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Probabilistric service life assessment of missile structures

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Probabilistic Service Life Assessment of Missile Structures

by

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Abstract

Tactical missiles that use solid rocket motors are stored under controlled thermal conditions or deployed at geographical locations under widely varying weather conditions that are best described as random processes. Depending on the storage conditions, a motor may be subjected to changing ambient temperature, solar radiation, wind, rain and snow, resulting in randomly varying induced stresses, strains and cumulative damage which may exceed the strength and strain capacities of the propellant. This could give rise to cracks that could induce failure upon firing. The motivation for service life assessment is the detection of propellant degradation and the prediction of the end life before it actually occurs.

An approach for probabilistic service life assessment of missile systems that incorporates structural degradation due to thermal loading cycles is developed. Probabilistic models for daily thermal load cycles is formulated and incorporated into structural response models for rocket motors. Probabilistic models for a missile structural capacity are proposed and calibrated from measured data. Performance functions for assessing the service life of stored missiles are developed. First Order Reliability Method (SORM) is used to calculate instantaneous reliability of a missile structure. Methodology for computing the time dependent reliability of a degrading missile structure is developed. Sensitivity of the time dependent reliability to system parameters is investigated. Example problems are used for demonstrations.

PROBABILISTIC SERVICE LIFE ASSESSMENT OF MISSILE STRUCTURES

By

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NAVAL APPLICATIONS OF MATERIAL TECHNOLOGY
5-7 MAY 2001

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PRESENTATION OUTLINE

- MOTIVATIONS/INTRODUCTION
- OBJECTIVE OF STUDY
- PROBABILISTIC MODELS FOR SERVICE LIFE ASSESSMENT
- DEMONSTRATION EXAMPLE
- LIMITATIONS OF APPROACH
- SUMMARY/CONCLUSIONS

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MOTIVATIONS/INTRODUCTION

- Solid Rocket Motors (SRMs) Constitute A Critical Component In Tactical Missiles
- Missiles Are Stored At Various Geographical Locations
- Thermal Load And SRMs Properties Exhibit A Great Deal Of Uncertainties
- Environmental Thermal Loads Degrade The Properties And Service Life Of A SRM
- Degradation Is In The Form Of Cracking And Debonding
- The Service Life Of Missiles Must Be Estimated To Avoid Catastrophic Failure Upon Ignition

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OBJECTIVES

- To Compute The Probability Of Failure Or Reliability Of Solid Rocket Motors
- To Determine The Sensitivity Of Reliability Or Probability Of Failure To The Uncertain Parameters
- To Determine The Contribution/Importance Of The Uncertain Parameters To Reliability

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MERITS OF PROBABILISTIC APPROACH

- Rational, Systematic And Utilizes Available Information On Uncertainties
- Sensitivity To Information
- It Allows For Quantification Of Reliability And Risk, Hence Provides A Framework For Application Of Risk Management Procedures
- Realistic For Decision Making
- Not Overly Conservative/Cost Effective
- Provides A Framework For Systematic Determination Of Overall System Reliability From The Reliability Of Individual Components

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RELATIONSHIP BETWEEN PROBABILISTIC AND DETERMINISTIC APPROACH

- Probabilistic And Deterministic Methods Are Not Competing But Complementary Methods Of Engineering Analysis And Decision Making
- Best Probabilistic Methods Are Those That Are Founded On Well-established Deterministic Procedures - Sound Deterministic Before Probabilistic

PROBABILISTIC MODELS FOR SERVICE LIFE ASSESSMENT

- Models for Thermal Loads
- Models for Critical Structural Responses
- Models for Structural Capacities
- Models for Performance Functions
- Algorithms for Computing Instantaneous Reliability
- Algorithms for computing Progressive Reliability

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MODELS FOR THERMAL LOAD

- **Maximum Daily Temperature Modeled by**

$$T(t) = T_m + T_y a(t) + T_d$$

- T_m = Mean Value of Yearly Temperature
- T_y = Yearly Amplitude of Temperature
- T_d = Peak Value of Daily Temperature
- $a(t) = sin(2\pi/\omega(t - t_0))$
- t - Day

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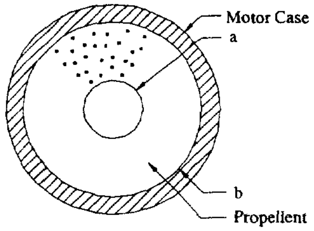
MODELS FOR CRITICAL STRUCTURAL RESPONSE

Schematic Diagram



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MODELS FOR CRITICAL STRUCTURAL RESPONSE

