’université Penn State pour développer un
modèle bistatique dépendant de la distance au moyen du logiciel Matlab. Un deuxième
atelier sur la modélisation de la réverbération est prévu pour mai 2008, mettant l’ac-
cent sur d’autres aspects, y compris l’écho de cible, le fouillis, la diffusion émanant
de bulles et de poissons et la diffusion du sous-sol du fond.
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1 Introduction

A Reverberation Modeling Workshop was held in Austin, Texas, 7–9 November 2006. Participants were asked to describe their models, and submit solutions to a number of problems. The problems were described in a number of documents [1] accessible over the internet. Twenty problems were specified. All were shallow water environments, with various geometries, and scattering functions. Each problem typically had three frequencies, three receiver depths, and several pulse bandwidths; a towed array was specified for 5 problems. All problems used a homogeneous infinite seabed. Doing all the problems would require hundreds or even thousands of runs, so people were selective. There were essentially 3 classes of problems: 2D computationally intensive physics-based models with realizations of rough bottoms, 3D flat-bottom (pseudo-monostatic) problems with scattering functions (Lambert’s rule, and perturbation theory), and 3D bistatic range-dependent problems, with a towed array. The most commonly solved was Problem 11: 3D, pseudo-monostatic, flat bottom, isospeed water, homogeneous half-space bottom, with Lambert scattering.

This paper presents an overview of the DRDC Atlantic normal-mode reverberation models OGOPOGO and NOGRP. Comparative results are presented for a number of the 3D range-independent problems, including bistatic geometry, and towed array beam patterns. Data files with time series for the solutions were submitted to the organizers to facilitate quantitative comparison with other models.

This Technical Memorandum (TM) is very similar to the paper [2] submitted for publication in the Workshop proceedings [3]. It contains a few corrections (in particular Figures 6, 9, and 12), as well as an additional Appendix listing the input and output data files. These files are included on the CD distribution of the TM.

2 Theoretical Approach

The reverberation calculations presented here are based on the normal mode approach of Ellis [4] and [5]. This is an extension of the approach first used by Bucker and Morris [6], but includes a time-dispersion effect using modal group velocities. Bistatic effects with towed array beam patterns can also be included. A four-slide summary is given in Appendix A.

The normal mode model is PROLOS, which is based on a two-ended shooting technique [7].
2.1 Reverberation with beam patterns

Following Ref. [4, Eq. (26)], but adding beam patterns [5], reverberation at time \( t \) for a short pulse of energy \( E_0 \) and a beam steered in the direction \( \beta_l \) can be written\(^1\) as

\[
R_0(t, \beta_l) = E_0 (2\pi)^3 \sum_{n=1}^{N} k_n^{-1} B_S(\theta_{nS}) [u_n(z_S) A_n(z_b)]^2 \\
\times \sum_{m=1}^{N} k_m^{-1} B_l(\theta_{mR}) [u_m(z_R) A_m(z_b)]^2 \\
\times C_{nm} r_{nm}^{-1} S_b(\theta_{nb}, \theta_{mb}) e^{-2(\delta_n + \delta_m) r_{nm}},
\]

where \( m \) refers to the source-scatterer path, \( n \) refers to the scatterer-receiver path, \( b \) refers to the boundary (surface or bottom), \( S_b \) is the scattering function at the boundary, \( z_b \) is the surface or bottom boundary depth, \( z_S \) and \( z_R \) are the source and receiver depths, \( B_S \) is the source beam pattern, \( B_l \) is the (effective) beam pattern of the \( l \)-th receiver beam, \( u_n, A_n, k_n, \delta_n \) and \( v_n \) are the mode functions, amplitudes, wave numbers, attenuations, and group speeds, \( \theta_{n[S,R,b]} \) are the equivalent ray angles of the modes at the source, receiver, and boundary depths, and \( r_{nm} = v_n t_n = v_m t_m \), where \( t = t_n + t_m \). The summations are over the number of modes. Note \( C_{nm} = \omega / (k_n + k_m) \), where \( \omega = 2\pi f \) is the angular frequency, is an area correction factor that basically reduces to half the average phase speed.

The effective receiver beam pattern, or reverberation response over a flat uniform bottom, [8] is the towed array beam pattern \( B_R(\theta, \phi; \beta_l) \) at vertical angle \( \theta \) integrated over all azimuth angles \( \phi \):

\[
B_{\text{eff}}(\theta) = \frac{1}{2\pi} \int_{0}^{2\pi} B_R(\theta, \phi; \beta_l) \, d\phi.
\]

The average up- and down-going beam pattern is needed for the normal-mode formulation:

\[
B_l(\theta) = \frac{1}{2} [B_{\text{eff}}(-\theta) + B_{\text{eff}}(\theta)].
\]

2.2 Reverberation with constant group velocity

Buried in Eq. (1) is an area calculation (Eq. (24) of Ref. [4]):

\[
dA_{mn} = \frac{1}{v_m + 1/v_n} \left( \frac{1}{c_m} + \frac{1}{c_n} \right)^{-1} t \, dr_{mn} \, d\phi,
\]

\(^1\)There should perhaps be a \( \rho_w^4 \) term for the water density in the denominator of this equation, as well as in Eqs. (5) and (6); see Section 7 for more discussion.
where \( c_m \) is the mode phase velocity.

If we use a constant mode group velocity \( c_g \) for the \( v_j \), and an average phase velocity \( c_p \) for the \( c_j \), then the area becomes \( dA = (c_g c_p t/4) \, dr \, d\phi \).

