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# **PyCoMod (Python Compartment Modelling) Programming Reference**

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## <span id="page-2-0"></span>**Abstract**

This Reference Document describes the PyCoMod (Python Compartment Modelling) code package. PyCoMod is used to build and run compartment models, such as susceptible-infectious-recovered (SIR) models of infectious disease. The package was initially developed to support analyses of the spread of Coronavirus Disease 2019 (COVID-19) in specific scenarios relevant to the Canadian Armed Forces (CAF) during the pandemic in 2020 and 2021. Over the course of multiple studies conducted during this period in collaboration with the Canadian Forces Health Services Group, the package evolved to include many features making it useful as a general modelling and simulation tool. The use of PyCoMod and its features will be described in detail in this Document.

## <span id="page-2-1"></span>**Significance to defence and security**

PyCoMod was developed to reduce the time and effort spent on creating, solving, and analyzing epidemiological compartment models. It was developed and used in multiple analyses that informed CAF decision making during the first two years of the COVID-19 pandemic. In addition to modelling COVID-19, PyCoMod can be used to develop models of other diseases relevant to military operations, such as vector-borne diseases (e.g., malaria), water-borne disease/illness (e.g., schistosomiasis, hepatitis), influenza, and others. Furthermore, PyCoMod compartment models fall into a category of general-purpose numerical models known as system dynamics. As a result, it is a tool that can be readily used outside the realm of epidemiological modelling, including areas such as logistics and resource management. As an open-source Python package hosted publicly on GitHub, it also highly suitable to collaborative development and modelling efforts.

## <span id="page-3-0"></span>**Résumé**

Le présent document de référence décrit le code du paquet PyCoMod (modélisation à compartiments Python). PyCoMod est utilisé pour créer et exécuter des modèles à compartiments, comme un modèle Susceptible-Infecté-Rétabli relatif à une maladie infectieuse. Le paquet a été élaboré initialement à l'appui des analyses sur la propagation de la COVID-19 dans le cadre de scénarios précis qui concernaient les Forces armées canadiennes pendant la pandémie en 2020 et en 2021. Au cours de multiples études effectuées pendant cette période en collaboration avec le Centre des services de santé des Forces canadiennes, le paquet a évolué et comprend maintenant de nombreuses fonctionnalités qui le rendent utile comme outil de modélisation et de simulation général. L'utilisation de PyCoMod et de ses fonctionnalités sera décrite en détail dans le présent document.

### <span id="page-3-1"></span>**Importance pour la défense et la sécurité**

PyCoMod a été mis au point dans le but de réduire le temps et les efforts consacrés à la création, à la résolution et à l'analyse de modèles épidémiologiques à compartiments. Le paquet a été élaboré et a ensuite été utilisé dans de multiples analyses qui ont orienté la prise de décisions au sein des Forces armées canadiennes pendant les deux premières années de la pandémie de COVID-19. En plus de modéliser la COVID-19, PyCoMod peut servir à élaborer des modèles d'autres maladies qui ont un rapport avec les opérations militaires, comme des maladies à transmission vectorielle (p. ex. paludisme), des maladies d'origine hydrique (p. ex. schistosomiase, hépatite), l'influenza, etc. En outre, les modèles à compartiments de PyCoMod font partie de la catégorie des modèles numériques généraux également connus sous le nom de « dynamique des systèmes ». Par conséquent, il s'agit d'un outil qui peut être facilement utilisé en dehors de la modélisation épidémiologique, y compris dans des domaines comme la logistique et la gestion des ressources. En tant que paquet Python libre hébergé publiquement sur GitHub, PyCoMod est parfaitement adapté aux efforts concertés d'élaboration et de modélisation.

## <span id="page-4-0"></span>**Table of contents**



# <span id="page-5-0"></span>**List of figures**



## <span id="page-6-0"></span>**List of tables**



## <span id="page-7-0"></span>**1 Introduction**

PyCoMod (Python Compartment Modelling) is a Python package for building and running compartment models derived from systems of differential equations such as the susceptible-infectious-recovered (SIR) model of infectious diseases. PyCoMod was developed to support analyses of the spread of the severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), which is the virus that causes Coronavirus Disease 2019 (COVID-19), in scenarios relevant to the Canadian Armed Forces (CAF) during the pandemic in 2020 and 2021 [\[1\].](#page-29-1)

The package uses object-orientated design to efficiently build and run compartment models. PyCoMod is not a model of a specific system; rather, it is Python package to create and analyze systems that can be well-represented by a set of compartments (equivalently, pools or stocks) interconnected with flows defined by mathematical expressions. Compartment models are an epidemiological application of a broader numerical modelling approach known as system dynamics.

In order to accommodate more-realistic scenarios and practical aspects of modelling and simulation, PyCoMod includes several capabilities that go beyond the basics of compartment modelling, including stochastic flows, nested models, dynamic model parameters, vectorized models, Monte Carlo simulation, and efficient simulation management using initialization files and multi-run automation.

The purpose of this Reference Document is to provide a coding reference for PyCoMod. It will describe the installation procedure and provide numerous code examples covering basic to advanced features of the package. For ease of reference, PyCoMod's built-in objects and functions are summarized in [Annex A.](#page-30-0) SIR models and extensions thereof will be used throughout the Document for illustrative purposes, but PyCoMod's applications are not limited to these types of models.

