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Analysis of Radar Pulse Width Effects on RCS Calibration and Signal to Noise Ratio

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Abstract

This document examines the effects of varying radar pulse width on both received signal power and peak signal to noise ratio for a simple pulsed carrier radar transceiver. The work was motivated by the use of mismatched radar pulse widths for calibration of radar cross section measurement data in the DRDC-O, Radar Cross Section Measurement System (DRMS). DRDC-O is Defence Research and Development Canada – Ottawa.

A simulated model of a radar receiver matched filter was implemented to predict receiver output. We confirm that the use of mismatched pulse widths has no effect on calibration if the pulse widths used for the calibrator and target observation are sufficiently large relative to the reciprocal of the receiver bandwidth With further analysis of SNR behaviour versus pulse width, we also note that any excess bandwidth over the optimum matched condition leads to a reduction in SNR

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1. Introduction

This report examines the effects of varying radar pulse width on both received signal power and peak signal to noise ratio for a simple pulsed carrier radar transceiver. The analysis is carried out by simulating various signal pulse widths and filter bandwidths. The output power and SNR was examined graphically. We confirm that the use of mismatched pulse widths has no effect on calibration if the pulse widths used for the calibrator and target observation are sufficiently large relative to the reciprocal of the receiver bandwidth. With further exploration and examination of SNR behaviour versus pulse width, we also note that any excess bandwidth over the optimum *matched condition* leads to a loss in SNR.

Section two introduces the theory of matched filters and how they maximize the SNR of a received signal This section defines a relationship between the input pulse and filter response function that will maximize SNR in the presence of white Gaussian noise.

Section three derives a rule of thumb from the theory of section two in the specific case where the input is a rectangular pulsed carrier waveform. This rule states that for maximum SNR, the bandwidth of the matched filter should equal the reciprocal of the radar pulse width.

Section four examines the effects of pulsed carrier pulse widths other than the reciprocal of receiver bandwidth on signal amplitude and maximum signal power. Through simulation we show that if the pulse width of the received signal exceeds the ideal matched pulse width, the output power remains constant. Under this condition different pulses widths can be used for calibration targets and measurement subjects without affecting the power or the power ratio.

Section five examines SNR behaviour with varying pulse widths and bandwidth through two more simulated cases. The first consist of a system with a constant pulse width of 1us. The bandwidth of this system is varied and a SNR plot is obtained vs. the different bandwidths. The second case examines the case where the bandwidth is constant and the pulse width is varied. The SNR versus pulse width is plotted for several values of bandwidth. These two cases serve to illustrate the effects of mismatched pulse widths and bandwidth on SNR and calibration for pulsed carrier radar.

The final section draws the conclusion that calibration is unaffected in the case where the differing pulse widths used for both the calibrator and target exceed the reciprocal of the bandwidth. We also conclude that maximum SNR is achieved when the matched condition is met, and that excess bandwidth of a factor N degrades SNR by the same factor. This last point suggests that reduction of excess bandwidth will yield improvements in SNR and extend the maximum range and sensitivity of radar system

2. Matched Filters

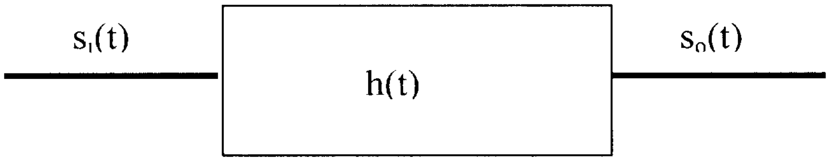

Figure 1. *Matched Filter with impulse response h(t)*

Figure 1is a block diagram of a matched filter with impulse response h(t). The input to the filter is the time signal $s_i(t)$ and the corresponding output is given by Equation 1.

convolution sum $$s_o(t) = s_i(t) * h(t) \qquad 1$$

The matched filter is designed such that its frequency response function, $H(j\omega)$, maximizes the peak-signal-to-mean-noise-power ratio (R_f). The key in using a matched filter is to find the transfer function of the filter ($H(j\omega)$) that will maximize the SNR, R_f For the complete derivation see [1]. Here we are more interested in the result which states that, in order to maximize R_f, $H(j\omega)$ must satisfy Equation 2, where G_a is a constant equal to the maximum filter gain, the factor $e^{-(j\omega)}$ is a phase shift and t_1 is the time where the maximum output is observed.

$$H(j\omega) = G_a S^*(j\omega) e^{-(j\omega t_1)} \qquad 2$$

In order to get a more intuitive view of the meaning of Equation 2, the impulse response of $H(j\omega)$ is found using the Fourier synthesis equation:

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega) e^{j\omega t} \, d\omega \qquad 3$$

Substituting Equation 2 into the synthesis equation and noting that $S^*(j\omega) = S(-j\omega)$:

$$h(t)=\frac{G_a}{2\pi}\int_{-\infty}^{\infty}S(j\omega)e^{[jw(t_1-t)]}d\omega=G_a s(t_1-t) \qquad 4$$

Thus, the impulse response of a matched filter that will maximize the SNR, R_f is the image of the real received signal reversed in time from time t_1 [1].

