Constitutive Modelling of Soils under High Strain Rates

Literature Review

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PWGSC Contract Number: W7701-135578/001/QCL
CSA: Grant McIntosh, Defence Scientist, 418-844-4000 ext. 4278

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Contract Report
DRDC-RDDC-2015-C072
March 2015
A REPORT

PRESENTED TO DRDC VALCARTIER

March 13, 2014

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LITERATURE REVIEW

by

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Introduction

In the study of soil behavior under extreme loading such as blasting, it is necessary to take into account the effect of strain rate on soil behavior. The strain rate is of overriding importance in such study, where the load is expected to act as a single pulse and stress produced by the blast pressure increases so rapidly that the material behaves significantly different from that under quasi-static loadings.

The modeling and experimentation of high strain rate phenomena are very important in determining the behavior of soils under critical events. It is widely accepted in the solid mechanics community that the strength, constitutive behavior, and overall response of soils are highly dependent on the strain rate. The dynamic properties of soils have been extensively investigated to predict the response of solids subjected to blasts, earthquakes, pile driving, meteor impact and many other transient loads. Despite existing studies, the behavior of soils at high strain rates and under various confining pressures, saturation, particle distribution, void ratios and other pertinent conditions have yet to be fully characterized.

This report contains a summary of the published studies in the field of high-strain rate soil dynamics. The contents of this report are organized as follows:

Chapter 1: General introduction on soil mechanics and deformation.

Chapter 2: Brief review about experiments used to investigate the response of soil behaviors under high strain-rate conditions.

Chapter 3: Review of the constitutive models used to simulate the time-dependent behavior of soils.

Chapter 4: Equation of state (EOS) used in the numerical simulation of soil under blast loading.

Chapter 5: Review of the numerical simulation of soil behaviors under blast or explode loading.
1.1 Phase relationship of soil

Soil is made up of solid particles, with spaces or voids in between. The assemblage of particles in contact is usually referred to as the soil matrix or the soil skeleton. In conventional soil mechanics, it is assumed that the voids are in general occupied partly by water and partly by air. This means that the soil is a three-phase material, comprising some solid (the soil grains), some liquid (the pore water) and some gas (the pore air). This is illustrated in Fig. 1-1, which shows the relative volumes of solid, liquid and gas. The phase relationships are important in characterizing the state of soil. They are described as follows.

The void ratio $e$ is defined as the ratio of the volume of voids $V_v$ to the volume of solids $V_s$, and it is conventionally given as:

$$
e = \frac{V_v}{V_s}$$ (1-1)

The saturation ratio $S_r$ is defined as the ratio of the volume of water to the volume of voids, and it is presented as:

$$S_r = \frac{V_w}{V_v}$$ (1-2)

The saturation ratio must lie in the range $0 \leq S_r \leq 1$, $S_r = 0$ for dry condition and and $S_r = 1$ for fully saturated.

The water content $w$ is defined as the ratio of the mass of water $m_w$ to the mass of soil solids $m_s$, and it is given as:
\[ w = \frac{m_w}{m_s} \]  

(1-3)

The density of the soil grains \( \rho_s \) is defined as \( \rho_s = m_s/V_s \), and it also can be expressed as \( \rho_s = G_s \rho_w \) where \( G_s \) is the particle relative density. \( G_s \) is the density of the soil particles relative to that of water and \( \rho_w \) is the density of water.

![Fig. 2-1 Soil as a three-phase material](image)

1.2 Strain rate effect on soil behavior

Under static loading, the pore pressure and drainage can be specified as drained or undrained. In rapid loading the drained condition is difficult to attain even in coarse grained soils, because the short time loading does not permit complete dissipation of the excess hydrostatic pore pressure. When sheared in the undrained condition, the excess hydrostatic pore pressure will be affected by the strain rate. Generally, the pore pressure decreases with the increase of strain rate (Whitman and Healy 1962, Richardson and Whitman 1963). It is suspected that the strain rate affects the nature of particle movements during shear deformation. During rapid strain the individual particles have less freedom to choose a path of least resistance than during slow strain. Hence, more particles are forced to override neighboring particles and this leads to an increase in dilation. As shown in Fig. 1-2, the pore pressure \( P_w \) has a lower magnitude in rapid loading than in slow loading for common triaxial tests.
The effect of the strain rate on the stress-strain curve is also shown in Fig. 1-2. The dashed curves represent the curve obtained in rapid loading and the solid curve from slow loading. In general, the secant modulus and tangent modulus also increase with the increase of strain rate.

In confined compressive tests, soil specimens are loaded axially, but expansion in the radial direction is limited. The axial stress $\sigma_1$ is the major principal stress. If stress increment is applied in large time interval, such test becomes a consolidation test. When loaded rapidly on the contrary, a curve of different shape will be obtained as shown in Fig. 1-3 (Heierli 1962). The value of the secant modulus increases rapidly with the increase of strain rate. The initial tangent modulus in rapid loadings is also substantially higher than that in slow loading.
The general behavior of the radial stress $\sigma_3$ during loading and unloading is similar to that in static tests. As shown in Fig 1-4, the increase of the radial stress $\sigma_3$ is proportional to the increase of the axial stress $\sigma_1$ during loading. However, when the axial stress $\sigma_1$ is reduced after reaching a maximum value, the decrease of radial stress $\sigma_3$ is smaller than that of the axial stress, in other way, the ratio of the radial stress to the axial stress ($K=\sigma_3/\sigma_1$) increases as $\sigma_1$ is reducing. Since in rapid loading conditions there is no opportunity for consolidation, the value of $K$ also depend on the saturation ratio of soils: for dry soils, $K$ varies in the range between $1/3$ and $1/2$ during loading (Wu 1971); and for saturated soils, the $K$ value is nearly unity.

1.3 Soil deformation under high strain rate monotonic loading

To characterize the stress wave propagating in a medium, properties of the medium should be studied first. The basic general characteristic of a medium is the relationship between pressure and relative volume.

Soils can be divided into two main groups; cohesive and noncohesive, based on the bonding condition between soil particles. Cohesive soils contains sufficient clay content to stick the mass together and have an internal strength. Noncohesive soils, such as sand and gravel, have no internal strength on their own. As a result in noncohesive soils, strength depends on friction between particles.
Generally, there are two main deformation mechanisms in soils under *hydrostatic loading* (Henrych 1979):

(a) Deformation of the soil skeleton: This deformation consists of elastic deformation of the bonds at the contact surfaces of grains at low pressure, and plastic deformation due to failure of bonds at high pressure.

(b) Deformation of all the soil phases: this deformation is determined by the volume compression of all three phases in the soil.

In the process of deformation, both of the mechanisms may occur simultaneously. At a certain stage of loading process, however, one of the mechanisms will dominate.

The deformation process in a dry soil is more complex compared to a saturated soil. It has the general form represented in Fig. 1-5(a) for dry soils and Fig.1-5(b) for saturated soils. Most of the voids in a dry soil are filled with air that has a higher compressibility as compared to the solid (minerals) and liquid (usually water) phases of the soil. Therefore, when a static or dynamic load is applied on a dry soil, the soil skeleton will resist the load and so the first deformation mechanism dominates. By increasing the pressure, the bonds between grains will start to fail.
(point A in Fig. 1-5(a)). Within the stage between point A and B, large displacements of the particles occur even with a slight change in pressure. The soil will be compressed and cease to transfer shear stresses, thus its behavior is similar to that of a liquid (after point C). In this stage the second mechanism becomes dominant and the first mechanism can be neglected.

In water-bearing soil (most of the voids are filled with water), the deformation mechanism of the soil depends on the loading rate. With a rapid dynamic loading, they have a higher resistance than the contact bonds of the skeleton grains. Under this loading condition, the second mechanism of deformation will be dominant (Fig. 1-5(b)).

The deformation mechanisms in the noncohesive soils are similar to the cohesive soils; however, at high stress level grains begin crushed and the skeleton can contract further more. Fig. 1-6 represents the stress- strain response of dry sand under uniaxial strain loading while lateral strains are prohibited.

Fig. 2-6 Stress-strain curve of dry sand under uniaxial strain loading condition (Omidvar et al. 2012).

There are usually three main mechanisms which govern the response of dry sands in the uniaxial strain loading condition: (1) elastic compaction of sand particles; (2) slippage and rearrangement of grains; and (3) grain crushing. Based on these three mechanisms, one can divide the response of dry sands into four distinct zones as presented in Fig. 1-6:
- **Zone 1:** In this zone, the applied stress is not enough to overcome the friction between individual particles and so all deformations corresponding to the elastic deformation between grains.

- **Zone 2:** The applied stress overcomes friction between grains and the soil particles start to slide and roll into the voids. In this stage the density of sand will change and inelastic deformation will commence.

- **Zone 3:** By increasing the applied stress, the sand grains rearrange and the contact points between grains will increase more. As a result, the soil will be more compacted and sliding and rolling of the grains will be more difficult. The hardening (lock-up) is thus observed in sand.

- **Zone 4:** In this stage, since the applied stress is really high, the grains begin to crush, leading to the hardening response of the material again.

In contrast to uniaxial compression tests, in a **triaxial test** the sample is allowed to deform laterally. In common triaxial test with a constant confining pressure, the soil can reach the critical state. Fig. 1-7 shows the difference between the response of sand in the uniaxial compression and triaxial compression. In triaxial compression test, after sample reaches the critical state the axial stress remains constant.

![Fig. 2-7 Stress strain response of sand in uniaxial and triaxial compression tests](image-url)
2 Experiments and tests

To apply high strain rate to the soil sample, several means such as a drop weight systems (Heierli 1962), gas-driven pistons (Sparrow and Tory 1966), hydraulic-driven pistons (Whitman 1970) and explosive-loaded pistons (Jackson Jr, Ehrigott et al. 1979) have been used. In addition, the split Hopkinson pressure bar (SHPB) is one of the powerful tools used to study soil behaviors under high strain rate loadings.

Generally, there are two main ideas of laboratory tests to study the behavior of soils under high strain rate (Omidvar et al. 2012):

(a) Reproducing the stress conditions at the propagation front by applying a uniform stress: The main consideration in this test is to ensure that the stress is distributed uniformly within sample. One can achieve this by controlling the rise time of applied stress or reducing the thickness of the sample. Heierli (1962) suggested that the thickness of the sample should be about one tenth of the wavelength of stress wave to achieve an acceptable uniformity of stress in the sample.

(b) Generating a stress wave in a long sample: In this method, in order to ignore the complications introduced by multiple reflected waves, a long sample of sand is prepared. This type of test is difficult; in addition, the sidewall friction affects the results of the test which should be taken into account in the data interpretation.

Based on the first idea discussed above, there are three common testing methods to study the behavior of sand under high strain rate: (1) One-dimensional strain compression test; (2) Triaxial strain compression test; and (3) Plate impact (plane shock wave) test.

2.1 Uniaxial and triaxial tests

Fig. 2-1 illustrates a one-dimensional strain compression test design in which the soil sample is placed inside a rigid jacket which confines the radial deformation. As shown in Fig.2-2, in the triaxial strain compression test a constant confining pressure is applied to the sample during the
test. The specimen is encapsulated into a fixture consisting of two punches and an elastomer sleeve. The sample assembly is then subjected to a fluid pressure. When a desired confining pressure is reached, an additional dynamic axial pressure is applied while the equipment control system maintains a constant fluid pressure. This additional pressure can have different rates to study the strain rate dependency of soils.

The stress-strain responses of sand in uniaxial strain compression ranging from quasi-static to high strain rates are presented in Fig. 2-3 (Omidvar et al. 2012). The effect of strain rate on the sand behaviors is obviously seen.