## <span id="page-8-0"></span>**2 Python Compartment Modelling Installation**

The PyCoMod package is publicly available on the Defence Research and Development Canada (DRDC) open science (OS) GitHub site:

https://github.com/DND-DRDC-RDDC/OS\_PyCoMod

To install PyCoMod directly from GitHub to a local Python environment (requires Git version control system), run the following from the command line:

pip install git+https://github.com/DND-DRDC-RDDC/OS\_PyCoMod.git

To install PyCoMod in Google Colab,<sup>1</sup> run the following in a code cell:

! pip install git+https://github.com/DND-DRDC-RDDC/OS\_PyCoMod.git

After installing the package, import PyCoMod into your code:

**import** pycomod **as** pcm

The examples that follow were tested in Google Colab and assume that PyCoMod has been installed and imported as above using the abbreviated name *pcm*. They are also assumed to be executed sequentially from start to finish so that earlier imports and definitions are available in later examples.

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<sup>&</sup>lt;sup>1</sup> [https://colab.research.google.com](https://colab.research.google.com/) (accessed date: 6 September 2023).

### <span id="page-9-6"></span><span id="page-9-0"></span>**3 Python Compartment Modelling examples**

This section will introduce PyCoMod's main features by way of a series of examples and descriptions starting with a basic SIR model and proceeding to more advanced modelling scenarios.

Note that these examples were designed to demonstrate the features of PyCoMod; they are not necessarily appropriate models for real situations.

### <span id="page-9-1"></span>**3.1 A simple susceptible-infectious-recovered model**

The SIR model compartmentalizes a population based on the disease state of each individual. There are three compartments (Susceptible [*S*], Infectious [*I*], and Recovered [*R*]) and two flows that move individuals from Susceptible to Infectious and from Infectious to Recovered [\[2\]](#page-29-2)–[\[4\].](#page-29-3) The compartments and flows are illustrated in [Figure 1,](#page-9-2) where variables  $S$ ,  $I$  and  $R$  represent the number of individuals in the Susceptible, Infectious, and Recovered compartments, respectively.



*Figure 1: Compartments and flows in the basic SIR model.*

<span id="page-9-2"></span>The flow of individuals from *S* to *I* is given by the rate

<span id="page-9-3"></span>
$$
F_{SI} = bS\frac{I}{N},\tag{1}
$$

where *b* is the transmission rate and *N* is the total population, equal to  $S + I + R$ .

The flow of individuals from *I* to *R* is given by the rate

$$
F_{IR} = gI,\tag{2}
$$

where g is the recovery rate and the reciprocal,  $g^{-1}$ , is the average time spent in the infectious compartment. This produces the following system of differential equations:

$$
\frac{dS}{dt} = -bS\frac{I}{N};\tag{3}
$$

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<span id="page-9-5"></span><span id="page-9-4"></span>

$$
\frac{dI}{dt} = bS\frac{I}{N} - gI; \text{ and} \tag{4}
$$

$$
\frac{dR}{dt} = gI. \tag{5}
$$

Given a population of size 100, where 5 individuals are infected (*I*) and the remaining 95 individuals are susceptible (*S*), we can model this simple system in PyCoMod with the following code:

```
class SimpleSIR(pcm.Model):
     def build(self):
        # Pools
        self.S = pcm.Pool(95)self.I = pcm.Pool(5)self.R = pcm.Pool(0) # Equations
         self.N = pcm.Equation(lambda: self.S() + self.I() + self.R())
         # Parameters
        self.b = pcm.Parameter(0.2)self.g = pcm.Parameter(0.1) # Flows
         self.Fsi = pcm.Flow(lambda: self.b() * self.S() * self.I() / self.N(),
                             src=self.S, dest=self.I)
         self.Fir = pcm.Flow(lambda: self.g() * self.I(),
                             src=self.I, dest=self.R)
         # Output
        self.set output('S', 'I', 'R')
```
The first two lines begin the definition of a custom class (i.e., a user-defined object type) that inherits properties from the PyCoMod base class for models and overrides the model's build method to define the elements of the SIR model. In this case, we create the three population compartments (S, I and R) using the PyCoMod Pool class (pool is the word used in PyCoMod for compartment) and specify the initial value of each pool  $(e.g., self.S = pcm.Pool(95))$ . We define the value N (the total population) as a PyCoMod Equation. Equations are defined by a function referencing other model elements, and we have used lambda functions [5] for their syntactical compactness. To obtain the value of a model element, we call the object by adding open- and close-parentheses; for example, the current number of susceptible individuals is obtained by self.S(). Using the PyCoMod Parameter class, we create and specify values for the model's parameters: the transmission rate, b, and recovery rate, g. Next, we define the movement between the compartments using the PyCoMod Flow class. Flows are defined by a function that returns the instantaneous flow rate. In this case, the flow functions correspond to the rate equations,  $\vec{F}_{SI}$  and  $\vec{F}_{IR}$ , defined in Equations [\(1\)](#page-9-3) and [\(2\).](#page-9-4) Flows must also specify a source pool and a destination pool using the src and dest named arguments. Note that when specifying source and destination pools, we reference the pool object itself rather than calling it (e.g., src=self.S, not src=self.S()). A final step in specifying the model is to let PyCoMod know which values we want to capture for output. This is done by calling the model's set\_output method and providing the names of the model elements that we want to track.

Having defined the SimpleSIR model class, we can now create an instance of it.

 $m =$  SimpleSIR()

We use another PyCoMod object called a RunManager to run it. The run manager keeps track of multiple models, run settings and outputs so that batches of runs can be automated. First, we create an instance of the run manager.

mgr = pcm.RunManager()

Now we can tell the run manager to run the SimpleSIR model. We can supply run settings (such as the duration in this example), and we must provide a label as a key to access the run results later.

```
mgr.run(m, duration=150, label='My run')
```
Finally, we can plot the results of the run using the PyCoMod Plotter. First, we create an instance of the plotter, which internally creates a Matplotlib Figure, and then we can plot outputs from the run on the figure axes. The result is shown is [Figure 2.](#page-11-1)

```
plt = pcm.Plotter(title='SIR Time Series', ylabel='Population', fontsize=14)
plt.plot(mgr['My run'], 'S', color='blue', label='S')
plt.plot(mgr['My run'], 'I', color='orange', label='I')
plt.plot(mgr['My run'], 'R', color='green', label='R')
plt.plot(mgr['My run'], 'S + I + R', color='black', label='Total')
```


*Figure 2: Plot of the S, I and R model outputs over time.*

<span id="page-11-1"></span>Each call to the plotter's plot function must specify a run and an output. The run is identified by indexing the run manager with the label that we specified when we ran the model (e.g., mgr ['My run']). The output must be one of the outputs that was specified in the model using set\_output. Outputs can be summed together in a plot, e.g.,  $S + I + R$  in the last line of the code above.

