Cognitive Radar Framework for Target Tracking

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COGNITIVE RADAR FRAMEWORK FOR TARGET TRACKING

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1-Abstract

Abstract – In this work we address the problem of radar waveform optimization and target tracking. We propose an algorithm for optimal waveform design and target tracking based on the Control theoretic approach, where the waveform parameters are adaptively designed by minimizing the tracking mean square error (MSE). In this work we take several approaches to enhance the radar tracking performance. In a first place the Kalman filter was used to estimate the target position which we use to optimize the waveform parameters. Experimental results demonstrated the ability of the proposed algorithm to track a flying target in the Cartesian space, by providing accurate estimates of the target position and the target speed cartesian vector as well as the radial speed. The algorithm adapted the waveform parameters on the fly based on the estimate vectors. In literature, the doppler effect theory was intensively used to estimate target velocity. Under certain conditions, such as tracking high speed targets or rough sea and weather conditions the doppler effect is less effective. Hence, in this first approach we introduce an algorithm that relies on the Kalman filter estimations independently of the doppler effect. A low pass filter with adaptive parameters in real time is applied to the estimated speed vector, and accurate speed estimations where extracted. Furthermore, we tackle the radar tracking problem from a realistic angle by admitting that the target motion can’t be described by on matrix like we have proposed using the Kalman filter, therefore we introduce the Interacting Multiple Model algorithm instead to estimate the target position. Through simulations, we demonstrate the good performance of the proposed algorithm and prove that waveform optimization can improve the tracking performance of the radar. Finally we consider harvesting information from two antennas instead of one, and using one of the data fusion algorithms, and the IMM algorithm, we were able to reduce the tracking error and provide a more robust and reliable solution to the tracking problem.

Keywords- Adaptive waveform design, Cognitive radar (CR), Kalman Filter, Moving average filter, MMSE filter, Neyman-Pearson Hypothesis decision, Target tracking Waveform optimization, Machine learning.

2-Introduction
Cognition is defined as the mental process involved in knowing, learning, and understanding things. This definition introduces three major ingredients that define a CR:
− The ability of the system to continuously interact with the environment and sense its landmarks including potential targets and obstacles; this makes phased array antennas a major component of the CRs for their ability to rapidly scan the environment.

− The ability to process the received echo intelligently and extract measurements about the target and the surrounding environment.

− The ability to extract the information about targets and environment and use it accordingly to make decisions concerning waveforms and target motion estimation.

Cognitive radars are mimicking in a way how the brain learns, and acts based on the senses, following a similar cycle: sense, learn, adjust, act. They continuously learn from the environment and make decisions to improve the tracking performance. A similar cycle, well-known as the Perception-Action Cycle (PAC), was addressed many times in literature explaining how the brain works or in the describing of some intelligent systems ([2][3][4]). Quoting [2], the neuroscientist Joaquin Fuster describes the perception-action cycle as “the circular flow of information from the environment to sensory structures, to motor structures, back again to the environment, to sensory structures, and so on, during the processing of goal-directed behavior”. Figure 1 explains the operating cycle of the brain in correlation to cognitive radars. In this work, we discuss all the steps of this closed-loop cycle that govern the performance of the CR. A system model is presented, and a further discussion with a focus on the estimation and waveform optimization process is covered.

In the literature, two main approaches for waveform selection are discussed: control theoretic and information theoretic. In this work, the criteria for waveform selection in the control theoretic approach is considered. Radar waveform parameters is determined mainly by minimizing the tracking mean square error (MSE).

![Figure 1. Brain/Cognitive Radar Perception-Action Cycle.](image)
The CRs have a closed-loop operating cycle. The system relies on the feedback from the receiver to gather knowledge about the target and the environment. This knowledge is then used to optimize the transmitted waveform and to improve, the detection, tracking, estimation, identification of the target. This concept was introduced for the first time in literature in 2006 by S. Haykin [1], where he wrote and we quoting [1] “The whole radar system constitutes a dynamic closed feed-back loop encompassing the transmitter, environment, and receiver.

The operating cycle of the CR (i.e. the above closed loop) starts with the illumination of the environment by the transmitter. The transmitted waveforms, bouncing off the environment (i.e. Target echo, Clutter, etc.) are then intercepted by the receiver. Useful information about the target and environment is extracted from the received echo, and then updates a library of information (memory block), to be used in the next cycle by the target estimator (TE) as a set of prior knowledge about the environment. In the light of the estimations provided by the TE, waveforms are optimized. Bayesian approaches are commonly considered to implement the target estimator.

In a CR, the extracted information come into play not only at the level of the receiver, but also at the transmitter level by changing on the fly the waveform and some relevant parameters such as pulse repetition frequency (PRF), pulse width, number of pulses N and the radar beaming schedule. This aspect is what distinguishes the CRs from classical adaptive radars that can only use the extracted information at the reception level.

Waveform optimization design has emerged within the signal processing community as an important research topic, due to its wide range of applications in many areas, such as communication systems, sonar, and in our case of interest to improve the performance of radar systems. In literature many design criteria are discussed, from which we mention the maximum signal to interference plus noise ratio (SINR) criterion [9], the Maximum detection probability criterion [14], The maximum Mutual information (MI) [8] criteria and minimizing the mean square error criteria (MMSE) [10], [11]. These design criteria approaches can be categorized into two types: the control theoretic approach where the objective is to develop a control model for continuously operating dynamic systems and the information theoretic method which focus more on studying the flow of information and extract more target information from the received measurements. In this paper, the control theoretic approach is employed the optimal waveform selection/design is determined by minimizing the tracking MSE.

3-System Model

A schematic model system of a CR is presented in Figure 2. It illustrates the unique closed-loop cycle that distinguishes CRs from conventional radar systems. The cycle starts with illuminating the environment following a predesigned schedule. Here phased array radars play a big role. Also known as electronically scanned array (ESA), they can create a beam that can be electronically...
steered to point in specific directions in a fast and precise manner. This gives cognitive radars the ability to rapidly and continuously interact with the environment. The received signal, having information about potential targets along with noise and clutter, is then processed and useful target information is extracted. So far, CRs are following the same linear cycle as conventional radar systems.

Three main blocks stand out in the system model and distinguish cognitive radars; the information library block, the target estimator (TE), and the waveform optimizer. In conventional radar systems, once the target information is extracted, they go to a GUI interface concluding the detection cycle. In a CR, target information is preserved and stored in a library, so it can improve the target estimation accuracy. The target estimator, taking in consideration the prior knowledge of the target, follows a Bayesian approach to track the target and define a set of parameters to optimize the next transmitted waveform. Bayesian estimator is discussed with further details in the next sections. And finally, the transmitter illuminates the environment once again after having the waveform parameters updated.

![Figure 2. CR system model.](image)

Let $S_k$ denote the transmitted waveform matrix where $k$ represents the $k$-th pulse ($k$-th Cycle), $Y_{i,k}$ denotes the received array matrix. We consider $N_t$ elements forming the phased array antenna, $S_k$ is transmitted by $N_t$ elements and received by $N_r$ elements at the receiver, we consider $L$ different snapshots forming the $k$-th pulse:

$$S_k = S_k(0) + S_k(1) + \cdots S_k(L), \quad l = 1 \ldots L, k = 1,2,3 \ldots$$

We define $G_{i,k}$ the Target impulse response of the $i$-th target in the $k$-th cycle ($k$-th pulse). $G_{i,k}$ is assumed to be a zero-mean Gaussian random vector.
We define \( C_k \) the Clutter impulse response. \( C_k \) is assumed to be a zero-mean Gaussian random vector.

Let \( N_k \) denote an additive white complex Gaussian noise matrix. \( N_k \in \mathbb{C}^{N_r \times L} \) where each element of \( N_k \) is a complex number where the real part is an independent zero mean Gaussian random variable with variance \( \sigma_n^2 \) and the imaginary part is also an independent zero mean Gaussian random variable with variance \( \sigma_n^2 \). The elements of \( N_k \) are considered to be independent and identically distributed.

We define \( Y_{i,k} \) to denote the received signal of the \( i \)-th target in the \( k \)-th cycle, thus:

\[
Y_{i,k} = S_k G_{i,k} + S_k C_k + N_k
\]  

\( \{1\} \)

4-Introduction to Phased Array Radars

Phased array radars have merged as an innovative concept in the world of radar tracking for its capacity to rapidly and precisely steer the beam which allows pointing to the direction of a moving target with a high performance. Phased array radars are substantial to our design of cognitive radar for that a continuous interaction and awareness of the environment must be satisfied.

A phased array radar, also known as electronically scanned array is mainly composed of many radiating elements equipped with phase shifters. The shape and direction of the radiation pattern is determined by the phases and amplitudes of the currents applied at each radiating element. A variable phase is applied at each radiating element in a way that the effective radiation pattern of the array is constructively amplified in a desired direction and destructively attenuated in the others.
Consider a linear array of $n$ radiating elements uniformly spaced as presented by figure 4. $d$ presents the element spacing parameter of the antenna.

The total radiated field of the array is the sum of the radiated fields of the $n$ elements:

$$\vec{E}_{Total} = \sum_{n=1}^{N} \vec{E}_{element} = E_{element} \cdot AF.$$ \hspace{1cm} \{2\}

Let $\vec{J}_1, \vec{J}_2, \ldots, \vec{J}_m, \ldots, \vec{J}_n$ be the currents exciting each radiating element, where

$$\vec{J}_m = a_m \vec{J}_1.$$ \hspace{1cm} \{3\}

$a_m$ is a complex number (magnitude and phase):

$$a_m = A_m e^{j\delta_m}$$ \hspace{1cm} \{4\}

where $A_m$ and $\delta_m$ are the respective magnitude and phase of the excitation current applied at the $m^{th}$ radiating element.
The radiated field of the \( m^{th}\) radiating element is defined as:

\[
\vec{E}_m = \vec{E}_1 \cdot a_m \cdot e^{jkr_m}\hat{r}
\]

Where \( \vec{E}_1 \) is the radiated field of the first radiating element of the array. Replacing \{5\} in \{2\} the total radiated field is then defined as:

\[
\vec{E}_{Total} = \sum_{m=1}^{N} \vec{E}_1 = (\sum_{m=1}^{N} a_m \cdot e^{jkr_m})\vec{E}_1 = AF(\theta) \cdot \vec{E}_1.
\]

Hence the array factor is defined as:

\[
AF(\theta) = \sum_{m=1}^{N} a_m \cdot e^{jkr_m} = \sum_{m=1}^{N} a_m \cdot e^{jkd\cos(\theta)}
\]

replacing \{5\} in \{8\}:

\[
AF(\theta) = \sum_{m=1}^{N} A_m \cdot e^{j\delta} \cdot e^{jkd\cos(\theta)} = \sum_{m=1}^{N} A_m \cdot e^{j\psi}
\]

Where \( \psi = kd\cos(\theta) + \delta \) is the total phase shift from one element to the next.

The direction of the maximum radiation corresponds to \( \psi = 0 \). For an electrical phase shift \( \delta \), the direction of the main lobe is then:

\[
\theta = \cos^{-1}\left(\frac{-\delta}{kd}\right)
\]

It is therefore possible by variation of the phase shift \( \delta \) to modify the direction of the main lobe \( \theta \).
Figure 5 illustrates the influence of the phase shift $\delta$ on the direction of the main beam.

