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The Phase Gradient Bearing Estimation Algorithm

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Scientific Literature
DRDC Atlantic SL 2013-210
November 2013

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IMPORTANT INFORMATIVE STATEMENTS

Abstract

A bearing estimation algorithm was developed as part of a Long Range Acoustic Bearing (LRAB) homing system for an Autonomous Underwater Vehicle (AUV). The Phase Gradient algorithm was designed to process data from a tri-axis cross-dipole, seven-element hydrophone array mounted in the nose-cone of the AUV. The algorithm estimates the bearing and elevation angles to a pulsed continuous wave (CW) signal from an acoustic beacon source. The algorithm directly estimates the three Cartesian components of the incoming signal wave-vector from estimated cross-spectra between the hydrophone elements. It is robust against hydrophone failure, and every hydrophone in the array is used to estimate each component of the wave-vector.

It was successfully implemented on the AUV's Acoustic Homing and Localization System (AHLS) processor, and is run in real time. In this paper the theoretical development of the Phase Gradient algorithm is presented, as well as the results from real applications of the algorithm in the AUV homing system.

Résumé

Un algorithme d'estimation de relèvement a été mis au point dans le cadre d'un système de ralliement acoustique à longue portée (LRAB) destiné à un véhicule sous-marin autonome (VSA). L'algorithme à gradient de phase a été conçu pour traiter les données d'une matrice d'hydrophones triaxiale à sept éléments à doublets croisés montée dans le cône avant du VSA. L'algorithme estime les angles de relèvement et d'élévation d'un signal d'onde entretenue (CW) pulsée provenant d'une balise acoustique source. L'algorithme estime directement les trois composantes cartésiennes du vecteur d'onde du signal entrant à partir des spectres croisés estimés entre les éléments hydrophones. L'algorithme résiste bien aux défaillances des hydrophones, et chaque hydrophone de la matrice est utilisé pour estimer chaque composante du vecteur d'onde.

L'algorithme a été implémenté avec succès avec le processeur AHLS (Acoustic Homing and Localization System) du VSA et a été exécuté en temps réel. Cet article présente le processus de développement théorique de l'algorithme de traitement de gradient de phase et donne les résultats d'applications réelles de l'algorithme dans le système de ralliement du VSA.

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INTRODUCTION

A bearing estimation algorithm was developed as part of a Long Range Acoustic Bearing (LRAB) homing system for an Autonomous Underwater Vehicle (AUV). The Phase Gradient algorithm was designed to process data from a tri-axis cross-dipole, seven-element hydrophone array mounted in the nose-cone of the AUV. The algorithm estimates the bearing and elevation angles to a pulsed continuous wave (CW) signal from an acoustic beacon source. The algorithm directly estimates the three Cartesian components of the incoming signal wave-vector from estimated cross-spectra between the hydrophone elements. It is robust against hydrophone failure, and every hydrophone in the array is used to estimate each component of the wave-vector. It was successfully implemented on the AUV's Acoustic Homing and Localization System (AHLS) processor, and is run in real time. In this paper the theoretical development of the Phase Gradient algorithm is presented, as well as the results from real applications of the algorithm in the AUV homing system.

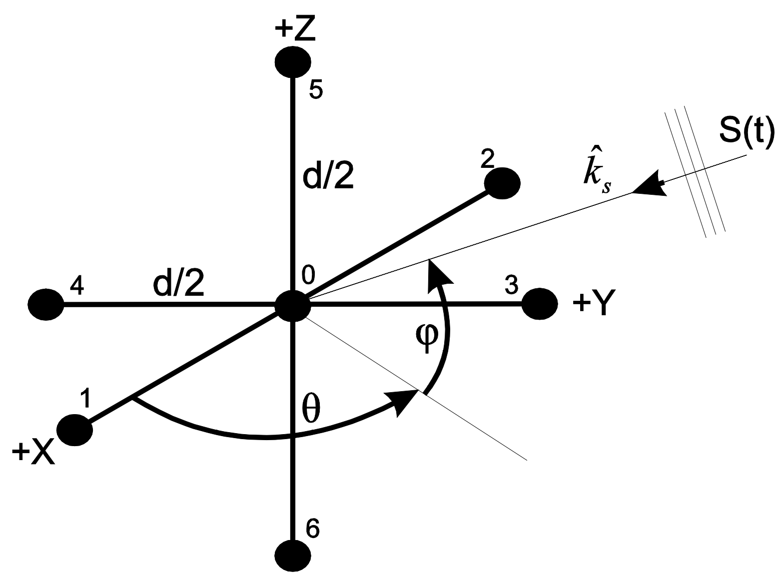

FIGURE 1. This diagram shows the hydrophone array layout for which the Phase Gradient algorithm was designed.

