Auroral clutter mitigation in an over-the-horizon radar using joint transmit-receive adaptive beamforming

R. J. Riddolls
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Abstract

New data is presented from an over-the-horizon radar experiment featuring simultaneous multiple transmit channels and multiple receive channels. A joint transmit-receive adaptive beamformer is demonstrated, whereby nulls are placed in the direction of auroral clutter echoes on both transmit and receive. It is shown that the level of the clutter suppression produced by the joint transmit-receive beamformer is the multiplicative product of separate adaptive beamformers on transmit and receive, in accordance with theoretical predictions. The joint transmit-receive adaptive beamformer demonstrates a useful technique for controlling auroral ionospheric clutter for an over-the-horizon radar airspace surveillance mission.

Résumé

Les nouvelles données sont présentées à partir d’un radar transhorizon expérimental, équipé de canaux multiples d’émission et de canaux multiples de réception simultanées. Un conformateur de faisceaux adaptatifs en émission-réception est installé, dans lequel des valeurs nulles sont placées dans la direction du clutter auroral, en émission et en réception. Il est démontré que le niveau d’élimination du clutter produit par le conformateur de faisceaux en émission-réception conjointe est le produit multiplicatif de conformateurs de faisceaux adaptatifs distincts en émission et en réception, conformément aux prévisions théoriques. Le conformateur de faisceaux adaptatifs en émission-réception conjointe se révèle être une technique utile pour contrôler un clutter ionosphérique auroral dans une mission de surveillance de l’espace aérien par radar transhorizon.
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Executive summary

Auroral clutter mitigation in an over-the-horizon radar using joint transmit-receive adaptive beamforming

R. J. Riddolls; DRDC Ottawa TM 2010-265; Defence R&D Canada – Ottawa; December 2010.

Introduction: Over-the-Horizon Radar (OTHR) is a radar technology that uses the bottom side of the earth’s ionosphere as a mirror to illuminate targets beyond the line-of-sight horizon. However, OTHR systems operating in northern latitudes are troubled by radar clutter consisting of strong echoes from plasma irregularities in the earth’s auroral ionosphere. DRDC has been investigating joint transmit-receive adaptive beamformers as a means to achieve a very strong suppression of the clutter echoes, well beyond the levels of suppression achievable with simple receive-side adaptive beamformers. This report aims to show that joint transmit-receive adaptive beamformers, applied to auroral OTHR clutter, achieve a level of clutter suppression corresponding to the multiplicative product of the levels of clutter suppression achieved by separate transmit-side and receive-side adaptive beamformers.

Results: First, it is shown theoretically that a joint transmit-receive adaptive beamformer factors into the product of a transmit adaptive beamformer and receive adaptive beamformer. This factorization considerably simplifies the prediction of performance for the joint beamformer. Second, it is shown that the improvement in signal-to-clutter ratio (SCR) of the joint beamformer is equal to the product of the SCR improvements offered by the individual transmit and receive adaptive beamformers. Third, an OTHR with two transmit channels and two receive channels has been tested in Ottawa, Canada. It has been demonstrated that adaptive beamforming with a four-channel joint transmit-receive adaptive beamformer yields a clutter suppression that is indeed the multiplicative product of the clutter suppression achieved with individual two-channel transmit-side and receive-side adaptive beamformers, in accordance with the theory.

Significance: Clutter sources in the auroral ionosphere are difficult to avoid in high-latitude OTHR systems because these systems must propagate signals through the ionosphere to achieve the intended long-range illumination of targets. However, receive-side-only adaptive beamforming is often insufficient to suppress the auroral clutter. This report shows that joint transmit-receive adaptive beamforming can vastly improve on the clutter suppression capability of receive-side-only adaptive beamforming. The ability to suppress auroral clutter in a high-latitude OTHR
system may ultimately allow Canada to pursue OTHR technology for the purpose of air target tracking.

**Future Plans:** Work to be completed in the next calendar year includes (1) implementing two arbitrary waveform generators to permit pulse compression to a range resolution of 50 km; (2) implementing a diesel generator, and making modifications to power supplies and transmitters, to allow average radar transmit power to be increased from 5 kW to 50 kW; and (3) setting up a bistatic configuration to allow all existing antennas to be used for transmit, which will improve transmit gain and allow the use of FMCW waveforms, which greatly simplifies the task of increasing average transmit power.
Auroral clutter mitigation in an over-the-horizon radar using joint transmit-receive adaptive beamforming