Figure 5. Influence of phase shift $\delta$ on the orientation of the main lobe of a uniform linear array of 10 isotropic elements with 0.5 m element spacing.

Now we consider a planar – array antenna system with $N_x$ elements on the x-axis direction and $N_y$ elements in the y-axis direction as illustrated by Figure 6.

Let $i_{mx,my}$ be the excitation current applied to the antenna element $A_{mx,my}$:

$$i_{mx,my} = i_{mx,my} e^{j \delta_{mx,my}}$$ \hspace{1cm} (12)

To obtain a maximum radiation in the direction $(\theta_0, \Phi_0)$ the excitation phase $\delta_{mx,my}$ takes the form:

$$\delta_{mx,my} = -m_x k d_x \sin(\theta_0) \cos(\Phi_0) - m_y k d_y \sin(\theta_0) \sin(\Phi_0)$$ \hspace{1cm} (13)
In a uniform planar array all radiating elements are excited with a current of the same amplitude: $I_{mx,my} = I_0$. The array factor is then written as follows:

$$AF(\theta, \Phi) = I_0 \sum_{m=1}^{N_x} e^{j(m-1)(kd_x \sin \theta \cos \Phi + \beta_x)} \sum_{n=1}^{N_y} e^{j(n-1)(kd_y \sin \theta \sin \Phi + \beta_y)}$$ \hspace{1cm} \{14\}

The normalized array factor is obtained as:

$$AF(\theta, \Phi) = \left\{ \frac{1}{N_x} \sin (\frac{N_x \psi_x}{2}) \right\} \left\{ \frac{1}{N_y} \sin (\frac{N_y \psi_y}{2}) \right\}$$ \hspace{1cm} \{15\}

Where:

$$\psi_x = kd_x \sin \theta \cos \Phi + \beta_x$$ \hspace{1cm} \{16\}

$$\psi_y = kd_y \sin \theta \sin \Phi + \beta_y$$ \hspace{1cm} \{17\}

Note that the array factor $AF(\theta, \Phi)$ of such a configuration is the product of two linear array factors along the x and y axis:

$$AF(\theta, \Phi) = AF(\theta, \Phi)_x N \cdot AF(\theta, \Phi)_y M$$ \hspace{1cm} \{18\}

The major lobe and the grating lobes are located at angles $\theta$ and $\Phi$ such that:

$$\begin{cases} kd_x \sin \theta \cos \Phi + \beta_x = \pm 2m \pi , & m = 0, 1, ... N_x \\
kd_y \sin \theta \sin \Phi + \beta_y = \pm 2n \pi , & n = 0, 1, ... N_y 
\end{cases}$$ \hspace{1cm} \{20\}

To steer the beam to the desired direction specified by $(\theta_0, \Phi_0)$ the progressive phases $\beta_x$ and $\beta_y$ must satisfy:
\[
\begin{align*}
\beta_x &= kd_x \sin \theta_0 \cos \Phi_0 \\
\beta_y &= kd_y \sin \theta_0 \sin \Phi_0
\end{align*}
\] 

\{21\}

One concern in the design of a phased array is the appearance of grating lobes. The grating lobes can be located at the directions specified by \((\theta_{mn}, \Phi_{mn})\) such that:

\[
\begin{align*}
\tan \Phi_{mn} &= \frac{\sin \theta_0 \cos \Phi_0 \pm \frac{n \lambda}{d_y}}{\sin \theta_0 \cos \Phi_0 \pm \frac{m \lambda}{d_x}} \\
\sin \theta_{mn} &= \frac{\sin \theta_0 \cos \Phi_0 \pm \frac{m \lambda}{d_x}}{\cos \Phi_{mn}} = \frac{\sin \theta_0 \cos \Phi_0 \pm \frac{n \lambda}{d_y}}{\sin \Phi_{mn}}
\end{align*}
\] 

\{22\}

A good design avoids the appearance of grating lobes, to do so, the elements spacing must be less than the wavelength \(\lambda\):

\[
\begin{align*}
d_x &< \lambda \\
d_y &< \lambda
\end{align*}
\] 

\{23\}

As illustrated in Figure 7, an element spacing exceeding the wavelength leads to the appearance of the undesired grating lobes whereas the satisfaction of \{23\} grants a good beam shape characterized by the absence of the grating lobes.

\[d_x = d_y = 2.\lambda\]

\[d_x = d_y = \lambda/2\]

Figure 7. Influence of the choice of element spacing.

4.1- Microstrip Patch Antenna design
The number of radiating elements used to form the planar array antenna will depend on the dimensions of these radiating elements. Also, the performance of the phased array antenna will depend heavily on the dimensions of the microstrip patch.

A microstrip patch is composed of a dielectric medium called the substrate having a particular value of dielectric constant laying in between a conducting patch and ground plane as illustrated in Figure 8.

The antenna operating frequency, the radiation efficiency, the microstrip patch dimensions, the choice of material (dielectric constant), are all interconnected parameters that can affect the performance of the radar.

In [3] a performance analysis was performed to search the optimum parameters in the design of a microstrip patch. Quoting [3], The study proposes the following considerations to attain higher radiation efficiency:

- “The relative dielectric constant of the dielectric material should be less than 3 (εr ≤ 3) in order to get higher radiation efficiency and directivity”

- “The operating (resonant) frequency of the microstrip antenna should be less than 10GHz or higher than 50GHz (10GHz ≥ fr ≥ 50GHz) in order to get improved radiation efficiency.”

- “The height (h) of the substrate should be near to 1mm to get the higher radiation efficiency. Again, at the higher operating frequency, for the lower height of the substrate, surface wave increases that in turn increases the losses and reduces the radiation efficiency. Thus, when the operating frequency is greater than 30GHz, then the radiation efficiency of the microstrip antenna increases with the increasing height of the substrate. Therefore, for the operating frequency less than 10GHz, the height should be h ≤ 1.5 mm. For the operating frequency greater than 30GHz, the height should be h ≥ 1.5 mm.”
The considerations suggested by [3] will be adopted to choose the resonant frequency and the dielectric medium for which the antenna is to be designed. Multiple substrates are usually used in the design of microstrip antennas with dielectric constants in the range $2.2 \leq \varepsilon_r \leq 12$. Usually one of the following substrates is considered:

<table>
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<th>Substrate</th>
<th>Dielectric Constant $\varepsilon_r$</th>
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<tbody>
<tr>
<td>AR4</td>
<td>4.8</td>
</tr>
<tr>
<td>Teflon (PTFE)</td>
<td>4.5</td>
</tr>
<tr>
<td>Fr-4</td>
<td>2.2</td>
</tr>
<tr>
<td>Rogers TMM 4</td>
<td>9.4</td>
</tr>
<tr>
<td>Taconic TLY-5</td>
<td>2.08</td>
</tr>
<tr>
<td>Alumina (96%)</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Table 1: Substrates Dielectric constant table

The practical dimensions of the patch can be calculated as follows:

$$W = \frac{c_0}{f_0} \sqrt{\frac{2}{\varepsilon_{r+1}}}, \quad \{24\}$$

Where:

$$L = \frac{1}{2f_0\sqrt{\varepsilon_{eff}}} - 2\Delta L, \quad \{25\}$$

where:
\[ \varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left[ 1 + 12 \frac{h}{W} \right]^{-1/2} \]  \tag{26}

\[ \Delta L = 0.41h \frac{\varepsilon_{eff} + 0.3}{\varepsilon_{eff} - 0.258} \left( \frac{W}{h} + 0.264 \right) \left( \frac{W}{h} + 0.8 \right) \]  \tag{27}

The dimensions of the ground plane define the dimension of the microstrip patch antenna. Knowing the dimensions of the patch antenna, the length and the width of ground plane are calculated using the following equations:

\[ \begin{cases} 
L_g = 6h + L \\
W_g = 6h + W 
\end{cases} \]  \tag{28}

Where,

- \( W \): width of the patch
- \( L \): Length of the patch
- \( W_g \): width of the ground plane
- \( L_g \): Length of the ground plane
- \( C_0 \): speed of light
- \( \varepsilon_r \): dielectric substrate constant
- \( f_0 \): operating frequency
- \( \varepsilon_{eff} \): Effective refractive index
- \( h \): height of substrate

### 4.2-Planar-array antenna design criteria

To guarantee the best performance of the cognitive radar an optimal design of the planar-array antenna should be investigated. An optimal design will provide a higher gain, absence of grating lobes, narrow beam-width …
Figure 9. investigates the effect of changing the element spacing ‘d’ on the array pattern. An increase in the element spacing prompts an increase in the array gain when \( \{23\} \) is satisfied. The result shown in Figure 9. validates the hypothesis proposed earlier in \( \{23\} \) which suggests that the element spacing must remain inferior to the wavelength \( \lambda \) to avoid the appearance of grating lobes.

![Figure 9: The effect of elements spacing on the array pattern.](image)

Table 2 summarizes the results of the simulation

<table>
<thead>
<tr>
<th>Elements spacing ( d )</th>
<th>Array Gain (dBi)</th>
<th>Grating Lobes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 ( \lambda )</td>
<td>6.5</td>
<td>Absence of grating lobes</td>
</tr>
<tr>
<td>0.5 ( \lambda )</td>
<td>13.27</td>
<td>Absence of grating lobes</td>
</tr>
<tr>
<td>0.75 ( \lambda )</td>
<td>15</td>
<td>Absence of grating lobes</td>
</tr>
<tr>
<td>1.5 ( \lambda )</td>
<td>11.38</td>
<td>Presence of grating lobes</td>
</tr>
</tbody>
</table>

These results suggest that an optimal design of the planar array should satisfy the following conditions:

- The ratio \( d/\lambda \) should be greater than 1: \( d/\lambda > 1 \)
- A higher element spacing corresponds to a higher gain.

Figure 10 investigates the effect of changing the number of radiating elements on the array pattern. For the same element spacing (\( d=0.75\lambda \)), an increase in the number of the radiating elements leads to an increase in the array gain.
Figure 10: The effect of the Number of radiating elements on the array pattern.

The choice of the optimal waveform depends heavily on the good choice of the following parameters; The Element spacing ‘d’, the number of radiating elements, the operating frequency, and the wavelength λ. Whether the design of the planar array imposes a specific operating frequency/the wavelength or a specific element spacing, the choice of the other should be defined in the light of the previous results. In other words, if the requirement is to design a planar array with a specific element spacing ‘d’ the choice of the operating frequency should be such that \( \lambda = \frac{d}{x} \) where \( x < 1 \). Taking \( x = 0.75 \), the operating frequency of the radar is then \( f_o = \frac{c}{\lambda} = 0.75 \frac{c}{d} \), where \( C \) is the speed of light.

In the design of the planar array, the choice of the number of elements is important as deducted form Figure 10. The number of radiating elements depends on the dimensions of the microstrip antenna elements (radiating elements) and the desired size of the planar array.

**4.3-Planar-array design**

Consider the design of a planar array with the following specifications:

- Array size: 2m by 2m
- Elements spacing: 5cm

The radar operating frequency can be chosen such that: \( f_o = \frac{c}{\lambda} = 0.75 \frac{c}{d} = 4.5 \text{Ghz} \) (considering \( d=0.75 \lambda \)). Using the equations \{24\} , \{25\} , \{28\} the dimensions of the microstrip patches that will form the planar array are calculated:

- \( W=19.39 \text{mm} \)
- \( L = 14.82 \text{ mm} \)
- \( W_g = 20 \text{ mm} \)
- \( L_g = 25 \text{ mm} \)
- \( h = 1 \text{ mm} \)

Note: The substrate considered is FR4 with **Dielectric Constant** \( \varepsilon_r = 4.8 \)
Figure 11: design of Microstrip Patch Antenna

Figure 12: Microstrip antenna element 3D response

Figure 13: Microstrip antenna element directivity (az=0)

Figure 14: Microstrip antenna element directivity (el=0)
Now that the dimensions of the microstrip antenna elements is known, the number of radiating elements can be defined in the light of the planar-array antenna size (2m x 2m) and the element spacing (5cm).