### <span id="page-11-0"></span>**3.2 Adding model elements**

To incorporate additional model elements, we simply add more pools, parameters, and flows to the model's build method. For example, we can expand the SIR model by incorporating an exposed compartment (*E*), thus creating a delay between the time of infection and the time of becoming symptomatic and infectious toward others, which models the virus' incubation period. This addition produces the common susceptible-exposed-infectious-recovered (SEIR) model [\[2\]](#page-29-2)–[\[4\]](#page-29-3) as seen i[n Figure 4.](#page-15-1)



*Figure 3: Compartments and flows in the basic SEIR model.*

<span id="page-12-0"></span>In the SEIR model, the parameter  $\alpha$  controls the flow from  $E$  to  $I$ , where the reciprocal,  $\alpha^{-1}$ , is the average incubation period for the disease*.*

This addition of the exposed compartment to the simple SIR example from Section [3.1](#page-9-1) is shown in the following code:

```
class SimpleSEIR(pcm.Model):
     def build(self):
         # Pools
        self.S = pcm.Pool(95)self.E = pcm.Pool(0)self.I = pcm.Pool(5)self.R = pcm.Pool(0) # Equations
         self.N = pcm.Equation(
                 lambda: self.S() + self.E() + self.I() + self.R())
         # Parameters
        self.b = pcm.Parameter(0.2)self.a = pcm.Parameter(0.1)self.g = pcm.Parameter(0.1) # Flows
         self.Fse = pcm.Flow(lambda: self.b() * self.S() * self.I() / self.N(),
                              src=self.S, dest=self.E)
         self.Fei = pcm.Flow(lambda: self.a() * self.E(),
                              src=self.E, dest=self.I)
         self.Fir = pcm.Flow(lambda: self.g() * self.I(),
                              src=self.I, dest=self.R)
         # Output
         self.set_output('S', 'E', 'I', 'R')
# Instantiate model
m = SimpleSEIR()
```
*# Run model*

```
mgr.run(m, duration=150, label='My run')
# Plot results
plt = pcm.Plotter(title='SEIR Time Series', ylabel='Population', fontsize=14)
plt.plot(mgr['My run'], 'S', color='blue', label='S')
plt.plot(mgr['My run'], 'E', color='red', label='E')
plt.plot(mgr['My run'], 'I', color='orange', label='I')
plt.plot(mgr['My run'], 'R', color='green', label='R')
plt.plot(mgr['My run'], 'S + E + I + R', color='black', label='Total')
```
### <span id="page-13-0"></span>**3.3 Stochastic model elements**

In PyCoMod, we can also introduce stochastic model elements and run Monte Carlo simulations. For example, two improvements to the simple SIR model would be to sample the transmission rate from a distribution reflecting the uncertainty in this parameter, and to make the flows stochastic and discrete. We show these changes below in a new model class called MonteCarloSIR.

```
import numpy as np
rng = np.random.default_rng()
class MonteCarloSIR(pcm.Model):
     def build(self):
         # Pools
        self.S = pcm.Pool(95)self.I = pcm.Pool(5)self.R = pcm.Pool(0) # Equations
         self.N = pcm.Equation(lambda: self.S() + self.I() + self.R())
         # Transmission rate parameters
        self.b m = pcm.Paramer(e.2)self.b s = pcm.Paramer (0.05) # Transmission rate random sample
         self.b = pcm.Sample(lambda: rng.normal(self.b_m(), self.b_s()))
         # Recovery rate parameter
        self.g = pcm.Parameter(0.1) # Flows
         self.Fsi = pcm.Flow(
                 lambda: rng.binomial(self.S(), self.b() * self.I() / self.N()),
                 src=self.S, dest=self.I)
         self.Fir = pcm.Flow(
                 lambda: rng.binomial(self.I(), self.g()),
                 src=self.I, dest=self.R)
         # Output
        self.set output('S','I','R')
```
m2 = MonteCarloSIR()

The first two lines, above, import  $NumPy<sup>2</sup>$  and instantiate its default random number generator (RNG). We now specify the transmission rate  $b$  in Equation [\(3\)](#page-9-5) with two parameters, a mean value  $b$  m and a standard deviation b\_s. Then we create a variable b for the transmission rate as a PyCoMod Sample, defined by a lambda function that calls NumPy's normal (or Gaussian) RNG, passing b\_m and b\_s as parameters. This will resample the transmission rate from the normal distribution at the start of each model run.

The flow  $F_{SI}$  has been updated such that, rather than being a deterministic rate, each susceptible person has a probability of remaining susceptible or being infected in one unit of time based on the number of infected people in the population and the transmission rate. Therefore, we use the binomial RNG to generate a discrete, random number of new infections that will move from the susceptible population to the infectious population in one time step: rng.binomial(self.S(), self.b()\*self.I()/self.N()). The flow  $F_{IR}$  has similarly been updated such that each infected person has a probability of recovering (or not) in each time step, again using the binomial RNG to generate a discrete, random number of people to move from the infectious compartment to the recovered compartment.

Lastly, we create an instance of the new model and call it m2. These modifications produce the same average behaviour as the deterministic model, but introduce variability based on the uncertainty in the transmission rate and the randomness of transmission and recovery events.