Following Ref. [4, Eq. 15], boundary reverberation at time \( t \) for a short pulse of energy \( E_0 \) can be written as

\[
R_0(t) = E_0 (2\pi)^3 (c_p/2) \sum_{n=1}^{N} k_n^{-1} [u_n(z_S)A_n(z_b)]^2 \times \sum_{m=1}^{N} k_m^{-1} [u_m(z_R)A_m(z_b)]^2 \frac{S_b(\theta_{nb}, \theta_{mb})}{r} e^{-2(\delta_n + \delta_m) r},
\]

where \( r = (c_g/2)(t - \tau) \). Note that this is basically Eq. (1) with \( r_{nm} = r \) and \( C_{nm} = c_p/2 \), and the beam patterns set to unity.

### 2.3 Coherent reverberation effects

Instead of summing the modes incoherently, one can sum them coherently. The equation is similar to Eq. 5. Basically one takes the squares off the terms between the summation signs, includes the neglected phase terms in the Hankel function, mode functions, and scattering function, and squares the magnitude of the resulting summation. This allows coherent propagation effects, so reverberation at convergence zones can be calculated; see Fig. 3 and Eq. (14) of Ref. [4]. NOGRP includes this option, but no results are presented here. There may be some slight coherent propagation effects for problems involving the summer and winter profiles.

### 2.4 Bistatic reverberation

The equation for bistatic reverberation is given in Ref. [5]. It is essentially the same as Eq. (1), except that the reverberation depends on azimuthal angle \( \phi \) (measured counter clockwise from the receiver, with the source at \( \phi = \pi \)), the scattering function \( S_b(\theta_m, \phi, \phi_{bis}) \) depends on the bistatic angle \( \phi_{bis} \) between the source-target-receiver, and one must use the actual 3-D beam patterns, not the effective beam patterns:

\[
R_0(t, \beta_i) = E_0 (2\pi)^2 \int_0^{2\pi} d\phi \sum_{n=1}^{N} k_n^{-1} B_S(\theta_{nS}, \phi) [u_n(z_S)A_n(z_b)]^2 \times \sum_{m=1}^{N} k_m^{-1} B_R(\theta_{mR}, \phi; \beta_i) [u_m(z_R)A_m(z_b)]^2 \times C_{nm}(\phi) \frac{S_b(\theta_{nb}, \theta_{mb}, \phi_{bis})}{r_{nm}' e^{-2(\delta_n + \delta_m) r_{nm}'}}.
\]
The range from the source to the scattering patch is $r'_{nm}$. The area correction $C'_{nm}(\phi)$ is similar to the term $\omega/(k_n + k_m) \approx c_p/2$ in the monostatic case, but more complicated. The incremental area is $dA_{nm} = r_{nm} dr_{nm} d\phi$, but, due to the azimuthal dependence and elliptical annuli $dr_{nm}$, depends on $r'_{nm}$ and other variables. In terms of the time element $d\tau$, the incremental area is

$$
\frac{v_n r'_{nm}}{r_{nm} + (v_n/v_m)r'_{nm} + L \cos \phi} r_{nm} d\phi d\tau
$$

where $L$ is the separation between source and receiver. The correction factor $C'_{nm}$ is the quantity in brackets [...]. When $L = 0$ and $m = n$, it reduces to $v_n/2 \approx c_g/2$; so it is inconsistent with the NOGRP calculation which is $c_p/2$. The area calculation in NOGRP uses the group velocity to determine the radius, and the phase velocity to determine the incremental area. The area calculation in OGOPOGO uses only the group velocity, not the phase velocity. The NOGRP approach seems to agree with the ray approach; it seems the OGOPOGO formulation should be modified to be consistent. However, for low angles the difference between $c_p$ and $c_g$ is quite small, so the inconsistency is an academic issue rather than a practical one.

### 2.5 Longer pulses

The above expressions for reverberation were for a very short pulse of energy $E_0$. For an extended pulse of duration $\tau_0$ and intensity $I_0(\tau)$, the modification is simply a convolution of the source pulse with the “impulse response” reverberation:

$$
R(t) = \int_0^{\tau_0} I_0(\tau) R_0^{(1)}(t - \tau) d\tau,
$$

where $R_0^{(1)}$ is the reverberation with unit energy $E_0 = 1$. A good approximation for pulses “not too long” is to use the reverberation at the mid-point of the pulse:

$$
R(t) = E_0 R_0^{(1)}(t - \tau_0/2).
$$

This simply introduces a slight time-shift in the reverberation, which is multiplied by the pulse energy.

### 2.6 Beam patterns

Some problems specified a 75-wavelength towed array with Hanning weights. Presumably this was an ideal continuous line array, but realistic arrays and our numerical models use discrete elements. We made the following suggestion to the organizers.
More than 150 elements are needed to avoid aliasing at endfire, and bandwidth requires even more; one-third wavelength spacing seemed like a good "operational" value. We settled on 202 elements, at a spacing of 2.25 m (at 250 Hz). This corresponds to a physical array of length $201 \times 2.25 = 452.25$ m or $199 \times 2.25 = 447.75$ m depending on whether one counts the two zero-weighted end elements; this brackets the specified array length of $75 \times 1500/250 = 450$ m in 1500 m/s water.

We also averaged the beam pattern intensity over the bandwidth of the source; this tends to fill in the nulls of the side lobes [8].

The ideal array also specified perfect left-right discrimination. Unfortunately, this is not realizable in practice. Our computer codes could have been modified to only integrate in one hemisphere, but we did not do that for the benchmarks lest we introduce errors into a working program. In symmetric environments, our results can be reduced by 3 dB to meet the specified requirements. We feel it would have been better to have specified a realistic benchmark problem.

### 2.7 Volume attenuation in the water

The volume attenuation in the water was specified to be the dB/km formula of Jensen et al. [9, Eq. 1.34]:

$$\alpha' \simeq 3.3 \times 10^{-3} + \frac{0.11F^2}{1 + F^2} + \frac{44F^2}{4100 + F^2} + 3.0 \times 10^{-4}F^2$$  \hspace{1cm} (9)

where $F = f/1000$ is the frequency in kHz. This is a "metric" version of the dB per kiloyard formula in Urick [10, p. 108], with coefficients rounded in the last decimal place. Both formulae are an extension of the original Thorp formula [11], and different from the Thorp formula in the Generic Sonar Model (GSM) [12].