Let $N_x$ and $N_y$ such that the planar array antenna is an $N_x \times N_y$ matrix of radiating elements.

The number of elements $N_x$ and $N_y$ can be found by solving the following equation that takes into consideration the planar array size and the element spacing:

$$\begin{align*}
D_x &= LN_x + d_x (N_x - 1) \\
D_y &= WN_y + d_y (N_y - 1)
\end{align*} \tag{29}$$

Where,

$D_x \times D_y$ represents the planar-array size ($D_x = D_y = 2000\text{mm}$).

d$x$, $d_y$ = the spacing between two adjacent elements ($d_x = d_y = 50\text{mm}$).

$W$ = width of the patch. ($W = 14.8\text{mm}$)

$L$ = Length of the patch. ($L = 19.4\text{mm}$)

Solving $\{29\}$, $D_x = 29$, $D_y = 33$.

Figure 15. 2m x 2m planar-array antenna design
Table 3. Planar array design summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Array Size</strong></td>
<td>2 meters by 2 meters</td>
</tr>
<tr>
<td><strong>Operating Frequency</strong></td>
<td>4.5 GHz</td>
</tr>
<tr>
<td><strong>Wavelength</strong></td>
<td>0.066m</td>
</tr>
<tr>
<td><strong>Number of Row elements</strong></td>
<td>29</td>
</tr>
<tr>
<td><strong>Number of Column elements</strong></td>
<td>33</td>
</tr>
<tr>
<td><strong>Row elements spacing</strong></td>
<td>0.05m</td>
</tr>
<tr>
<td><strong>Column elements spacing</strong></td>
<td>0.05m</td>
</tr>
<tr>
<td><strong>Mirostrip antenna specifications</strong></td>
<td></td>
</tr>
<tr>
<td>Substrate used</td>
<td>FR4</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>4.8</td>
</tr>
<tr>
<td>Patch length</td>
<td>19.39 mm</td>
</tr>
<tr>
<td>Patch width</td>
<td>14.82 mm</td>
</tr>
</tbody>
</table>

5-Kalman filter: The optimal MMSE target estimator (literature review)

The system model proposed in this work, illustrated in Figure 2, introduces the target estimator as a substantial element to the Cognitive radar design. The interest in Estimation theory has recently expanded to serve in many cutting-edge technology applications. Now estimation theory has been greatly related to many topics of great interest like machine learning, neural networks, pattern recognition, particle tracking ...

In this work a waveform optimization solution is investigated such that the Mean Square error is minimized. Such a solution falls within the control theoretic approach. Hence a Bayesian estimator that leads to the minimum mean square error will be considered.

The target estimator will use the information library that contains information about the target such as target position (XYZ coordinates), target range, target speed, azimuth angle
and elevation angle to estimate the next state of the target. The choice of the Bayesian estimator rather than any classical estimator comes from the fact that the parameters (target state parameters) are considered as random variables rather than unknown constants.

In 1960 R.E.Kalman introduced for the first time a recursive solution to the discrete-data linear filtering problem which proved to be an optimal solution for state estimation, tracking and control. In literature Kalman filter was introduced many times as one of the rare algorithms that are provably optimal for the Bayesian state estimation. In [5] Kalman filter introduced in comparison to other Bayesian estimators and described as the best algorithm in its domain. In [6] Kalman filter was used to predict a shallow water flow with the aid of water level measurements, in which the high capability of the filter was proven. Quoting [6] “The filter has been tested extensively using field data. The results show excellent filter performance”.

In this work the Kalman filter will be considered. The performance of the TE will play a big role in the waveform optimization decision for that the cognitive radar will adaptively change the parameters of the waveform in the light of the new estimation of the target state.

5.1- Kalman Filter outline

Ever since first introduced in 1960, Kalman filter has been extensively used in control theory for state estimation. It has been provably optimal in the recurrent estimation of a set of parameters changing over time.

The following notation will be respected for the rest of this section:

- $z_k$: observation vector at time $k$
- $x_k$: state vector at time $k$
- $A$: state transition matrix
- $B$: update mapping matrix
- $H$: Observation matrix
- $x_{k|k-1}$: estimate of state $x$ at time $k$ based on time $k-1$
- $x_{k-1|k-1}$: estimate of $x_{k-1}$ based on previous interval prediction and correction
- $x_{k|k}$: corrected estimate based on update and measurement
The Kalman filter addresses the problem of estimating the state of a controlled process through indirect or uncertain measurements. To model the dynamic system, we assume that the state evolves as a first order Markov random process, in other words the state at the next time period relies only on the current state of the system. Hence, we can assume that the system can be modeled by the state transition equation:

\[ x_k = Ax_{k-1} + Bu_k + w_{k-1} \]  \{30\}

Where \( x_{k+1} \) is the state at time \( k+1 \), \( A \) is the state transition matrix which relates the state at the previous time \( k-1 \) to the states at the current step \( k \), \( B \) is the update mapping matrix also called the input transition matrix which relates the control input to the state, \( u_k \) is the input control vector and \( w_k \) is the process noise. We also assume that the state observation can be governed through a measurement model that can be represented by the following observation equation:

\[ z_k = Hx_k + v_k \]  \{31\}

Where \( Z_k \) denotes the measurement vector made at time \( k \), \( x_k \) is the state at time \( k \), \( H \) is the observation matrix and \( v_k \) is an additive measurement noise.

The Kalman filter presents an optimal solution to the described model, However, it comes with several assumptions; the process and measurement noise random processes are assumed to be independent, zero-mean white-noise processes and with normal probability distributions

\[ p(w) \sim N(0, Q_w), \]
\[ p(v) \sim N(0, Q_v), \]

Where \( Q \) and \( R \) represent the process noise covariance and the measurement noise covariance. Also \( x_k \) is assumed to be a random process whose mean and covariance matrix vary according to a dynamic model, the state model evolves as a first order Markov random
process, hence the sufficient statistic for estimating the state $x_k$ is only the previous state $x_{k-1}$.

The Kalman filter mathematical model consists of two main phases, the prediction phase and the correction phase. In the estimation phase only the previous state $x_{k-1}$ is taken into consideration, an estimation of state $x_k$ is then calculated based on $x_{k-1}$ without considering the measurement vector $Z_k$. In the correction phase the measurement vector $Z_k$ is used to correct the predicted state estimation. Furthermore, we break the sequential estimation problem into five steps as illustrated in Figure 16 which is explained in detail in the next section.

![Figure 16. Kalman filter sequence outline](image)

Step 1: Prediction of current state

Step 2: Prediction of State Covariance

Step 3: Kalman Gain Matrix

Step 4: state estimate correction

Step 5: Posterior state covariance

5.2- Kalman filter steps derivation

**Step1: Prediction of current state**

The prediction of current state is given by the following:

$$x_{k|k-1} = E[x_k | Z^k]$$

$$= E[Ax_{k-1} + Bu_k + w_{k-1}|Z^{k-1}]$$
= AE[x_{k-1}|Z^k] + BE[u_k|Z^{k-1}] + E[w_{k-1}|Z^{k-1}]

Using the fact that the process noise is zero-mean white-noise processes (p(w)~N(0,Q),), the term E[w_{k-1}|Z^{k-1}] is then equal to zero. Also, because u_k is independent of prior data the second term cancels out: E[u_k|Z^{k-1}] = E[u_k] = 0. The prediction of current state equation becomes:

\[ x_{k|k-1} = A x_{k-1|k-1}. \]  \{33\}

**Step2: Covariance of prediction**

In parallel with the state prediction we process the state covariance matrix \( M_{k|k-1} \) defined as:

\[ M_{k|k-1} = E[(x_k - x_{k|k-1})(x_k - x_{k|k-1})^T] \]  \{34\}

\[ = E[(x_{k-1} - x_{k-1|k-1})(x_{k-1} - x_{k-1|k-1})^T] \]

\[ = E[(Ax_{k-1} + Bu_k - Ax_{k-1|k-1})(Ax_{k-1} + Bu_k - Ax_{k-1|k-1})^T] \]

\[ = AE\left[(x_{k-1} - x_{k-1|k-1})(x_{k-1} - x_{k-1|k-1})^T\right]A^T + BE[u_ku_k^T]B^T \]

\[ = AM_{k-1|k-1}A^T + BQB^T \]

**Step3: Kalman Gain**

We define the Kalman gain coefficient as:

\[ K = \frac{E[(x_k - x_{k|k-1})(z_k - Hx_{k|k-1})^T]}{E[(z_k - z_{k|k-1})(z_k - z_{k|k-1})^T]}, \]  \{35\}

we recall that \( z_k = Hx_k + v_k \) Where \( p(v)\sim N(0,Q_v) \),

\[ K = \frac{E[(x_k - x_{k|k-1})(x_k - x_{k|k-1})^T]H^T}{HE\left[(s_k - s_{k|k-1})(s_k - s_{k|k-1})^T\right]H^T + Q_v} \]

\[ = \frac{M_{k|k-1}H^T}{HM_{k|k-1}H^T + Q_v} \]
Intuitively thinking the Kalman gain coefficient suggests that when the measurements are accurate (meaning the magnitude of R is small) the state estimate depends mostly on the measurements. And when the state is accurate (meaning \(hp-h\) is small in comparison to R) the state estimate depends mostly on the prediction based on the previous state.

**Step4: State estimate correction**

The corrected estimate \(x_{k|k}\) is based on the update and the measurement and defined as:

\[
x_{k|k} = x_{k|k-1} + K(z_k - z_{k|k-1})
\]

\[
= x_{k|k-1} + K(z_k - Hx_{k|k-1}) \tag{36}
\]

**Step5: Posterior state covariance**

The last step of the Kalman filter algorithm is the update of the error covariance

\[
M_{k|k} = E[(x_k - x_{k|k})(x_k - x_{k|k})^T] \tag{37}
\]

\[
= E[(x_k - x_{k|k-1} - K(z_k - Hs_{k|k-1}))(x_k - x_{k|k-1} - K(z_k - Hs_{k|k-1}))^T]
\]

After expanding the previous equation and grouping the four terms we get:

\[
M_{k|k} = (I - KH)M_{k|k-1}. \tag{38}
\]

5.3- KF application in the Cognitive radar

The Kalman filter will be used to estimate the next state vector that defines the system, the estimations will be used by the cognitive radar to optimize the waveform.