R. J. Riddolls; DRDC Ottawa TM 2010-265; R & D pour la défense Canada – Ottawa; décembre 2010.

Introduction: Le radar transhorizon (OTHR) est un sorte de radar qui utilise la couche inférieure de l’ionosphère de la Terre comme un miroir pour éclairer des cibles au-delà de la ligne d’horizon visible de la Terre. Toutefois, les systèmes OTHR qui fonctionnent dans des latitudes nordiques sont perturbés par le clutter radar qui consiste en de forts échos provenant d’irrégularités du plasma dans l’ionosphère aurorale de la Terre. RDDC a effectué des recherches sur les conformateurs de faisceaux adaptatifs en émission-réception conjointes comme moyen de parvenir à une élimination efficace du clutter, bien au-delà des niveaux d’élimination que l’on peut obtenir avec des conformateurs de faisceaux adaptatifs en émission-réception simple. Ce rapport a pour objectif de démontrer que des conformateurs de faisceaux adaptatifs en émission-réception conjointe, appliqués au clutter OTHR auroral, parviennent à un niveau d’élimination du clutter correspondant au produit multiplicatif des niveaux de suppression du clutter atteints par les conformateurs de faisceaux adaptatifs séparés côté émission et côté réception.

Résultats: Tout d’abord, il est démontré théoriquement qu’un conformateur de faisceaux adaptatifs en émission-réception conjointe constitue un facteur dans le produit d’un conformateur de faisceaux adaptatifs en émission et d’un conformateur de faisceaux adaptatifs en réception. Cette factorisation simplifie considérablement la prévision du rendement du conformateur de faisceaux conjoints. Ensuite, il est démontré que l’amélioration du rapport signal / clutter (SCR) du conformateur de faisceaux conjoints est égal au produit des améliorations offertes par les conformateurs de faisceaux adaptatifs en émission et en réception simple. Enfin, un OTHR avec deux canaux d’émission et deux canaux de réception a été testé à Ottawa, au Canada. Il a été démontré qu’un conformateur de faisceaux adaptatifs en émission-réception conjointes à quatre canaux parvient à une élimination du clutter qui est réellement le produit multiplicatif de l’élimination du clutter atteint avec les conformateurs de faisceaux adaptatifs à deux canaux individuels côté émission et côté réception, conformément à la théorie.

Importance: Les sources du clutter dans l’ionosphère aurorale sont difficiles à éviter dans les systèmes d’OTHR sous les hautes latitudes, parce que ces systèmes doivent diffuser des signaux dans l’ionosphère afin de parvenir à l’illumination voulue de
cibles à longue portée. Toutefois, la formation de faisceaux adaptatifs côté réception seulement est souvent insuffisante pour éliminer le clutter auroral. Le présent rapport démontre que le conformateur de faisceaux adaptatifs en émission-réception conjointe peut améliorer considérablement la capacité d’élimination du clutter d’un conformateur de faisceaux adaptatifs en réception seulement. La capacité de supprimer le clutter auroral dans un système OTHR à haute latitude pourra éventuellement permettre au Canada d’approfondir la technologie des OTHR pour mieux suivre les cibles aériennes.

**Projets futurs:** Les travaux à effectuer au cours de la prochaine année civile comprennent : 1) la réalisation de générateurs de forme d’onde arbitraire pour permettre la compression des impulsions à une résolution d’une portée de 50 km ; 2) la mise en place d’une génératrice diesel, et la réalisation de modifications aux alimentations électriques, afin de permettre une puissance d’émission radar moyenne qui puisse être augmentée de 5 kW à 50 kW ; 3) la réalisation d’une configuration bistatique pour permettre d’utiliser toutes les antennes existantes en émission, ce qui augmentera le gain d’émission et permettra d’utiliser les formes d’ondes entretenues modulées en fréquence (FM-CW), ce qui simplifiera grandement la tâche d’augmentation de la puissance moyenne d’émission.
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1 Introduction

Over-the-Horizon Radar (OTHR) uses the bottom side of the earth’s ionosphere as a mirror to illuminate targets beyond the line-of-sight horizon. This strategy allows for surveillance coverage to a range of about 2000 nautical miles (NM). Sometimes the term High Frequency (HF) sky wave radar is used to distinguish the technology from HF Surface Wave Radar (HFSWR), which illuminates over-the-horizon targets using the diffraction of waves around the curved surface of the earth. HFSWR is a shorter range technology, achieving coverage to about 200 NM. Both HF sky wave radar and HFSWR are high-frequency over-the-horizon radar systems. However, in common practice the term OTHR is normally used only in the context of sky wave radar systems, and the term HFSWR is used to refer to surface wave systems. In keeping with common practice, in this report we use the term OTHR in reference to HF sky wave radar systems.