### 2.8 Scattering functions

#### 2.8.1 Lambert’s rule

Many of the problems used Lambert’s rule for bottom scattering:

$$S_B(\theta_i, \theta_s, \phi) = \mu \sin \theta_i \sin \theta_s,$$  \hspace{1cm} (10)

where $\theta_i$ and $\theta_s$ are the incident and scattered grazing angles, with Mackenzie’s [13] canonical strength factor $10 \log \mu = -27$ dB. Note that this function has no azimuthal dependence $\phi$, and peaks in the vertical rather than in the specular direction, so may not be realistic for highly bistatic reverberation calculations [14].
2.8.2 Pierson-Moskowitz surface scattering

Problems 8, 9 and 10 involved reverberation from a rough ocean surface, with a scattering function derived from first order perturbation theory and a Pierson-Moskowitz surface spectrum for 10 m/s winds. The scattering function is given by Ref. [1]:

\[ S_S(\theta_i, \theta_s, \phi) = \alpha \frac{\sin^2 \theta_i \sin^2 \theta_s \exp \left(-\beta g^2 \frac{U^4}{k^2 \Omega} \right)}{\pi \Omega}, \] (11)

where \( k = \frac{\omega}{c(0)} \) is the wavenumber at the water-surface interface, \( c(z) \) being the sound speed profile, \( \beta = 0.74 \), \( g = 9.81 \text{ m/s}^2 \), \( U \) is the wind speed in m/s at a height\(^2\) of 19.5 m, and

\[ \Omega = (\cos \theta_i - \cos \theta_s \cos \phi)^2 + \cos^2 \theta_s \sin^2 \phi. \] (12)

In-plane, \( \phi = 0 \) corresponds to forward scattering, and \( \phi = \pi \) corresponds to backscatter, or \( \Omega = (\cos \theta_i + \cos \theta_s)^2 \).

It is interesting that Eq. (11) has a frequency-dependent “facet” peak near the specular, but with a (unphysical) frequency dependent “hole” at specular where \( \Omega = 0 \) and the scattering function goes to zero. For pure backscatter, \( \theta_s = \theta_i \), away from the steep angles the function has a \( \sin^4 \theta_i \) dependence, vs. the \( \sin^2 \theta_i \) dependence of Lambert’s rule. Figure 1 illustrates a number of these features, and a comparison with Lambert’s rule for scattering. For scattering in the backward direction (\( \phi = \pi \)) the in-plane grazing angle is the scattered grazing angle \( \theta_s \); for forward scatter (\( \phi = 0 \)) the in-plane grazing angle is \( 180^\circ - \theta_s \).

The coherent reflection loss was also given [1] for the Pierson-Moskowitz surface spectrum:

\[ R(\theta_i) = \exp(-2k^2 \sigma_S^2 \sin^2 \theta_i), \] (13)

where \( \sigma_S \) is the rms surface roughness.

Our normal mode formulation [7] could not handle the coherent reflection coefficient directly, but includes a perturbation formula for the contribution to the modal attenuation coefficient using the Kuperman-Ingenito formula [15] for a rms surface roughness:

\[ \delta^S_n = \frac{\sigma_S^2 \gamma_n(0)}{2 \rho_w k_n} \left[ \frac{du_n(0)}{dz} \right]^2, \] (14)

where \( \gamma_n^2(z) = \left[ \frac{\omega^2}{c^2(z)} - k_n^2 \right]^2 \), \( \sigma_S = (U^2/2g)\sqrt{\alpha/\beta} \approx 0.53 \text{ m} \) is the rms surface roughness, and \( \rho_w \) is the water density. These are valid in the small wave height approximation.

\(^2\)The present standard height for measurement of wind speed is 10 m, so there is a slight difference.
Figure 1: Pierson-Moskowitz scattering kernels. The curves that extend beyond 90 degrees [usually labelled “fwd...” for forward scatter, or “back” for backward scatter] are scattering from an incident angle of 20 degrees; the curves that end at 90 degrees [labelled “bks”] are backscattering where the incident angle equals the scattered angle.

There is some question as to whether we should be including the coherent reflection losses in the reverberation calculations. We performed the reverberation calculations with and without the coherent losses of Eq. (14).

Equation (13) may not be applicable at low grazing angles. Ainslie [16] has expressions which show that the low-angle reflection loss depends on the sine of the grazing angle rather than on \( \sin^2(\theta_i) \). Chapman [17] has modal attenuation coefficients that could replace Eq. (14).

2.8.3 Rough bottom scattering

Problems 5, 6 and 7 involved reverberation due to rough bottom scattering. The bottom scattering function was derived from perturbation theory, with the scattering function given by [1]:

\[
S_B(\theta_i, \theta_s, \phi) = \left(\frac{k_w^2}{4}\right) G^2(k_i, k_s) P_{2D}(k_i - k_s),
\]  

where \( k_w = \omega/c_w \) is the wavenumber at the water-bottom interface, \( k_i = k_w \cos \theta_i \), \( k_s = k_w \cos \theta_s \),

\[
G(k_i, k_s) = a(k_i, k_s) \left[1 + \Gamma(k_s)\right] \left[1 + \Gamma(k_i)\right] + b(k_i, k_s) \left[1 - \Gamma(k_s)\right] \left[1 - \Gamma(k_i)\right].
\]
Γ(k) is the Rayleigh reflection coefficient for a flat bottom

\[
\Gamma(k) = \frac{\rho \sqrt{1 - k^2/k_w^2} - k_b/k_w \sqrt{1 - k^2/k_b^2}}{\rho \sqrt{1 - k^2/k_w^2} + k_b/k_w \sqrt{1 - k^2/k_b^2}},
\]