We define a state vector as the smallest vector that can fully describe our system. We consider \(x_k\) the state vector that describes a moving target in the XYZ coordinate system. At any time \(k\), the state vector provides a full description of the target position \((P_x, P_y, P_z)\), to fully describe the system, the position vector is not enough. the velocity vector is also needed; hence we define the state vector \(x_k\) such that:

\[
x_k = [P_x, P_y, P_z, V_x, V_y, V_z]
\]

We also define the observation vector \(z_k\) that describes the target position at any time \(k\):

\[
z_k = [P_x, P_y, P_z]
\]
We recall that:

State equation \[ x_k = Ax_{k-1} + Bu_k + w_{k-1} \]

Observation Equation \[ z_k = Hx_k + v_k \]

To fully describe the filter model, we define the following matrices: state transition matrix A, the update mapping matrix B, the update covariance Q, the Observation matrix H, the covariance matrix of measurement noise N and the initial covariance of prediction M:

The movement of the target is defined by the current position of the target, its speed V and acceleration \( \mathcal{A} \) and governed by the following equation:

\[
\begin{align*}
P_{x_k} &= P_{x_{k-1}} + dV_x + d\mathcal{A}_x \\
P_{y_k} &= P_{y_{k-1}} + dV_y + d\mathcal{A}_y \\
P_{z_k} &= P_{z_{k-1}} + dV_z + d\mathcal{A}_z
\end{align*}
\]

Hence, we define the state transition matrix as following:

\[
A = \begin{bmatrix}
1 & 0 & 0 & d & 0 & 0 \\
0 & 1 & 0 & 0 & d & 0 \\
0 & 0 & 1 & 0 & 0 & d \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

where d is the sampling time.

We assume that the initial state is know therefore we define the initial covariance of prediction as:

\[
M = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

The Matrix M will be reevaluated at each iteration according to step 4.

The outside world might be affecting the system, in other words some changes are not related to the state itself, for example it a safe assumption that the target is receiving a command by pushing the throttle which cause it to accelerate. This additional information is covered by the control matrix B and the control vector \( u_k \)
In this simulation we are trying to estimate the position of the target \((P_x, P_y, P_z)\), which the same vector that is being measured therefore the observation matrix \(H\) is defined as:

\[
H = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

And finally, we define the covariance matrix of measurement noise as:

\[
N = \begin{bmatrix}
0.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.1 & 0 & 0 & 0 \\
\end{bmatrix}
\]

5.4- Simulations and results

The Kalman Filter algorithm is used to estimate the position of a target \((P_x, P_y, P_z)\), the target initial position and velocity are known and defined as:

\[
\begin{align*}
\text{Target initial position} : & \ (P_{x0} = 3532 \ m, \ P_{y0} = 800 \ m, \ P_{z0} = 10 \ m) \\
\text{Target Velocity} : & \ (V_{x0} = -60 \ m/s, \ V_{y0} = -80 \ m/s, \ V_{z0} = 0 \ m/s)
\end{align*}
\]

the results of simulation are illustrated in the following figures.
Figure 17. Kalman filter estimation of X position of the target

Figure 18. Kalman filter estimation of Y position of the target

Figure 19. Kalman filter estimation of Z position of the target
The Kalman filter has been considered as the optimal solution to data prediction problems and has been frequently documented as the best mean squared error minimizer. Once more, the Kalman filter is showing good results through the previous simulation. Figure 17-19 demonstrate the capacity of the Kalman filter algorithm to track the position of the moving target and minimizing the mean square error over the time. Under the assumption that the process and measurement noise random processes are independent, zero-mean white-noise processes and with normal probability distributions, the Kalman filter presents an optimal solution to the cognitive radar system design. The target estimator block (refer to Figure 2.), will be a Kalman filter estimator. The waveform optimization will be based on the TE estimations of the target position. Hence the waveform parameters will be decided on the fly by minimizing the tracking mean squared error.

6- waveform optimization

In this work the waveform parameters will be determined by minimizing the mean squared error on the light of the target estimation performed by the Kalman filter. Through the target estimation, we can anticipate the change in the target state, and therefore we can change the waveform parameters accordingly before the change happens. The main waveform parameters that will be taken into play are: the radar beam schedule, where the scan area will be reduced and defined precisely to point the beam to the exact direction of the target instead of scanning the entire scan area. The scan grid is to be adaptively changed by estimating the azimuth and elevation angles of the target. Also, the pulse repetition frequency, the number of pulses N and the duty cycle will be adaptively changed based on the estimated range of the target. The pulse width and sampling frequency will depend on the range resolution.

6.1-PRF

The pulse repetition frequency (PRF) of the radar is the number of transmitted pulses per second. We define the pulse repetition time (PRT) as the time between two consecutive pulses. To avoid interference between the target echo and the next pulse, the radar must
wait long enough until the first pulse echo returns to the receiver. As illustrated in figure 20, the pulse repetition time must exceed the time it takes the first pulse to travel the two-way path between the radar and target. Hence the PRF of the radar will be adaptively changed according to the estimated target range:

\[
PRF = \frac{C}{2R_{est}} \quad \{39\}
\]

Where \(R_{est}\) is the estimated target range and \(C\) is the speed of light.

![Figure 20. Pulse repetition frequency](image)

6.2-Pulse width

The radar pulse width is defined as the time the radar transmitter is active during each cycle. It is inversely related to the radar pulse bandwidth: \(PW = \frac{1}{PBW}\). The radar pulse bandwidth defines the radar range resolution which is the radar ability to distinguish the echo coming from two close targets according to the following relation: \(PBW = \frac{C}{2R_{res}}\), where \(R_{res}\) the range resolution is the distance between two targets under which the radar
can not distinguish the two targets and will treat them as one target. The cognitive radar can adaptively choose the PW in case two close targets are detected such that:

\[ PW < \frac{2D_{targets}}{C} \quad \{40\} \]

Where \(D_{targets}\) is the distance between the two targets.

![Figure 21. Range resolution](image)

In Figure 21 for the same pulse width the radar can distinguish the two different targets when the distance between them is higher than the range resolution specific to that pulse width. Whereas without adapting the pulse width in the second scenario the radar is unable to distinguish the two targets and treats them as one target.

### 6.3-duty cycle

Duty cycle is defined as the fraction of time that the system is in active state, hence we can define it as the product of the radar pulse width and the radar pulse repetition frequency. By adaptively changing the radar PRF and the radar PW as explained earlier, the duty cycle is adaptively changed as well.
6.4- Number of pulses N (coherent pulse integration)

To distinguish the echo bouncing of a target from a random noise, the signal to noise ratio must exceed a certain level that can be derived from the radar receiver operating characteristics (ROC) curve. Based on the design specifications, for a specific radar probability of detection ($P_d$) and probability of false alarm ($P_{fa}$) we can find the required Signal to noise ratio at the receiver $SNR_{min}$.

![Figure 22. Radar receiver operating characteristics curve](image)

However, often radars cannot achieve the required signal to noise ratio $SNR_{min}$ for detection with a single pulse. To solve this problem several pulses are integrated. For this design a coherent pulse integration is considered. Ideally the integration of N pulses multiplies the signal to noise ratio by N. figure 23 illustrates the principle of coherent integration. By adding N coherent pulses (the amplitude and phase of signal is the same from pulse to pulse), the resulting signal is N times greater than that of one signal pulse. The noise signal however has a random amplitude and phase therefore the noise signals can add up constructively or destructively.
The signal to noise ratio (SNR) can be calculated and compared to the required signal to noise ratio $SNR_{\text{min}}$ using the radar range equation:

$$SNR = \frac{P_t G^2 \lambda^2 N \sigma}{(4\pi)^3 R^4}$$  \[41\]

Where:

- $P_t$ transmitter power
- $G$ antenna gain
- $\lambda$ radar wavelength
- $N$ number of integrated pulses
- $\sigma$ Target radar cross section in square meter
- $R$ Target range

Figure 23. Principle of coherent pulse integration
**SNR\_min**  The minimum required signal to noise ratio at the receiver.

In case the Signal to noise ratio is below the required threshold $SNR_{\text{min}}$, a pulse integration is required. For a certain range $R$, knowing the transmitter power and the $SNR_{\text{min}}$ from the ROC curve we can solve for the number of pulses that should be integrated to achieve a good radar detection performance as follow:

$$N = \frac{R^4(4\pi)^3SNR_{\text{min}}}{P_tG^2\lambda^2\sigma} \quad \{42\}$$

Hence the cognitive radar can adaptively change the number of integrated pulses in the light of the estimations of the target range. Moreover, we develop an expression for a quantitative test that makes a comparison to the minimum SNR required at the receiver and decides the number of pulses to integrate accordingly:

$$\tilde{T}_N = \frac{P_tG^2\lambda^2N\sigma}{(4\pi)^3(\lvert \hat{\theta}_k \rvert)^4} \geqsum_{N+1}^{N} ROC(P_D, P_{fa})$$

Where ROC () is a function that takes $P_D$ and $P_{fa}$ as an input and returns the convenient SNR, AND $\lvert \hat{\theta}_k \rvert$ is the estimated target range.

$\tilde{T}_N$ can be summarized in the following algorithmic steps:

1. Find $SNR_{\text{min}}$: $ROC(P_D, P_{fa})$
2. Calculate SNR for the known $P_t$ and the estimated range [41]
3. Compare SNR to $SNR_{\text{min}}$:
   - If $\text{SNR}>SNR_{\text{min}}$ \( \Rightarrow \) $N=1$.
   - Else ($\text{SNR}<SNR_{\text{min}}$)
     - Compute the new SNR using eq. 41
     - If $\text{SNR} > SNR_{\text{min}} \Rightarrow \text{Break} (N)$
     - Else

**6.5- radar scan scheduling**
One of the aspects of the cognitive radar is its capacity to locate the target and to keep track of it through its capacity to estimate its next position. By knowing the target estimated position precisely, the radar doesn’t have to scan the entire scan area and wait for the echo coming from all the scanned angles, instead the cognitive radar can focus the scan on the estimated angle of the target. Taking advantage of the phased array to rapidly and precisely steer the beam towards a specific angle, cognitive radars focus the time and energy on the estimated direction of the target. The estimated scan angle can be deduced by converting the target cartesian coordinates to the spherical coordinates system.

\[
\hat{\theta} = \frac{\hat{p}_y}{|\hat{p}_y|} \cos^{-1} \frac{\hat{p}_x}{\sqrt{\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2}} \cos \hat{\phi} \tag{9}
\]

\[
\hat{\phi} = \frac{\hat{p}_z}{\hat{p}_x} \tag{10}
\]

\[\theta = \frac{\hat{p}_y}{|\hat{p}_y|} \cos^{-1} \frac{\hat{p}_x}{\sqrt{\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2}} \cos \phi \]

\[
\phi = \frac{\hat{p}_z}{\hat{p}_x}
\]

**Figure 24.** Target coordinates estimation

### 6.6- Detection threshold

In radar application, processing the received signal often means deciding between two hypotheses, one suggests that the received signal consists of noise only whereas the second suggests that the received data consists of signal bouncing off from a target plus noise. In 1933 Jerzy Neyman and Egon Pearson [7] introduced for the first time a theory that was named after them and sets a foundation for hypothesis testing. Using the Neyman-pearson theory, the required threshold for detection for a radar can be decided for a given probability of false alarm.

The neyman-pearson lemma theory models the system observation according to one of two distributions:
\[ \begin{align*}
H_0: x &= w \sim p(x, H_0) \\
H_1: x &= s + w \sim p(x, H_1)
\end{align*} \]  \{43\}

where \( p(x, H_0) \) and \( p(x, H_1) \) are the probability distribution of \( X \) under \( H_0 \) and \( H_1 \) hypothesis.