The overall aim of OTHR research in Canada is to achieve a National Airspace Surveillance (NAS) capability. An NAS capability is currently cost-prohibitive using conventional line-of-sight microwave radar technology due to the large number of systems that would have to be deployed to adequately track low-flying targets. However, previous efforts to operate OTHR in Canada were challenged by the problem of auroral clutter [1]. The motivation for the work described in this report is thus to find a suitable clutter suppression strategy for OTHR operation in Canada [2]. In particular, receive-side sampled apertures and transmit-side waveform-diverse apertures provide adaptive beamforming opportunities that were not available to the previous efforts in the early 1970s. This current work aims to demonstrate the benefits of exploiting these new opportunities.

Recent studies have demonstrated the phenomenon of adaptive transmit-side beamforming in an OTHR system [3]. Adaptive beamforming on the radar transmit side requires multiple independent transmit channels that retain their identity through signal scattering and reception. The reception of multiple transmit channels at multiple receive elements in OTHR systems comprises a Multiple-Input Multiple-Output (MIMO) architecture [3]. The transmit elements are closely spaced (approximately half-wavelength spacing) [4], and thus this implementation of MIMO is sometimes called coherent MIMO to distinguish it from incoherent MIMO, which uses large transmit element spacing to achieve spatial diversity with respect to the angular scintillation in a target radar cross section [5]. Furthermore, one could achieve the same goal of transmit beamforming by the non-MIMO technique of time-multiplexing the transmit elements (see [6] for example). However, a time-multiplexed scheme leads to performance degradation when the echoes to be cancelled have limited temporal correlation. Finally, it should also be noted that the MIMO technique does not improve the signal-to-noise ratio (SNR) of the system, and thus is generally only applicable in
high-SNR situations where the overall performance is limited by the signal-to-clutter ratio (SCR) of the system [7], as in the OTHR auroral clutter case.

Transmit-side adaptive beamforming has been demonstrated by the suppression of transponder signals [3], multimode propagation [8], and auroral clutter [9]. These studies used a single receive element to clearly prove that the signal suppression was due to the transmit-side beamforming. The effort described in this report takes a further step by providing a demonstration of joint transmit-receive adaptive beamforming for the suppression of auroral clutter. It is shown in this report that both theory and experiment suggest the transmit and receive signal suppression performances combine multiplicatively. This result represents a major improvement over single-sided adaptive beamforming.

This report is organized as follows. In Section 2, a review is done of adaptive signal processing to extract unknown signal parameters from a background of clutter. The receive-only case is considered initially. Then, the concept is extended to the radar transmit and receive sides, where it is shown that the joint adaptive beamformer factors into the product of transmit-side and receive-side beamformers. In Section 3, we look at performance metrics and show that the improvement in signal-to-clutter ratio (SCR) in the joint beamformer is equal to the multiplicative product of the SCR improvements in the transmit and receive sides of the radar. With this in mind, we proceed to calculate performance levels for the simplest case of a two-element transmit array and two-element receive array. Results are provided in terms of both the eigenvalues of the separate transmit and receive covariances, and the azimuthal angular locations of the clutter source and target. In Section 4, an experiment is described that attempts to demonstrate the hypothesis that the SCR improvements on the transmit and receive sides combine multiplicatively for the joint beamformer. We have conducted an experiment where the transmit and receive sides are symmetric, which allows one to confirm the multiplicative hypothesis by looking at the SCR improvement of receive-side-only beamforming versus joint transmit-receive beamforming. Finally, a conclusion is briefly drawn in Section 5 about the direction of future research.
2 Beamformer derivations

This section provides the theoretical overview of the optimal beamformers used in this study. There are two subsections. In the first subsection, we derive the optimal spatial filter, referred to here as a beamformer, for processing signals on the receive side of the radar. The second subsection extends the concept to the case of a joint transmit-receive adaptive beamformer.

2.1 Receive-only adaptive beamforming

We consider a plane wave radar target echo with complex amplitude $d$. This echo is mapped onto a spatial array of antennas by an array manifold vector denoted $v$. We consider multiple spatial observations $u$ of $d$ in the presence of clutter $n$:

$$u = v^H d + n.$$  \hspace{1cm} (1)

We ignore noise in this formulation, under the assumption that target detection is strongly clutter limited. In particular, we assume the clutter is sufficiently strong, and sufficiently spatially uncorrelated, such that we will not suppress the clutter down to the noise level in the radar system. Now, assuming that the Doppler shift due to motion of the target has been removed from the echo through Doppler processing, $d$ can be viewed as a constant non-random, but unknown quantity. We seek a set of beamformer weights $w$ that produce an output $y$ that is an estimate of $d$:

$$y = w^H u.$$  \hspace{1cm} (2)