![Fig. 2-1 A schematic of one-dimensional compression test: (a) Cross-section view; (b) Plan view](image1)

![Fig. 2-2 Triaxial Compression (Lu 2010)](image2)
2.2 Shock wave tests

Shock wave tests are another group of tests used to study the behavior of soils under extremely high strain rate. In this test, impact of a flat plate on a target disc produces the shock wave which propagates in the soil sample as a planar shock wave. Fig. 2-4 shows a schematics of the target assembly used by Chapman et al. (2007). As can be seen from the figure, the shock wave is transmitted to the sample by means of an impact of flyer on the anvil plate.

Fig. 2-3 Stress-strain responses of sand in uniaxial compression from static and high strain rate tests (Omidvar et al. 2012)

Fig. 2-4 A schematic of the target assembly used by Chapman et al. (2007)
Since the amplitude of shock waves exceeds the dynamic flow strength of the soil substantially, the shear stresses produced in the soil may be neglected as compared with the hydrostatic components. In this case, soil can be considered as a gas or liquid and based on this assumption the equations of conservation of mass, energy and momentum can be written as (Meyers 1994):

\[ U_s \rho_0 = (U_s - U_p) \rho \]  \hspace{1cm} (2-1)

\[ E - E_0 = \frac{1}{2} (P + P_0)(V - V_0) \]  \hspace{1cm} (2-2)

\[ P - P_0 = \rho_0 U_s U_p \]  \hspace{1cm} (2-3)

in which \( \rho \), \( E \), \( P \) and \( V \) are the density, energy, pressure and specific volume of soil (inverse of volume behind the shock wave); and \( \rho_0 \), \( E_0 \), \( P_0 \) and \( V_0 \) are the density, energy, pressure and specific volume of soil in front of the shock wave. Also, \( U_s \) and \( U_p \) are the shock and particle velocities, respectively. From the tests, on can obtain a linear relationship between \( U_s \) and \( U_p \) which is called Hugoniot relation:

\[ U_s = C_0 + SU_p \]  \hspace{1cm} (2-4)

where \( C_0 \) is the speed of sound and \( S \) is a constant. The above relations, combined with the equations for the material Equation of State to determine full material response.

Zhixiong et al. (2001) have done a series of plate explosive impacts and determined stress histories using piezoelectric crystal gauges pre-positioned at different depths of clayey and sandy soils (Fig.2-5). Using the Lagrangian analysis, they obtained the empirical soil dynamic constitutive relation from the measured stress histories by fitting a stress- depth- time surface.
2.3 Split Hopkinson Pressure Bar (SHPB) tests

The split Hopkinson pressure bar (SHPB) was originally developed by Kolsky (1949), and is the most common experimental method used in the study of engineering materials under high strain rate (HSR) loading (Field et al. 2004). Tests are performed by placing a specimen between two long bars that are strong enough to remain elastic, termed incident bar and transmitting bar. A third bar, called the striker bar, is propelled, typically using a gas gun, to strike the incident bar. The impact generates an elastic compressive wave in the incident bar, which then travels towards the specimen. At the interface between the sample and incident bar, part of the wave is transmitted into the sample and then the transmitting bar, and part of it is reflected back to the incident bar. By placing strain gauges on the incident and transmitting bars, the incident, reflected and transmitted waves are measured.

In order to consider inertial effects in the soil specimen, the Lagrangian analysis has been used to correct the initial portion of the stress-strain response (Seaman 1974, Felice et al. 1987). Other corrections and modifications to the testing and measurement methods can be found in Chen and Song (2010) and Kaiser (1998, 1998). A summary of SHPB tests on sand is presented in Table 2-1 (Omidvar et al. (2012)). In SHPB tests, measurement of lateral stresses and strains allows for constitutive laws to be established for behavior of sand.
SHPB setups have been recently modified to control the lateral stress during testing. Pierce and Charlie (1990) applied confining pressure in the SHPB test by placing the sample in a deformable membrane and applying a confining pressure using a thin layer of pressurized water between the membrane and the confining cylinder. Martin (2007), Kabir and Chen (2011) have recently reported successful triaxial SHPB tests on sand with confining pressures of up to 150 MPa. In these tests confining pressure is applied by means of two pressure chambers; a radial confining pressure chamber encasing the sample, and an axial confining chamber at the opposite end of the transmission bar. The two chambers apply equal pressures to the sample in the axial and radial directions to produce hydrostatic confinement. Dynamic deviatoric stress is then applied by the striker bar.

The effect of confinement on stress-strain response is shown in Fig. 2-6 (Omidvar et al. (2012)). Similar to static loading, confinement results in a stiffer soil response, due to the increase in frictional resistance of sand as a result of increase in ambient stress. Moreover, the stress-strain curves over a wide range of high confining pressures indicate that the specimen regain strength following an initial yield at lower strains.

In addition, Kabir et al. (2010) performed triaxial SHPB tests at high confining pressures of up to 150 MPa, where grain crushing during the consolidation phase has been reported to dominate the response in shear. Triaxial SHPB tests suggest that the increasing the strain rate from 500/s up to 1000/s has negligible effect on the stress-strain response of sand. As will be discussed in the following sections, some of these observations are in contrast with the observations from conventional triaxial tests performed on sand.

Omidvar et al. (2012) have concluded the effect of the strain rate to the soil dynamic behaviors, of which some important points were given as:

(a) Modulus: either under uniaxial compression or under triaxial compression test conditions, the dynamic modulus ratio increases gradually with the increasing of strain rate.

(b) Strength: Similar as the effect on modulus, the strength of sand also increases with the increasing of strain rate. Triaxial compression tests have shown an increase of up to 30% in shear
strength of dry sand. Shock wave tests show that the strength of sand is related to the grain constituent minerals.

(c) Confining pressure: in the triaxial SHPB tests with the confining pressures in the range of 25–150 MPa, the strength increases with the increasing of confining pressure, and this relationship is not affected by increasing the loading strain rate. HSR Triaxial tests with the low confining pressure show that the dilatancy rate of the sample is affected by confining pressure.

(d) Saturation: the response of saturated sand under HSR loading is thoroughly undrained, even if drainage is fully permitted. In triaxial tests, fully saturated loose sand tends to expand and may exhibit very high increase in strength. Dense saturated sand expands under HSR loading, with showing limited effects on shear strength.

(e) Particle breakage: SHPB and uniaxial compression tests indicate that particle breakage decreases under the HSR loading, and the test results show that particle breakage degree decreases by up to 25% in HSR loading compared to static loading.

Fig. 2-6 Effect of confining pressure on HSR stress-strain response of dry sand from triaxial HPB tests (Omidvar et al. 2012)

However, the effect of strain rate on the soil behaviors is still not well understood at present. Comprehensive experiment investigations are necessary for understanding the effects of confining pressure, initial void ratio, particle shape, size, gradation, surface texture and mineralogy on the soil behaviors. For example, nowadays it is an very interesting work to
research the effect of different saturated degree on the sand dynamic behaviors under HSR loading.

Table 2-1 Summary of SHPB tests on sand (Omidvar et al. 2012)

<table>
<thead>
<tr>
<th>Reference</th>
<th>Peak pressure</th>
<th>Soils tested</th>
<th>Saturation</th>
<th>Max strain rate</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fletcher and Poorooshab (1968)</td>
<td>0.84 MPa</td>
<td>Fine grained soils</td>
<td>Partially saturated</td>
<td>200/s</td>
<td>One of the first SHPB studies on soils reported; inertial effects not clearly considered</td>
</tr>
<tr>
<td>Gaffney and Brown (1984)</td>
<td>500 MPa</td>
<td>Clayey silty sand</td>
<td>Partially saturated</td>
<td>5000/s</td>
<td>Negligible strain rate effects observed</td>
</tr>
<tr>
<td>Gaffney et al. (1985)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Felice (1986)</td>
<td>760 MPa</td>
<td>Clayey silty sand</td>
<td>Dry/Partially saturated</td>
<td>5000/s</td>
<td>Effect of saturation studied</td>
</tr>
<tr>
<td>Felice et al. (1987)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ross et al. (1986)</td>
<td>10.3 MPa</td>
<td>20/40 and 50/80 silica sand</td>
<td>Dry/Partially saturated</td>
<td>NA</td>
<td>Long samples up to 15 cm tested; Wave propagation speed measured</td>
</tr>
<tr>
<td>Pierce (1989)</td>
<td>4.5 MPa</td>
<td>50/80 silica sand; 20/30 Ottawa sand</td>
<td>Dry, partially, fully saturated</td>
<td>NA</td>
<td>Wave propagation speed studied; effect of saturation considered</td>
</tr>
<tr>
<td>Pierce and Charlie (1990)</td>
<td>760 MPa</td>
<td>Eglin, Tyndall and Ottawa 20-30 sand</td>
<td>Partially saturated</td>
<td>2000/s</td>
<td>Effect of saturation studied</td>
</tr>
<tr>
<td>Charlie et al. (1990)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semblat et al. (1999)</td>
<td>750 MPa</td>
<td>Fine sand</td>
<td>Dry</td>
<td>1245/s</td>
<td>Lateral stresses measured; Effect of lateral confinement studied</td>
</tr>
<tr>
<td>Semblat et al. (1995)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bragov et al. (2004)</td>
<td>500 MPa</td>
<td>Fine quartz sand</td>
<td>Dry</td>
<td>NA</td>
<td>Lateral stresses measured; Plate load tests reported as well</td>
</tr>
<tr>
<td>Bragov et al. (2004)</td>
<td>400 MPa</td>
<td>Fine sand</td>
<td>Dry</td>
<td>NA</td>
<td>Wave speed propagation studied; numerical model applied; lateral stress</td>
</tr>
<tr>
<td>Martin (2007)</td>
<td>35 MPa</td>
<td>Poorly graded fine quartz</td>
<td>Dry/Partially saturated</td>
<td>470/s</td>
<td>Effect of saturation studied; pulse shaping techniques used</td>
</tr>
<tr>
<td>Martin et al. (2009)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Song et al. (2009)</td>
<td>31 MPa</td>
<td>Fine silica sand</td>
<td>Dry</td>
<td>1450/s</td>
<td>Effect of lateral confining studied by using different confining tubes</td>
</tr>
<tr>
<td>Luo et al. (2011)</td>
<td>360 MPa</td>
<td>Fine quartz sand</td>
<td>Dry</td>
<td>680/s</td>
<td>Sample preparation and relative density carefully controlled; lateral stresses recorded</td>
</tr>
<tr>
<td>Lu et al. (2009)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kabir (2010)</td>
<td>150 MPa</td>
<td>Fine silica sand</td>
<td>Dry</td>
<td>470/s</td>
<td>Triaxial SHPB tests performed; Pulse shaping techniques used</td>
</tr>
<tr>
<td>Kabir et al. (2010)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3 Constitutive equations

The high dependency of the soil properties upon the loading conditions make it extremely difficult to develop a unified model for the deformation of soils, especially for soils under the high-rate blast loading. This is the main reason that various constitutive equations for soils under this kind of loading were proposed.

In this chapter, two classes of constitutive equations are explored: the Three-phase model and the Viscoplastic models. Because of the viscoplastic cap models are widely used, they are reviewed in a separate part in this chapter.

3.1 Three-phase soil model

As mentioned in Chapter one, soil consists of mineral particle, air and water. Each of these three phases has its own properties, making it difficult to predict the deformation in the soil. Generally, there are two different deformation mechanisms (Henrych 1979, Wang and Lu 2003, Wang et al. 2004): (a) deformation of solid skeleton; (b) deformation of all the soil phases.