\( H_0 \) is considered as the default model and is called the null hypothesis. \( H_0 \) suggests that the data present at the radar receiver consist of noise only. Whereas \( H_1 \) is called the alternative hypothesis and suggests that the data at the receiver consists of more than noise, it consists of the noise plus a signal having valuable information about the target. The NP goal is to decide a threshold that maximizes the probability of choosing \( H_1 \) when the data were in fact generated by \( H_1 \). Hence the probability of correctly choosing \( H_1 \) is the subject of investigation, which is denoted as the probability of detection: \( P_D = P(H_1; H_1) \). Furthermore, if the test chooses \( H_1 \) whereas the data were actually generated by \( H_0 \) is called a false alarm or false-positive: \( P_{FA} = P(H_1; H_0) \). Our goal is to maximize the probability of detection \( P_D \) under the constraint of a given probability of false alarm \( P_{FA} \) by choosing the optimal threshold \( \gamma \) as illustrated in Figure 2.4. \( \gamma \) achieves the largest \( P_D \) under the constraints \( P_{FA} \). Once more, we work under the assumption that the noise \( (W) \) follows a normal distribution with a zero mean and a variance \( \sigma^2 \).

\[
W \sim N(0, \sigma^2),
\]

\[
p(x, H_0) \sim N(0, \sigma^2)
\]

\[
p(x, H_1) \sim N(\mu, \sigma^2), \mu > 0.
\]

Figure 25. Neyman-Pearson Hypothesis decision
The likelihood ratio test takes the form:

\[
\frac{p(x,H_1)}{p(x,H_0)} \overset{\text{H}_1}{\overset{\text{H}_0}{\gtrless}} \gamma \tag{44}
\]

Under the previous assumptions the likelihood test is expressed as following:

\[
\frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(\frac{(\mu-x)^2}{2\sigma^2}\right) \overset{\text{H}_1}{\overset{\text{H}_0}{\gtrless}} \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(\frac{(-\mu)^2}{2\sigma^2}\right) \overset{\text{H}_0}{\overset{\text{H}_1}{\gtrless}} \gamma \tag{45}
\]

Which we can simplify under the previous assumptions to the form:

\[
x \overset{\text{H}_1}{\overset{\text{H}_0}{\gtrless}} \frac{2\sigma^2 \log \gamma + \mu}{2} = \gamma' \tag{46}
\]

The \(P_D\) and \(P_{FA}\) can then be expressed like follows:

\[
P_{FA} = Pr\{x > \gamma'; H_0\} = \int_{\gamma'}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(\frac{(-x)^2}{2\sigma^2}\right) dx = Q\left(\frac{\gamma'}{\sqrt{\sigma^2}}\right) \tag{47}
\]

Where the Q-function is an invertible function called the tail distribution function of the standard normal distribution and defined as follows:

\[
\text{If } Y \sim N(\mu, \sigma^2), \text{the standard normal distribution is } X = \frac{Y - \mu}{\sigma} \text{ and } Pr\{Y > y\} = Pr\{X > x\} = Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left(\frac{(-x)^2}{2}\right) dx. \tag{48}
\]

Similarly, the probability of detection can be expressed as follows:

\[
P_D = Pr\{x > \gamma'; H_1\} = \int_{\gamma'}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(\frac{(-x)^2}{2\sigma^2}\right) dx = Q\left(\frac{\gamma' - \mu}{\sqrt{\sigma^2}}\right) \tag{49}
\]

Using \{47\} and \{49\} the probability of detection can be expressed in a more general definition:

\[
P_D = Pr\{x > \gamma'; H_1\} = Q\left(\frac{\gamma' - \mu}{\sqrt{\sigma^2}}\right) \overset{\text{H}_1}{\overset{\text{H}_0}{\gtrless}} Q^{-1}(P_{FA}) - \frac{\mu^2}{\sigma^2} \tag{50}
\]

Where \(\frac{\mu^2}{\sigma^2}\) the signal to noise ratio.

Hence, for a fixed \(P_{FA}\), using the Neyman-Pearson the optimal threshold \(\gamma\) is decided such that the probability of detection \(P_D\) is maximized. Furthermore, if N signals are integrated, the probability of detection is:
\[ P_D = Pr\{x > \gamma'; H_1\} = \int_{\gamma'}^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x)^2}{2N\sigma^2}\right) dx = Q\left(\frac{\gamma' - N\mu}{\sqrt{\sigma^2}}\right) = Q\left(Q^{-1}(P_{FA}) - \frac{N\mu^2}{\sigma^2}\right) \]

This implies that the probability of detection increases if \(N\) increases which again agrees that higher probability of detection is attainable for higher signal to noise ratio as illustrated in Figure 26.

![Figure 26. Receiver operating characteristic for different SNR](image)

On the light of the previous results, in our waveform optimization algorithm, the detection threshold will be adaptively changed for each cycle, to maximize the probability of detection under the constraint of a fixed probability of false alarm and taking in consideration the number of pulses integrated \(N\) which affects the SNR threshold of detection.

6.7- Waveform optimization approach
Initialization
(Tx) Transmit $S_0$
(Rx) Receive $X_0$

$p(x,H_1) \geq H_1 \propto H_0$

Estimation $\hat{\psi}$
Estimated angle, range, elevation...

MMSE Estimation $\hat{\psi}$

Next waveform design:
- PRF
- Radar Scan Schedule
- Target Nature decision
- Number of pulses to integrate $N$

(Tx) Transmit $S_k$
(Rx) Receive $X_k$

$p(x,H_1) \geq H_1 \propto H_0$

Target Lost

Update information library

Target Tracking And Waveform design

Initial Active Scanning

Target Estimation (MMSE)

Radar Speed Data Processor (RSDP)
(Table.5)

Figure.25- Flowchart of CR tracking and waveform design algorithm
7-Kalman Filter application in target tracking and waveform optimization

7.1- Tracking and waveform design algorithm

The problem of adaptive waveform design in the context of cognitive radars is a growing subject of interest and has been widely investigated in literature in the recent years. Different approaches have been considered including designing the waveform in a way that maximizes the mutual information and detection probability as proposed in [8] where the information theoretic approach has been applied to estimate and track parameters of multiple targets. Moreover, another approach is used in [9] where the proposed waveform design is built up on maximizing the signal to noise ratio. In this work the waveform is optimized by minimizing the mean square error. A similar approach was considered in [10], [11]. In [10] Huleihel et al introduces a model for a cognitive radar for which he defines a system transfer function that depends on an unknown random vector \( \theta \) which is the subject of estimation. The vector \( \theta \) may consist of target range, amplitude, direction … quoting [10] “we are interested in the design of the transmit signal matrix at the kth step, given history observations such that at each step we aim to optimize the estimation performance of \( \theta \) in terms of MSE. His solution intends to minimize the conditional (depends on the measurement vector or observation history) MMSE matrix that he defines as: 

\[
\Sigma_k = E[(\hat{\theta}_k - \theta)(\hat{\theta}_k - \theta)^T | X^{k-1}]
\]

where \( \hat{\theta}_k \) is the MMSE estimator of \( \theta \) at step \( k \) and \( X^{k-1} \) is the observation vector at step \( k-1 \). In [11], a particle filter was applied to estimate the system state and a cost function, inspired by the work of Huleihel et al [10], was expressed to optimize the mean square error. In both [10] and [11], the focus was on decreasing the angle estimate variance so that the power is steered towards the direction of the current estimate of the target. In this work, we introduce a similar tracking and waveform design algorithm, inspired by the work done in [10] and [11].

We define the system state vector \( \psi \) that describes the target in a tracking scenario at any step \( k \). We define the state vector as the smallest vector that fully describes the system state. The waveform parameters are to be defined in such a way that the estimation mean square error of \( \theta \) is minimized, hence we consider \( \hat{\psi}_k \) an MMSE estimator such that the following MSE matrix is minimized:

\[
\text{MSE} = E[(\hat{\psi}_k - \psi)(\hat{\psi}_k - \psi)^T] .
\]

The MMSE estimator \( \hat{\theta}_k \) is then defined as the estimator achieving minimal MSE: 

\[
\hat{\theta}_k = \arg \min_{\hat{\theta}_k} \text{MSE} .
\]

In this approach we define the state vector \( \theta \) as the target position and speed in the Cartesian coordinate system as it can fully describe the target at any step \( k \).

\[
\psi_k = [P_x, P_y, P_z, V_x, V_y, V_z]
\]
Table. 4-CF Tracking and waveform optimization design

Initialization: k=0
1- $s_0$ initial illumination of environment
2- $x_0$ receive signal
3- Hypothesis decision: $\frac{p(x_0|H_1)}{p(x_0|H_0)} \geq \frac{H_1}{H_0}$ $\gamma$  $\Rightarrow$ If $H_0$ go back to step 1 else if $H_1$ go to next step
4- Initial Measurement vector $Z_0 = [P_x, P_y, P_z]$;
5- Initial state vector $\Psi_{initial} = [P_x, P_y, P_z, V_x, V_y, V_z]$ ;
6- $K+=1$;
7- TE (\(\hat{\Psi}_k\)): Figure.2
   7.1- prediction of current state: $\Psi_{k|k-1} = A \Psi_{k-1|k-1}$
   7.2- covariance prediction: $M_{k|k-1} = AM_{k-1|k-1}A^T + BQB^T$
   7.3- Kalman gain: $K = \frac{M_{k|k-1}H^T}{HM_{k|k-1}H^T+Q_v}$
   7.4- state estimate correction: $\Psi_{k|k} = \Psi_{k|k-1} + K(Z_k - H\Psi_{k|k-1})$=[$\hat{P}_x, \hat{P}_y, \hat{P}_z, \hat{V}_x, \hat{V}_y, \hat{V}_z$]
   7.5-Update Posterior state covariance: $M_{k|k} = (I-KH)M_{k|k-1}$
8- Waveform design
   8.1- $\hat{R} = \sqrt{\hat{P}_x^2 + \hat{P}_y^2 + \hat{P}_z^2}$
   8.2- PRF = $\frac{c}{2\hat{R}}$
   8.3- Find the optimal N: $\hat{T}_K = \frac{P_r\sigma^2 \lambda^3 Q}{(4\pi)^2(|\hat{\Psi}_k|)^2} \geq N_{+1} ROC(P_D, P_f) \ (B), Table. 1$
   8.4- Compute the new detection threshold: $\gamma' = \sqrt{\sigma^2 Q^{-1}(P_f, \lambda)\sqrt{N}}$
   8.5- Update scan angles: [$\theta_k, \phi_k$]:
      8.5.1- $\phi_k = \frac{\hat{P}_y}{\hat{P}_x}$
      8.5.2- $\theta_k = \frac{\hat{P}_x}{\hat{P}_y} \cos^{-1} \frac{\hat{P}_x}{\sqrt{\hat{P}_x^2 + \hat{P}_y^2 + \hat{P}_z^2 \cos \phi}}$
9- $s_k$ illumination of environment with the optimized waveform.
10- $x_k$ receive signal
11- Hypothesis decision: $\frac{p(x_k|H_1)}{p(x_k|H_0)} \geq \frac{H_1}{H_0}$ $\gamma$  $\Rightarrow$ If $H_0$ go back to step 1 else if $H_1$:Update $Z_k$

Update information library.
Go back to step 6
In the proposed algorithm the control theoretic approach criteria is considered. The target position estimation is mainly performed in a way that minimizes the tracking mean square error (MMSE). Our algorithm shows a great performance in this regard as presented in Figure 28, where the target position is estimated along the three cartesian axes while the tracking error is minimized. Once more the Kalman filter proves to perform well in the control theory for state estimation. Knowing the target estimated coordinates, the target range and angles can be easily estimated using simple geometry rules.
**Figure 2.9** Target range estimations with adaptive waveform parameters
The waveform optimization algorithm relies heavily on two parameters, the target range using which the pulse repetition frequency and the number of integrated pulses is decided adaptively, and the target angle which defines the new radar scan schedule as we design the new scan grid for the next emission to be centered at the estimated target angle and have a width equal to the radar beam width. The great performance of the algorithm in minimizing the tracking error and providing precise estimations of the target range and angle helps deciding the optimal radar parameters for each cycle. Figure 30 shows that the algorithm estimates the right angle of the target with a half of a degree precision. With such a precision, the CR can concentrate the power and time at the direction of target.