Various criteria can be established to find the optimal beamformer weights. In signal detection problems, where detection algorithms have to find a target signal among competing clutter, a reasonable criterion is to maximize the signal-to-clutter ratio. By inspection of Equation (1) this ratio is given by

$$\text{SCR} = \frac{E[w^H v d (w^H v d)^H]}{E[w^H n (w^H n)^H]},$$  \hspace{1cm} (3)

where $E$ denotes expected value. Note that the SCR contains two factors of $w$ in each of the numerator and denominator. Thus maximizing SCR does not constrain the overall gain of $w$. Thus we need to impose an additional constraint on $w$ that specifies its gain. A very reasonable constraint is that the expected value of the output of the beamformer is equal to the target signal, or in other words

$$E(y) = d.$$  \hspace{1cm} (4)

In view of Equations (1) and (2) we see that this constraint corresponds to

$$w^H v = 1.$$  \hspace{1cm} (5)
Under this constraint, Equation (3) becomes
\[
\text{SCR} = \frac{|d|^2}{E[w^Hn(w^Hn)^H]}.
\] (6)

Clearly, to maximize the SCR we need to minimize the denominator of the previous expression:
\[
w_0 = \arg\min_w E[w^Hn(w^Hn)^H]
\] (7)
\[
= \arg\min_w w^HRw,
\] (8)

where we have used the definition \( R = E(nn^H) \). To minimize \( w^HRw \) under the constraint \( w^Hv = 1 \), we use the method of Lagrange multipliers and find the minimum of the auxiliary function by setting its gradient to zero:
\[
\nabla[w^HRw + \lambda(w^Hv - 1)] = 0,
\] (9)

where
\[
\nabla = \begin{bmatrix}
\frac{\partial}{\partial a_1} + i\frac{\partial}{\partial b_1} \\
\frac{\partial}{\partial a_2} + i\frac{\partial}{\partial b_2} \\
\vdots \\
\frac{\partial}{\partial a_N} + i\frac{\partial}{\partial b_N}
\end{bmatrix}
\] (10)

and
\[
w = \begin{bmatrix}
a_1 + ib_1 \\
a_2 + ib_2 \\
\vdots \\
a_N + ib_N
\end{bmatrix},
\] (11)

with \( N \) being the number of beamformer weights. Computing the gradient in Equation (9), we find that
\[
Rw + \lambda v = 0.
\] (12)

Assuming \( R \) has an inverse, we can solve this equation for \( w \):
\[
w = -\lambda R^{-1}v.
\] (13)

In order to satisfy the constraint \( w^Hv = 1 \), we need
\[
\lambda = -(v^HR^{-1}v)^{-1},
\] (14)

thus the optimal radar beamformer weights are given by
\[
w = \frac{R^{-1}v}{v^HR^{-1}v}.
\] (15)

This result is often referred to as the Minimum Variance Distortionless Response (MVDR) beamformer [10] or sometimes the Minimum Variance Unbiased Estimator (MVUE) [11].
2.2 Joint transmit-receive adaptive beamforming

In this subsection we consider the possibility of transmit-side adaptive beamforming. Multiple transmit channels are realized by transmitting distinguishable waveforms from each of \( M \) transmit elements and separating the individual transmitted signals received at each of \( N \) receive elements using appropriate signal processing. The signals corresponding to each transmit channel can then be linearly combined to form an a posteriori transmit beam pattern.

The presence of \( M \) distinguishable waveforms adds complexity to the estimation problem. The observations at the receive array can now be written as

\[
u = \mathbf{v}_R \sum_{i=1}^{M} v_{Ti} \mathbf{d} + \sum_{i=1}^{M} \mathbf{n}_i,
\]

where \( \mathbf{v}_R \) is the receive array manifold vector, \( v_{Ti} \) is the \( i \)th element of the transmit array manifold vector, and \( \mathbf{n}_i \) is the clutter related to emissions from the \( i \)th transmit element. We continue to ignore noise in this formulation, and assume that the system is strongly clutter limited. In particular, we continue to assume that the clutter is sufficiently strong, and sufficiently spatially uncorrelated, such that we will not cancel the clutter down to the noise level in the radar system.