BACKGROUND THEORY

Consider a planar acoustic signal incoming to a hydrophone array from a fixed direction of arrival, as shown in Figure 1. Define $s(t)$ as the time domain signal present at hydrophone #0 due to the plane wave arrival, in the absence of any noise. Define the vector $\hat{k}_s$ as the unit vector that points in the direction of travel of the incoming plane wave. We define $x_i(t), i = 1\ldots6$ as the time domain signals measured at each hydrophone, including noise. The time domain signal measured on the i^th^ hydrophone, in terms of $s(t)$ is given by

$$s_n(t) = s\left(t - \frac{\hat{k}_s \circ \vec{r}_n}{c}\right) + n_n(t). \tag{1}$$

Here we have defined $\vec{r}_i$ as the position vector to the i^th^ hydrophone, measured from the coordinate system origin located at hydrophone #0, and *c* as the speed of sound in water. The term $n_i(t)$ represents the noise present on the i^th^ hydrophone that is unrelated to the incoming signal. The measured hydrophone time series $x_i(t)$ are delayed or advanced versions of $s(t)$ with noise added, the delay time depending on the direction of arrival of the incoming signal.

If we take the Fourier Transform of Equation 1, we obtain the frequency domain representation of the hydrophone outputs, given by

$$\begin{aligned} X_i(f) &= S(f)\exp\left(-j\vec{k}_s \circ \vec{r}_i\right) + N_i(f) \\ &= \left|S(f)\right|\exp\left(-j\vec{k}_s \circ \vec{r}_i + j\angle S(f)\right) + N_i(f). \end{aligned} \tag{2}$$

The frequency domain representation of the time signals is denoted by capital letters, and *j* is the imaginary unit. The Argument of $S(f)$ is denoted by $\angle S(f)$. The wavevector $\vec{k}_s$ of Equation 2 is a function of frequency and for the geometry of Figure 1 is given by

$$\vec{k}_s = \left|\vec{k}_s\right|\hat{k}_s = -\frac{2\pi f}{c}\left(\cos\phi\cos\theta, \cos\phi\sin\theta, \sin\phi\right). \tag{3}$$

From Equation 2 we can see that for a given frequency *f*, the spatial phase at any position vector $\vec{r}$ of the incoming signal, neglecting noise, is given by

$$\psi(\vec{r}) = -\vec{k}_s \circ \vec{r} + \angle S(f). \tag{4}$$

The phase angle $\angle S(f)$ does not depend upon the spatial position $\vec{r}$. If we can estimate the gradient of the spatial phase $\psi(\vec{r})$, we will have an estimate of the wavevector, since $\nabla\psi(\vec{r}) = -\vec{k}_s$. Knowing the wavevector at frequency f gives us the arrival angles of the incoming signal, for that frequency component. If the incoming signal is broad-band, then the wavevector and associated arrival angles can be estimated for each frequency in a set of discrete frequencies within the signal's bandwidth.

PHASE GRADIENT ESTIMATION

To estimate the Cartesian components of the phase gradient $\nabla\psi(\vec{r})$ we use averaged cross-spectra between hydrophone pairs. For the x-axis direction, the phases measured at hydrophones 1 and 2 are given by $\psi_i = -\vec{k}_s \circ \vec{r}_i + \angle S(f),\, i = 1\ldots 2$. Taking the average cross-spectra between hydrophones 1 and 2, and assuming the cross-spectra of the noise terms with other noise terms and signal terms are zero, we obtain

$$\left\langle X_1(f)X_2^*(f)\right\rangle = \left\langle S_1(f)S_2^*(f)\right\rangle e^{j(\psi_1-\psi_2)}. \tag{5}$$

Here $\langle\ \rangle$ represents taking the expectation value, and $*$ means complex conjugate. If we take the Argument of the cross-spectra between hydrophones 1 and 2, and divide by their separation *d*, we get an estimate of the x-component of $\nabla\psi(\vec{r})$. Dropping the explicit frequency dependence, we get

$$\frac{\angle\left\langle X_1X_2^*\right\rangle}{d} = \frac{\psi_1-\psi_2}{d} \approx \frac{\partial\psi}{\partial x}. \tag{6}$$