To test our algorithm’s capacity of adaptively changing the number of pulses to integrate coherently $N$, we set the radar parameters and the target track scenario such that the number of the integrated pulses must be increased to achieve the minimum signal to noise ratio (SNR) for detection. According to Figure 31, for a radar transmit power of 1000 watts the maximum attainable range is about 12100 meters for one pulse. As the target exceeds the maximum range, the CR intelligently increases the number of integrated pulses $N$ to attain the minimum required signal to noise ratio.

Table 5. Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Tx Power</td>
<td>1000w</td>
</tr>
<tr>
<td>Frequency</td>
<td>4.5Ghz</td>
</tr>
<tr>
<td>Target initial range</td>
<td>120500m</td>
</tr>
<tr>
<td>Initial number of pulses</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 3.1 - effect of pulse integration on radar maximum range

We test the CR's algorithm in the scenario where the target exceeds the maximum range that corresponds to the Max Tx power. The simulation result is illustrated in Figure 3.2. To reach the minimum signal to noise ratio, the CR intelligently increases the number of pulses to integrate after exceeding the maximum range for N=1.

Figure 3.2. CR adaptively changing the number of integrated pulses

We simulate the capacity of our algorithm to keep track of the SNR in comparison to the required SNR and to guarantee that the CR can reach the minimum signal to noise ratio required for detection at all time. In case the SNR under certain radar parameters is below the required SNR, CR change the number of integrated pulses accordingly to guarantee that the calculated SNR is above the required value. Figure 3.3 shows that the SNR required decreases as the number of integrated pulses increase, hence, whenever the achieved SNR is below the required SNR our algorithm adaptively increases the number of pulses to integrate, and contrarily, if the target is approaching, the algorithm can decrease the number of integrated pulses while keeping the SNR below the required value to avoid the unnecessary use of valuable energy. Figure 3.2 illustrates the results of the simulation to test the algorithm in the scenario of an increasing range and decreasing range and the target decision concerning the number of integrated pulses N to guarantee a signal to noise ratio above the required value.
The pulse repetition frequency (PRF) of the radar is continuously adapted to the target range. An example of simulation of our algorithm is presented in Figure 35 where the PFR is adapted in response to an increase of the target range.
Optimizing the radar parameters improves the performance of CRs for example deciding the right number of pulses N guarantees the minimum required SNR for detection, also adaptively changing the PRF with a varying target range helps the radar to avoid the interference between pulses, also redefining the radar scan schedule reduces drastically the target revisit time and improves the precision of tracking.

Figure 36. Radar tracking with waveform optimization

CR’s do not only interrogate the surrounding environment through previous observations, but also through information stored in external databases. For instance, in our algorithm we consider a database containing the locations of surrounding friendly units. We continually compare the locations provided by the external database with the measured location of the tracked target and decide the nature of the target accordingly. The target is assumed an enemy target if the target observed location doesn’t match any of the locations in the database. In this context CR’s can play a role in the avoidance of friendly fire incidents and the proper classification of targets intercepted by radar systems.

Figure 37. CR Target Classification
Figures 38. Target coordinates do not match the external database information. The target is presumed Enemy target.

Figures 39. Target coordinates match the external database information. The target is presumed Friendly target.

CR’s not only track the target position but also the target speed. We designed the MMSE estimator (KF) to estimate the next state vector that is defined as: \( x_k = [P_x, P_y, P_z, V_x, V_y, V_z] \) where \([P_x, P_y, P_z]\) denotes the target position vector and \([V_x, V_y, V_z]\) denotes the target speed vector. We propose applying a low pass filter (Moving average filter) to analyze the estimated speed data points stored in the CR library. The longer the target is tracked the bigger is the moving average window which results in smoother estimations, less vulnerable to the sudden fluctuations of the MMSE speed estimations. In case of target trajectory change or in other words a change in the target speed vector, it is necessary to restart the Moving average filter to avoid the resistance to follow the new changes in the speed vector caused by the old data points. Therefore, every change in the speed vector must be detected and the moving average window is redefined such that the data points prior to the speed change point are excluded. The filtered speed estimates along the three cartesian axes is calculated as following:

\[
\bar{V}_n = \frac{1}{n-\alpha} \Sigma_{n+1}^{n} V_n
\]

Where \( \alpha \) denotes the index at which a speed change is detected. \( \alpha \) is initialized at 1 and is adaptively changed in real time. To detect the change of speed, we keep comparing the output of the Kalman filter to the output of the moving average filter. A big deviation of the Kalman filter estimation from the running average indicates a change in the speed vector. We propose the following algorithm to accomplish the tasks proposed above where the following notations have been followed:

\( \widetilde{\alpha}_x \): The index at which \( V_x \) has changed
\( \widetilde{\alpha}_y \): The index at which \( V_y \) has changed
\( \widetilde{\alpha}_z \): The index at which \( V_z \) has changed
\( x_{k|k} \): The output of the Kalman filter

\( y \): speed deviation threshold

\( \bar{V}_x, \bar{V}_y, \bar{V}_z \): The output of the Moving Average filter applied to the estimated \( V_x, V_y, V_z \).

**Table 6: Filtering the Speed estimation algorithm.**

<table>
<thead>
<tr>
<th>Initialization: ( n=1; \bar{\alpha}_x = 1; \bar{\alpha}_y = 1; \bar{\alpha}_z = 1; )</th>
</tr>
</thead>
<tbody>
<tr>
<td>While (tracking) {</td>
</tr>
</tbody>
</table>
| 1) \( \psi_{k|k} = [\bar{P}_x, \bar{P}_y, \bar{P}_z, \bar{V}_x, \bar{V}_y, \bar{V}_z] \)  
| \( \bar{V}_x = \left( \frac{1}{n - \bar{\alpha}_x} \right) \sum_{n}^{\bar{V}_x} \)  
| \( \bar{V}_y = \left( \frac{1}{n - \bar{\alpha}_y} \right) \sum_{n}^{\bar{V}_y} \)  
| \( \bar{V}_z = \left( \frac{1}{n - \bar{\alpha}_z} \right) \sum_{n}^{\bar{V}_z} \)  
| \{ If \( |\bar{V}_x - \bar{V}_l| > \gamma \) \( (\bar{\alpha}_x = n; \) \}  
| \{ If \( |\bar{V}_y - \bar{V}_l| > \gamma \) \( (\bar{\alpha}_y = n; \) \}  
| \{ If \( |\bar{V}_z - \bar{V}_l| > \gamma \) \( (\bar{\alpha}_z = n; \) \}  
| 4) \( n+1 \); |
| } |

**Figure 40.** illustrates the results of simulation of the above algorithm.
8- Interacting Multiple Model (IMM) Algorithm:

In the previous section we have introduced the Kalman filter as the optimal MMSE algorithm for target tracking, however it assumes that the target motion can be predicted and therefore defined as the state transition matrix which is used in the prediction phase. In the previous section we have considered that the target follows a uniform motion with a constant speed, we defined it as following:

\[ A = \begin{bmatrix}
1 & 0 & 0 & d & 0 & 0 \\
0 & 1 & 0 & 0 & d & 0 \\
0 & 0 & 1 & 0 & 0 & d \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \]

This assumption is obviously far from the truth because the target motion can be considered random and unpredictable. Flying targets are not often governed by one uniform motion model but rather can go through a diversity of arbitrary maneuvers, therefore we are proposing 4 different combinations of motion models which allows the radar to adapt to the continues change of the target dynamics matrix. For this reason, The IMM algorithm has been widely used for tracking flying targets. It tackles the tracking problem with a realistic assumption by modeling the target as a moving object with random motions and behaviors. Hence multiple models are derived to describe potential target maneuvers and motions. In this section the following kinematic models were considered to describe the target motion: the constant velocity motion model (CV)[20], the Singer Model [21] and the 3D coordinated turn model (3DCT) [22].

8.1- The IMM algorithm

The IMM approach is based on the idea that describing the dynamics of a maneuvering target using one model may not be resourceful. Therefore, the maneuver scenario is depicted as a hybrid system where multiple motion models are combined. In this approach, \( n \) models can be predefined to match the motion of the target and the state estimate vector is computed under every proposed model at time \( k \) using \( n \) filters. **Fig. 41** presents the sequence outline of the IMM algorithm.
The state vector $\psi$ is defined to fully describe the target at any step $k$ as:

$$\psi_k = [P_x, V_x, A_x, P_y, V_y, A_y, P_z, V_z, A_z]$$  \hspace{1cm} (18)

where $P_x, P_y, P_z$ represents respectively the target position along the x, y, and z axes, $V_x, V_y, V_z$ are the corresponding velocities and $A_x, A_y, A_z$ are the corresponding accelerations.

**Fig. 4.1.** IMM sequence outline

The IMM algorithm is a multi-model-based algorithm, where each model can be a unique combination of one or more kinematic models that describes different possible target motions and manoeuvres. To model the dynamic system, we assume that the state evolves as a first order Markov random process, in other words the state at the next time period relies only on the current state of the system. Hence, we model the target motion by:

$$\psi_k = A\psi_{k-1} + w_k$$  \hspace{1cm} (19)
Where $\psi_{k-1}$ is the state at time $k-1$, $A$ is the state transition matrix which relates the state at the previous time $k-1$ to the states at the current step $k$, and $w_k$ is the process noise. We also assume that the state observation can be governed through a measurement model that can be represented by the following observation equation:

$$z_k = H\psi_k + v_k \quad (20)$$

Where $z_k$ denotes the measurement, vector made at time $k$, $\psi_k$ is the state at time $k$, $H$ is the observation matrix and $v_k$ is an additive measurement noise. In this work we propose three different models by combining at least 2 or more of the following kinematic models:

**8.1.1-Constant Velocity Model (CV)**

The CV model assumes that the target motion is characterized by a constant speed (the target velocity derivative is a white noise). The State transition matrix $A$ can be defined as:

$$A_{CV} = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & T & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & T & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

**8.1.2-Constant Acceleration Model (CA)**

The CV model assumes that the target motion is characterized by a constant speed (the target velocity derivative is a white noise).