One main assumption in soil mechanics is that the water phase is incompressible, and the solid particles do not deform during the loading process. This means the total external load $\sigma$ is exerted onto the soil skeleton, and water and gas phases. The stress borne by water and gas phases is named the pore pressure $p_w$, and the stress borne by soil skeleton is named effective stress $\sigma'$ which can be written as:

$$\sigma' = \sigma - p_w$$  \hspace{1cm} (3-1)

The effective stress concept for fully saturated soils is acceptable in the case of static and slow transient loading.

However, in case of high strain rate loading such as shock wave, the water and gas cannot drain from the voids, and they will deform with the soil skeleton. In this case, application of effective
stress in modeling of the soil behavior is questionable. Therefore, it is important to derive new models which are appropriate for high strain rate loading.

Many researchers have proposed the three-phase constitutive equations for soils (Nagel and Meschke 2007, Nagel and Meschke 2010, Zhou and Meschke 2013). A comprehensive framework to define the constitutive behavior of unsaturated soils is developed within the theory of mixtures applied to three-phase porous media by Loret and Khalili (2000). They developed elastic and elastic-plastic constitutive equations considering each of the three phases is endowed with its own strain and stress. Furthermore, Wang et al. (2004) and Wang and Lu (2003) established a conceptual model of soil under shock loading. They implemented the concept introduced by Kandaur (Henrych 1979) in which he considered soil particles as the rectangular blocks that constituted the soil structure (Fig. 3-1). As can be inferred from Fig. 3-1, the voids between solid particles can be filled with water or air.

Moreover, An et al. (2011) developed a model for finite-element simulations of explosions in soil with various degrees of saturation. They developed EOS models for the three phases of the soil based on Kandaur’s concept and integrated it with the viscoplastic cap model previously developed by Tong and Tuan (2007). The EOS is reviewed in detail in the chapter 4 as a separate part.
3.2  Viscoplastic models

The actual behavior of many practical problems is governed by not only plastic but also rheological effects. Both sciences of plasticity and rheology are concerned with the description for important mechanical properties of materials. The method of plasticity is limited to quasi-static processes that are sufficiently short and creep and relaxation effects do not occur. However, according to research concerning the description of dynamic properties of materials, it has been shown that the application of theory of plasticity, in which rheologic effects are disregarded, leads to too large discrepancies between theoretical and experimental results (Perzyna 1966).

The simultaneous description of rheologic and plastic behavior of materials is called viscoplasticity. The viscous properties of the material introduce a time dependence of the state of stress, while the plastic properties further leads to the dependence on the loading path. The fundamental problems of viscoplasticity are the determination of an adequate yield criterion for a material and the establishment of suitable constitutive equations. In the following section, different viscoplastic constitutive models are reviewed.

3.2.1 Differences between plasticity and viscoplasticity

General formulation of the constitutive equations of classical plasticity can be stated as:

\[
\sigma = D(\varepsilon, \sigma, \dot{\varepsilon}) \dot{\varepsilon}
\]  

(3-2)

where \( \sigma \) and \( \dot{\varepsilon} \) are stress and strain rates respectively and \( D(\varepsilon, \sigma, \dot{\varepsilon}) \) is the stiffness operator.

In the context of classical plasticity, usually the strain rate is expressed as a function of state \((\varepsilon, \sigma)\) and the stress rate. This means the above equation can be expressed as:

\[
\dot{\varepsilon} = C(\varepsilon, \sigma, \dot{\sigma}) \dot{\sigma}
\]  

(3-3)
where $\overline{C}(\varepsilon, \sigma, \dot{\sigma})$ is the compliance operator.

The plasticity theories based on the above equations are rate-independent theories. In contrast to these plasticity theories, viscoplasticity theories include rate effects in the body of their formulation. Viscoplasticity theories are expansion of classical plasticity theories which may be applied to the soil constitutive laws to take into account the strain or stress rate effects.

For example, the material behavior can be described by a quasilinear rate-type constitutive equation as follow (Cristescu and Suliciu 1982):

$$\dot{\sigma} = \varphi(\varepsilon, \sigma) \dot{\varepsilon} + \psi(\varepsilon, \sigma)$$  \hspace{1cm} (3-4)

where $\varphi(\varepsilon, \sigma)$ represents the instantaneous elastic and/or plastic response and $\psi(\varepsilon, \sigma)$ expresses the rate of viscoplastic strain in the material. Fig. 3-2 shows stress-strain curve in an impact experiment for a given section in a bar. A possible solution for the above equation has been represented in Fig. 3-2 and is named the *dynamic curve*. The strain at each point of this curve (such as point A) is the sum of three different values of instantaneous elastic, plastic, and viscoplastic strains.

![Fig. 3-2 Different possible decomposition of the total strain at a given section of a bar in an impact experiment (Cristescu and Suliciu 1982)](image-url)
Furthermore, to better understand the difference between viscoplastic and plastic solutions, one can examine Fig. 3-3 (An 2010). This figure is in the space of first invariant of stress ($I_1$) and second invariant of deviatoric stress ($J_2$). As can be inferred, the plastic solution ($\sigma_{n+1}$) must lie on one of the yield surfaces. While the viscoplastic solution ($\bar{\sigma}_{n+1}$) may be outside of the yield surface due to rate effect. It can be said that the plasticity may be considered as a special case of viscoelasticity when the rate of strain is low enough to be neglected. It is worth mentioning that Fig. 3-3 belongs to a group of viscoplastic models which is called *overstress viscoplasticity model*.

As Perzyna (1966) pointed out, we shall make a distinction between an elastic-viscoplastic material and the elastic/viscoplastic one. In the first case, both elastic and plastic parts of material show viscous behavior while in the second one, the viscous behavior can be only seen in the plastic part. The determination of the yield criterion for an elastic-viscoplastic material is very difficult. In comparison to elastic-viscoplastic material, in elastic/viscoplastic material the initial yield condition can remain the same as in flow theory. The only difference is the change of the flow surface during the deformation process.

In general, to model the time-dependent stress-strain behavior of a viscous soil, under the condition of the elastic/viscoplastic case, the total rate of strain $\dot{\varepsilon}$ can be defined as:
\[ \dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^{ip} + \dot{\varepsilon}^{vp} \]  

(3-5)

where \( \dot{\varepsilon}^e, \dot{\varepsilon}^{ip} \) and \( \dot{\varepsilon}^{vp} \) are elastic, plastic and viscoplastic rate of strains, respectively. Again, it is worth mentioning here the viscoelastic behavior of material has been ignored in this equation. Cristescu and Suliciu (1982) and Yin et al. (2002) have pointed out that the separation of inelastic strain rate into instantaneous plastic rate and the viscoplastic strain rate has no sound physical basis. However, based on Perzyna’s theory (1966), the total strain rate should be divided into two parts as follows:

\[ \dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^{vp} \]  

(3-6)

where the irreversible part of strain are embedded in \( \dot{\varepsilon}^{vp} \). The elastic strain rate \( \dot{\varepsilon}^e \) can be easily calculated by:

\[ \dot{\varepsilon}^e = C \dot{\sigma} \]  

(3-7)

where the \( C \) is elastic compliance matrix and \( \dot{\sigma} \) is the stress rate vector. The fundamental task in this case is defining a suitable constitutive equation for the viscoplastic part of the above equations.

In the following, a brief review of three viscoplastic models and their algorithm aspects will be given. Perzyna and Duvaut-Lions models will be presented first. These two models are based on overstress (Fig. 3-3) viscoplastic methods in which the stress state is permitted to exist outside of the yield surface. Another group of models discussed here is based on the consistency approach, where the viscoplastic regularization is achieved by introducing a rate-dependent yield surface.

### 3.2.2 Perzyna viscoplastic model

Perzyna’s theory (1966) is the basis of most viscoplasticity formulations. In this theory, the viscous behavior of a material is considered as a time-rate flow rule in the body of the
formulation. The flow rule is assumed to be associative such that the viscoplastic potential is identical or at least proportional to the yield surface (Chen and Baladi 1985). As mentioned before, this model is based on the overstress method in which the plastic flow is constrained by the plastic relaxation equation. In the Perzyna model, the viscoplastic strain rate is defined similarly to the plastic standard rate in the standard plasticity:

\[
\dot{\varepsilon}^{vp} = \dot{\lambda} \frac{\partial f}{\partial \sigma}
\]  

(3-8)

in which \( f \) is yield function. However, Perzyna modified the plastic multiplier as:

\[
\dot{\lambda} = \gamma <\phi(f)>
\]  

(3-9)

in which \( \gamma \) is a viscosity constant of the material (or fluidity parameter) and \( \phi(f) \) is an arbitrary function of the yield function. This function is a dimensionless viscous function and \( <\phi(f)> \) is a ramp function which is defined as \((\phi(f)+|\phi(f)|)/2\). Two popular forms of the viscoplastic flow function are as follows (Zienkiewicz, Humpheson et al. 1975, Akai, Adachi et al. 1977):

\[
\phi(f) = \left(\frac{f}{f_0}\right)^N
\]  

(3-10)

\[
\phi(f) = \exp\left(\frac{f}{f_0}\right)-1
\]  

(3-11)

In which \( N \) is an exponent and \( f_0 \) is a normalizing constant with the same unit as \( f \). As can be seen here, two parameters (\( \gamma \) and \( N \)) are necessary to define the viscous behavior of the material under different rates of loading.
This theory has been used extensively to model time-dependent stress-strain behavior of soils and rocks (Yin, Zhu et al. 2002, Yin 2013). Katona (1984) mentioned the reasons of using Perzyna theory as: (1) the formulation is well-accepted and well-used; (2) the incorporation of the inviscid cap model is relatively straightforward; (3) the generality of the time-rate flow rule offers the capability of simulating time-dependent behavior over a wide range of loading; (4) techniques for parameter identification are feasible; (5) the formulation is readily acceptable to a numerical algorithm suitable for a finite element procedure.

### 3.2.3 Duvaut-Lions Viscoplastic model

Another alternative formulation of rate-dependent plasticity has been proposed by Duvaut and Lions (1972). In this model the strain rate is defined by the difference in response between the viscoplastic material and the rate-independent material (Simo, Ju et al. 1988, Loret and Prevost 1990). This is in contrast with the Perzyna model in which the value of the yield surface participates in the determination of the viscoplastic strain rate. This model can be combined with a yield surface which has an apex or is non-smooth (Wang 1995). In this model, the viscoplastic strain rate $\dot{\varepsilon}_{vp}$ and the hardening law $\dot{\kappa}$ are defined as:

\[
\dot{\varepsilon}_{vp} = \frac{1}{\eta} (D_e)^{-1} : [\sigma - \bar{\sigma}] 
\]  

(3-12)

and

\[
\dot{\kappa} = -\frac{1}{\eta} (\kappa - \bar{\kappa}) 
\]  

(3-13)

In which $\bar{\sigma}$ is the contribution of a rate-independent material (Fig. 3-3) which can be viewed as a projection of the current stress to the yield surface. $\eta$ is a viscous parameter which depends on strain and strain-rate.
3.2.4 Consistency viscoplastic model

Another group of viscoplastic models has been proposed recently by Ponthot (1995), Wang et al. (1997), and Winicki et al. (2001) in which rate independent and damage models extend to rate dependent material behavior. In the overstress viscoplastic models such as Perzyna and Duvant-Lions models, the current stress state can be outside of the yield surface and so the standard Kuhn-Tucker conditions are not applicable. However, in the consistancy viscoplastic model, the strain rate contribution is incorporated through a rate-dependent yield surface.