(17)

and

\[
a(k_i, k_s) = \frac{1}{\rho - 1} \cos \theta_i \cos \theta_s \cos \phi + 1 - \kappa^2/\rho,
\]

(18)

\[
b(k_i, k_s) = \sin \theta_s \sin \theta_i (\rho - 1),
\]

(19)

\[
P_{2D}(k_i - k_s) = \frac{\sigma_B^2 K_L}{2\pi(K_L^2 + K^2)^{3/2}} = \frac{\sigma_B^2 l^2}{2\pi(1 + K^2 l^2)^{3/2}},
\]

(20)

where \(l\) is the correlation length, \(K_L \equiv 1/l\), \(\kappa = k_b/k_w\), and \(K^2 = k_w^2 \Omega\). The wavenumber in the bottom \(k_b\) is a complex number including the bottom absorption, and \(\Omega\) is defined as with the surface scattering, Eq. (12).

Two bottoms were specified; in the organizer’s description they were a “typical-rough” bottom (with a correlation length of 400 m, and \(\sigma_B = 0.1\sqrt{10} \approx 0.316 \text{ m}\)), and a “rough-rough” bottom (with a correlation length of 10 m, and \(\sigma_B = 0.1\sqrt{2} \approx 0.141 \text{ m}\)). Note that the typical rough bottom has a larger rms roughness than the rough-rough bottom, but the longer correlation length gives rise to a smoother interface with large-scale undulations. The “spectral strength” in Eq. (20) is proportional to \(\sigma_B^2 l^2\), so is higher for the rough-rough bottom by a factor of 8 (\(\approx 9 \text{ dB}\)). Figures 2 and 3 illustrate some of the features of the scattering functions, which are roughly proportional to frequency. Note in this case there is a peak, but no hole, near specular.

**Figure 2:** Reflection loss (upper figure) and scattering kernels for a typical-rough bottom at 250 Hz (lower figure). The in-plane angle is the same as for Fig. 1.
The coherent reflection loss was also given [1]:

\[ R(\theta_i) = \Gamma(k_w \cos \theta_i) \exp\left(-2k_w^2\sigma_B^2 \sin^2 \theta_i\right), \]  

(21)

where \( \sigma_B \) is the rms surface roughness, and \( \Gamma \) is the reflection coefficient for the flat bottom. As with the surface scattering, our normal mode formulation could not handle the reflection coefficient directly, but uses the Kuperman-Ingenito formula [15] for rms bottom roughness to give a contribution to the modal attenuation coefficient:

\[ \delta^B_n = \frac{\sigma_B^2 \gamma_n(h^-)}{2\rho_w k_n} \left\{ \left[ \frac{du_n(h^-)}{dz} \right]^2 + \left[ \gamma_n(h^-)u_n(h^-) \right]^2 \right\}, \]  

(22)

where \( \gamma_n^2(z) = \left[ \frac{\omega^2}{c^2(z)} - k_n^2 \right]^2 \), \( \sigma_B \) is the rms bottom roughness, and \( h^- \) indicates that the functions are to be evaluated just above the water-bottom interface. These are valid in the small wave height approximation.

There is some question as to whether we should be including the coherent reflection losses in the reverberation calculations. We performed the reverberation calculations with and without the coherent losses of Eq. (22).

### 3 Numerical Techniques

The code is all standard Fortran 77 or Fortran 95, with various system dependent “features” [bugs] weeded out over the years. The normal mode code was originally Digital Equipment Corporation DEC 20 (c. 1978) or VAX VMS (c. 1988), converted to HP Unix and DEC Alpha VMS (c. 1994), then to g77 on Linux or Mac OSX (c. 2003),
and recently (c. 2006) to g95 running on MacBook (Intel) OSX. The reverberation codes OGOPOGO (c. 1988) and NOGRP (c. 1994) followed similar migration paths. (Except for the date and time extensions to Fortran 77, the identical code compiles in g77 and g95). 3

The computation first involves running the normal-mode program POPP, a variant of PROLOS, which basically calls the normal mode subprogram MODES, and writes out a binary file of mode information. The reverberation codes OGOPOGO (bistatic) and NOGRP (monostatic) read this binary file. An extension of NOGRP to handle beam patterns, Rosella, was used to do the reverberation calculations with beam patterns. Effective patterns for some monostatic calculations were calculated directly in Rosella, or separately in auxiliary programs OCTBM3 and B3D2.

Reverberation output (as time-reverberation pairs) was written to ASCII “flat” files, which can be trivially read in and plotted using Matlab or other software.

3.1 Code changes required for the Workshop problems

For problems with the Pierson-Moskowitz spectrum and bottom roughnesses from perturbation theory, additional routines were coded and incorporated into NOGRP. Other than that, all changes were minimal.

A minor extension was required to the MODES subprogram to use the Jensen et al. volume attenuation Eq. (9), since it had previously used Urick’s equation for the Thorp volume absorption, with a 0.9144 yard-to-meter conversion factor.

A beam pattern bandwidth option in OGOPOGO was changed to allow non-integer “one-third-octave” bandwidths.

Increases in array sizes were required in NOGRP and OGOPOGO to accommodate 250 modes, and to the azimuthal integrations to calculate the narrow beam patterns for the 75-wavelength array.

3The recent g95 modifications to OGOPOGO have not yet been checked for back-compatibility with g77. Though portability has improved considerably over the years, one must be cognizant of the numerical analyst’s lament: “There is nothing so humbling as trying to port your code to another machine.” —Prof. Pat Keast, c. 1987.
4 Advantages/Disadvantages/Known Limitations

The obvious advantages, disadvantages, and known limitations are given in slide 3 of the presentation reproduced in Appendix A.