- **Model can be:**
  - **Analytical**
  - **Experimental**
  - **Numerical**

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MODELS FOR CRITICAL STRUCTURAL RESPONSE

- **Critical Structural Responses are:**
  - **Bore Strain / Stress (Bore Cracking)**
  - **Bondline Stress (Debonding)**
- **Analytical Models are Employed in Study**

MODELS FOR CRITICAL STRUCTURAL RESPONSE

- **Bore Strain Formula**

$$\varepsilon_\theta = \log_e\{1+\varepsilon_\theta^1(a)\}$$

- **Bore Stress Formula**

$$\sigma_{rc}(a) = 2E_p\alpha_R\lambda^2\Delta T$$

- **Bondline Stress Formula**

$$\sigma_r(b) = E_p\alpha_R\left(\lambda^2-1\right)\Delta T$$

MODELS FOR CRITICAL STRUCTURAL RESPONSE

- **Where**

$$\varepsilon_\theta^1 = \left(3/2\right)\alpha_R \lambda^2 \Delta T$$

$$\alpha_R = \alpha_p - \frac{2}{3}(1+\nu_c)\alpha_c$$

$$\lambda = \frac{b}{a}$$

$$\Delta T = T - T_{sfree}$$

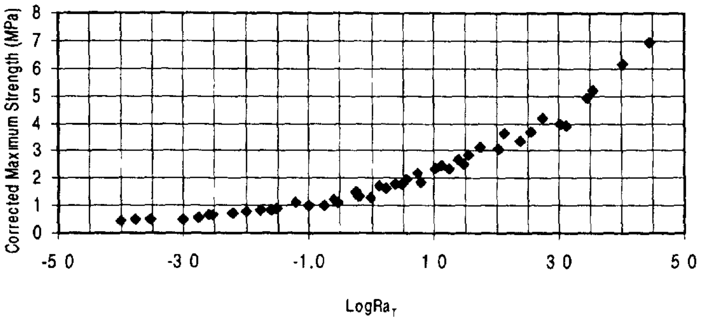
- T – Actual Temperature
- T_{sfree} - Stress Free Temperature
- b – Outside Diameter
- a – Inner Diameter,
- α_p - Propellant's Thermal Coefficient of Expansion
- α_c - Case's Thermal Coefficient of Expansion
- ν_c - Case's Poisson's Ratio

MODELS FOR CRITICAL STRUCTURAL CAPACITY

- **Capacities are Temperature / Loading Rate Dependant**
- **It has Variability**
- **Non-Linear Least Square Piecewise Technique Used to Fit Curves to Capacities**

MODELS FOR CRITICAL STRUCTURAL CAPACITY

Typical Stress Capacity Plot



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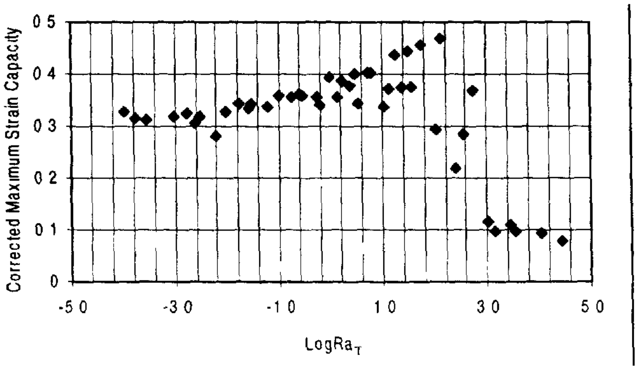
MODELS FOR CRITICAL STRUCTURAL CAPACITY