The consistency viscoplastic models can be divided into two groups: continuous viscoplasticity and consistent viscoplasticity. The prior model was proposed by Ponthot (1995). He (Ponthot 1995) used a yield function $F(\sigma, \kappa, \dot{\lambda})$ from which the consistancy condition can be derived when $\dot{F} = 0$. He defines the viscoplastic multiplier as:

$$\dot{\lambda} = \frac{\psi(F)}{\eta}$$  \hspace{1cm} (3-14)

in which $\psi(F)$ is the overstress and $\eta$ is viscosity. The viscoplastic strain rate, controlled by the viscoplastic consistency condition, is defined as:

$$\dot{\varepsilon}^{vp} = \dot{\lambda} m$$  \hspace{1cm} (3-15)

where $m$ is the gradient of the yield surface. Ponthot introduced the rate dependent yield condition as:

$$\ddot{F}(\sigma, \kappa, \dot{\lambda}) = F(\sigma, \kappa) - \psi^{-1}(\dot{\lambda} \eta) = 0$$ \hspace{1cm} (3-16)

The expansion of viscoplastic consistency condition is:
\[
\dot{F} = \pi : \dot{\sigma} + \dot{r} \dot{\lambda} + \ddot{s} \dddot{\lambda} = 0 \quad (3-17)
\]

where

\[
\pi = \frac{\partial F}{\partial \sigma} \quad (3-18)
\]

\[
\ddot{r} = \left( \frac{\partial F}{\partial \kappa} - \frac{\partial \psi^{-1}(\dot{\lambda} \eta)}{\partial \kappa} \right) h \quad (3-19)
\]

\[
\dddot{s} = -\frac{\partial \psi^{-1}(\dot{\lambda} \eta)}{\partial \dot{\lambda}} \quad (3-20)
\]

and

\[
h = \frac{\dot{\kappa}}{\lambda} \quad (3-21)
\]

In the second group of consistency viscoplastic models (i.e. consistent viscoplastic models), considering an argument similar to above, Wang (1995) introduced the viscoplastic yield surface as:

\[
\dot{F} = \dot{F}(\sigma, \kappa, \dot{\kappa}) = 0 \quad (3-22)
\]

in which the rate of state variable \( \dot{\kappa} \) is included in the formulation as an independent state variable. In this case, the viscoplasticity consistency condition can be stated as:

\[
\dot{F} = \ddot{\pi} : \dot{\sigma} + \ddot{r} \dot{\lambda} + \dddot{s} \lambda = 0 \quad (3-23)
\]

where
\[ \bar{\Pi} = \frac{\partial \hat{F}}{\partial \sigma} + \frac{\partial \hat{F}}{\partial \kappa} \frac{\partial h}{\partial \sigma} \]  

(3-24)

\[ \hat{\kappa} = \left( \frac{\partial \hat{F}}{\partial \kappa} + \frac{\partial \hat{F}}{\partial \kappa} \frac{\partial h}{\partial \kappa} \right) \hat{\lambda} \]  

(3-25)

\[ \hat{s} = -\frac{\partial \hat{F}}{\partial \kappa} h \]  

(3-26)

In both formulations, the consistency differential equations are same; however, \( n \), \( r \), and \( s \) are slightly different.

Heeres et al. (2002) compared the consistency model with the Perzyna model. They proved that the Perzyna model and the consistency model, to a certain extent, are identical during the progression of loading, while their responses are different during unloading. They also proposed a new implicit numerical scheme for the consistency models which yields a higher convergence rate compared to the one used for the Perzyna model (Heeres, Suiker et al. 2002), Zaera and Fernandez (2006).

### 3.2.5 Algorithm aspects of viscoplastic models

In this section, general algorithms which have been proposed by Wang (1995) or the overstress viscoplastic models (Perzyna and Duvaut-Lions) and the consistency viscoplastic model will be presented. By integrating from strain rate over a time step of \( \Delta t \) from \( t \) to \( t + \Delta t \), incremental strain and stress can be determined as:

\[ \Delta \varepsilon = \Delta \varepsilon^e + \Delta \varepsilon^{vp} \]  

(3-27)

\[ \Delta \sigma = D^e (\Delta \varepsilon - \Delta \varepsilon^{vp}) \]  

(3-28)

Based on Euler’s method, the viscoplastic incremental strain \( \Delta \varepsilon^{vp} \) and internal variable vector \( \Delta \kappa \) can be approximated as:
\[ \Delta \varepsilon^v = [(1-\theta)\dot{\varepsilon}^v_t + \theta \dot{\varepsilon}^v_{t+\Delta t}] \Delta t \]  

(3-29)

\[ \Delta \kappa = [(1-\theta)\dot{\kappa}_t + \theta \dot{\kappa}_{t+\Delta t}] \Delta t \]  

(3-30)

where \( \theta \) is the adjustable integration parameter \((0 \leq \theta \leq 1)\). The integration scheme is explicit when \( \theta = 0 \) and is fully implicit if \( \theta = 1 \).

(a) Perzyna viscoplastic model

The viscoplastic strain rate at the end of the time interval is expressed in a limited Taylor series expansion as (In the on-step Euler integration scheme):

\[ \dot{\varepsilon}^v_{t+\Delta t} = \dot{\varepsilon}^v_t + \left[ \frac{\partial \dot{\varepsilon}^v}{\partial \sigma} \right] \Delta \sigma + \left[ \frac{\partial \dot{\varepsilon}^v}{\partial \kappa} \right] \Delta \kappa = \dot{\varepsilon}^v_t + G_t \Delta \sigma + h_t \Delta \kappa \]  

(3-31)

where

\[ G_t = \gamma \left[ \frac{\partial \phi}{\partial \sigma} m^T + \phi \frac{\partial m}{\partial \sigma} \right], \quad h_t = \gamma \left[ \frac{\partial \phi}{\partial \kappa} m^T + \phi \frac{\partial m}{\partial \kappa} \right] \]  

(3-32)

By substituting above equation in Eq. 3-29:

\[ \Delta \varepsilon^v = (\dot{\varepsilon}^v_t + \theta G_t \Delta \sigma + \theta h_t \Delta \kappa) \Delta t \]  

(3-33)

The stress-strain relationship can be found by substituting Eq. 3-33 into Eq. 3-28:

\[ \Delta \sigma = D^v \Delta \varepsilon - \Delta q \]  

(3-34)

where
\[ D^{vp} = [(D^s)^{-1} + \theta \Delta t G_i]^{-1} \]  \hspace{1cm} (3-35)

is the tangent viscoplastic stiffness matrix and

\[ \Delta q = D^{vp}(\dot{\varepsilon}^{vp} + \theta h_i \Delta \kappa)\Delta t \]  \hspace{1cm} (3-36)

Based on above equations, the Euler stress-update algorithm for Perzyna viscoplasticity is presented in Table 3-1.

Table 3-1 The Euler stress-update algorithm for Perzyna viscoplasticity (in iteration \( I+1 \)) (Wang 1995)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \Delta \varepsilon = B \Delta a ]</td>
<td></td>
</tr>
<tr>
<td>[ \sigma^{vp} = \sigma_i + D^{vp} \Delta \varepsilon ]</td>
<td></td>
</tr>
<tr>
<td>If ( f(\sigma^{vp}, \kappa_i) \geq 0 ): we are in the plastic state</td>
<td></td>
</tr>
<tr>
<td>(1) [ G_i = \gamma \left[ \frac{\partial \phi}{\partial \sigma} m^T + \frac{\partial m}{\partial \sigma} \right] ]</td>
<td></td>
</tr>
<tr>
<td>(2) [ h_i = \gamma \left[ \frac{\partial \phi}{\partial \kappa} m^T + \frac{\partial m}{\partial \kappa} \right] ]</td>
<td></td>
</tr>
<tr>
<td>(3) [ D^{vp} = [(D^s)^{-1} + \theta \Delta t G_i]^{-1} ]</td>
<td></td>
</tr>
<tr>
<td>(4) [ \Delta q = D^{vp}(\dot{\varepsilon}^{vp} + \theta h_i \Delta \kappa)\Delta t ]</td>
<td></td>
</tr>
<tr>
<td>(5) [ \sigma_{i+\Delta t} = \sigma_i + D^{vp} \Delta \varepsilon - \Delta q ]</td>
<td></td>
</tr>
<tr>
<td>(6) [ f(\sigma_{i+\Delta t}, \kappa_i + \Delta \kappa^{(i)}) \geq 0 ]</td>
<td></td>
</tr>
<tr>
<td>(7) [ \dot{\varepsilon}^{vp}<em>{i+\Delta t} = \gamma \phi(\sigma</em>{i+\Delta t}, \kappa_i + \Delta \kappa^{(i)})m ]</td>
<td></td>
</tr>
</tbody>
</table>
(8) \( \mathbf{k}^{(i+1)}_{t+\Delta t} = \sqrt{\left(\mathbf{\dot{e}}^{vp}_{t+\Delta t}\right)^T A \mathbf{\dot{e}}^{vp}_{t+\Delta t}} \)

(9) \( \Delta \mathbf{k}^{(i+1)} = [(1 - \theta)\mathbf{k}_{t} + \theta \mathbf{k}^{(i+1)}_{t+\Delta t}]\Delta t \)

Else, we are in elastic state: \( \sigma_{t+\Delta t} = \sigma^\prime \)

\( B \) : Strain-displacement matrix; \( A = \text{diag}\left[\frac{2}{3} \ 2/3 \ 2/3 \ 1/3 \ 1/3 \ 1/3\right] \).

In the above scheme, the current viscoplastic state is calculated based on the gradient of the yield surface at time \( t \). However, in a fully implicit integration method, the gradient of the yield surface at time \( t + \Delta t \) is used to determine the viscoplastic state of material. This means:

\[
\Delta \mathbf{e}^{vp} = \Delta \lambda \ m_{t+\Delta t} \tag{3-37}
\]

This viscoplastic problem can be solved using the same algorithm for inviscid problems under the condition that the following residual, \( r \), is reduced to zero during a local iteration:

\[
r = \phi(\sigma_{t+\Delta t}, \lambda_{t+\Delta t}) - \frac{\Delta \lambda}{\gamma \Delta t} \rightarrow 0 \tag{3-38}
\]

Substituting eq. 3-37 into eq. 3-28 yields:

\[
\Delta \sigma = D^\prime[\Delta \mathbf{e} - \Delta \lambda \ m_{t+\Delta t}] \tag{3-39}
\]

The i-th iterative improvement of \( \Delta \sigma^{(i)} \), \( \Delta \mathbf{e}^{(i)} \) and \( \Delta \lambda^{(i)} \) are denoted by \( \delta \sigma \), \( \delta \mathbf{e} \) and \( \delta \lambda \) respectively. By differentiating Eq. 3-35, the variation of \( \Delta \sigma^{(i)} \) as a function of the variation of \( \Delta \mathbf{e}^{(i)} \) and \( \Delta \lambda^{(i)} \) during iteration (i) will be:
$$\delta \sigma = P \delta \varepsilon - P(m + \Delta \lambda \frac{\partial m}{\partial \lambda}) \delta \lambda$$ \hspace{1cm} (3-40)

In which $P$ is a pseudo-elastic material stiffness matrix:

$$P = [(D^e)^{-1} + \Delta \lambda \frac{\partial m}{\partial \sigma}]^{-1}$$ \hspace{1cm} (3-41)

All variables without specification refer to the $i$-th local iteration during the global iteration ($I$) at the current time step $t + \Delta t$. To determine $\Delta \lambda$, a Newton-Raphson iteration method can be used. By substitution of the differentiation of Eq. 3-38 into Eq. 3-40:

$$\delta \lambda = \frac{1}{\alpha} [(\frac{\partial \phi}{\partial \sigma})^\top P \delta \varepsilon - \delta \sigma]$$ \hspace{1cm} (3-42)

where

$$\alpha = (\frac{\partial \phi}{\partial \sigma})^\top P(m + \Delta \lambda \frac{\partial m}{\partial \lambda}) + \frac{1}{\gamma \Delta t} \frac{\partial \phi}{\partial \lambda}$$ \hspace{1cm} (3-43)

The tangent stiffness matrix can be achieved by substituting Eq. 3-42 into Eq. 3-40:

$$D^T = P - \frac{1}{\alpha} P(m + \Delta \lambda \frac{\partial m}{\partial \lambda})(\frac{\partial \phi}{\partial \sigma})^\top P$$ \hspace{1cm} (3-44)

The iterative implicit stress update for the Perzyna model is presented in Table 3-2.