Since the models are normal-mode based, their main strengths are for shallow-water environments at low frequencies; computation time increases as the square of the frequency. Surface and bottom scattering can be handled, as well as volume reverberation in the water and bottom [18]; the model has recently been extended to handle scattering from a subbottom interface. The models handle beam pattern effects, including towed arrays, which are important for model-data comparisons. The NOGRP model employs effective beam patterns [8], which are useful in flat bottom monostatic environments. The OGOPOGO model includes a 3-D scattering function; the azimuthal dependence and near-specular “facet” term can be important for bistatic geometries [14]. OGOPOGO also includes the option for calculating the time dependence of the target echo [18] at a specified range.

The main limitation of the present models is that they are range independent, so limited to flat bottoms. Using normal modes for propagation means that they do not apply at ranges less than a few water depths. The group velocity correction is only an approximation to the time dependence. Like most reverberation models, they assume an empirical scattering function and the single-scattering approximation.

5 Speed

The NOGRP/OGOPOGO models were developed to allow comparisons with measured reverberation data. They were meant to be practical in the same sense as the Generic Sonar Model, using empirical scattering functions and towed array beam patterns. They do not use physics-based scattering, so are not benchmark models in that sense. However, there are very few reverberation benchmark predictions available, so in some sense the results published in 1995 [4] have been treated by some people as a form of benchmark. The normal mode formulation is a good check on the validity of the energy flux methods. The time dependence using group velocity is clearly an approximation.

The models were run on a MacBook Pro laptop: a 2 GHz Intel Core Duo with 2 GB of SDRAM under Mac OS X 10.4.8. The g95 compiler dates from April 2006. At that time a demo version of a commercial Fortran 95 compiler from Intel ran about 50% faster. All compiler switches were default. For a single run the Utility Monitor shows the g95 compiler only accesses one of the two processors of the Core Duo; two
Table 1: Typical run times for the reverberation calculations

<table>
<thead>
<tr>
<th>Problem</th>
<th>250 Hz</th>
<th>1000 Hz</th>
<th>3500 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.47 s</td>
<td>6.9 s</td>
<td>83 s</td>
</tr>
<tr>
<td>5r</td>
<td>-</td>
<td>-</td>
<td>1 m 39 s</td>
</tr>
<tr>
<td>6r</td>
<td>-</td>
<td>-</td>
<td>1 m 32 s</td>
</tr>
<tr>
<td>7r</td>
<td>-</td>
<td>-</td>
<td>1 m 12 s</td>
</tr>
<tr>
<td>8</td>
<td>0.26 s</td>
<td>3.8 s</td>
<td>46 s</td>
</tr>
<tr>
<td>11</td>
<td>0.12 s</td>
<td>1.5 s</td>
<td>18 s</td>
</tr>
<tr>
<td>12</td>
<td>0.11 s</td>
<td>1.5 s</td>
<td>18 s</td>
</tr>
<tr>
<td>13</td>
<td>0.10 s</td>
<td>1.2 s</td>
<td>13 s</td>
</tr>
<tr>
<td>15</td>
<td>2.4 s</td>
<td>90. s</td>
<td>56 m</td>
</tr>
<tr>
<td>15-bm</td>
<td>12. s</td>
<td>6 m 32 s</td>
<td>-</td>
</tr>
</tbody>
</table>

Jobs can run in parallel, however, without slowing each other.

The run times for a particular problem generally depend on the number of modes, which are proportional to water depth and frequency; for the 100 m isospeed water the number of modes are 16, 63, and 220, at frequencies of 250, 1000, and 3500 Hz respectively. The mode calculation in PROLOS depends approximately linearly on frequency. The reverberation calculations depend on the square of the number of modes.

The run times are also proportional to the number of output reverberation points. For the monostatic problems reverberation was calculated every 0.1 s out to 60 s. For the bistatic problems the time increment was increased to 0.5 s. The bistatic problems also require an azimuthal integration; without beam patterns a 1° increment was used, for the narrow beams this had to be decreased by a factor of 4.

Table 1 shows some typical run times for the reverberation calculations. The longer run times in Problems 5 and 8, compared to Problem 11, are due to the more complex scattering functions, and the (inefficient) coding of the bistatic function including the azimuthal dependence (which is unnecessary for monostatic calculations). The times for 3500 Hz for problems 5r, 6r and 7r (rough reflection loss) include all 3 frequencies. The run times for the bistatic Problem 15 increase faster than the expected square of the frequency; the beam pattern calculations take about 4 times longer as expected.
6 Problem Results

6.1 Brief description of the environments

The OGOPOGO/NOGRP model was used to solve the 3-D problems in range-independent environments. In these cases the water depth was 100 m, with a source at 30 m depth, and receivers at depths of 10, 50 and 90 m. (To allow for comparison with monostatic models, a receiver depth of 30 m was also included in many of the calculations presented here). Three sound speed profiles were used: isospeed, summer, and winter. In all cases the sound speed at the bottom was 1500 m/s; the sound speeds at the surface were 1500 m/s, 1530 m/s, and 1490 m/s respectively. A density of 1.0 g/cm$^3$ was used in the water. The bottom was a half-space with a sound speed of 1700 m/s, attenuation of 0.5 dB/wavelength, and bottom density ratio of 2.0.

Calculations were done at frequencies of 250 Hz, 1000 Hz, and 3500 Hz. There was considerable confusion in the specification [1] for the pulse length, pulse strength, and bandwidth; a pulse of unit energy was used in the calculations here. (See Section 6.3 for more discussion.)

6.2 Summary of problems attempted

Problems 11, 12, and 13 were the simplest, using Lambert’s rule for bottom scattering, in isospeed, summer, and winter sound speed profiles over a bottom halfspace.