Typical Strain Capacity Plot



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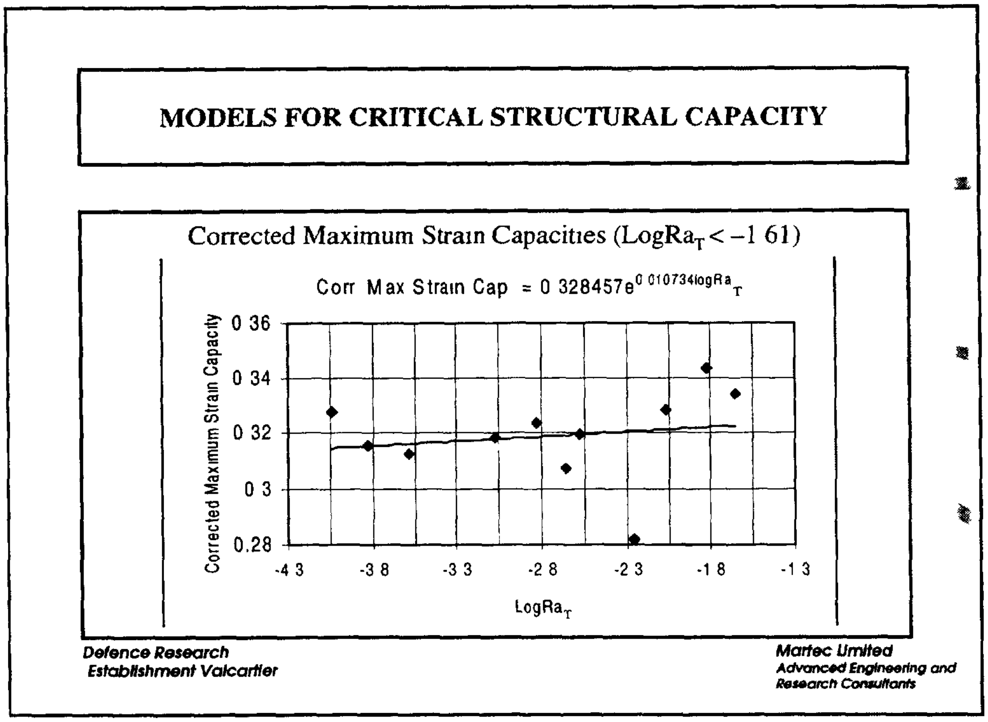

MODELS FOR CRITICAL STRUCTURAL CAPACITY

Corrected Maximum Strain Capacities (LogRa$_T$ < –1 61)

Corr Max Strain Cap = 0 328457e$^{0\ 010734\mathrm{logRa}_T}$

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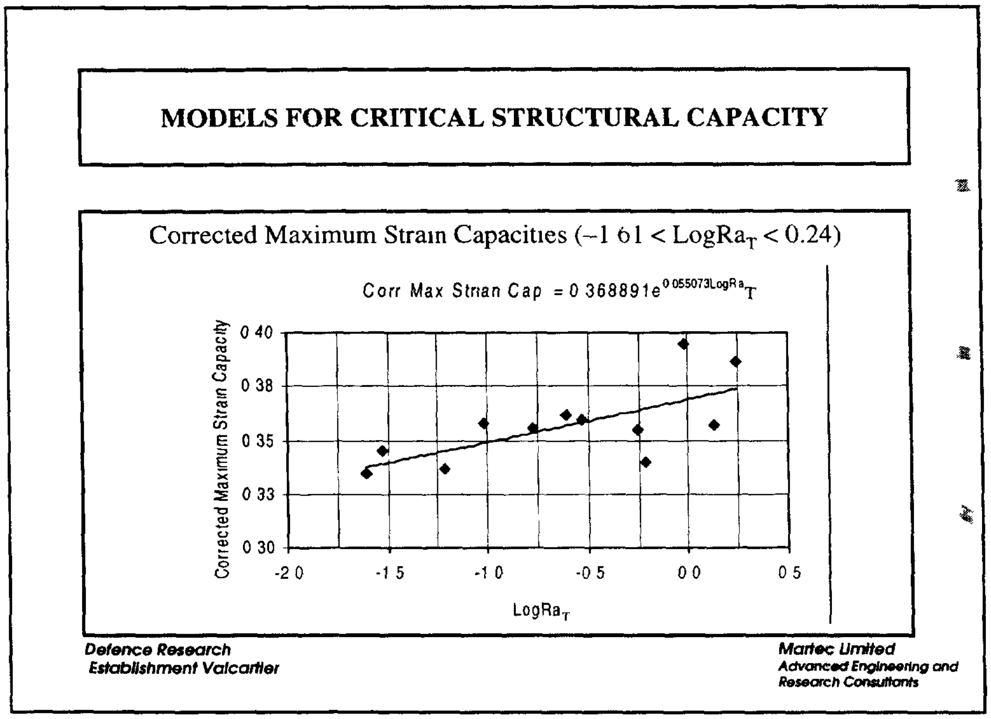

MODELS FOR CRITICAL STRUCTURAL CAPACITY

Corrected Maximum Strain Capacities (–1 61 < LogRa$_T$ < 0.24)