<table>
<thead>
<tr>
<th>Table 3-2 Iterative implicit stress update for Perzyna model (Wang 1995)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \varepsilon = B \Delta a$</td>
</tr>
</tbody>
</table>

31
\[ \sigma^{tr} = \sigma_t + D^{\varepsilon} \Delta \varepsilon \]

If \( f(\sigma^{tr}, \lambda_t) \geq 0 \): we are in the plastic state

(1) Local iteration

\[ \Delta \lambda^{(0)} = 0, \quad \Delta \sigma_t^{(0)} = \sigma_t + D^{\varepsilon} (\Delta \varepsilon - \Delta \lambda^{(0)} m^{tr}) \]

\[ r^{(0)} = \phi(\sigma_t^{(0)}, \lambda_t + \Delta \lambda^{(0)}) - \Delta \lambda^{(0)} (\gamma \Delta t)^{-1} \]

(2) \[ P^{(i)} = [(D^{\varepsilon})^{-1} + \Delta \lambda \frac{\delta m}{\delta \sigma}]^{-1} \]

(3) \[ \alpha = (\frac{\delta \phi}{\delta \sigma})^T (i) P^{(i)} (m + \Delta \lambda \frac{\delta m}{\delta \lambda})^T (i) + \frac{1}{\gamma \Delta t} - (\frac{\delta \phi}{\delta \lambda})^T (i) \]

(4) \[ \Delta \lambda^{(i+1)} = \Delta \lambda^{(i)} + r^{(i)} \alpha^{-1} \]

(5) \[ \Delta \sigma_t^{(i+1)} = \sigma_t + D^{\varepsilon} (\Delta \varepsilon - \Delta \lambda^{(i+1)} m^{(i)}) \]

(6) \[ r^{(i+1)} = \phi(\sigma_t^{(i+1)}, \lambda_t + \Delta \lambda^{(i+1)}) - \Delta \lambda^{(i+1)} (\gamma \Delta t)^{-1} \]

(7) \[ |r^{(i+1)}| > \delta \quad i \leftarrow i + 1 \quad \text{go to (2)} \]

Else, we are in elastic state: \( \sigma_t = \sigma^{tr} \)

(b) Duvaut- Lions viscoplastic model

The stress update is carried out in two steps for this model; first the inviscid back-bone stress \( \bar{\sigma} \) is updated; then the viscoplastic response is determined. In the first step, an Euler backward algorithm is used for the return mapping of the stress onto the yield surface. The tangential stiffness relation yields:
\[ \Delta \bar{\sigma} = \overline{D}^{vp} \Delta \varepsilon \]  

with the elasto- plastic stiffness tensor of:

\[ \overline{D}^{vp} = D^e - \frac{D^e \bar{m} \bar{m}^T D^e}{h + \bar{m}^T D^e \bar{m}} , \quad \bar{n} = \frac{\partial f}{\partial \bar{\sigma}} \]  

(3-46)

Now, based on Eq. 3-12, the current viscoplastic strain rate can be written as:

\[ \dot{\varepsilon}_{vp}^{i+\Delta} = \frac{1}{\eta} (D^e)^{-1} \left[ \varepsilon_{vp}^{i+\Delta} - \bar{\sigma}_{vp}^{i+\Delta} \right] = \frac{1}{\eta} (D^e)^{-1} \left[ \Delta \sigma - \Delta \bar{\sigma} + \sigma_{vp}^{i} \right] \]  

(3-47)

in which \( \sigma_{vp}^{i} = \sigma_{i} - \bar{\sigma} \) is the viscous stress at the beginning of the time step. The viscoplastic strain can be achieved using Eq. 3-29:

\[ \Delta \varepsilon_{vp}^{i} = [(1 - \theta)\dot{\varepsilon}_{vp}^{i} + \frac{\theta}{\eta} (D^e)^{-1} (\Delta \sigma - \Delta \bar{\sigma} + \sigma_{vp}^{i})] \Delta t \]  

(3-48)

Using Eq.3-28 and Eq. 3.48:

\[ \Delta \sigma = D^{vp} \Delta \varepsilon - \Delta q \]  

(3-49)

where

\[ D^{vp} = \frac{\eta}{\eta + \theta \Delta t} \left[ D^e + \frac{\theta \Delta t}{\eta} \overline{D}^{vp} \right] \]  

(3-50)
\[ \Delta q = \frac{\eta \Delta t}{\eta + \theta \Delta t} [(1 - \theta) D^e \dot{\varepsilon}_t^{vp} + \frac{\theta}{\eta} \sigma_t^{vp}] \]  

(3-51)

Based on above equations, and considering that the current viscosity parameter \( \eta \) can be a function of the viscoplastic strain rate, Wang (1995) proposed the scheme presented in Table 3-3 for a one-step implicit stress-update algorithm for the Duvaut- Lions model.

Table 3-3 A one-step implicit stress-update algorithm for the Duvaut- Lions model (Wang 1995)

\[ \Delta \varepsilon = B \Delta a, \ \sigma^{vp} = \sigma_t + D^e \Delta \varepsilon \]

If \( f(\sigma^{vp}, \kappa_i) \geq 0 \): we are in the plastic state

1. \( \bar{\sigma}_{t+\Delta t} = \bar{\sigma}_t + \bar{D}^{vp} \Delta \varepsilon \)

2. \( \bar{\kappa}_{t+\Delta t} = \bar{\kappa}_t + \bar{\kappa}_{t+\Delta t} \Delta t \)

3. Local iteration: \( \eta^{(i)}_{t+\Delta t} = \eta_t \)

4. \( D^{vp} = \frac{\eta^{(i)}}{\eta^{(i)} + \theta \Delta t} [D^e + \frac{\theta \Delta t}{\eta^{(i)}} \bar{D}^{vp}] \)

5. \( \Delta q = \frac{\eta^{(i)}}{\eta^{(i)} + \theta \Delta t} [(i - \theta) D^e \dot{\varepsilon}_t^{vp} + \frac{\theta}{\eta^{(i)}} \sigma_t^{vp}] \)

6. \( \sigma_{t+\Delta t} = \sigma_t + D^{vp} \Delta \varepsilon - \Delta q \)

7. \( \dot{\varepsilon}_{t+\Delta t}^{vp} = \frac{1}{\eta^{(i)}} (D^e)^{-1} [\sigma_{t+\Delta t} - \bar{\sigma}_{t+\Delta t}] \)

8. \( \kappa_{t+\Delta t}^{vp} = \sqrt{(\dot{\varepsilon}_{t+\Delta t}^{vp})^T A \dot{\varepsilon}_{t+\Delta t}^{vp}} \)

9. \( \kappa_{t+\Delta t} = \bar{\kappa}_{t+\Delta t} - \eta^{(i)} \bar{\kappa}_{t+\Delta t} \)
(10) \[ \tau^{(i)} = \dot{\sigma}(\kappa_{t+\Delta t}) - 3G\eta^{(i)} \dot{\kappa}_{t+\Delta t} \]

(11) \[ \eta^{(i+1)} = \eta^{(i)} + \frac{s \dot{\sigma} \tau^{(i)}}{3G s \dot{\sigma} \dot{\kappa}_{t+\Delta t} - \alpha(s - 3G \eta^{(i)})} \]

(12) if \( r > \delta \) \( i \leftarrow i + 1 \) go to (4)

Else, we are in elastic state: \( \sigma_{t+\Delta t} = \sigma^r \)

\( \dot{\sigma} \): An equivalent viscous stress

\[ s = \frac{\partial \dot{\sigma}}{\partial \kappa} \quad (3-52) \]

\[ \alpha = \frac{3\theta \Delta t}{2(\eta + \theta \Delta t)^2} [(D^r - D^{\text{vis}}) \Delta \varepsilon - (1 - \theta)D^r \dot{\varepsilon}_t^{\text{vis}} \Delta t + D^{\text{vis}}] A \tau^{\text{vis}} \quad (3-53) \]

In which \( \tau^{\text{vis}} \) is deviatoric viscous stress.

(c) Consistency viscoplastic model

In this model the plastic flow is defined as:

\[ \dot{\varepsilon}^{\text{vis}} = \dot{\lambda} m \quad (3-54) \]

By discretising consistency condition of the rate-dependent yield surface, the plastic flow is achieved as:

\[ f(\sigma^{(i+1)}, \lambda^{(i+1)}, \dot{\lambda}^{(i+1)}) \approx f + n^{\top} \delta \sigma + r \delta \lambda + s \delta \dot{\lambda} = 0 \quad (3-55) \]

Wang (1995) assumed that the rate of the history parameter can be related to viscoplastic multiplier via \( \kappa = \alpha \dot{\lambda} \). By this assumption, he used a local Newton-Raphson iteration process to define \( \Delta \lambda \). Using the differentiation of Eq.3-28 to Eq. 3-30:
\[ \delta \sigma = D^e \delta \varepsilon - D^e [\dot{\lambda} (\frac{\partial m}{\partial \sigma}) \delta \sigma + \frac{\partial m}{\partial \lambda} \delta \lambda + \frac{\partial m}{\partial \dot{\lambda}} \delta \dot{\lambda}] + m \delta \dot{\lambda}] \theta \Delta t \] (3-56)

\[ \delta \lambda = \theta \Delta t \delta \dot{\lambda} \] (3-57)

Using two later equations:

\[ \delta \sigma = P \delta \varepsilon - P \overline{n} \delta \lambda \] (3-58)

\[ P = [(D^e)^{-1} + \theta \dot{\lambda} \Delta t \frac{\partial m}{\partial \sigma}]^{-1} \] (3-59)

\[ \overline{n} = m + \theta \dot{\lambda} \Delta t \frac{\partial m}{\partial \lambda} + \dot{\lambda} \frac{\partial m}{\partial \dot{\lambda}} \] (3-60)

By substituting Eq. 3-58 into Eq. 3-55, \( \delta \lambda \) is expressed as:

\[ \delta \dot{\lambda} = \frac{n^T P \delta \varepsilon + f}{n^T P \overline{n} + r + s(\theta \Delta t)^{-1}} \] (3-61)

Also, the tangential viscoplastic stiffness matrix can be determined by substituting Eq. 3-61 into Eq. 3-58 as:

\[ D^{vp} = P - \frac{P \overline{n} n^T P}{n^T P \overline{n} + r + s(\theta \Delta t)^{-1}} \] (3-62)