The State transition matrix $A$ can be defined as:
\[ A_{CA} = \begin{bmatrix}
1 & 0 & 0 & T & 0 & 0 & \frac{\tau^2}{2} & 0 & 0 \\
0 & 1 & 0 & 0 & T & 0 & 0 & \frac{\tau^2}{2} & 0 \\
0 & 0 & 1 & 0 & 0 & T & 0 & 0 & \frac{\tau^2}{2} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & T & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & T \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix} \]

8.1.3-Horizontal coordinated turn (HCT)

The HTC model assumes that the target maneuver is mainly in the horizontal plane with minor to negligible vertical maneuvers. Also, the target motion is regarded as a constant speed and constant turn rate motion. The model dynamics matrix \( A \) is defined as:

\[ A_{HCT} = \begin{bmatrix}
1 & 0 & 0 & \frac{\sin(\omega T)}{\omega} & \frac{\cos(\omega T) - 1}{\omega} & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -\frac{\cos(\omega T) - 1}{\omega} & \frac{\sin(\omega T)}{\omega} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos(\omega T) & -\sin(\omega T) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sin(\omega T) & \cos(\omega T) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix} \]

8.1.4-Singer Model

The singer model assumes that the target acceleration is a zero-mean stationary first-order Markov process. In this model the target acceleration varies exponentially with respect to the manoeuvre time constant \( \tau \). The Singer model dynamics matrix \( A \) is defined as:
Unlike the HCT model, the 3DCT model assumes that the target is performing manoeuvres in the 3D space rather than in the horizontal plane. The 3DCT model dynamics matrix $A$ is defined as:

$$A_{\text{Singer}} = \begin{bmatrix}
1 & 0 & 0 & T & 0 & 0 & \tau^2(-1 + \frac{1}{\tau} T + e^{-\frac{r}{\tau}}) & 0 & 0 \\
0 & 1 & 0 & 0 & T & 0 & 0 & \tau^2(-1 + \frac{1}{\tau} T + e^{-\frac{r}{\tau}}) & 0 \\
0 & 0 & 1 & 0 & 0 & T & 0 & 0 & \tau^2(-1 + \frac{1}{\tau} T + e^{-\frac{r}{\tau}}) \\
0 & 0 & 0 & 1 & 0 & 0 & \tau(1 - e^{-\frac{r}{\tau}}) & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \tau(1 - e^{-\frac{r}{\tau}}) & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \tau(1 - e^{-\frac{r}{\tau}}) \\
0 & 0 & 0 & 0 & 0 & e^{-\frac{r}{\tau}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & e^{-\frac{r}{\tau}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{-\frac{r}{\tau}} & 0
\end{bmatrix}$$

9.1.5 - The three-dimensional coordinated turn (3DCT)

We propose the following three models where three different target motion states are considered. The first proposed model is the CV model where the target motion is considered uniform with constant velocity. The second model is CV-CA, which assumes that the target motion is uniform with constant speed and acceleration. The third model is CV-HCCT-HCCCT where HCCT stands for Horizontal Clockwise Coordinated Turn in which we consider a positive turn rate $\omega$ whereas HCCCT stands for Horizontal counter-clockwise coordinated turn with a negative turn rate $\omega$. Finally the fourth proposed model is Singer-3DTR where we assume that the target is performing maneuvers in the 3D space with varying acceleration.
8.2- IMM based tracking and waveform optimization algorithm

1- Model-conditioned reinitialization (i=1,2,…M):

Predicted model probability: \( \mu_{k|i-1} = \sum_j \pi_{ji} \mu_{k-1|i} \)

Mixing Weight: \( \mu_{k-1|i} = \pi_{ji} \mu_{k-1|i} / \mu_{k|i-1} \)

Mixing estimate: \( \tilde{\psi}_{k-1|i-1} = \sum_j \tilde{\psi}_{k-1|i-1} \mu_{k-1|i} \)

Mixing covariance: \( \tilde{P}_{k-1|i-1} = \sum_j [P_{k-1|i-1} + (\tilde{\psi}_{k-1|i-1} - \tilde{\psi}_{j-1|i-1})(\tilde{\psi}_{k-1|i-1} - \tilde{\psi}_{j-1|i-1})'] \mu_{k-1|i} \)

2- Model-conditioned Filtering (i=1,2…M)

Prediction of current state: \( \psi_{k|i} = A \psi_{k|i-1} \)

Covariance prediction: \( P_{k|i} = AP_{k|i-1}A^T + BQB^T \)

Filter gain: \( K = P_{k|i-1}A^T/H P_{k|i-1}A^T + Q_v \)

State estimate correction: \( \hat{\psi}_{k|i} = \psi_{k|i-1} + K(Z_k - H \psi_{k|i-1}) \)

Update Posterior state covariance: \( P_{k|i} = (I-KH)P_{k|i} \)

3- Model probability update (i=1,2…M)

Model likelihood: \( L_k^i = N(Z_k - H \psi_{k|i-1}; 0; HP_{k|i-1}H^T + Q_v) \)

Model probability: \( \mu_k^i = \mu_{k|i-1}L_k^i / \sum_j \mu_{k|i-1} L_k^j \)

4- Estimate fusion:

Overall estimate: \( \hat{\psi}_{k|i} = \sum_j \hat{\psi}_{k|i} \mu_k^i \)

Overall covariance: \( P_{k|i} = \sum_j [P_{k|i} + (\tilde{\psi}_{k|i} - \hat{\psi}_{k|i})(\tilde{\psi}_{k|i} - \hat{\psi}_{k|i})'] \mu_k^i \)

5- Waveform design

Range update: \( \hat{R} = \sqrt{\hat{\psi}_{k|i}[0]^2 + \hat{\psi}_{k|i}[1]^2 + \hat{\psi}_{k|i}[2]^2} \)

Pulse repetition frequency update: \( PRF = \frac{c}{2 \hat{R}} \)

Find the optimal number of pulses to integrate N:

\( T_{N} = \frac{P_s G^2 \lambda^2 N \sigma}{(4\pi)^3 (|\theta_k|)} \sim N \sum ROC(P_{D}, P_{FA}) \)

Compute the new threshold: \( \gamma' = \sqrt{\sigma^2 Q^{-1}(P_{FA})} \sqrt{N} \)

Update scan angles: \( [\theta_k, \phi_k] \):

\( \phi_k = \phi = \frac{P_s}{P_x} \)

\( \theta_k = \tilde{\theta} = \frac{P_s}{|P_s|} \cos^{-1} \frac{P_s}{\sqrt{P_s^2 + P_s^2 + P_s^2 \cos \phi}} \)
In this section, we design a hybrid trajectory to test the proposed algorithm in different scenarios including uniform motion with changing velocity and steep turn maneuvers with constant turn rate. We observe that the three models exhibit good trucking performance in general. Model 2 results in smaller trucking error during the linear segments but model 1 and 3 prove superiority during the turn maneuvers.

**Figure 42.** Tracking the target cartesian position
Figure 43. Tracking error along the X axis

Figure 44. Tracking error along the Y axis
The simulation results show a good performance in general. All the models exhibit similar results during the uniform motion segments, whereas during target steep turns, the first model exhibits higher tracking error because it doesn’t count for turn maneuvers is the model description.

9- The effect of waveform optimization on radar tracking performance

In this section we focus on studying the effect of waveform optimization ion the target tracking performance. Facing the change in the target dynamics, it is necessary to design the radar parameters adaptively to guarantee a better performance. To counterbalance the target motion model uncertainty, we are proposing the IMM-based algorithm for target tracking and waveform optimization to allow the cognitive radar to adapt to the continuous changes of the target dynamics. In this approach the target state vector estimate resulting from the IMM algorithm is taken to adaptively change the radar parameters. To test the effect of waveform optimization on the tracking performance, we are considering the following kinematic models as the interacting models; the constant Velocity Model (CV), the singer model, and the 3DCT model. The simulation initial parameter are decided as follow:

<table>
<thead>
<tr>
<th>Table column subhead</th>
<th>Subhead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Tx Power</td>
<td>1000 W</td>
</tr>
<tr>
<td>Number of integrated pulses</td>
<td>1</td>
</tr>
<tr>
<td>Target initial coordinates</td>
<td>[105000; 46000; 20]</td>
</tr>
<tr>
<td>Target initial range</td>
<td>100700 meters</td>
</tr>
</tbody>
</table>

9.1-Results
In general, we observe that the tracking performance was significantly increased by optimizing the radar waveform parameters. The IMM-based tracker exhibits lower tracking error when the proposed waveform parameters were adaptively changed during the constant velocity linear motion segments, as well as maneuver segments. A major observation is the avoidance of target track loss as a result of waveform optimization. The target tracking loss occurs when the target reaches the maximum attainable range without having the capacity to adaptively change the number of integrated pulses $N$. In the proposed algorithm, we keep track of the SNR level and adaptively change the number of integrated pulses $N$ to guarantee reaching the minimum SNR required for detection. As a result, during the simulation the CR successfully increases the number of pulses $N$ when the target reaches the maximum range, whereas the target was lost without doing so. Fig. 9 shows the update of the $N$ during the simulation.
Fig. 46- Tracking target X position

Fig. 47- Tracking error along X axis
**Fig. 48.** Tracking target Y position

**Fig. 49.** Tracking error along Y axis
The tracking performance of the proposed algorithm is summarized in Table 6.

**Table 6: Target Tracking Performance with and without waveform optimization**

<table>
<thead>
<tr>
<th></th>
<th>IMM</th>
<th>IMM+W O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average tracking error (m) (X position)</td>
<td>209 m</td>
<td>78 m</td>
</tr>
<tr>
<td>Average tracking error (m) (Y position)</td>
<td>1255 m</td>
<td>147 m</td>
</tr>
<tr>
<td>Track loss (range)</td>
<td>121100 m</td>
<td>NA</td>
</tr>
</tbody>
</table>

Typically, radar antennas are constrained with a beam width of a few degrees. Although this is suitable for early warning roles, it doesn’t reach the required precision for target tracking which is in the orders of a tenth of a degree. To attain such a beam width, it is essential to use large array antennas which is usually impractical. Alternatively, by refining the radar scan step size, the radar can have better estimates of the target direction from which the returned echo with the maximum signal strength was intercepted. The radar can hereby estimate the target elevation and azimuth angle of the flying target with a tracking error as low as the scan step size, which can be in the order of a tenth of a degree. Fig. 10 shows the performance difference in the target azimuth angle estimation with and without waveform optimization. The improvement of target angle estimation improves drastically the target position estimation which is demonstrated in Fig. 4.

![Tracking Target Angle](image)

**Fig. 50.** Target angle estimation with and without radar parameter optimization
The waveform optimization algorithm successfully adapts the PRF to the target range to keep it always within the maximum unambiguous range. Targets residing at ranges beyond the maximum unambiguous range appear to the radar at ranges closer than they truly are due to range folding phenomena. Fig. 12 shows a false echo intercepted when the target exceeded the maximum unambiguous range when the simulation was run without waveform optimization. Hence, waveform optimization can improve the target tracking and detection by avoiding false range measurements due to range folding.

![Fig. 12](image)

**Fig. 12** - Target Response interference without waveform optimization

In summary, we have shown that optimizing the radar parameters improves the performance of CRs, for instance deciding the right number of pulses N guarantees the minimum required SNR for detection, which helps the radar avoid track loss that might occur when the target exceeds the maximum attainable detection range for a fixed N and specified transmit power. Also, adaptively changing the PRF with a varying target range helps the radar avoid false echoes due to range folding. Furthermore, redefining the radar scan schedule reduces drastically the target revisit time and unnecessary energy loss, also refining the radar scan step size increases the precision of angle measurements.

**10-Sensor data fusion**

**10.1-Introduction**
Data fusion is a growing technology within the research community, it involves the process of integrating more than one data source, in order to produce a solution more reliable and robust. The data provided by any kind of sensor is always susceptible to distortion due to variable levels of noise and can be marked with some levels of impreciseness and uncertainty, hence, combing data provided by multiple sensors can increase robustness and reliability and decrease uncertainty of the whole system.