Because the phase varies linearly in Equation 6, we can take the finite difference as an exact value for $\partial\psi/\partial x$. Performing the same operations with the y-axis using hydrophones 3, 4, and the z-axis using hydrophones 5, 6 we get all the components of $\nabla\psi(\vec{r})$. To obtain the direction of arrival of the incoming signal, we take

$$\frac{\nabla\psi}{\left|\nabla\psi\right|} = -\hat{k}_s = \hat{\upsilon} = \left(\upsilon_x, \upsilon_y, \upsilon_z\right), \tag{7}$$

where $\hat{\upsilon}$ is a unit vector pointing in the direction of arrival. The arrival angles θ, ϕ of Figure 1 are calculated from

$$\theta = \arctan\left(\frac{\upsilon_y}{\upsilon_x}\right),\ \ \phi = \arctan\left(\frac{\upsilon_z}{\sqrt{\upsilon_x^2+\upsilon_y^2}}\right). \tag{8}$$

For a broad band signal, we can average the components of $\hat{\upsilon} = (\upsilon_x, \upsilon_y, \upsilon_z)$ over the frequency bins of the signal, before applying Equation 8 to obtain the arrival angles. Alternatively, we can estimate the angles using Equation 8 at each frequency bin in the spectrum, and then create a histogram of the number of frequency bins with resulting bearings that lie within a range of bearing angles. We can then read off the bearing angles with the largest number of counts as the angles corresponding to the direction of arrival.

Phase Gradient with Improved Averaging

To take advantage of the symmetry of the crossed-dipole array, a modified cross-spectral method was developed. This method uses more hydrophones for calculating the required phase differences, resulting in better averaging and more stable bearing estimates. If we write $\vec{k}_s$ in Cartesian coordinates $\vec{k}_s = (k_x, k_y, k_z)$, then for the x-axis direction we define a 'new' cross-spectrum as

$$\begin{aligned}\langle X_1 X_2 \rangle_{\text{new}} &= \left\langle X_1 \left(X_0 + X_3 + X_4 + X_5 + X_6\right)^* + \left(X_0 + X_3 + X_4 + X_5 + X_6\right) X_2^* \right\rangle \\ &= 2\langle \text{SS}^* \rangle \left(1 + 2\cos(k_y \frac{d}{2}) + 2\cos(k_z \frac{d}{2})\right) e^{j\frac{1}{2}(\psi_1 - \psi_2)}.\end{aligned} \tag{9}$$

Here we have taken the cross-spectrum of hydrophone #1 with the sum of all the hydrophones in the y-z plane, then added to that, the cross-spectrum of the sum of all hydrophone in the y-z plane with hydrophone #2. In Equation 9 the phase contributions from the y and z-axis directions combine to create a real-valued amplitude factor depending on k_y, k_z because of the array symmetry, leaving only half the phase difference between hydrophones #1 and #2 as the Argument of the complex exponential. Using the 'new' cross-spectrum ensures that all the array hydrophones are used in the estimate of the phase difference in each axis direction. Similar to Equation 6, we can calculate the x-component of $\nabla \psi(\vec{r})$ from

$$\frac{2\angle \langle X_1 X_2^* \rangle_{\text{new}}}{d} = \frac{\psi_1 - \psi_2}{d} \approx \frac{\partial \psi}{\partial x}. \tag{10}$$

For the y and z-axis directions we can perform similar operations to Equations 9 and 10, obtaining the remaining components of $\nabla \psi(\vec{r})$. We then apply Equations 7 and 8 to obtain the direction of arrival. The Phase Gradient bearing estimation algorithm with improved averaging is what was used in the actual AUV homing system.

The original design of the Phase Gradient algorithm was for the 3-d cross-dipole array layout of Figure 1. The algorithm has been generalized to other array shapes that also possess high geometrical symmetry. The algorithm with improved averaging was successfully adapted for use with a 4-element tetrahedral hydrophone array [1].

Phase Quality Measure

To quantify the stability of the cross-spectrum phase estimates over the FFT time segments used for the spectral averaging, we define a Phase Quality measure. If the estimated phases are not reasonably stable over the FFT time segments, the resulting bearing estimates will be unstable. With N equal to the number of FFTs that are averaged when calculating the 'new' cross-spectrum, we define the Phase Quality for the x-direction Q_x, and for the x-y plane Q_{xy} from

$$
\begin{aligned}
\langle X_1 X_2 \rangle_{\text{new}} &= \left\langle X_1 \left(X_0 + X_3 + X_4 + X_5 + X_6 \right)^* + \left(X_0 + X_3 + X_4 + X_5 + X_6 \right) X_2^* \right\rangle \\
&= \frac{1}{\mathrm{N}} \sum_{i=1}^{N} \left(Z_{1i} + Z_{2i} \right), \\
Z_{1i} &= \left\{ X_1 \left(X_0 + X_3 + X_4 + X_5 + X_6 \right)^* \right\}_i, \quad Z_{2i} = \left\{ \left(X_0 + X_3 + X_4 + X_5 + X_6 \right) X_2^* \right\}_i, \\
Q_x &= \frac{\left| \sum_{i=1}^{N} \left(Z_{1i} + Z_{2i} \right) \right|}{\sum_{i=1}^{N} \left(\left| Z_{1i} \right| + \left| Z_{2i} \right| \right)}, \quad Q_{xy} = \min(Q_x, Q_y).
\end{aligned}
\tag{11}
$$