If the transmitted waveforms are separable, we can recast the model as

\[
\begin{bmatrix}
\mathbf{v}_R v_{T1} \\
\mathbf{v}_R v_{T2} \\
\vdots \\
\mathbf{v}_R v_{TM}
\end{bmatrix} \mathbf{d} +
\begin{bmatrix}
\mathbf{n}_1 \\
\mathbf{n}_2 \\
\vdots \\
\mathbf{n}_M
\end{bmatrix}
\]

\[= (\mathbf{v}_T \otimes \mathbf{v}_R) \mathbf{d} + \mathbf{n}_C \]

\[= \mathbf{v}_C \mathbf{d} + \mathbf{n}_C,
\]

where \( \otimes \) is the Kronecker product, \( \mathbf{v}_C \) is a combined transmit-receive manifold vector, and \( \mathbf{n}_C \) is a combined transmit-receive clutter vector whose covariance can be written as an \( MN \times MN \) matrix given by

\[
\mathbf{R}_C = \mathbf{R}_T \otimes \mathbf{R}_R.
\]

with \( \mathbf{R}_T \) being the covariance of the clutter across transmit elements and \( \mathbf{R}_R \) being the covariance of the clutter across receive elements (denoted simply \( \mathbf{R} \) in the previous subsection).

Equation (19) is of the same form as Equation (1) and therefore an MVDR beamformer can be devised for the combined transmit-receive system, with the beamformer
weights given by

\[ w_C = \frac{R_C^{-1}v_C}{v_C^H R_C^{-1}v_C} \]  

(21)

\[ = \frac{(R_T \otimes R_R)^{-1}(v_T \otimes v_R)}{(v_T \otimes v_R)^H (R_T \otimes R_R)^{-1}(v_T \otimes v_R)}. \]  

(22)

Using the identities

\[(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}\]  

(23)

\[(A \otimes B)^H = A^H \otimes B^H\]  

(24)

\[(A \otimes B)(C \otimes D) = AC \otimes BD,\]  

(25)

it follows immediately that

\[ w_C = \frac{R_T^{-1}v_T}{v_T^H R_T^{-1}v_T} \otimes \frac{R_R^{-1}v_R}{v_R^H R_R^{-1}v_R} \]  

(26)

\[ = w_T \otimes w_R. \]  

(27)

Thus the transmit and receive beamformers can be formulated independently, with the joint beamformer being computed as the Kronecker product of the individual beamformer weights.


3 Performance assessment

In this section we look at the SCR improvement offered by a joint transmit-receive adaptive beamformer, and how this relates to the eigenvectors of the covariance matrix and the azimuthal angles of the target signal and clutter source.

3.1 Array gain of joint beamformer

Let us first consider the receive-side-only adaptive beamformer, where $N$ is the number of array elements. Since the derived beamformer does not affect signal levels, the SCR improvement relates entirely to the suppression of the clutter. The SCR improvement, denoted as the array gain [10] is given by the input clutter level divided by the output clutter level:

$$G = \frac{E(n^H n)/N}{E[w^H n (w^H n)^H]}.$$  (28)

The input clutter level, in the numerator, assumes the clutter has the same variance at all elements. It is calculated as the inner product of $n$ with itself, divided by the number of elements $N$. The output mean-square clutter level is in the denominator, and captures the effect of the beamformer. Plugging in the MVDR weights we find that

$$G = v^H R^{-1} v E(n^H n)/N.$$  (29)

We rewrite this expression with the following notation

$$G = v^H \rho^{-1} v,$$  (30)

where $\rho$ is an asymptotic clutter covariance matrix that is normalized so that it contains ones on the diagonal. Thus the improvement in SCR depends on both the plane wave orientation with respect to the array as specified by $v$ and the correlation of the clutter across the elements, as specified by $\rho$.

In a similar manner, one can extend the notion of array gain to a joint transmit-receive adaptive array. The array gain for the combined system is

$$G_C = v_C^H \rho_C^{-1} v_C$$
$$= (v_T \otimes v_R)^H (\rho_T \otimes \rho_R)^{-1} (v_T \otimes v_R).$$  (31)

Applying the identities of Equations (23) through (25) we find that

$$G_C = (v_T^H \rho_T^{-1} v_T) \otimes (v_R^H \rho_R^{-1} v_R)$$
$$= G_T G_R.$$  (33)
Thus the array gains combine in a multiplicative manner. Thus one can check the performance of a joint beamformer $\mathbf{w}_C$ against the combined effects of the separate transmit-side ($\mathbf{w}_T$) and receive-side ($\mathbf{w}_R$) beamformers. This is the key hypothesis to be verified in the experiments carried out in this study.

### 3.2 Eigenanalysis

Since the array gain can be factored into transmit ($G_T$) and receive ($G_R$) portions, we can examine these portions individually. We now look at the structure of $G_T$ or $G_R$, which we will denote as simply $G$ in this subsection.