The implicit- stress update algorithm for consistency viscoplastic model is presented in Table 3-4.
Table 3-4 The implicit stress update algorithm for consistency viscoplastic model (Wang 1995)

\[ \Delta \varepsilon = B \Delta \sigma \]

\[ \sigma^{pr} = \sigma_t + D^{pr} \Delta \varepsilon \]

If \( f(\sigma^{pr}, \dot{\lambda}, \dot{\lambda}_p) \geq 0 \) : we are in the plastic state

1. Local iteration

\[ \Delta \lambda_t^{(0)} = 0, \quad \dot{\lambda}_t^{(0)} = 0 \]

\[ \sigma_t^{(0)} = \sigma^{pr}, \quad f_t^{(0)} = f^{pr} \]

2. \( P^{(i)} = [(D^{pr})^{-1} + \theta \dot{\lambda}_t^{(i)} \Delta t \frac{\partial m^{(i)}}{\partial \sigma}]^{-1} \)

3. \( \bar{n} = (m + \theta \dot{\lambda}_t \Delta t \frac{\partial m^{(i)}}{\partial \lambda} + \dot{\lambda} \frac{\partial m^{(i)}}{\partial \lambda})^{(i)} \)

4. \( \delta \lambda = \Delta \lambda_t^{(i)} + \frac{f^{(i)}}{n^T P \bar{n} + r + s(\theta \Delta t)^{-1}} \)

5. \( \dot{\lambda}_t^{(i+1)} = \frac{\Delta \lambda_t^{(i+1)}}{\theta \Delta t} - \frac{1 - \theta}{\theta} \dot{\lambda}_t \)

6. \( \Delta \varepsilon^{vp} = [(1 - \theta) \dot{\varepsilon}_t^{vp} + \theta \dot{\lambda}_t \Delta \lambda_t^{(i+1)} m^{(i)}_{t+\Delta t}] \Delta t \)

7. \( \sigma_t^{(i+1)} = \sigma_t^{r} + D^{r}[\Delta \sigma - \Delta \varepsilon^{vp}] \)

8. \( f^{(i+1)} = f(\sigma_t^{(i+1)}, \lambda_t, \Delta \lambda_t^{(i+1)}, \dot{\lambda}_t^{(i+1)}, \dot{\lambda}_t) \)

9. If \( |f^{(i+1)}| > \delta \): \( i \leftarrow i + 1 \) go to (2)

Else, we are in elastic state: \( \sigma_{t+\Delta t} = \sigma^{pr} \)
3.3  **Viscoplastic cap model**

The cap model is a group of inviscid plasticity models which is frequently used for simulation of geological material behavior. These models were introduced by Drucker et al. (1957) to improve the model proposed by Drucker and Prager (1952). To overcome the deficiencies of Drucker-Prager model (e.g. predicting dilatancy is higher than that in experiments), Drucker et al. proposed a second function which can harden and soften (Resende and Martin 1985).

3.3.1  **Theory of viscoplastic cap models**

In the principal stress space, the volumetric yield surface can be visualized as a cap put on the top of the shear yield surface cone (Fig.3-4) (Puzrin 2012). The shape of the cap in the principle stress is usually considered elliptical (Resende and Martin 1985), Sandler et al. (1974), Sandler and Rubin (1979) and Rubin (1991) while some other researchers such as (Bathe, Snyder et al. 1980) considered it as a plane. The cap model, proposed by Rubin (1991), is illustrated in Fig. 3-5.

![Fig. 3-4 A schematic of the cap model](image)

This group of models have also been used to simulate the behavior of soil under explosive loading conditions (An, Tuan et al. 2011). The original two invariant associative cap models proposed by DiMaggio and Sandler (1971), Sandler et al. (1974) and Sandler and Rubin (1979) are independent of time while most of geological materials show significant time-dependent behavior. Thus, some viscoplastic formulations have been suggested so far in the literature to
introduce time dependency in the context of cap model formulation (Katona 1984, Tong and Tuan 2007).

Generally, the cap model consists of three parts:

(i) A shear failure surface showing increasing shear stress with increasing mean stress:

\[ f(I_1, \sqrt{J_2}) = \sqrt{J_2} - F_v(-I_1) = \sqrt{J_2} - [A - C \exp(B \sqrt{J_2})] = 0 \]  

\[ F_v = q - p \tan \beta - d = 0 \]  

\[ f(I_1, \sqrt{J_2}) = \sqrt{J_2} - F_v(-I_1) = \sqrt{J_2} - [\alpha - \gamma \exp(-\beta \sqrt{J_2}) + \theta I_1] = 0 \]  

in which \( I_1 \) = first stress invariant; \( J_2 \) = the second deviatoric stress invariant; \( q \) = deviatoric stress and \( p \) = hydrostatic stress; \( A, B, \alpha, \gamma, \beta \) and \( \theta \) are material constants.

(ii) A curved "cap" intersecting both the shear failure surface and the mean stress axis; this surface is defined as follow in Katona’s model:

![Diagram of cap model](image)
\[ f(I_1, \sqrt{J_2}, \kappa) = \sqrt{J_2} - \frac{1}{R} \sqrt{[X(\kappa) - L(k)]^2 - [I_1 - L(\kappa)]^2} = 0 \]


in which \( R \) = material parameter which is the ratio of the horizontal axis of the elliptical cap to the vertical axis of the elliptical cap; and \( \kappa \) = hardening parameter related to the actual viscoplastic volumetric change.

In Eq. (3-66), \( X \) and \( L \) can be expressed as:

\[ X = \frac{1}{D} \ln(1 + \frac{\varepsilon^p}{W}) - X_0 \]  

(3-67)

\[ L = \begin{cases} \kappa & \text{if } \kappa > 0 \\ 0 & \text{if } \kappa \leq 0 \end{cases} \]  

(3-68)

where \( D, W, X_0 = X(\varepsilon^p = 0) \) are the material parameters for the cap yield surface, and the hardening parameter (for Tong and Tuan (2007)) can be expressed as:

\[ \kappa = X - R[\alpha - \gamma \exp(\beta I_1) + \theta I_1] \]  

(3-69)

The cap surface proposed by Resende and Martin (1985) has two parts; A compression surface which can be defined mathematically as:

\[ f_c = \sqrt{(p - p_a)^2 + \left(\frac{Rq}{1 + \alpha - \alpha/cos\beta}\right)^2} - R(d + p_a \tan\beta) = 0 \]  

(3-70)

where \( \alpha \) is a numerical parameter defining a smooth transition yield intersection between the cap and failure surfaces, and \( p_a \) is an evolution parameter for volumetric plastic, strain-driven hardening/softening of the cap.
The second part of this surface is a **transition surface** allowing a smooth intersection between the cap and failure surfaces (see Fig. 3-7) which can be expressed as:

\[
f_c = \sqrt{(p - p_a)^2 + [q - (1 - \frac{\alpha}{\cos \beta})(d + p_a \tan \beta)]^2} - \alpha(d + p_a \tan \beta) = 0
\]  

(3-71)

(iii) A cutoff region which is tension cutoff criterion (see Fig. 3-6):

\[
f = I_1 - (-T) = 0
\]  

(3-72)

in which \(T\) is a material constant representing the threshold of volumetric tension stress at which abrupt stress releases occur due to tension damage.

Fig. 3-6 Static yield surface of cap model (Tong and Tuan 2007)
Different algorithms have been employed to simulate cap models numerically (Hughes and Taylor (1978), Katona (1984) and Simo et al (1988)). Katona (1984) introduced a numerical simulation algorithm that relies heavily on the implicit procedure developed by Hughes (1978). This algorithm involves the solution of a nonlinear system of equations for each iteration within a given time step. An algorithm (referred to as return mapping algorithm) proposed by Simo et al (1988) results in the solution of a single nonlinear scalar equation. General numerical solutions for different viscoplastic models have been discussed in previous section.

3.3.2 Parameter identification of the Viscoplastic cap model

Parameters identification and model calibration of the cap models is a nonunique process that is especially difficult for viscoplastic cap models, since the loading schedules of most laboratory tests are not designed in a way that one can identify viscous and instantaneous responses separately. Some researchers use a trial and error procedure which is not preferable to determine the parameters. In general four main groups of parameters are needed for above cap models:

- Elastic parameters
- Shear failure surface parameters
- Cap model parameters
- Required parameters for rate dependent behavior of material

3.3.2.1 Elastic parameters

To define the behavior of the material in the elastic region, Young’s modulus and Poisson’s ratio are needed. These parameters can be determined using the laboratory tests such as uniaxial and triaxial tests and/or the resonant frequency test. The assumption made here is that the elastic properties are not rate dependent.

3.3.2.2 Shear failure surface parameters

There are different models proposed by researchers for the shear failure surface and each model consists of different parameters. For example, in the failure surface proposed by Resende and Martin (1985), only two parameters should be identified: material cohesion and internal friction angle. Katona (1984) and Tong and Tuan (2007)’s failure surfaces need three and four material parameters respectively, which can be determined by curve fitting to triaxial test results. These parameters are rate independent as well.

3.3.2.3 Cap model parameters

The plastic parameters \( D, W \) and \( X_0 \) can be defined via the loading-unloading Isotropic Compressive (IC) tests, which is shown as Fig. 3-8. The symbol \( e_0 \) is the void ratio of the material, which is used to calculate the value of \( e_0^p \). After defined \( D, W \) and \( X_0 \), the parameter \( R \) can be determined via IC and Common Triaxial Compressive (CTC) tests.
The stress path of the test is shown in Fig. 3-9. Firstly, the soil is loaded from point O to point D along the IC stress path. The plastic volumetric strain occurred at point D is defined as

$$\varepsilon_{vD}^p = W\{1 - \exp[-D(I_1 - X_0)]\}$$  \hspace{1cm} (3-73)

Then the soil is loaded from point D to point E (when the failure occurs) along the CTC stress path, and the cap yield surface intersects the axis $I_1$ at point F. The volumetric strain of the soil at point E can be calculated as $\varepsilon_{vE}^p = \varepsilon_{vD}^p + \Delta\varepsilon_{v}^p$, where $\Delta\varepsilon_{v}^p$ can be directly measured in the CTC test. The horizontal axis of F can be expressed as:

$$I_{1F} = \frac{-\ln(\varepsilon_{vE}^p/W)}{D} + X_0$$  \hspace{1cm} (3-74)
Thus, the parameter $R$ can be determined as

$$R = \frac{(I_{1E} - I_{1})}{\sqrt{J_{2E}}}$$

(3-75)

where $I_{1E}$ and $\sqrt{J_{2E}}$ both can be directly measured in the CTC test.

3.3.2.4 Required parameters for rate dependent behavior of material

As mentioned above, there are different viscoplastic models and each of them has its parameters. For instance, in the Perzyna model two parameters should be determined which will be discussed here. The Perzyna viscoplastic multiplier is expressed as $\dot{\lambda} = \gamma <\phi(f)>$ in which

$$\phi(f) = (\frac{f}{f_o})^N \text{ or } \phi(f) = \exp(\frac{f}{f_o}) - 1$$

(3-76)

As can be seen here, two parameters ($\gamma$ and $N$) are necessary to define the viscous behavior of the material under different rate of loading. These parameters can be achieved by curve fitting or numerical methods (Katona (1985)).
4 Equation of state (EOS)

In numerical simulation of hypervelocity impacts or dynamic response of materials at extremely high rates, the total stress tensor is usually decomposed into a deviatoric stress tensor and a hydrostatic pressure: \( \sigma_{ij} = \sigma'_{ij} - p \delta_{ij} \), where \( \sigma_{ij} \) is the total stress, \( \sigma'_{ij} \) the deviatoric stress, \( p \) the hydrostatic pressure, and \( \delta_{ij} \) the Kronecker delta.