3. Filter Design Parameters for Pulsed Carrier Radar

The use of a rectangular[1] pulsed carrier of length τ leads to a rather interesting rule of thumb to maximize SNR in radar pulse practice. The receiver bandwidth (usually applied at IF), or the bandwidth of the matched filter (B), must be approximately equal to the reciprocal of the pulse width (τ). This arises from the Fourier transform properties of a rectangular pulse, which are illustrated in Figure 2.

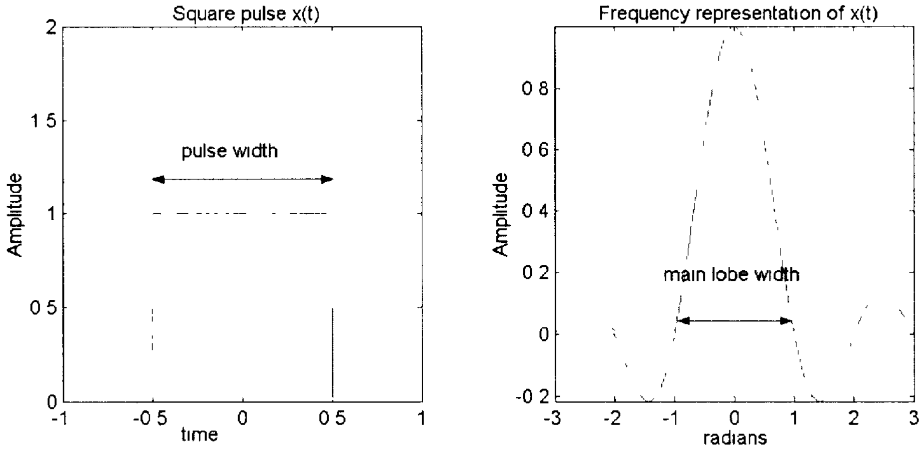

***Figure 2.** Square pulse and corresponding Fourier transform*

The width of the main lobe for the corresponding square pulse with pulse width τ is 2/τ Hz. A widely used approximation for the bandwidth is half of this value, or 1/τ Hz. From the above discussion, since the impulse response of the matched filter is just a shifted and reversed version of the received signal, then the matched filter would have a square wave impulse response and the above H(jω) would resemble the right hand graph of Figure 2 The bandwidth B of the matched filter would thus be 1/τ Hz and the product of the bandwidth and pulse width would be unity (Bτ = 1).

---

[1] Here we assume an ideal rectangular pulse for discussion purposes. The results hold for realistic pulse shapes, which will be band limited

4. Effects of Non-Matching Pulse Widths on Calibration

As mentioned in section 2, SNR is a ratio of peak signal power to mean noise power The peak signal power is dependent on the matched filter characteristics. This power value also plays a key role in the calibration of radar data as demonstrated in Equation 5, the equation used to calculate RCS for a typical RCS measurement analysis [3].

$$RCS_t = 20 \bullet \log\left(\left[\frac{R_t - RangeCor - 0.15P_t}{R_c - RangeCor - 0.15P_c}\right]^2 \bullet \frac{<coh_t>}{<coh_c>} \bullet \sqrt{Rcs_c}\right) \quad 5$$

Thus the calibrated target's RCS is related to the target return power (P_t) and the calibrators return power (P_c). In this equation *RangeCor* is a range correction factor, $<coh_t>$ is a coherent average of the target amplitude, $<coh_c>$ is a coherent average of the calibration target amplitude, and RCS_c is the known RCS of the calibration target.