We begin with an eigendecomposition of $\rho$:

$$\rho = \mathbf{Q}\Lambda\mathbf{Q}^H,$$

where $\mathbf{Q}$ contains the normalized eigenvectors of $\rho$ in its columns and $\Lambda$ is a diagonal matrix containing the corresponding eigenvalues. Since $\rho$ is a Hermitian matrix, we can write $G$ as

$$G = (\mathbf{v}^H\mathbf{Q})\Lambda^{-1}(\mathbf{Q}^H\mathbf{v}).$$

Let us define $\mathbf{q}_k$ as the $k$th eigenvector and $\lambda_k$ as its corresponding eigenvalue (in descending order of size). We can then write

$$G = \sum_{k=1}^{N} \frac{|\mathbf{v}^H\mathbf{q}_k|^2}{\lambda_k}.$$

The interpretation of this expression is that the gain is large when $\mathbf{v}$ is in the direction of the eigenvectors with the smallest eigenvalues. Intuitively, this means that the target signal should arrive from a different direction than the clutter in order for $G$ to be large. Furthermore, it should be noted that we have so far ignored noise in the radar system. This is equivalent to assuming that even the smallest eigenvalues in the system are dominated by the clutter. This is a reasonable assumption for strong clutter with weak spatial correlation in a system with a low number of degrees of freedom. For example, in the experiment described later in this report, the transmit and receive sides of the radar each have only two degrees of freedom. However, when the number of degrees of freedom in the system becomes much larger than unity, this assumption must be revisited.

Before proceeding to the experimental results, we consider a theoretical analysis of the array gain behaviour for a system with a low number of degrees of freedom. Let us consider a 2-element beamformer oriented along the $x$ axis, with half-wavelength spacing between elements, and with the target and clutter signals arriving at azimuthal
angles $\phi$ and $\theta$, respectively, with respect to the array axis. The array manifold vector for the target signal is

$$v = \begin{bmatrix} 1 \\ e^{-i\pi \cos \phi} \end{bmatrix}. \quad (37)$$

We assume the clutter consists of a “crinkly” wavefront originating in the earth’s auroral region. The normalized covariance matrix for this signal is given by

$$\rho = \begin{bmatrix} 1 & \rho^* \\ \rho & 1 \end{bmatrix}, \quad (38)$$

where $\rho$ is the correlation coefficient of the clutter signal at the first and second array element locations. We model the clutter autocorrelation as exponential in the direction perpendicular to the propagation, and complex exponential in the direction of propagation [12], such that

$$\rho = \exp(-\alpha |\sin \theta| - i\pi \cos \theta), \quad (39)$$

where $\alpha$ is a correlation decay rate. The eigendecomposition of $\rho$ can be written

$$Q = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -e^{-i\pi \cos \theta} \end{bmatrix}, \quad (40)$$

and

$$\Lambda = \begin{bmatrix} 1 + e^{-\alpha|\sin \theta|} & 0 \\ 0 & 1 - e^{-\alpha|\sin \theta|} \end{bmatrix}. \quad (41)$$

If we insert these expressions into Equation (36), we find that

$$G = \frac{2 \cos \left[ \frac{\pi}{2} (\cos \phi - \cos \theta) \right]^2}{1 + e^{-\alpha|\sin \theta|}} + \frac{2 \sin \left[ \frac{\pi}{2} (\cos \phi - \cos \theta) \right]^2}{1 - e^{-\alpha|\sin \theta|}}. \quad (42)$$

Each term is the inner product of $v$ with an eigenvector, divided by the corresponding eigenvalue. If $\alpha \ll 1$ (good correlation), then there is good eigenvalue separation and the second term on the right side of the previous expression will dominate. Expanding the denominator in Taylor series, we find that

$$G \approx \frac{2 \sin \left[ \frac{\pi}{2} (\cos \phi - \cos \theta) \right]^2}{\alpha |\sin \theta|}. \quad (43)$$

A number of observations can be made about this expression. First, the numerator suggests that we need good angular separation between the target signal ($\phi$) and clutter ($\theta$), in order to get good array gain. Second, for modest separations between target signal and clutter angles, the difference $\cos \phi - \cos \theta$ is larger when $\phi, \theta \approx \pi/2$ than when $\phi, \theta \approx 0$. In other words, small angular separations favor broadside operation with respect to the target signal and clutter rather than endfire operation.
Finally, the third observation is that the factor of $\sin \theta$ in the denominator suggests a large improvement when the clutter is at endfire ($\theta \approx 0$), which competes with the broadside-favouring term in the numerator. One could speculate that a two-dimensional array on transmit or receive may be able to exploit both possibilities. However, the results that will be presented in the following section only address one-dimensional transmit and receive arrays. The two-dimensional case will be examined in a later report.
4 Experiment

This section discusses an experiment. The apparatus is presented in the first subsection. The second subsection presents the results.