The deviatoric stress is governed by the constitutive model while the pressure is governed by the equation of state (EOS). Because of this decomposition, codes simulating hypervelocity impacts are called hydro-codes and LS-Dyna is such a commercial code.

4.1 Historical Development of EOS

The EOS stems from thermodynamics. The most common EOS is that for ideal gas with constant entropy:

\[
p\left(\frac{V}{V_0}\right)^\gamma = c
\]

(4-1)

where \( V_0 \) is the initial specific volume, \( V \) is the current specific volume, \( \gamma \) is the isentropic index, and \( c \) is a constant. Using this equation, one can find the pressure given the specific volume or density of the gas.

4.1.1 Bridgeman Equation

For solids, the first EOS was developed based on hydrostatic compression, which was done under the constant temperature (Bridgeman (1949)). Based on tests of tens of solids up to pressure of 10 GPa, the Nobel Prize laureate Bridgman proposed the isothermal EOS for solids:

\[
\frac{V_0 - V}{V_0} = ap - bp^2
\]

(4-2)

Using this equation, the Lagrange bulk modulus can then be calculated as:
$K(p) = \frac{dp}{d\Delta} = -V_0 \frac{dp}{dV} = \frac{1}{a - 2bp}$ (4-3)

where $\Delta$ is the volumetric strain, $a$ and $b$ are material constants. This equation correctly predicts that the bulk modulus increases with the pressure.

Further assuming that $2bp / a << 1$, Eq. 4-3 can be simplified to:

$$K(p) = \frac{1}{a} (1 + \frac{2b}{a} p)$$ (4-4)

### 4.1.2 Murnaghan Equation

From the definition of Euler bulk modulus and assume that there is a similar relation as that shown in Eq. 4-4, i.e.

$$k(p) = -V \frac{dp}{dV} = k_0 (1 + \eta p)$$ (4-5)

Invoking the initial condition: $V(p = 0) = V_o$, Eq. 4-5 can be integrated to yield:

$$p = \frac{k_0}{n} \left\{ \left( \frac{V_0}{V} \right)^n - 1 \right\}$$ (4-6)

where $n = k_0 \eta$.

Eq. 4-6 is the isentropic EOS or Murnaghan EOS describing the state of solids during a process where entropy is constant. It is noted that the Murnaghan EOS has a similar form to the isentropic EOS for an ideal gas (Eq. 4-1) and $n$ bears the similar physical meaning of the isentropic index $\gamma$. 

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4.1.3 Mie-Grüneisen Equation

The isothermal EOS (Bridgeman Equation) and isentropic EOS (Murnagham Equation) are for special cases of thermodynamic processes, a general form of EOS is thus desired for general cases. In general, the pressure is a function of internal energy $E$ and specific volume $V$,

$$ p = p(E,V) \quad (4-7) $$

Introducing the Grüneisen parameter as:

$$ \Gamma = V \left( \frac{\partial p}{\partial E} \right)_V \quad (4-8) $$

Noted that based on thermodynamics law,

$$ dE = TdS - pdV \quad (4-9) $$

For isochoric processes, the volume is kept constant, and thus

$$ \left( \frac{\partial E}{\partial T} \right)_V = T \left( \frac{\partial S}{\partial T} \right)_V C_V \quad (4-10) $$

where $C_V$ is the isochoric specific heat. Furthermore, the isochoric process satisfies:

$$ dV = \left( \frac{\partial V}{\partial p} \right)_T dp + \left( \frac{\partial V}{\partial T} \right)_p dT = 0 \quad (4-11) $$

Then,
\[
\left( \frac{\partial p}{\partial T} \right)_V = -\left( \frac{\partial V}{\partial T} \right)_p \left/ \left( \frac{\partial V}{\partial p} \right)_T \right. = -V \left( \frac{\partial p}{\partial V} \right)_T \cdot \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p = k_T \alpha
\] (4-12)

where \( \alpha \) is the isobaric expansion coefficient and \( k_T \) is the isothermal bulk modulus.

Hence from Eq. 4-8 and 4-11,

\[
\Gamma = V \left( \frac{\partial p}{\partial E} \right)_V = V \left( \frac{\partial p}{\partial T} \right)_V \left/ \left( \frac{\partial E}{\partial T} \right)_V \right. = \frac{k_T \alpha V}{C_V}
\] (4-13)

Further assuming that the Grüneisen parameter depends only on \( V \), i.e., \( \Gamma = \gamma(V) \).

Integration of Eq. 4-8 yields,

\[
p = p_K(V) + \frac{\gamma}{V} \left[ E - E_K(V) \right]
\] (4-14)

where index \( K \) denotes the reference state at \( K_0 \) and this is the famous Mie-Grüneisen Equation.

### 4.2 Shock Wave Hugoniots and Mie-Grüneisen EOS Parameters from Shock Hugoniots

Shock wave tests are usually done by the Explosive lens or planer impact and the planer impact is the most common methods. As mentioned in Chapter one, the governing equations for the shock wave are:

\[
\rho_0 U_s = \rho(U_s - u_p)
\] (4-15a)

\[
p_H = \rho_0 U_s u_p
\] (4-15b)
\[ E_H - E_0 = \frac{p_H}{2} (V_0 - V_H) \]  

(4-15c)

where \( U_S \) is the shock wave velocity, \( u_P \) is the particle velocity, \( H \) denotes the shocked state, and \( 0 \) indicates the initial state.

Fig. 4-1 Shock Wave Structure

There are totally 5 physical variables involved in Eq. 4-15. One relation (i.e., shock Hugoniot) between any 2 out of these 5 physical variables can be established from shock wave tests. One more equation is needed to solve the system and that equation is the EOS. The common shock Hugoniot is given between \( U_S \) and \( u_P \):

\[ U_S = C_0 + s u_P \]  

(4-16)

From Equations 4-15b and 4-16, we have:

\[ p_H = \rho_0 (C_0 + s u_P) u_P \]  

(4-17)

Denote \( \mu = \frac{\rho}{\rho_0} - 1 = \frac{V_0}{V} - 1 \) and use Equation 15a, one can have another shock Hugoniot between specific volume and shock pressure:
\[ p_H = \rho_0 C_0^2 \frac{\mu}{1 + \mu} \left(1 - s \frac{\mu}{1 + \mu}\right)^2 \]  

(4-18)

Taking advantage of the knowledge of the state achieved after the shock, we use the shocked state as the reference state in the Mie-Grüneisen equation:

\[ p - p_H = \frac{\gamma}{V} (E - E_H) \]  

(4-19)

Substitution of Equation 4-15c into Equation 4-19 yields,

\[ p = p_H + \frac{\gamma}{V} (E - E_0) - \frac{\gamma}{V} \frac{p_H}{2} (V_0 - V) \]  

(4-20)

This is the Mie-Grüneisen equation used in the most hydro-codes. It is obvious that to use this equation, one would need the shock Hugoniot that determines the shock pressure (e.g., Eq. 4-18).
4.3 EOS Models in LS-DYNA

In addition to Mie-Grüneisen equation, there are different types of EOS provided in LS-DYNA (Hallquist 2006), here are some of the models:

4.3.1 Linear Polynomial

This polynomial EOS, linear in the internal energy per initial volume, $E$, is given by

$$p = C_0 + C_1 \mu + C_2 \mu^2 + C_3 \mu^3 + (C_4 + C_5 \mu + C_6 \mu^2)E$$

(4-21)

where $C_0$, $C_1$, $C_2$, $C_3$, $C_4$, $C_5$, and $C_6$ are constants, $\mu = (1/V) - 1$, $V = V_0 / V$ is the relative volume.

In expanded elements, the coefficients of $\mu^2$ are set to zeroes (i.e. $C_2 = C_6 = 0$). Internal energy, $E$, is increased according to an energy deposition rate versus time curve.

The linear polynomial EOS may be used to model gas with the gamma law equation of state. This may be achieved by setting: $C_0 = C_1 = C_2 = C_3 = C_6 = 0$ and $C_4 = C_5 = \gamma - 1$, where $\gamma$ is the ratio of specific heats. Then, the pressure is given by $p = (\gamma - 1) \frac{\rho}{\rho_0} E$.

Note that the units of $E$ are the units of pressure.

4.3.2 JWL High Explosive

The JWL EOS defines pressure as a function of relative volume, $V$, and internal energy pre initial volume, $E$, as

$$p = A \left(1 - \frac{\omega}{R_1 V}\right) e^{-R_1 V} + B \left(1 - \frac{\omega}{R_2 V}\right) e^{-R_2 V} + \frac{\omega E}{V}$$

(4-22)
where $\omega$, $A$, $B$, $R_1$, and $R_2$ are input parameters. This EOS is normally used for determining the pressure of the detonation products of high explosives in applications involving metal acceleration. Input parameters for this equation are given by Dobratz (1981) for variety of high explosive materials.

### 4.3.3 Sack “Tuesday” High Explosive

Pressure of detonation products is given in terms of the relative volume, $V$, and internal energy per initial volume, $E$, as (Woodruff 1973):

$$p = \frac{A_3}{V^{A_4}} e^{-A_5} \left(1 - \frac{B_1}{V}\right) + \frac{B_2}{V} E$$  \hspace{1cm} (4-23)

where $A_1$, $A_2$, $A_3$, $B_1$, and $B_2$ are input parameters.

### 4.3.4 Grüneisen

The Grüneisen EOS with cubic shock velocity-particle velocity defines pressure for compressed material as

$$p = \frac{\rho_0 C^2 \mu \left[1 + \left(\frac{1 - \gamma_0}{2}\right)\mu - \frac{a}{2} \mu^2\right]}{1 - (S_1 - 1)\mu - S_2 \frac{\mu^2}{\mu + 1} - S_3 \frac{\mu^3}{(\mu + 1)^2}} + (\gamma_0 + \alpha\mu)E$$  \hspace{1cm} (4-24)

and for expanded materials as: $p = \rho_0 C^2 \mu + (\gamma_0 + \alpha\mu)E$, where $E$ is the internal energy per initial volume, $C$ is the intercept of the $U_s-u_p$ curve, $S_1$, $S_2$, and $S_3$ are the coefficients of the $U_s-u_p$ curve, $\gamma_0$ is the Gruneisen gamma, and $a$ is the first order volume correction to $\gamma_0$. Constants $C$, $S_1$, $S_2$, $S_3$, $\gamma_0$, and $a$ are all input parameters. The compression is defined in terms of the relative volume, $V$, as: $\mu = \frac{V_0}{V} - 1$. 
It is noted that the form is different from that shown in Eq. 4-20.

### 4.3.5 Ratio of Polynomials

The ratio of polynomials EOS defines the pressure as

\[
p = \frac{F_i + F_2E + F_3E^2 + F_4E^3}{F_5 + F_6E + F_7E^3} (1 + \alpha \mu) \tag{4-25}
\]

where \( F_i = \sum_{j=0}^{n} A_j m^i \)

\[
\begin{align*}
  n & = 4 & \text{if } i < 3, \\
  n & = 3 & \text{if } i \geq 3, \text{ and } \mu = \frac{\rho}{\rho_0 - 1}.
\end{align*}
\]

In expanded zoned, \( F_1 \) is replaced by \( F'_1 = F_1 + \beta \mu^2 \). Constants \( A_j \), \( \alpha \), and \( \beta \) are input parameters.