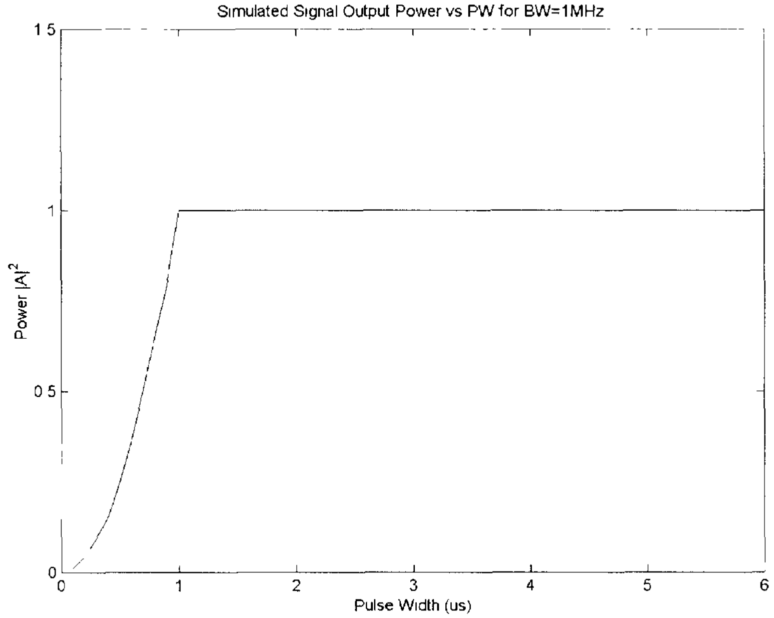

Figure 3. Signal amplitude squared vs. pulse width

Figure 3 demonstrates the simulated behaviour of the square of the output signal amplitude as the pulse width of the input signal (to a 1MHz bandwidth, matched filter) is varied. MATLAB was used to numerically compute the convolution of the pulse and filter impulse response. The figure plots the maximum amplitude squared of the output of the matched filter $|A|^2$, versus the varying pulse widths. Since,

$$MaxPower \propto |A|^2$$

from the figure, it is observed that as the pulse width of a received signal is increased, the output power first increases, until the maximum is reached when the matched condition is met ($B*\tau=1$). Past this point, any increase in pulse width has no effect on peak power. From this fact and Figure 3, we can state that the pulse width used when measuring a calibration target has no effect on RCS calibrations as long as the pulse widths used for both target and calibrator are larger than 1/BW.

5. Use of Non-Matching Pulse Widths and Bandwidths

This section consists of an examination of the effects of varying pulse widths and bandwidths on SNR rather than signal output amplitude. Although there is a relationship between the two, the results here provide a better understanding of the matched filter condition than the above analysis.

5.1. Effect of varying the bandwidth with constant pulse width

Now let us assume that we have a filter whose bandwidth is not equal to $1/\tau$, but much greater. Note that $B>>1/\tau$ implies that $B\tau>>1$. This would imply that the corresponding impulse response would be a square pulse with a much smaller pulse width and corresponding frequency response with a much broader main lobe. The broader frequency response introduces more noise to the output, thus reducing the SNR [1]. Figure 4 demonstrates the behaviour of signal to noise ratio when the bandwidth is varied with a constant pulse width

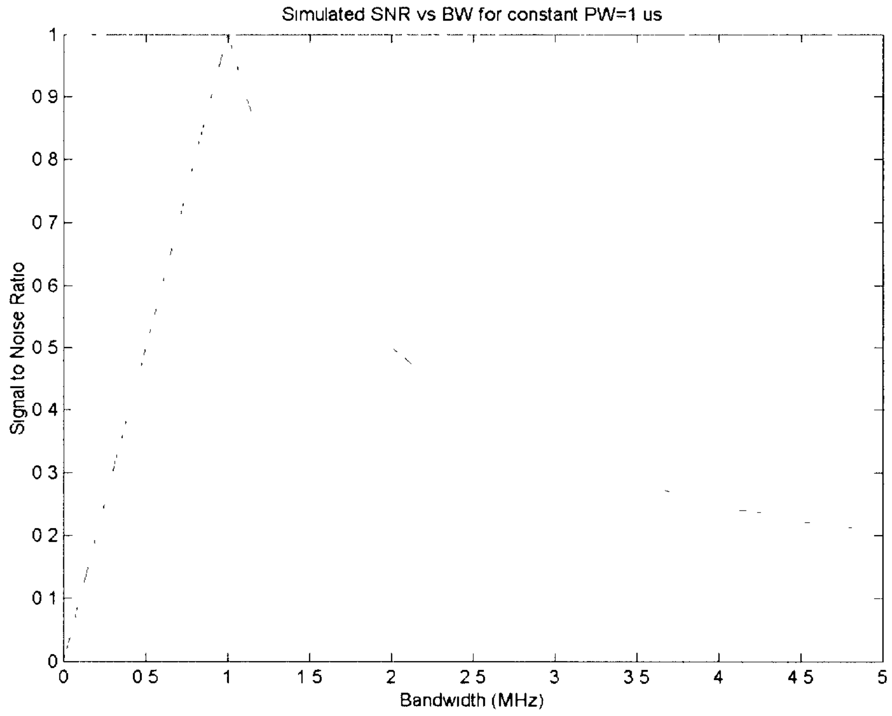

Figure 4. SNR vs. bandwidth for constant unity pulse width

For the example above (i.e. $B\tau >> 1$) the SNR value is to the right of the peak value of SNR and thus, is much lower. The relationship for SNR was found by making the following assumptions:

The peak output power is proportional to the square of the peak output amplitude. The average noise power is proportional to

$$\int |h(t)|^2 dt$$.(from [1] and Parseval's relation [3])

SNR is defined as the ratio of these two entities. Figure 4 was obtained by varying the bandwidth and numerically simulating peak output power and average noise power using MATLAB. This simulation demonstrates that, as the bandwidth of the filter is broadened, the SNR first increases because of the increase in output signal power. Then, since the output power cannot be increased once the matched condition has been achieved, the SNR decreases proportionately to the increase in noise (due to the broadening filter bandwidth).

5.2. Effect of varying the pulse width with constant bandwidth

In this section, we examine the effect of varying the pulse width, once the bandwidth of the filter has been set. Figure 5 demonstrates the results of the simulations cases.

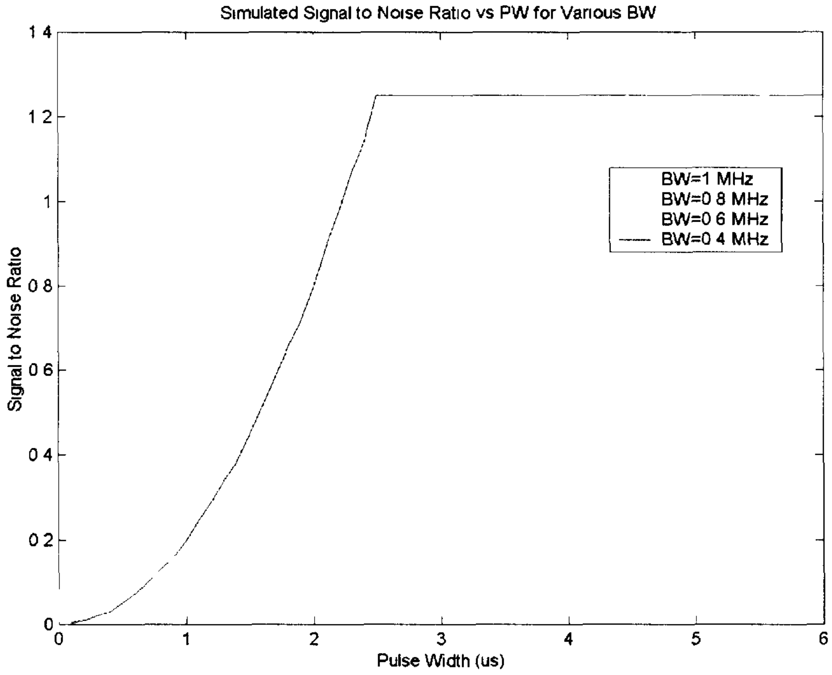

Figure 5. SNR vs. pulse width for different filter BW

These behaviour curves were obtained by assuming that the peak power of the output signal is proportional to the square of the output amplitude and that the mean noise is as in case 1.

From the simulation result of Figure 5 we notice that initially, increasing pulse width leads to increasing SNR. The point at which the SNR reaches its maximum is when the relationship $B*\tau = 1$ is met. If the pulse width is increased past this point, the SNR remains unchanged This can be explained by recalling that the filter bandwidth is set, thus the noise is constant, and the output power cannot be increased by increasing the pulse width, once the matched condition has been obtained (as stated in section 4).

If the bandwidth of the filter is decreased, the behaviour of the curve remains the same, but the SNR is higher. This is so because the noise, which is proportional to the bandwidth of the filter, has decreased.

6. Conclusion

The amplitude, or maximum power of a signal, is unaffected by increasing the pulse width of the radar signal past the matched pulse width. This is an important result because it confirms that calibration of target RCS can be carried out with differing pulse widths, as long as the matching condition is exceeded

However, signal to noise degradation occurs for a pulsed carrier radar as the bandwidth of the IF filter is increased beyond $1/\tau$, the reciprocal of the (shortest) pulse width. This degradation is due to the excess noise that is passed by the IF filter. We can also conclude that the pulse width of the transmitted signal only affects the SNR if it is less than 1/B. For larger pulse widths, the SNR is unchanged.

These conclusions were reached by examining matched filter theory and a rule of thumb used to approximate the matched filter for pulsed carrier radars. Numerical simulation using MATLAB was used to illustrate the behaviour of SNR for conditions where either the pulse width or bandwidth is held constant while the other parameter was varied.

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