4.1 Apparatus

An experimental OTHR has been set up in Ottawa, Canada. A photograph of the site, with antennas drawn in, is shown in Figure 1. The OTHR features an array of six Beverage antennas. A Beverage antenna is a horizontal long-wire antenna that supports a travelling wave mode. As the wave travels along the wire it radiates at a low elevation angle in the direction of propagation. Beverage antennas are selected for this experiment because they require neither a ground plane nor a tall tower. It is difficult to lay a ground plane at the radar site due to swampy conditions and rapid

Figure 1: Experiment layout.
vegetation growth. It was not possible to install towers during the funding cycle for
the experiment due to the long lead times for securing approval to install tall struc-
tures. The length of each Beverage antenna is 300 m, and each is suspended at 6.5 m
above the ground using poles at 75-m intervals. The antennas are oriented in order
to achieve maximum gain toward magnetic north, with the intent of obtaining strong
auroral ionospheric clutter echoes. The element gain is 4 dBi at 20-degree elevation.
There is sufficient space on the field to install a total of six Beverage antennas at
\(\sim 30\)-m spacing between the antennas, which is one half-wavelength spacing at the
operating frequency of \(\sim 5\) MHz. In the experiment described here, two of the anten-
nas were used for transmit, two were used for receive, and two were not connected.
The number of active antennas was the bare minimum number of four in the interest
of producing experimental results within the calendar year. Future experiments may
turn the six-element Beverage array into a dedicated transmit facility, with a receive
facility to be built elsewhere. This would allow the use of Frequency-Modulated
Continuous-Wave (FMCW) waveforms. In addition, the area to the left of the array
as seen in Figure 1 is sparsely populated with trees and could possibly be cleared in
order to increase the aperture of the transmit array.

The basic radar waveform consists of 7.5-ms square pulses of 16-kW power at a pulse
repetition frequency (PRF) of 40 Hz. This results in a duty cycle of 30% and an aver-
age power of \(\sim 5\) kW. The average power was limited to 5 kW due to the availability
of utility power. The radar shelter is provided with about 20 kW of power, but the
conversion efficiency from AC power to RF power is only about 25%. A 200-kW gen-
erator was purchased in November 2010, which should ultimately allow an increase
in RF power to about 50 kW once other issues with power supply and amplifier up-
grades are sorted out. The 7.5-ms square pulse provides a range resolution of 1125
km, and thus all auroral echoes are grouped into one range resolution cell. Plans are
in place to replace the current pulse generator with an arbitrary waveform genera-
tor, which would allow the realization of pulse compression waveforms. The current
OTHR frequency authorization permits bandwidths of up to 3 kHz, corresponding to
50-km range resolution cells.

Two distinguishable waveforms are realized by pulse-modulating two carrier frequen-
cies spaced by 20 Hz (half the PRF), amplifying each to 8 kW, and sending one out
each of the two transmit antennas. Since the carrier spacing is 20 Hz, after receive-
side Doppler processing, the two waveforms appear as distinct signals in the Doppler
spectrum and can be separated by partitioning the radar Doppler spectrum in two
[13].
4.2 Results

An experiment was run on 12 Oct 2010. The radar had a fixed frequency assignment during the experiment. Thus instead of adjusting frequency to find the location of auroral echoes, with a fixed frequency one has to wait until ionospheric conditions permit the propagation of the radar beam to a location of space with auroral scatterers. On 12 Oct, good spread echoes (at least \( \pm 5 \) Hz) were obtained at 2352 UTC. A total of 25 seconds of data were taken. The recorded data were organized into the four joint transmit-receive channel combinations corresponding to the elements of \( \mathbf{u}' \) in Equation (17). The channels were then combined using various beamforming strategies to try to reduce the clutter.