10.2- Sensor fusion Algorithms

Data fusion is a very broad term that encompasses many algorithms and techniques form which we can state the following:

- Probabilistic data fusion: which relies on the probability density function of the observation. The essence of this method is the application of the Bayes estimator to combine fragments of information coming from multiple sources. This technique is often referred to as “Bayesian fusion “
- Central limit theorem (CLT) : In probability, the CLT suggests that any large number of observations can be averaged by a normal distribution. For example if we toss a dice and log which one of the 6 faces it landed on , and repeat the process for a big number of times the result will indicate that the probabilities of landing the dice on each face are equal.
- The Kalman filter : The use of the Kalman filter (or any of the variations of the Kalman filter) is also so common in data fusion , for its simplicity and ease of implementation.

In this work we are using the Extended Kalman filter to apply data fusion on the radar tracking problem.

10.3- Problem description and System Model:

In radar tracking, counting on one source of data might lead to unwanted system behaviors like target track loss, or false target classification and identification. Also radar data can always be affected by some degrees of uncertainty and impreciseness due to noisy environment and physical limitations of the radar systems. Therefore, data fusion can be used in radar applications like target trucking, which might
increase tracking performance and decrease levels of uncertainty. Hence, the application of data fusion on radar systems can lead to more robust, reliable and certain systems.

In this approach we will consider two phased array antennas, feeding the same radar central unit that we called the radar Cognitive unit (RCU). The RCU will be responsible of collecting the data from both antennas and target tracking. In this work the IMM algorithm will be considered for target tracking and the Extended Kalman filter will be considered for data fusion. Figure xx shows the model of the proposed system:

The positions of the two antennas must be known because the received echo has target information relative to the position of the transmitter and the receiver, therefore the received coordinates must be transformed to one coordinate system in the origin of which the RCU is placed. Let $z_1 = [Px1, Py1, Pz1]$ be the target position in respect to antenna number 1 and $z_2 = [Px2, Py2, Pz2]$. The radar cognitive unit will convert the two target data pieces $z_1$ and $z_2$ to one coordinate system:

\[
\begin{align*}
    z_1 &= [Px1 - x_1, Py1 - y_1, Pz1 - z_1] \\
    z_2 &= [Px2 - x_2, Py2 - y_2, Pz2 - z_2]
\end{align*}
\]
The extended Kalman filter is one extension of the classic Kalman filter that can be applied to nonlinear systems. It is a very well-established data fusion approach that has been extensively studied and tested both theoretically and in real-life applications.

We can assume that the system can be modeled by the state transition equation:

$$
x_k = Ax_{k-1} + Bu_k + w_{k-1}
$$

Where $x_{k+1}$ is the state at time $k+1$, $A$ is the state transition matrix which relates the state at the previous time $k-1$ to the states at the current step $k$. We also assume that the state observation can be governed through a measurement model that can be represented by the following observation equation:

$$
z_k = Hx_k + v_k
$$

Where $Z_k$ denotes the measurement, vector made at time $k$, $x_k$ is the state at time $k$, $H$ is the observation matrix and $v_k$ is an additive measurement noise. The measurement vector $Z_k$ will be composed of the data coming from both antennas $z_1, z_2$.

The movement of the target is simplified in this work and can be described by the following equations:

$$
\begin{align*}
P_{x_k} &= P_{x_{k-1}} + dV_x \\
P_{y_k} &= P_{y_{k-1}} + dV_y \\
P_{z_k} &= P_{z_{k-1}} + dV_z
\end{align*}
$$

Hence, the state transition matrix is defined as following:

$$
A = \begin{bmatrix}
1 & 0 & 0 & d & 0 & 0 \\
0 & 1 & 0 & 0 & d & 0 \\
0 & 0 & 1 & 0 & 0 & d \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

The control vector $u_k$ and the control matrix $B$ are ignored.

10.4.1 - The Extended Kalman Filter sequence outline:
10.5- Simulation and Results

10.5.1- Simulation

In this section we will use the same simulation initial parameters as the previously. For the IMM algorithm we are considering the same models as well; the constant Velocity Model (CV), the singer model, and the 3DCT model.

The simulation initial parameter:

```
<table>
<thead>
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</tr>
<tr>
<td>Target initial range</td>
<td>100700 meters</td>
</tr>
</tbody>
</table>
```

10.4.1- Results:
Fig. 54 - Target tracking using IMM algorithm and data fusion

Fig. 55 - Target tracking performance during uniform motion segment
Fig. 56 - Target tracking performance during turn maneuvers

Fig. 57 - Target tracking Error Along X axis
Cognitive radars are intelligent systems that interact with the surrounding environment constantly in the learning process to the necessary changes to the radar parameters in an autonomous way to guarantee the best performance. In this context CR’s are considered self-controlled and self-adjusted systems, where all acts are preceded by a full awareness of the surrounding environment. In this section we address the aspects that substantiates the superiority of CR’s over conventional radars.

CR’s provide a tactical supremacy over the target, as it is governed by an algorithm that estimates the target position an act based on those estimations before the target state changes. Instead of the system lagging the target as in Conventional radars, CR’s are designed to lead the target. In Figure.29 and Figure.30 we have demonstrated the good performance of our algorithm to provide accurate estimation of the target state. With such a precision, the CR can concentrate the
power and time at the direction of the target. Hence CR’s prove a superiority over conventional radars in terms of saving valuable energy and reducing the target revisit time, which means CR’s can provide updates about the target faster and more frequent than conventional radars. Figure 41 and Figure 42 illustrates this comparison.

![Figure 59: CR target revisit time](image)

**Figure 59:** CR target revisit time

![Figure 60: Conventional radar target revisit time](image)

**Figure 60:** Conventional radar target revisit time.
With such a reliable radar system proving to provide accurate estimations about the target position with a high update frequency, cognitive radars are always a step ahead of the target which gives the operating unit an advantage over the enemy target. In this context CR’s can be used for more than just surveillance, but also to better the unit defense systems.

CR’s also provide an example of adequate usage of valuable energy, as it refines the radar scan schedule and redirects the transmit energy accurately towards the target, avoiding the unnecessary use of energy in other directions. Figure 43 illustrates a comparison between CR’s and Conventional radars in terms of the percentage of energy used to successfully intercept a target.

![Comparison between CR’s and Conventional Radars](image)

**Figure 61.** % of energy used to successfully intercept the target

In Table 7 we summarize the aspects of eminence of CR’s.

<table>
<thead>
<tr>
<th>Capability</th>
<th>CR’s</th>
<th>Conventional Radars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar scan schedule</td>
<td>Cr’s using the capacity to estimate the target position aims and illuminates the target accurately without unnecessarily scanning other angles</td>
<td>Predefined: Loss of energy and radar timeline due to unnecessary illumination of angles not containing any targets.</td>
</tr>
<tr>
<td>PRF</td>
<td>Adapted based on the target estimated range which eliminates the risk of pulses interference.</td>
<td>Fixed: possibility of pulses interference.</td>
</tr>
</tbody>
</table>
10.Conclusion
In this paper, we have considered the control theoretic approach to introduce an algorithm for adaptive radar waveform optimization and target tracking. We have introduced the Kalman filter algorithm as the backbone of our algorithm. The target track estimations were performed in a cartesian environment where the tracking mean square error is minimized. The simulations proved the good performance of our algorithm in the target track and speed estimation as well as the capacity to adaptively changing the radar parameters, however, the performance varies along with the target range, it is more suitable for short to medium range targets. Hence it is recommended to consider in applications such as the control of radar-controlled weapons or short-range surveillance.

11.References


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In this work we address the problem of radar waveform optimization and target tracking. We propose an algorithm for optimal waveform design and target tracking based on the Control theoretic approach, where the waveform parameters are adaptively designed by minimizing the tracking mean square error (MSE). In this work we take several approaches to enhance the radar tracking performance. In a first place the Kalman filter was used to estimate the target position which we use to optimize the waveform parameters. Experimental results demonstrated the ability of the proposed algorithm to track a flying target in the Cartesian space, by providing accurate estimates of the target position and the target speed cartesian vector as well as the radial speed. The algorithm adapted the waveform parameters on the fly based on the estimate vectors. In literature, the Doppler Effect theory was intensively used to estimate target velocity. Under certain conditions, such as tracking high speed targets or rough sea and weather conditions the Doppler Effect is less effective. Hence, in this first approach we introduce an algorithm that relies on the Kalman filter estimations independently of the Doppler Effect. A low pass filter with adaptive parameters in real time is applied to the estimated speed vector, and accurate speed estimations where extracted. Furthermore, we tackle the radar tracking problem from a realistic angle by admitting that the target motion can’t be described by on matrix like we have proposed using the Kalman filter, therefore we introduce the Interacting Multiple Model algorithm instead to estimate the target position. Through simulations, we demonstrate the good performance of the proposed algorithm and prove that waveform optimization can improve the tracking performance of the radar. Finally we consider harvesting information from two antennas instead of one, and using one of the data fusion algorithms, and the IMM algorithm, we were able to reduce the tracking error and provide a more robust and reliable solution to the tracking problem.

Dans ce travail, nous abordons le problème de l'optimisation de la forme d'onde radar et du suivi de la cible. Nous proposons un algorithme pour la conception optimale du signal et le suivi de la cible, basé sur l'approche théorique du contrôle, dans lequel les paramètres de la forme d'onde sont conçus de manière adaptative en minimisant l'erreur de suivi. Dans ce travail, nous adoptons plusieurs approches pour améliorer les performances de suivi radar. Dans un premier temps, le filtre de Kalman a été utilisé pour estimer la position cible utilisée pour optimiser les paramètres de forme d'onde. Les résultats expérimentaux ont démontré la capacité de l'algorithme proposé à suivre une cible volante dans l'espace cartésien, en fournissant des estimations précises de la position de la cible et de la vitesse cible du vecteur cartésien, ainsi que de la vitesse radiale. L'algorithme a adapté les paramètres de forme d'onde à la volée en fonction des vecteurs d'estimation. Dans la littérature, la théorie de l'effet Doppler a été utilisée de manière intensive pour estimer la vitesse de la cible. Dans certaines conditions, telles que la poursuite d'objets à grande vitesse ou des conditions météorologiques difficiles, l'effet Doppler est moins efficace. Par conséquent, dans cette première approche, nous introduisons un algorithme qui repose sur les estimations du filtre de Kalman indépendamment de l'effet Doppler. Un filtre passe-bas avec des paramètres adaptatifs en temps réel est appliqué au vecteur vitesse estimé et des estimations de vitesse précises sont extraites. De plus, nous abordons le problème de poursuite radar sous un angle réaliste en admettant que le mouvement de la cible ne peut pas être décrit comme sur une matrice, comme nous l’avons proposé avec le filtre de Kalman. Nous introduisons donc l'algorithme Interacting Multiple Model pour estimer la position de la cible. Au moyen de simulations, nous démontrons les bonnes performances de l'algorithme proposé et prouvons que l'optimisation des formes d'ondes peut améliorer les performances de suivi du radar. Enfin, nous envisageons de collecter des informations à partir de deux antennes au lieu d'une, et en utilisant l'un des algorithmes de fusion de données, et l'algorithme IMM, nous avons pu réduire l'erreur de suivi et fournir une solution plus robuste et fiable au problème de suivi.