We can now run the model in Monte Carlo mode using the run manager's run mc method, passing the number of replications (reps) in the run settings, and giving the run a new label.

mgr.run mc(m2, duration=150, reps=100, label='My run - mc')

We can plot the results of a Monte Carlo run using the plotter's plot mc method. The optional interval parameter specifies the percentile range from the distribution of outputs to be displayed. An interval of 50 means the middle 50% of the distribution, or the inter-quartile range. An interval of 90 would display the region from the 5th to 95th percentile. The result is shown in [Figure 4.](#page-15-1)

```
plt = pcm.Plotter(title='SIR Time Series - Monte Carlo', ylabel='Population',
 fontsize=14)
plt.plot_mc(mgr['My run - mc'], 'S', color='blue', interval=50, label='S')
plt.plot_mc(mgr['My run - mc'], 'I', color='orange', interval=50, label='I')
plt.plot_mc(mgr['My run - mc'], 'R', color='green', interval=50, label='R')
plt.plot_mc(mgr['My run - mc'], 'S + I + R', color='black', interval=50,
            label='Total')
```
l

<sup>2</sup> [https://numpy.org](https://numpy.org/) (accessed date: 6 September 2023).

SIR Time Series - Monte Carlo



<span id="page-15-1"></span>*Figure 4: Plot of the median and inter-quartile range for S, I and R over time from a Monte Carlo simulation where the transmission rate and infection and recovery events are stochastic.*

### <span id="page-15-0"></span>**3.4 Nested models and model initialization**

PyCoMod models support nesting, so any PyCoMod model can be used as an element inside another model. For example, if we have two sub-populations with different transmission dynamics and a certain degree of mixing between them, we can create a new model, MixSIR, that contains two instances of the MonteCarloSIR model defined previously in Section [3.3.](#page-13-0)

```
class MixSIR(pcm.Model):
     def build(self):
         # Sub models
         self.GrpA = MonteCarloSIR()
         self.GrpB = MonteCarloSIR()
         # Transmission parameter between groups
        self.b mix = pcm.Parameter()
         # Cross-infection flows
         self.Fsi_GrpA = pcm.Flow(
                 lambda: rng.binomial(self.GrpA.S(),
                                       self.b mix() * self.GrpB.I()
                                                      / self.GrpB.N()),
                  src=self.GrpA.S, dest=self.GrpA.I)
         self.Fsi_GrpB = pcm.Flow(
                 lambda: rng.binomial(self.GrpB.S(),
                                       self.b mix() * self.GrpA.I()
                                                      / self.GrpA.N()),
                 src=self.GrpB.S, dest=self.GrpB.I)
         # Output
         self.set_output('GrpA','GrpB')
```

```
m3 = MixSIR()
```
In the code above, the two sub-populations, GrpA and GrpB, are both defined as instances of the MonteCarloSIR model. Each group behaves internally as before according to its parameters and initial conditions, but we introduce the possibility of cross-infection between these groups. The cross-infections occur with a different transmission rate, b mix, defined as a Parameter in the MixSIR model. The cross-infection flows result in new infections within each group as a result of an interaction with an individual from the infectious population in the other group. Note that in order to save the output from a sub-model, the sub-model must be listed in the parent model's output list, self.set\_output(GrpA, GrpB); then all elements of the sub-model will be accessible when plotting.

While GrpA and GrpB are the same model, we will supply them with different parameter values and initial conditions. Previously, we specified these values while defining the model, but it is often preferable to separate model inputs from the model itself. Therefore, we can supply the inputs for the model at run-time using a Python dictionary. For the MixSIR model, the initialization dictionary would look something like init mix below.

```
init GrpA = \{ 'S': 95, 'I': 5, 'R': 0, 'b'm': 0.2, 'b's': 0.05, 'g': 0.1 \}init_GrpB = {'S': 30, 'I': 0, 'R': 0, 'b_m': 0.3, 'b_s': 0.05, 'g': 0.1}
init_model = { 'b_mix': 0.05, 'GrpA': init_GrpA, 'GrpB': init_GrpB }init run = \{ 'reps': 100, 'end': 150 \}init_mix = {'run':init_run, 'model':init_model}
```
The initialization dictionary consists of two entries: run contains a dictionary of run inputs, and model contains a dictionary of model inputs. In this case, the supplied run inputs are the number of repetitions (reps) and the end time. The model dictionary contains keys corresponding to the names of the model elements, and values to be used to initialize each element. The only model elements that accept input are pools, parameters, and sub-models. The entry value for a pool is the initial population of the pool. The entry value for a parameter is the parameter's value which is a constant. To initialize a sub-model, such as GrpA above, the entry value is another dictionary designed to initialize the sub-model, init\_GrpA = {'S': 95, 'I': 5, 'R': 0, 'b\_m': 0.2, 'b\_s': 0.05, 'g': 0.1}. Hence, nested models are initialized with equivalently nested dictionaries. In this example, GrpA is given the same initialization values as in the MonteCarloSIR model while GrpB is a smaller population (Size 30) with a higher mean transmission rate, but with no initial infections. We then run the model using the dictionary to set both the model inputs and the run inputs.

```
mgr.run mc(m3, init=init mix, label='My run - mix')
```
We can then plot the Monte Carlo simulation of GrpA, as shown i[n Figure 5.](#page-17-0)

```
plt = pcm.Plotter(title='SIR Time Series - Monte Carlo - GrpA',
                   ylabel='Population', fontsize=14)
plt.plot_mc(mgr['My run - mix'], 'GrpA.S', color='blue',
             interval=50, label='S')
plt.plot mc(mgr['My run - mix'], 'GrpA.I', color='orange',
             interval=50, label='I')
plt.plot mc(mgr['My run - mix'], 'GrpA.R', color='green',
             interval=50, label='R')
plt.plot_mc(mgr['My run - mix'], 'GrpA.S + GrpA.I + GrpA.R', color='black',
             interval=50, label='Total')
```