The adaptive beamforming results are shown in Figure 2. There are three traces. The top trace is the Doppler spectrum of a single channel of data consisting of TX1 transmitting and RX1 receiving (see Figure 1). The plotted data correspond to the measured signal \( |u'_1|^2 \) in the notation of Equation (17). Since the transmit and receive arrays are broadside to the direction of the magnetic pole, we postulate that the clutter signal is also broadside to the arrays. To best demonstrate the production of a broadside null while maintaining power on a target that is well-separated in

![Figure 2: Auroral clutter Doppler spectrum for different adaptive beamforming schemes. Top trace is single-channel data. Middle trace is receive-only beamforming. Bottom trace is joint transmit-receive beamforming.](image)
azimuth from the clutter, we inject an endfire synthetic target at -5.9 Hz into the array data. The injected target signal is very strong in Figure 2, for the purpose of showing that the MVDR beamformer does in fact maintain target amplitude, while suppressing the clutter. We also assume the target signal is well-confined in Doppler and any Doppler-spreading appears only in the clutter. This Doppler spreading arises due to the velocity distribution of the clutter-producing plasma irregularities in the auroral ionosphere, as shown in [9]. However, the target signal itself could spread in Doppler if these plasma irregularities were to occur somewhere along the ray path of the target signal.

For the other two traces shown in Figure 2, we estimate the adaptive beamformer weights using four Doppler bins adjacent to the Doppler cell under test. It is clear that there is not sufficient sample support here to very accurately estimate the asymptotic covariance matrix, so achievable array gain in each of the transmit and receive portions of the array may fall a little bit short of what is predicted by the eigenanalysis in the previous section. In particular, in experiments with better range resolution, one might desire to use adjacent range samples, as these represent another dimension of radar data that would vastly improve the amount of information by which a covariance estimate could be formed. Another possibility is to use diagonal loading when estimating the covariance (see [14] for example).

The second trace in Figure 2 shows the Doppler spectrum of receive-only adaptive beamforming, namely $|w_R^H u'_1 u'_2|^2$. The adaptive weights in this case were on the order of $w_R = [0.5 -0.5]^T$, which is consistent with a broadside null. The broadside clutter is thus suppressed whereas the endfire target is maintained. The level of suppression is about 15 dB in the middle of the plot and 10 dB or less on the left and right sides. Note there exists a patch of low-correlation auroral clutter between approximately -6 Hz and -4 Hz, for which the suppression is less effective.

The third trace shows the Doppler spectrum of joint transmit-receive adaptive beamforming $|w_C^H u'|^2$, again with the endfire target unaffected. In this case the adaptive weights were on the order of $w_C = [0.25 -0.25 -0.25 0.25]^T$. We see that the joint transmit-receive beamforming increases the level of clutter suppression to around 30 dB near the center of the plot, tapering to about 20 dB on the left and right sides. This represents an approximate doubling (in dB scale) of the clutter suppression that was achieved by receive-side-only beamforming. Since the radar transmit and receive sides are symmetric, this joint beamformer result is consistent with the multiplicative product of receive and transmit beamforming predicted by the theory. Furthermore, we see that the beamformer preserves the level of the target signal, as expected in a MVDR beamformer. Finally, we speculate that the tapering in the amount of clutter suppression on either side of the Doppler spectrum arises due to the finite noise floor in the system, which was not included in the theory. Better experimental results could be obtained with a higher clutter-to-noise ratio (CNR). CNR improvements
could be realized by increasing the average transmit power, which is being planned for future experiments.
5 Conclusion

The experiment shows evidence that a joint transmit-receive adaptive beamformer can suppress clutter to a degree equal to the multiplication of individual transmit and receive MVDR beamformers. This evidence is in agreement with the theoretical idea that the joint MVDR beamformer is separable into transmit and receive factors. Work to be completed in the next calendar year includes (1) implementing two arbitrary waveform generators to permit pulse compression to a range resolution of 50 km; (2) implementing a diesel generator, and making modifications to power supplies and transmitters to allow average radar transmit power to be increased from 5 kW to 50 kW; and (3) setting up a bistatic configuration to allow all six Beverage antennas to be used on transmit, resulting in an array gain of 12 dBi. In particular, the last modification allows the use of FMCW waveforms, which greatly simplifies the task of increasing average transmit power.
References


**Auroral clutter mitigation in an over-the-horizon radar using joint transmit-receive adaptive beamforming**

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Technical Memorandum

**DRDC Ottawa TM 2010-265**

**Unlimited announcement**
New data is presented from an over-the-horizon radar experiment featuring simultaneous multiple transmit channels and multiple receive channels. A joint transmit-receive adaptive beamformer is demonstrated, whereby nulls are placed in the direction of auroral clutter echoes on both transmit and receive. It is shown that the level of the clutter suppression produced by the joint transmit-receive beamformer is the multiplicative product of separate adaptive beamformers on transmit and receive, in accordance with theoretical predictions. The joint transmit-receive adaptive beamformer demonstrates a useful technique for controlling auroral ionospheric clutter for an over-the-horizon radar airspace surveillance mission.

radar
sky wave
high frequency
ionosphere
clutter
adaptive
antenna
array
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