Problems 5, 6 and 7 were for rough bottom scattering, with isospeed, summer, and winter sound speed profiles, respectively. The bottom scattering function was derived from perturbation theory; two bottoms were specified: a typical-rough bottom ($\sigma_B = 0.316$ m, and correlation length of 400 m), and a rough-rough bottom ($\sigma_B = 0.141$ m, and correlation length of 10 m). Since there are two scattering functions, each problem is really two problems. In addition, there is the question whether or not to include the coherent reflection loss in the reverberation calculations; so in reality each problem became 4 problems. A full set of calculations was done using the flat-bottom reflection loss; using the modal attenuation coefficients for the rough bottoms, calculations were done only for the 50 m receiver.

Problems 8, 9 and 10 were for isospeed, summer, and winter sound speed profiles, respectively, but for a rough ocean surface. The scattering function derived from first order perturbation theory and a Pierson-Moskowitz surface spectrum for 10 m/s winds. The issue of whether or not to include the surface losses arises here as well. A full set of calculations was done using the flat-surface reflection loss; using the modal attenuation coefficients for the rough surface, calculations were done only for the 50 m receiver.
Problem 15 was for a bistatic environment, with a towed array receiver separated from the source by 10 km. The environment was isospeed, with Lambert’s rule for scattering, so is essentially Problem 11. There are really two quite different issues here: (i) beam patterns effects, and (ii) bistatic reverberation. Results are presented for (i) beam patterns in the monostatic case (essentially Problem 11 with beam patterns), (ii) bistatic reverberation without beam patterns, and (iii) bistatic reverberation with a selection of beam patterns. The bistatic calculations were quite time consuming, so not done at all frequencies and depths.

The results for Problems 11–13 and Problem 15 presented at the Workshop in November are given in Appendix B. Results with a slightly different pulse length and intensity chosen to give $E_0 = 1$, along with results for the rough bottom cases (Problems 5–8), and the Pierson-Moskowitz surface scattering (Problems 8–10) appear in this section. NOGRP was used for most of the problems; OGOPOGO was used for the bistatic problems; for the monostatic problem with beam patterns, a variant of NOGRP, Rosella, was used.

### 6.3 Initial pulse

Due to the confusion about the pulse, results here are presented using $I_0\tau_0 = E_0 = 1$, i.e., 0 dB. The pulse length $\tau_0 = 1/B_W$, where $B_W$ is the bandwidth, i.e., $f_c/20$ in most cases. The intensity is then specified to be $I_0 = 1/\tau_0$, giving “unit energy”. The table below shows the values used in the calculations.

<table>
<thead>
<tr>
<th>Center Frequency</th>
<th>Bandwidth</th>
<th>Pulse length</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 Hz</td>
<td>12.5 Hz</td>
<td>0.08 s</td>
<td>10.9691 dB</td>
</tr>
<tr>
<td>1000 Hz</td>
<td>50. Hz</td>
<td>0.02 s</td>
<td>16.9897 dB</td>
</tr>
<tr>
<td>3500 Hz</td>
<td>175. Hz</td>
<td>0.005714 s</td>
<td>22.4304 dB</td>
</tr>
</tbody>
</table>

Only the $f_c/20$ bandwidth was used. It gave quite short pulse lengths compared to what one would normally use anyway; the $f_c/5$ bandwidth would be greater, and give rise to even shorter pulse lengths. From Eq. (8) one sees that the only difference of the wider bandwidth and shorter pulse length is an insignificant time shift in the reverberation.

### 6.4 Results of the calculations

Results for the problems are presented graphically for each problem. Typically each figure contains 4 graphs of reverberation vs time, one for the 4 receiver depths at each frequency and a 4th graph comparing results for all three frequencies for the 50-m
receiver depth. As a visual aid in comparing results, the 4th graph often includes the 250 Hz prediction for Problem 11.

The figure captions often only contain a file name plt_nnc.pdf, where nn refers to the problem number and c is a qualifier: t (typical rough bottom), r (rough-rough bottom), rb (rough bottom comparisons), or rs (rough surface comparisons). Brief comments are included under each problem.

6.4.1 Problem 5

Figure 4 shows the results at three frequencies and all receiver depths for Problem 5 (isospeed profile) with the typical-rough bottom; the 4th graph compares the 3 frequencies for the 50-m receiver depth. Figure 5 shows the result for Problem 5 with the rough-rough bottom. The coherent reflection loss is not included in the mode calculations.

Figure 6 compares results for the 50-m receiver with and without the Kuperman-Ingenito reflection loss included in the modal calculations.\textsuperscript{4}

6.4.2 Problem 6

Figures 7, 8, and 9 show similar results for the summer sound speed profile.

6.4.3 Problem 7

Figures 10, 11, and 12 show similar results for the winter sound speed profile.

6.4.4 Problem 8

Figure 13 shows the results at three frequencies and all receiver depths for Problem 8 (isospeed profile) with the Pierson-Moskowitz surface scattering; the 4th graph compares the 3 frequencies for the 50-m receiver depth. The coherent reflection loss is not included in the mode calculations.

Figure 14 compares results for the 50-m receiver with and without the Kuperman-Ingenito reflection loss included in the modal calculations.

\textsuperscript{4}In November 2007 Kevin LePage, John Perkins, and Eric Thorsos noticed some discrepancies between the NOGRP predictions and those from other models and correctly suggested that the original rms roughnesses in the bottom reflection loss were reversed. As a result, Figures 6, 9, and 12 in this paper are corrected versions of those in the Workshop paper [2].
6.4.5 Problem 9
Figures 15 and 16 show similar results for the summer sound speed profile.

6.4.6 Problem 10
Figures 17 and 18 show similar results for the winter sound speed profile.

6.4.7 Problem 11
Figure 19 shows the results at three frequencies and all receiver depths for Problem 5 (isospeed profile) with Lambert bottom scattering; the 4th graph compares the 3 frequencies for the 50-m receiver depth.