*Figure 5: Plot of the median and inter-quartile range for S, I and R over time for GrpA.*

<span id="page-17-0"></span>Similarly, we can plot what happens to GrpB, as shown in [Figure 6.](#page-17-1)

```
plt = pcm.Plotter(title='SIR Time Series - Monte Carlo - GrpB',
                   ylabel='Population', fontsize=14)
plt.plot_mc(mgr['My run - mix'], 'GrpB.S', color='blue',
             interval=50, label='S')
plt.plot_mc(mgr['My run - mix'], 'GrpB.I', color='orange',
             interval=50, label='I')
plt.plot_mc(mgr['My run - mix'], 'GrpB.R', color='green',
             interval=50, label='R')
plt.plot mc(mgr['My run - mix'], 'GrpB.S + GrpB.I + GrpB.R', color='black',
             interval=50, label='Total')
```


*Figure 6: Plot of the median and inter-quartile range for S, I and R over time for GrpB.*

<span id="page-17-1"></span>Note in the above code that to specify the output we want to plot in a nested model, we use dot-notation to navigate the sub-models. E.g., GrpB.S plots the susceptible population within GrpB.

### <span id="page-18-4"></span><span id="page-18-0"></span>**3.5 Initialization files**

Initialization dictionaries are useful when we want to set up the model in Python code, but it is often more practical to specify the initialization data in a separate file. This allows different model setups to be saved and edited by hand. For this purpose, PyCoMod models can also be initialized from an Excel file. The Excel file template to initialize a particular model can be generated by the model itself by calling write excel init and providing a file name.

```
m3.write excel init('init mix.xlsx')
```
In Google Colab, the initialization file will be written to the temporary session storage and can be downloaded, modified and re-uploaded. In a local Python environment, the file is written to local storage.

The Excel initialization file is structured in a similar way to the initialization dictionary. The first tab contains run inputs, the second tab contains the top-level model inputs, and subsequent tabs contain sub-model inputs if sub-models are present. In the case of the example provided in Section [3.4,](#page-15-0) there are four tabs as shown in [Figure 7.](#page-18-1)



*Figure 7: Tab structure of an Excel initialization file.*

<span id="page-18-1"></span>The content of the run tab is shown in [Figure 8](#page-18-2) and always consist of the following run settings:

- *t*—the initial simulation time (usually 0),
- *date*—the initial simulation date,
- *dt*—the simulation time step,
- *end*—the simulation end time, and
- *reps*—the number of replications for Monte Carlo runs.

	date	end	reps
	0 2023-02-03 00:00:00	300	

*Figure 8: Content of the run tab.*

<span id="page-18-2"></span>The model tab contains the initialization inputs for the elements of the top-level model. In this case, they are *GrpA*, *GrpB*, and *b\_mix*, which are shown in [Figure 9.](#page-18-3)

GrpA	<b>GrpB</b>	b mix	out	
model.GrpA   model.GrpB			$0.05$ GrpA	
			GrpB	

*Figure 9: Content of the model tab.*

<span id="page-18-3"></span>We can edit the value for the cross-infection parameter *b* mix, here.

Because *GrpA* and *GrpB* are sub-models, the value under these labels is the name of the tab that contains the initialization data for that sub-model. So under *GrpA*, the value is *model.GrpA* which is the name of the third Excel tab. Tab names contain the full path from the model hierarchy to avoid naming collisions in the event that two sub-models have the same name. It should not be necessary to change the sheet-name entry under a sub-model within the *model* tab. In the *model.GrpA* tab, shown in [Figure 10,](#page-19-1) we find the inputs for the elements of the *GrpA* sub-model: *S*, *I*, *R*, *b\_m*, *b\_s*, and *g*.

		В						
			R	b_m	bs	g	out	
2	95		0	0.2	0.05	$0.1$ S		
3								
							R	

*Figure 10: Content of the model, GrpA tab.*

<span id="page-19-2"></span><span id="page-19-1"></span>The same applies to the *GrpB* sub-model tab. Each model tab also contains an *out* entry which is used to list the desired outputs for the model or sub-model. This has the same function as calling set output within the model definition. Recall that the outputs of a sub-model will only be saved if the parent model includes the sub-model in its output list.

We can edit the values in the Excel file, for example, by changing *b* mix to 0.025 (cutting the transmission rate between the two populations in half) and then saving the changes.

In Google Colab, we then must upload the edited file to the session storage.

Now we can run the model using the Excel file to initialize it.

mgr.run mc(m3, init='init mix.xlsx', label='My run - mix - xls')

The output can then be viewed in the same way as shown in the previous section.

### <span id="page-19-0"></span>**3.6 Dynamic model parameters**

It is often necessary to adjust model parameters over time. This can be accomplished using PyCoMod's equation class. For example, we might want to modify the SimpleSIR model to make the transmission rate decay over time, reflecting increasing adherence to public health measures. So, we could replace the transmission parameter *b* with an equation implementing an exponential decay (i.e.,  $b(t) = 0.2(0.98)^t$ ).