### 4.3.6 Ignition and Growth Model

A JWL EOS defines the pressure in the unreacted high explosive as

\[
p_e = A_e \left( 1 - \frac{\omega_e}{R_{ee} V_e} \right) e^{-R_{ee} V_e} + B_e \left( 1 - \frac{\omega_e}{R_{ee} V_e} \right) e^{-R_{ee} V_e} + \frac{\omega_e E}{V_e} \tag{4-26}
\]

where \( V_e \) is the relative volume, \( E_e \) is the internal energy, and the constants \( A_e \), \( B_e \), \( \omega_e \), \( R_{ee} \), and \( R_{ee} \) are input constants. Similarly, the pressure in the reaction products is defined by another JWL form

\[
p_p = A_p \left( 1 - \frac{\omega_p}{R_{pp} V_p} \right) e^{-R_{pp} V_p} + B_p \left( 1 - \frac{\omega_p}{R_{pp} V_p} \right) e^{-R_{pp} V_p} + \frac{\omega_p E}{V_p} \tag{4-27}
\]

The mixture of unreacted explosive and reaction products is defined by the fraction reacted \( F \) (\( F = 0 \) implies no reaction, \( F = 1 \) implies complete conversion from explosive to products). The
pressures and temperature are assumed to be in equilibrium, and the relative volumes are
assumed to be additive:

\[ V = (1 - F) V_e + F V_p \]  \hspace{1cm} (4-28)

The rate of reaction is defined as

\[
\frac{\partial F}{\partial t} = I (FCRIT - F)^y \left( V_e^{-1} - 1 \right)^3 \left[ 1 + G \left( V_e^{-1} - 1 \right) \right] + H (1 - F)^y F^x P^z \left( V_p^{-1} - 1 \right)^m
\]  \hspace{1cm} (4-29)

where \( I, G, H, x, y, z, \) and \( m \) (generally \( m=0 \)) are input constants.

The JWL EOS and the reaction rates have been fitted to one- and two-dimensional shock
initiation and detonation data for four explosives: PBX-9404, RX-03-BB, PETN, and cast TNT.
The details of calculation method are described by Cochran and Chan (1979). The detailed one-
dimensional calculations and parameters for the four explosives are given by Lee and Tarver
(1980). Two-dimensional calculations with this model for PBX 9404 and LX-17 are discussed by

4.3.7 Tabulated Compaction

Pressure is positive in compression, and volumetric strain \( \varepsilon_v \) is positive in tension. The tabulated
compaction model is linear in internal energy per unit volume. In the loading phase, pressure is
defined by

\[ p = C(\varepsilon_v) + \gamma T(\varepsilon_v) E \]  \hspace{1cm} (4-30)
In the compacted states, the bulk unloading modulus depends on the peak volumetric strain. The volumetric strain is given by the natural logarithm of the relative volume. Unloading occurs at a slope corresponding to the bulk modulus at the peak (most compressive) volumetric strain, as shown in Fig. 4-3. Reloading follows the unloading path to the point where unloading began, and then continues on the loading path described by Eq. 4-30.

### 4.3.8 Tabulated model

The tabulated EOS model is linear in internal energy. Pressure is defined by

$$p = C(\varepsilon_v) + \gamma T(\varepsilon_v) E$$  \hspace{1cm} (4-31)

The volumetric strain is also given by the natural logarithm of the relative volume. Up to 10 points and as few as 2 may be used when defining the tabulated functions. The pressure is extrapolated if necessary. Loading and unloading are along the same curve unlike EOS of tabulated compaction model.
5 Numerical simulations

The studies of the numerical simulation are reviewed as three parts: software review, implementation of constitutive models, existing research results.

5.1 Available Software

Publically available studies on the numerical simulation of soil behavior under high strain-rates (blasting, explosions, or detonations) are not as extensive as compared with those conducted on constitutive models. Most numerical simulations are conducted with several kinds of software: ABAQUS/Explicit, LS-DYNA, and AUTODYN.

(a) The ABAQUS system includes ABAQUS/Standard, a general-purpose finite element program; ABAQUS/Explicit, an explicit dynamics finite element program; and the Visualization module, an interactive post processing program that provides displays and output lists from output database files written by ABAQUS/Standard and ABAQUS/Explicit.

(b) ANSYS AUTODYN software, used in many applications, is an explicit analysis tool for modeling nonlinear dynamics of solids, fluids and gases as well as their interaction. The software has been developed specifically for analyzing non-linear, dynamic events such as impacts and blast loading of structures and components. The solver types supported by the program include Lagrange, Shell, Euler, ALE (Arbitrary Lagrange Euler) and SPH (Smooth Particle Hydrodynamics).

(c) LS-DYNA is a general purpose nonlinear finite element program compatible with distributed and shared memory solvers using Linux, Windows, and UNIX. LS-DYNA. is suitable to investigate phenomena involving large deformations, sophisticated material models, and complex contact conditions for structural dynamics problems. LS-DYNA allows switching between explicit and implicit time stepping schemes. Disparate disciplines, such as coupled thermal analyses, Computational Fluid Dynamics (CFD), fluid-structure interaction, Smooth Particle Hydrodynamics (SPH), Element Free Galerkin (EFG), Corpuscular Method (CPM), and the Boundary Element Method (BEM) can be combined with structural dynamics. For pre- and post-processing, LS-DYNA comes with the LS-PrePost tool. LS-PrePost can be utilized to generate inputs and visualize numerical results. The software package LS-OPT for optimization and robust design is also supplied with LS-DYNA.
5.2 Implementation of constitutive models

It is very important to implement the developed constitutive models in the selected analysis software. The user’s model, developed by researchers continuously, can be supported in all of the ABAQUS/Explicit, LS-DYNA and AUTODYN software. Because most of works in this project will be done using LS-DYNA, we mainly focus on the implementation of user’s model in the LS-DYNA.

For implementing the user material models, LS-Dyna provides object files and multiple source routines; only one of them is used for the UMAT. The developer needs to modify the supplied source routine by adding their own subroutines. After that the users compile the modified source file to link with the object files that were provided. The procedure of the implementation is given in Appendix A, and the subroutine umat code is given in Appendix B.

5.3 Existing research results

Studies on the numerical simulation of soil behavior under blast loading are rather limited. Higgins et al. (2013) have presented a viscoplastic constitutive model for simulating the high strain-rate behavior of sands. The model parameters are determined for Ottawa and Fontainebleau sands with split Hopkinson pressure bar tests up to a strain rate of 2000/s. This constitutive model was implemented in a finite-element analysis software ABAQUS to analyze underground tunnels in sandy soil subjected to internal blast loads. Figure 5-1 shows the flowchart of the viscoplastic cutting plane algorithm used in this study. The inputs to the algorithm at any time $t$ are the current values of stress ($\sigma_{ij}$), strain ($\epsilon_{ij}$) and hardening variables $\zeta_i$, all denoted with a superscript $t$, the applied strain increment and the time increment $dt$ ($dt$ is controlled from outside of the algorithm either by the user or by the FE analysis software).
Figure 5-2 shows the typical FE mesh by William et al. (2013). In order to save on the computation time, only one half of the actual domain was analyzed by imposing a symmetry boundary condition along the left vertical boundary of the mesh. The top horizontal boundary was free to displace, while the bottom horizontal boundary was restrained against both vertical
and horizontal displacements. Vertical displacements were allowed along the left and right vertical boundaries but not horizontal displacements. The bottom horizontal boundary and the right vertical boundary were located at sufficient distances so that they had no impact on the results of the analysis. The results were obtained at a time when the stress wave from the blast was far from these boundaries. The mesh for the 5 m deep tunnel consists of 1624 elements and 1718 nodes. The blast was simulated using the JWL equation-of-state model. It was found that the type and relative density of sand and the depth of tunnel influence the propagation of the blast-induced stress waves through the soil. The wave speed was found to be greater in Fontainebleau sand than in Ottawa sand. The rate of decrease of the maximum mean stress in soil with increase in distance from the tunnel was comparable for both the sands while the decrease of the maximum shear stress in soil was faster for Ottawa sand.

Grujicic et al. (2008) presented a procedure and the results of a CU-ARL sand model parameterization analysis. The CU-ARL sand model was incorporated into a transient non-linear dynamics computer program and its use in the simulation of a number of buried-land mine blast scenarios along with experimental results were also given.

Fig. 5-2 Typical finite-element mesh used in the analysis of tunnels (Higgins et al. 2013).
Fig. 5-3 Typical temporal simulation of material deformation during landmine detonation: (a) in case of dry sand; (b) in case of fully saturated sand (Grujicic et al. 2008).

Grujicic et al. (2008) also present the temporal simulation of material deformation during landmine detonation in case of dry sand and fully saturated sand, separately (shown in Figure 5-3). The software used in these work is AUTODYN.

The viscoplastic cap model developed by Tong and Tuan (2007) to address the high strain rate effects on soil behavior, has been used by An et al. (2011) for finite-element analysis of blasts due to explosives embedded in soil. The revised cap model comprises a Gruneisen equation of state for each of the three phases: soil, water and air. The equations of state for solid, water, and air can be integrated with the viscoplastic cap model to simulate behaviors of soil with different degrees of water saturation. An et al. (2011) incorporated this model into the LS-DYNA software as user-supplied subroutines for numerical simulations.
Taking advantage of symmetry, An et al. (2011) established only one quarter finite model of the test setup (as shown in Fig. 5-4) using the LS-DYNA software. The finite-element model,
containing a 110-cm air volume above and a 90-cm soil volume below the soil surface, meshed with 6,400 eight-node solid elements. Fine mesh was generated for the explosive and for the air and soil volumes surrounding the C4 where high strain gradients are anticipated. The soil ejecta heights between high speed video and numerical simulation at time = 420 and 1040 μs since detonation for tests in dry sand and in saturated sand are compared in Fig. 5-5(a) and (b), respectively. It shows that the ejecta heights predicted by the revised cap model agree fairly well with the experimental data. Meanwhile, it also shows that the revised cap model is adequate for blast loading behavior simulations for soils.

Other researchers also obtained some interesting results of the numerical simulation for dynamic behavior of soils, which can be seen from the works by Nagy et al. (2010), Yang et al. (2010), Lu et al. (2005), Feldgun et al. (2008; 2008) and Karinski et al. (2009), etc.
Conclusion

This report has considered the fundamental aspects of the simulation of soils under blast loading. Accurate and effective numerical modeling depends on several factors; namely, the selection of an appropriate constitutive model and equation of state to define the stress-strain behaviour of the soil in question, parameterization of the constitutive models and EOS using carefully designed experiments over a range of strain rates, and numerical implementation of the selected theoretical models in a computer sub-routine.

What is immediately apparent from the literature review is that there is no unifying constitutive model that succeeds in capturing the behaviour of soils under all possible conditions. Saturation; particle shape, size, and distribution; cohesion; and strain rate all effect the constitutive models and their components (flow rule, yield surface, rate parameters). Other effects, such as particle crushing under high strain rates, are even less understood.

Moving forward in the investigation of soil and its dynamic properties, the class of viscoplastic constitutive models will be employed. The viscoplastic constitutive models have been demonstrably successful in the modeling of complicated material behaviour problems that occur over orders of magnitude in strain rates. The class of models is also versatile enough that it can be combined with many of the other tenets of plasticity such as various flow rules and cap models.

In the next major phase of the project, experimentation and theoretical modeling of the soil will occur simultaneously. It is important to note that no specific constitutive model has been selected a priori; rather, selection of a specific constitutive model will follow from the initial experimental data obtained from the tests. Depending on the type of model selected, additional physical testing or computer calibration may be required to establish additional parameters.
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