6.4.8 Problem 12
Figure 20 shows similar results for the summer sound speed profile.

6.4.9 Problem 13
Figure 21 shows similar results for the winter sound speed profile.
Figure 4: File plt_05t.pdf

Figure 5: File plt_05r.pdf
Figure 6: File plt_05rb_new.pdf

Figure 7: File plt_06t.pdf
Figure 8: File plt_06r.pdf

Figure 9: File plt_06rb_new.pdf
**Figure 10:** File plt_07t.pdf

**Figure 11:** File plt_07r.pdf
Figure 12: File plt_07rb_new.pdf

Figure 13: File plt_08.pdf
Figure 14: File plt_08rs.pdf

Figure 15: File plt_09.pdf
Figure 16: File plt_09rs.pdf

Figure 17: File plt_10.pdf
Figure 18: File plt_10rs.pdf

Figure 19: File plt_p11.pdf
Figure 20: File plt.p12.pdf

Figure 21: File plt.p13.pdf
6.4.10 Problem 15 – omnidirectional receiver

Figure 22 shows the bistatic reverberation predictions for an omnidirectional source and receiver at a separation of 10 km. Since the depth dependence is minimal, only the 50-m receiver is plotted, along with the monostatic calculations from Problem 11. The bistatic calculations were performed with OGOPOGO; the monostatic calculations were performed with NOGRP. The OGOPOGO calculations include the (dispersed) main blast arrival from about 6.7 to 7 s; reverberation is calculated every 0.5 s from 0.5 s to 60 s. By about 15 s the two results are indistinguishable, since the ellipses have essentially become circles. The two reverberation codes are completely different, indicating that the numerical integration around the ellipse is reasonably accurate.

![Figure 22: File plt_15.pdf. Bistatic and monostatic reverberation comparisons for an omnidirectional receiver.](image)

6.5 Beam patterns

Figure 23 shows the effective beam patterns for a 75-wavelength array of 202 elements and a selection of steering angles from broadside. Figure 24 shows corresponding monostatic reverberation predictions at 250 Hz. The reverberation predictions are explained [8] by the effective beam patterns. Note that these calculations are for a realizable line array; for the idealized array with left-right discrimination the results should be reduced by 3 dB.
6.5.1 Problem 15 – directional receiver

Problem 15 was run with a selection of narrow beams: 0 (broadside), 30, 60, 70, 80 and 90 (endfire). The beam patterns are for a line array at 250 and 1000 Hz only; 202 elements at 2.25 and 0.5625 m spacing, with Hanning weightings. Runs were for receiver depth 50 m only, since the results are insensitive to depth. Figure 25 shows reverberation from a selection of these beams, including a comparison with the broadside beam from the monostatic calculation; the two curves converge reasonably well by 20 s. All curves seem to have an unexplained glitch in the level at 8.5 s. Note that the broadside beam looks in the direction of the source, so sees the full intensity of the main blast; the other beams are steered away from the main blast, so what appears on the side lobes is essentially numerical noise.

Figure 26 shows similar curves at 1000 Hz; the direct path dropouts are not as pronounced.
Figure 24: File mono_bms.pdf at 250 Hz.

Figure 25: File plt_p15_250_bms.pdf
Figure 26: File plt_p15_1000_bms.pdf
6.6 Comparisons

A number of comparisons presented at the Workshop in November are given in Appendix B. Some of them have been discussed and others are updated here.

6.6.1 Receiver depth dependence

The problems do not show much depth dependence. Only when the receiver is close to the ocean surface will the effect be strong; see Ellis [4, Fig. 7].

6.6.2 Frequency dependence

Most of the frequency dependence in range is due to the Thorp absorption. While it is necessary that practical models include it, for most benchmark problems it may be better not to include it.

6.6.3 SSP dependence

The effect of the sound speed profiles is generally quite small. Figure B.2 in Appendix B shows only a 5 dB effect at 60 s for the Lambert bottom scattering problems with receiver at mid-depth. Figure 27 here shows a comparison for surface scattering (Problems 8, 9 and 10) at 1000 Hz and receiver depth at 10 m, where one would expect the biggest effect — which is $\sim$15 dB at 40 s.

6.6.4 Scattering function dependence

Lambert scattering is quite different from the Pierson-Moskowitz surface scattering, and rough bottom scattering derived from perturbation theory. The overall strength of the scattering seems to be the dominant effect on the reverberation; how the detailed shape of the scattering function affects the time dependence of reverberation is not as obvious. The fourth graph in Figures 4, 5, and 13 includes the corresponding prediction at 250 Hz from Lambert’s rule with an isospeed sound speed profile.

6.6.5 Model dependence

The NOGRP and OGOPOGO models agree reasonable well with the Generic Sonar Model (GSM) [12]. Figure B.6 from Appendix B shows a comparison for Problem 11 — 1 to 2 dB difference from 2 to 60 s. Note that a number of defaults in GSM have to be overridden to ensure the models are doing similar calculations — see the GSM run stream in Appendix C.
Results presented at the Workshop indicated good agreement for Problem 11. Mike Ainslie from TNO found very good agreement between a number of models (including NG for NOGRP) for Problem 11; see Figure 28.

### 6.6.6 Short-range effects

Figure B.5 in Appendix B indicated that the steeper angles were important at times less than a second, and the fathometer returns were important to 0.5 second. Figure 29 is an update which includes a mode calculation with a false bottom (5000 m/s); even these steep angles do not fully reproduce the reverberation shown by GSM.

Figure 28, courtesy of Mike Ainslie of TNO, shows the short-range dependence of various models. It also indicates that the steeper angles are important at times less than 2 s, and fathometer returns are visible to 0.5 s or more.
**Figure 28**: Ainslie’s comparison of various models at short ranges.