**class ModSIR**(pcm.Model):

```
 def build(self):
    # Pools
   self.S = pcm.Pool(95)self.I = pcm.Pool(5)self.R = pcm.Pool(0) # Equations
    self.N = pcm.Equation(lambda: self.S() + self.I() + self.R())
    # Parameters
   self.b = pcm.Equation(lambda: 0.2 * (0.98) **self.t())self.g = pcm.Parameter(0.1) # Flows
```

```
DRDC-RDDC-2023-D111 13
```

```
 self.Fsi = pcm.Flow(lambda: self.b() * self.I() * self.S() / self.N(),
                      src=self.S, dest=self.I)
 self.Fir = pcm.Flow(lambda: self.g() * self.I(),
                     src=self.I, dest=self.R)
 # Output
 self.set_output('S', 'I', 'R', 'b')
```
 $m4 = ModSIR()$ 

The current simulation time can be accessed and used in the equation for b by calling the special variable self.t. We can view the modified transmission rate over time by including b in the list of outputs, running the model, and then plotting it; see [Figure 11.](#page-20-0)

```
mgr.run(m4, duration=150, label='Mod SIR')
plt = pcm.Plotter(title='Dynamic transmission rate',
                   ylabel='Value', fontsize=14)
plt.plot(mgr['Mod SIR'], 'b', color='blue', label='Transmission rate')
```


*Figure 11: A dynamic transmission rate modelled with an exponential decay equation.*

<span id="page-20-0"></span>Sometimes we want a parameter to change to specific values at specific times, in other words, a step function. It is possible to implement a step function as a PyCoMod Equation, but this is not trivial. As this is a common requirement in modelling and simulation, PyCoMod provides a built-in equation sub-class called Step. For example, we can change the ModSIR model from Section [3.6](#page-19-0) such that the transmission rate increases and decreases at certain times, reflecting specific public health measures coming into and out of force.

 $self.b = pcm.Step([0.2, 0.13, 0.2], [0, 7, 21])$ 

When initializing the PyCoMod step object, we provide a list of values and a corresponding list of times. In this case, the transmission rate is initially 0.2 at time 0, it then reduces to 0.13 on day 7 for a period of two weeks, after which it returns to 0.2 on day 21, shown in [Figure 12.](#page-21-0) Note that the default time unit in PyCoMod is 1 day.





*Figure 12: A dynamic transmission rate modelled as a step function.*

<span id="page-21-0"></span>In the above examples, the numerical constants used to define the changing transmission rate, b, could be replaced with PyCoMod Parameters which would register them as model inputs allowing them to be adjusted via an initialization dictionary or initialization file. This is an important advantage of using parameters rather than literals in a model.

In the case of the Step class, we need two vectors, and PyCoMod Parameter objects support vector inputs. So, we can create a parameter b\_v for the values of the transmission rate and a parameter b\_t for the times at which they will be applied.

```
self.b v = pcm.Paramerer([0.2, 0.13, 0.2])self.b t = pcm.Paramerer([0, 7, 21])self.b = pcm.Step(self.b_v(), self.b_t())
```
The initialization dictionary for this model would then specify lists for the values of  $b_$  v and  $b_$  t.

```
init mod = {'run': {'end': 150},
              'model':{'S': 95, 'I': 5, 'R': 0, 'b_v': [0.2, 0.13, 0.2],
                     'b_t': [0, 7, 21], 'g': [0, 1]
```
If we would rather create an Excel initialization file for this model, we will see two vector inputs for the parameters b\_v and b\_t, shown in [Figure 13.](#page-21-1)

D	E
bν	bt
0.2	0
0.13	
0.2	21

*Figure 13: Vector inputs in the initialization Excel file for the transmission rate step function.*

<span id="page-21-1"></span>Whichever method is used, we can now edit the timing and magnitude of changes to the transmission rate. The size of the vector is not restricted to the initial dimension of three in this example. More values and times can be added so long as there is always a corresponding time for each value.

The PyCoMod Impulse is another type of dynamic function similar to Step. The Impulse class generates specified values at specified times, but only at those times. In other words, the impulse value is held only for the timestep that contains the impulse time, otherwise it returns 0 or an optional default value. For example, the transmission rate in our model could be 0.2 under normal circumstances, but on certain dates there may be events that are expected to result in elevated transmission.

self.b = pcm.Impulse([0.5, 0.5, 0.5], [10, 25, 45], 0.2)

When initializing PyCoMod Impulse, we provide a list of impulse values, a list of impulse times, and an optional default value. In this case, it produces an elevated transmission rate of 0.5 on days 10, 25 and 45, but it otherwise produces the nominal rate of 0.2, shown in [Figure 14.](#page-22-1)



*Figure 14: A dynamic transmission rate modelled as an impulse function.*

<span id="page-22-1"></span>The same approach as described above can be used to set these values using an initialization dictionary or Excel file.

### <span id="page-22-0"></span>**3.7 Initial flows**

In some cases, it may be useful to incorporate flows into establishing the initial state of the system. For example, we may not know that there are exactly 5 initial infections in the population, as in the preceding examples. Instead, we may only know that there is a 5% chance that any given person is infected, based on some larger population statistics. To model this situation, we can place the entire population in the susceptible, S, compartment, and use a stochastic initial flow to move a random number of them to the infectious, I, compartment based on the aforementioned 5% probability.

```
class MonteCarloSIR2(pcm.Model):
     def build(self):
         # Pools
        self.S = pcm.Pool(100)self.I = pcm.Pool(0)self.R = pcm.Pool(0) # Equations
         self.N = pcm.Equation(lambda: self.S() + self.I() + self.R())
         # Transmission rate parameters
        self.b_m = pcm.Parameter(0.2)
```

```
self.b s = pcm.Parameter(0.05) # Transmission rate random sample
         self.b = pcm.Sample(lambda: rng.normal(self.b_m(), self.b_s()))
         # Recovery rate parameter
        self.g = pcm.Parameter(0.1) # Flows
         self.Fsi = pcm.Flow(
                 lambda: rng.binomial(self.S(), 
                                      self.b() * self.I() / self.N()),
                 src=self.S, dest=self.I)
         self.Fir = pcm.Flow(
                 lambda: rng.binomial(self.I(), self.g()),
                 src=self.I, dest=self.R)
         # Initial flow
        self.Pi = pcm.Parameter(0.05) self.Fsi_init = pcm.Flow(lambda: rng.binomial(self.S(), self.Pi()),
                                   src=self.S, dest=self.I, init=True)
         # Output
         self.set_output('S','I','R')
m5 = MonteCarloSIR2()
```
In the above code, note that the S pool is initialized to contain the whole population, and I and R are empty. Toward the end of the model definition, we have added a parameter, P<sub>1</sub>, for the 5% probability of initial infection, and the initial flow Fsi init. This flow uses a binomial RNG to move a random number of individuals from *S* to *I* using the probability *Pi*. To flag this flow as an initial flow, we set the optional init parameter to *True*. This flow will now only be executed once at the start of each run.