The Phase Quality measure is calculated for each axis direction. The Phase Quality for horizontal bearings is taken as being the minimum quality over each axis direction in the horizontal x-y plane, and lies between 0 (poor) and 1 (perfect) quality. From experiments it has been established that a received beacon SNR above 6 dB, combined with a horizontal Phase Quality measure over 0.65, gave stable bearing estimates to the beacon source from the AUV.

ALGORITHM APPLICATION

The LRAB acoustic array was incorporated into the nose-cone of the AUV that was used to carry out Arctic bathymetry missions. The array consisted of seven omni-directional hydrophones in a 3-d cross-dipole pattern, as shown in Figure 2. The diameter of the LRAB array was 40 cm. Voltage data from each hydrophone was collected at a sample rate of 8000 Hz, simultaneously sampled with a 16-bit A/D converter. Each array hydrophone had a sensitivity of -202 dB re 1V/μPa. The Phase Gradient algorithm computations were carried out in the AUV's AHLS onboard processor. For spectral estimation we used 8192 sample (1.024 second) time windows that were processed using the Fast Fourier Transform (FFT), with 50% overlap. We used 15 averages to calculate the required averaged cross-spectra for the algorithm, giving us a processing gain of ~12 dB.

FIGURE 2. Seven-element hydrophone array mounted in the AUV nose cone, which is flooded during operation.

The LRAB array and the AHLS were extensively tested during a trial that took place in December 2009 at the Canadian Forces Maritime Experimental and Test Ranges (CFMETR) facility, Nanoose Bay, Canada. At this trial a CW tonal beacon at a frequency of 1361 Hz and strength of 180 dB re 1μPa @ 1m was used. Accurate bearings to the beacon source from the AUV were estimated using the Phase Gradient algorithm, at ranges up to 29 kilometers.

During an experiment in the Canadian Arctic in March 2010, the AUV was programmed to carry out under-ice deep-water bathymetry measurements, and was required to run under-ice missions over 300 Km in length. The AUV ran its missions along the sea floor at an altitude of 100 m from the bottom, in water depths of approximately 2 Km. The vehicle's real-time position was estimated from its onboard Inertial Navigation System (INS). The vehicle was programmed to go into a 'homing' mode when its estimated distance to the pre-programmed destination position was less than 50 Km.

At the destination ice camp, a strong acoustic beacon was placed into the water at a depth of 150 m, to which the AUV was to home in to. The acoustic source was a single CW beacon at a frequency of 1361 Hz, with a source level of 189 dB re 1μPa @1m. The beacon was turned on well in advance of the estimated vehicle arrival time, to ensure that it would be on when the vehicle went into homing mode. Once the vehicle switches into homing mode, it looks for the beacon source to be detected. A valid detection occurs when the beacon signal SNR is above 6 dB, and simultaneously when the cross-spectrum phases have a horizontal Phase Quality greater than 0.65. The AUV was programmed to use the horizontal bearing returned from the LRAB system, after a valid beacon detection, to guide the AUV to the mission destination.

One under-ice AUV mission is highlighted in Figure 3, which shows the AUV track for a multi-day mission. From the figure we can see that the AUV went into homing mode at a range of ~50 Km, then immediately obtained valid beacon detection and changed course towards the beacon. The AUV self-noise at the beacon frequency was approximately 82 dB re 1μPa/√Hz, and it was assumed that this would be well above the local ambient noise level. The water depth was ~ 2.1 Km. Transmission Loss (TL) calculations were performed using the locally measured upward-refracting sound velocity profile, between the source and the AUV. The estimated TL values were near 95 dB @ 50 Km for this environment. Using these parameters, we estimated the signal SNR for detection at the AUV to be 189 +12 – 95 – 82 = 24 dB @ 50 Km, well above the required 6 dB SNR for detection. The AUV was able to detect the beacon source at ~50 Km range once in homing mode, and it successfully homed in to the beacon.

The LRAB homing system was vital to the successful recovery of the vehicle at its mission end. The destination ice camp (and the man-made recovery ice hole) could potentially drift up to 20 km/day, and the AUV missions were between 1-3 days in duration.