**Figure 29**: Short time reverberation from 0.01 s to 1.5 s, including fathometer returns calculated using GSM for a pulse of duration 0.03 s.
7 Future Directions

A few theoretical “bugs” appeared in the writing of this report:

- Preston [19] noted that the expression for the normal-mode pressure [7, 9] includes a $\rho_w(z_s)$ term in the denominator; this factor did not appear in the reverberation formulation of Ellis [4]. While the effect is small in “dB land”, if one uses $\rho_w = 1.027$ instead of $\rho = 1.0$ for seawater, the $\rho_w^4$ factor could reduce the reverberation by $\sim 0.5$ dB. The reverberation formulation needs to be checked out; the calculations presented here assume $\rho_w = 1.0$.

- The bistatic area calculation in OGOPOGO is based on taking the derivative of the range determined from the group velocities, instead of the combination of phase and group velocities done in ray models [20] and the monostatic formulation of Ellis [4, Eqs. (11 and 23)]. Using the group velocity to determine the ellipse, and the phase velocity to determine the incremental area, seems to agree with the ray approach.

- Ainslie pointed out that there is a better expression [16] for the Peirson-Moskowitz surface loss at low grazing angles, and furthermore, that Chapman [17] had produced an expression for the normal-mode attenuation coefficients. It is yet another example of the prophet not being heralded in his own country (or even his own laboratory), that Chapman’s expressions, although present in the code at one time, did not get carried forward as part of the PROLOS/MODES model. This option will be re-incarnated.

Options for additional bottom and surface scattering functions should be added to the codes.

The direct arrival in OGOPOGO was fine for a small number of modes, but is quite cumbersome when there are many modes; an alternate implementation should be inserted.

The codes have rudimentary target echo and clutter capability; e.g., see [21]. Further development of these features is anticipated.

Development of a bistatic range-dependent adiabatic mode model seems feasible. DRDC Atlantic has a monostatic $N \times 2D$ range-dependent reverberation code SWAMI; one option in it uses adiabatic modes [22]. Based on the success at this Workshop of Preston’s Matlab-based reverberation code [19] for flat bottoms, the extension seems feasible. The limit of validity of adiabatic modes remains an unanswered question at this point. The success of David Fromm’s BiKr model [23] on some of the Workshop problems will guide our decision.
Additional exercise of the models will be done. In collaboration with scientists at TNO and NURC, a more careful look at Problem 11 was presented at the November 2007 meeting of the Acoustical Society of America [24]. Some interesting mode effects are apparent at long times, and a journal manuscript is in preparation. Further work is anticipated for problems at the second Reverberation Modeling Workshop in May 2008.

8 Conclusions

8.1 Comments regarding OGOPOGO and NOGRP

The models seemed to work quite well. The problems did not “break” them, except that array sizes had to be increased for the very narrow beams, and higher frequencies.

The Lambert rule scattering was explored quite well at the Workshop, and our model results seemed in good agreement with others.

The group velocity effect is not very significant for the reverberation calculations, and adds complexity to the coding and increases computation time. The time dispersion will be more important for clutter feature and target echo calculations.

The rough surface and bottom perturbation theory scattering functions are recent additions, and have not been checked closely with predictions from other models. The reflection losses need to be checked out further, as well as more investigation of the Pierson-Moskowitz scattering function. [Additional note for the TM: At a special session “Underwater Reverberation Measurements and Modeling” at the November 2007 meeting of the Acoustical Society of America, Perkins and Thorsos [25] presented preliminary comparisons between the models for a number of problems. The NOGRP predictions for Problems 5–10 compared well with other models, separated in two groups with and without the rough surface loss correction. It was a pleasant surprise to see that the Kuperman-Ingenito [15] perturbation loss formula worked as well as it did.]

8.2 More general comments on reverberation modelling

Appendix D contains some general comments drafted following the Workshop. It mentions a number of unresolved modelling issues, and some suggestions for a potential future Workshop.
Acknowledgements

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Useful discussions were held with Jim Theriault and Gary Brooke. John Preston flagged the problem with the density. Exchanges with Mike Ainslie on Problem 11 were very useful, and he had valuable insights about the Pierson-Moskowitz spectrum.

Kevin LePage, John Perkins and Eric Thorsos, in comparing various model results for Problems 5–7, correctly suggested I had originally reversed the rms roughnesses for the reflection loss calculations.
References


Annex A: Four-slide Model Summary

This is a four-slide summary of the OGOPOGO model presented at the Workshop.

**Figure A.1: Four-slide model summary for OGOPOGO.**
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Annex B: Results presented at Workshop 7–9
November 2006

2006 ONR Reverberation Modelling Workshop: Results, including beam pattern effects, using bistatic normal mode model OGOPOGO and fast monostatic equivalents

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B.1 Results to 4 Nov 2006

Here are a selection of results for the 3-D Lambert bottom scattering problems 11 (isospeed), 12 (summer), 13 (winter), and 15 (bistatic). Beam pattern effects are included for problem 15 (and its monostatic counterpart problem 11).

I used my normal mode bistatic reverberation model (OGOPOGO) and a fast monostatic version of it (NOGRP, soon to be renamed MeeShee). I also ran GSM (version F) in monostatic mode (with both source and receiver at 30 m depth) for comparison.

The calculations used a 0.1 s pulse, with 10 dB source, to give unit energy. Then as I understood from the definition of the source pulse, I [usually] used the 2.34/f0 factor (-20.29 dB, -26.31 dB, and -31.75 dB at 250, 1000, and 3500 Hz) to adjust the levels. The “final” word of October 31 says this is off by ∼3 dB.

The following are a selection of results so far, showing a number of comparisons:

Depth dependence
Frequency dependence
SSP dependence
Model dependence
Model dependence at short ranges
Bistatic calculations
Effective beam patterns
Monostatic calculations with beam patterns
Bistatic calculations with beam patterns