If we run this model, we can see that the initial state of the system is now uncertain, and there is more variability in the outcome, shown in [Figure 15,](#page-24-1) compared to the first MonteCarloSIR model in Section [3.3.](#page-13-0)

```
mgr.run_mc(m5, duration=150, reps=100, label='My run - mc2')
plt = pcm.Plotter(title='SIR Time Series - Monte Carlo',
                   ylabel='Population', fontsize=14)
plt.plot_mc(mgr['My run - mc2'], 'S', color='blue',
                 interval=50, label='S')
plt.plot_mc(mgr['My run - mc2'], 'I', color='orange',
                 interval=50, label='I')
plt.plot mc(mgr['My run - mc2'], 'R', color='green',
                 interval=50, label='R')
plt.plot_mc(mgr['My run - mc2'], 'S + I + R', color='black',
                 interval=50, label='Total')
```


<span id="page-24-1"></span>*Figure 15: The result of the model where the initial state is controlled by a stochastic initial flow from S to I.*

### <span id="page-24-0"></span>**3.8 Vectorization**

In PyCoMod, the values held by model elements can be vectors. As with vector parameters introduced previously in Section [3.6,](#page-19-0) pools can also be initialized with a list of values and are stored internally as NumPy arrays. Many mathematical operations in NumPy are seamlessly compatible with vector values. NumPy's RNG functions are also compatible with vector inputs. In many cases a model developed for scalar values will be compatible with vector values with little or no changes. This feature is useful for modelling multiple isolated or semi-isolated populations in parallel, such as a training setting in which students are divided into parallel cohorts. Note that a familiarity with how NumPy handles vectors in mathematical expressions is necessary to build vectorized models.

For example, we can vectorize the MonteCarloSIR2 model from Section [3.7](#page-22-0) simply by changing the pool initial values to lists. In this case, the susceptible population is initialized to 10 cohorts containing 10 individuals each, and the infectious and recovered populations are initialized to 10 empty cohorts. Note that the S, I and R pools must all have the same number of cohorts. The rest of the model accommodates the vectorized populations without any changes. So rather than a single SIR model of 100 people, we have 10 parallel SIR models of 10 people each.

```
class VecSIR(pcm.Model):
```

```
 def build(self):
    # Pools
   self.S = pcm.Pool([10] * 10)self.I = pcm.Pool([0] * 10)self.R = pcm.Pool([0] * 10) # Equations
    self.N = pcm.Equation(lambda: self.S() + self.I() + self.R())
    # Transmission rate parameters
   self.b_m = pcm.Parameter(0.2)self.b_s = pcm.Parameter(0.05) # Transmission rate random sample
    self.b = pcm.Sample(lambda: rng.normal(self.b_m(), self.b_s()))
    # Recovery rate parameter
   self.g = pcm.Parameter(0.1)
```

```
 # Flows
 self.Fsi = pcm.Flow(
         lambda: rng.binomial(self.S(), self.b() * self.I() / self.N()),
         src=self.S, dest=self.I)
 self.Fir = pcm.Flow(
         lambda: rng.binomial(self.I(), self.g()),
         src=self.I, dest=self.R)
 # Initial flow
self.Pi = pcm.Parameter(0.05) self.Fsi_init = pcm.Flow(lambda: rng.binomial(self.S(), self.Pi()),
                           src=self.S, dest=self.I, init=True)
 # Output
 self.set_output('S','I','R')
```

```
m6 = VecSIR()
```
If we plot the result, we can see the protective effect of dividing the population into isolated cohorts, [Figure 16.](#page-25-0) Note that when we plot a model output that is vectorized, it is the sum of the vector that is shown on the figure.

```
mgr.run_mc(m6, duration=150, reps=100, label='My run - vec')
plt = pcm.Plotter(title='SIR Time Series - Monte Carlo',
                   ylabel='Population', fontsize=14)
plt.plot_mc(mgr['My run - vec'], 'S', color='blue',
             interval=50, label='S')
plt.plot_mc(mgr['My run - vec'], 'I', color='orange',
             interval=50, label='I')
plt.plot_mc(mgr['My run - vec'], 'R', color='green',
             interval=50, label='R')
plt.plot_mc(mgr['My run - vec'], 'S + I + R', color='black',
             interval=50, label='Total')
```


<span id="page-25-0"></span>*Figure 16: The result of a vectorized model where all populations are divided into 10 cohorts.*