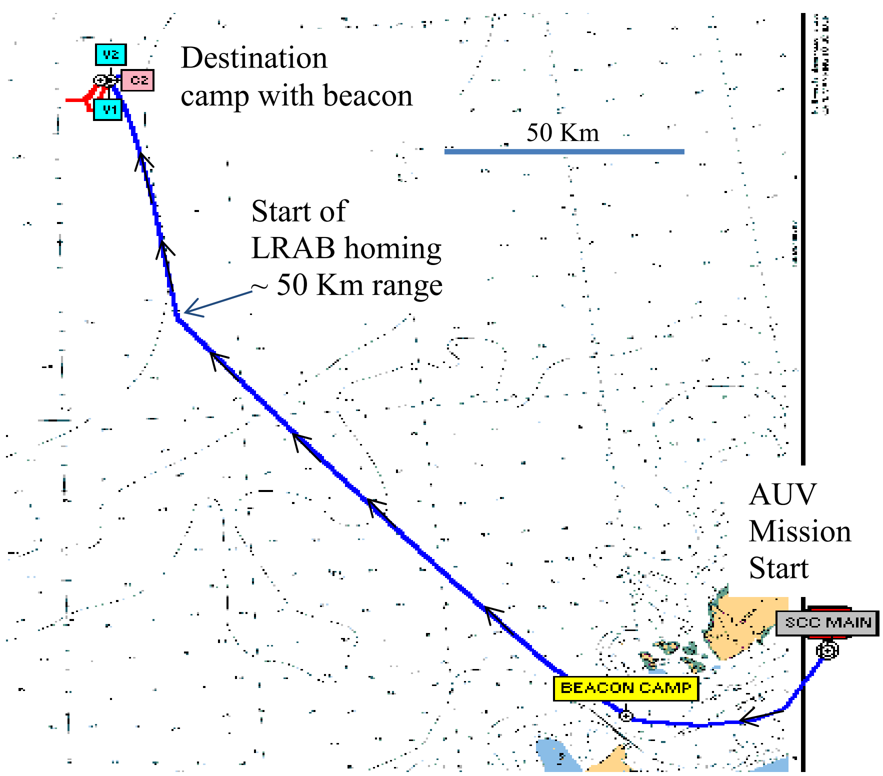

FIGURE 3. AUV track during an Arctic under-ice bathymetry mission, homing using the Phase Gradient algorithm.

CONCLUSION

The Phase Gradient bearing estimation algorithm is computationally efficient and performed well as part of an AUV homing system. The algorithm does not search over beam angles for maximum beam-power; instead it estimates the components of the incoming signal wavevector directly by estimating the gradient of the phase of the signal across the hydrophone array. This dramatically decreases the amount of signal processing and time required to generate a bearing estimate at the frequency of interest, or at all frequencies in a broad-band signal. The algorithm has been successfully used to track narrow and broad-band acoustic targets in an efficient way using both the 7-element 3-d cross-dipole array, and a 4-element tetrahedral array.

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3. TITLE (The complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S, C or U) in parentheses after the title.)

The Phase Gradient Bearing Estimation Algorithm

4. AUTHORS (last name, followed by initials – ranks, titles, etc. not to be used)

Lucas, C.; Heard, G.; Pelavas, N.

5. DATE OF PUBLICATION (Month and year of publication of document.)	6a. NO. OF PAGES (Total containing information, including Annexes, Appendices, etc.)	6b. NO. OF REFS (Total cited in document.)
November 2013	18	1

7. DESCRIPTIVE NOTES (The category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.)

Scientific Literature

8. SPONSORING ACTIVITY (The name of the department project office or laboratory sponsoring the research and development – include address.)

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9a. PROJECT OR GRANT NO. (If appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant.)	9b. CONTRACT NO. (If appropriate, the applicable number under which the document was written.)
	11cu23

10a. ORIGINATOR'S DOCUMENT NUMBER (The official document number by which the document is identified by the originating activity. This number must be unique to this document.)	10b. OTHER DOCUMENT NO(s). (Any other numbers which may be assigned this document either by the originator or by the sponsor.)
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phase gradient bearing estimation algorithm; Long Range Acoustic Bearing; LRAB; homing system; autonomous underwater vehicle; AUV; hydrophone array; pulsed continuous wave; CW; acoustic beacon; incoming signal; wave vector; hydrophone failure; acoustic home and localization system; AHLS; acoustic signal; phase quality measure; FFT; Canadian Forces Maritime Experimental and Test Ranges; CFMETR; Nanoose Bay

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