However, it is usually not realistic to assume that populations are perfectly isolated, so we can introduce a potential for spread between cohorts. At the end of the model definition, we add the parameter b\_mix which is the smaller rate of transmission between cohorts (one tenth the nominal transmission rate within cohorts), and we add the flow Fsi mix which creates new infections within each cohort as a result of mixing between cohorts. When a susceptible person is in a mixed setting (e.g., a hallway where cohorts share the same space), the probability that they encounter an infectious person is given by the total proportion of infectious people in the population, hence the modified term  $self.I()$ .sum()/ $self.N()$ .sum() appears in the flow equation. The addition of .sum() returns the sum of the vector, in other words, the sum across the cohorts.

```
class VecSIR(pcm.Model):
```

```
 def build(self):
         # Pools
        self.S = pcm.Pool([10] * 10)self.I = pcm.Pool([0] * 10)self.R = pcm.Pool([0] * 10) # Equations
         self.N = pcm.Equation(lambda: self.S() + self.I() + self.R())
         # Transmission rate parameters
        self.b m = pcm.Paramerer(0.2)self.b s = pcm.Parameter(0.05) # Transmission rate random sample
         self.b = pcm.Sample(lambda: rng.normal(self.b_m(), self.b_s()))
         # Recovery rate parameter
        self.g = pcm.Parameter(0.1) # Flows
         self.Fsi = pcm.Flow(
                 lambda: rng.binomial(self.S(), self.b() * self.I() / self.N()),
                 src=self.S, dest=self.I)
         self.Fir = pcm.Flow(
                 lambda: rng.binomial(self.I(), self.g()),
                 src=self.I, dest=self.R)
         # Initial flow
        self.Pi = pcm.Parameter(0.05) self.Fsi_init = pcm.Flow(lambda: rng.binomial(self.S(), self.Pi()),
                                   src=self.S, dest=self.I, init=True)
         # Mixing
        self.b_mix = pcm.Parameter(0.02) self.Fsi_mix = pcm.Flow(
                 lambda: rng.binomial(self.S(),
                                       self.b_mix() * self.I().sum()
                                                    / self.N().sum()),
                 src=self.S, dest=self.I)
         # Output
         self.set_output('S','I','R')
m6 = VecSIR()
```
If we plot the output, we can see the effect of the limited degree of mixing between cohorts in [Figure 17.](#page-27-0)



<span id="page-27-0"></span>*Figure 17: The result of a vectorized model where all populations are divided into 10 cohorts and a limited degree of mixing between cohorts occurs.*

## <span id="page-28-0"></span>**4 Conclusion**

PyCoMod was developed to reduce the time and effort required to build, solve, and analyze detailed, stochastic compartment models. Although initially developed to model COVID-19 transmission in CAF operational settings, PyCoMod can be used to develop a variety of models outside this realm. The most obvious example is employing PyCoMod to model other infectious diseases, whether airborne (e.g., influenza), water-borne (e.g., schistosomiasis, hepatitis) or vector-borne (e.g., malaria, dengue). Several such diseases would be of concern to military operations around the world. Applying PyCoMod to vector-borne diseases would likely involve modelling both the human population and the vector (e.g., mosquitoes) in tandem [\[6\].](#page-29-5) Many diseases relevant to the military have previously been modelled using compartment models [\[7\],](#page-29-6) however they have typically been analyzed in a civilian context and may not consider aspects of the population or environment that are particular to a CAF population, such as the unique facilities and conditions of a deployment or exercise. PyCoMod's features were developed specifically to efficiently model such unique characteristics of populations operating in an infectious disease setting. These features can be employed by other researchers in creating compartment models tailored to other unique populations or situations.

Outside the realm of epidemiology, compartment models fall into the more general class of modelling known as system dynamics, which are used in a wide range of fields including, for example, personnel management and training logistics [\[8\]](#page-29-7). PyCoMod's capabilities are not exclusive to disease modelling and may be applied wherever system dynamics models are used, and its unique features may be beneficial over existing tools in those fields. The package has been made publicly available on DRDC's GitHub page [\[9\]](#page-29-8) in order to give access to as many interested parties as possible. This was especially beneficial during the COVID-19 pandemic given the heightened need for researchers to be able to collaborate remotely.

## <span id="page-29-0"></span>**References**

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## **Annex A Python Compartment Modelling function and object reference table**

<span id="page-30-2"></span>For ease of reference, the main elements of the PyCoMod package are summarized in [Table A.1](#page-30-2) t[o Table A.4.](#page-32-1)

*Table A.1: PyCoMod classes.*

<span id="page-30-1"></span><span id="page-30-0"></span>



### *Table A.2: Model methods.*



#### *Table A.3: RunManager methods.*

<span id="page-31-1"></span><span id="page-31-0"></span>



<span id="page-32-1"></span><span id="page-32-0"></span>

# <span id="page-33-0"></span>**List of symbols/abbreviations/acronyms/initialisms**





13a. ABSTRACT (When available in the document, the French version of the abstract must be included here.)

This Reference Document describes the PyCoMod (Python Compartment Modelling) code package. PyCoMod is used to build and run compartment models, such as susceptible-infectious-recovered (SIR) models of infectious disease. The package was initially developed to support analyses of the spread of Coronavirus Disease 2019 (COVID-19) in specific scenarios relevant to the Canadian Armed Forces (CAF) during the pandemic in 2020 and 2021. Over the course of multiple studies conducted during this period in collaboration with the Canadian Forces Health Services Group, the package evolved to include many features making it useful as a general modelling and simulation tool. The use of PyCoMod and its features will be described in detail in this Document.

13b. Résumé (when available in the document, the French version of the abstract must be included here)

Le présent document de référence décrit le code du paquet PyCoMod (modélisation à compartiments Python). PyCoMod est utilisé pour créer et exécuter des modèles à compartiments, comme un modèle Susceptible-Infecté-Rétabli relatif à une maladie infectieuse. Le paquet a été élaboré initialement à l'appui des analyses sur la propagation de la COVID-19 dans le cadre de scénarios précis qui concernaient les Forces armées canadiennes pendant la pandémie en 2020 et en 2021. Au cours de multiples études effectuées pendant cette période en collaboration avec le Centre des services de santé des Forces canadiennes, le paquet a évolué et comprend maintenant de nombreuses fonctionnalités qui le rendent utile comme outil de modélisation et de simulation général. L'utilisation de PyCoMod et de ses fonctionnalités sera décrite en détail dans le présent document.