Corr Max Strain Cap = 0 368891e$^{0\ 055073\mathrm{LogRa}_T}$

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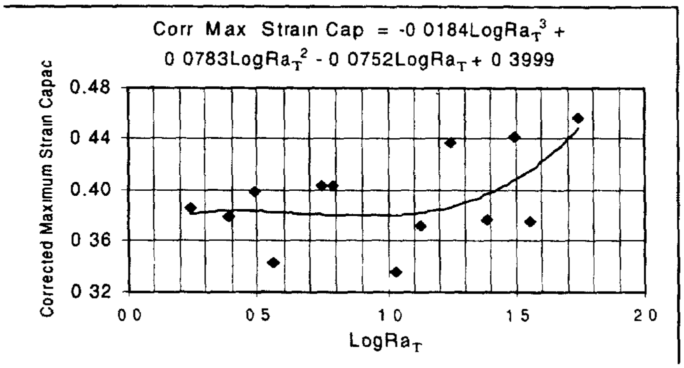
MODELS FOR CRITICAL STRUCTURAL CAPACITY

Corrected Maximum Strain Capacities (0.24 < LogRa_T < 1.74)



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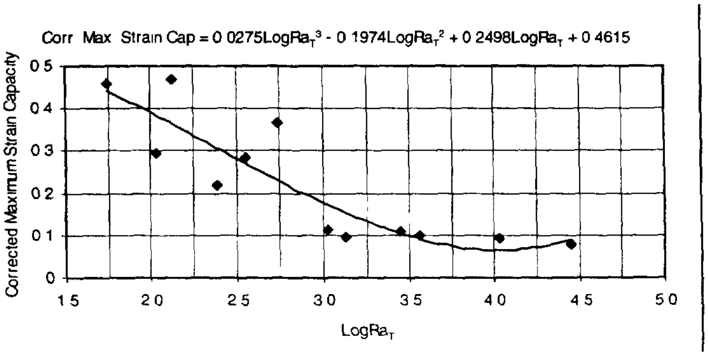
MODELS FOR CRITICAL STRUCTURAL CAPACITY

Corrected Maximum Strain Capacities (LogRa_T> 1.74)



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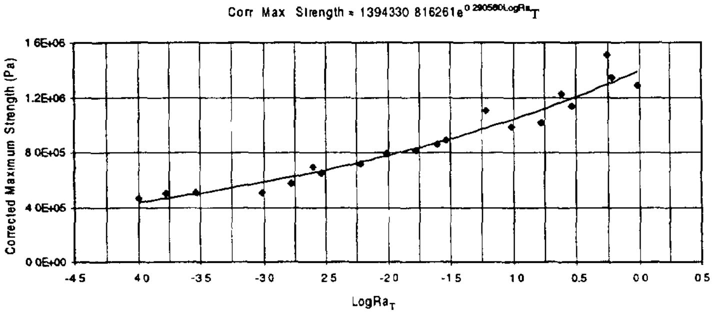
MODELS FOR CRITICAL STRUCTURAL CAPACITY

Corrected Maxımum Strength ($LogRa_T < -0.02$)



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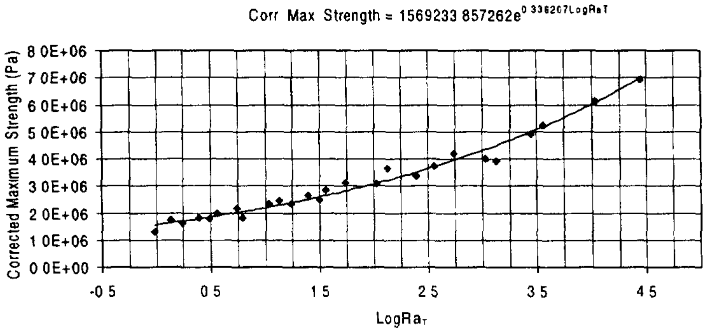
MODELS FOR CRITICAL STRUCTURAL CAPACITY

Corrected Maximum Strength ($LogRa_T > -0.02$)



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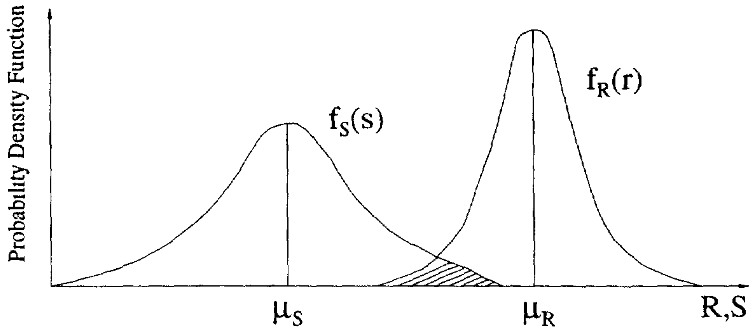
MODELS FOR PERFORMANCE FUNCTION



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MODELS FOR PERFORMANCE FUNCTION

◆ General Performance Functions Can Be Developed

$$g(X(T)) = R(X(t)) - L(X(t))$$

◆ Instantanious and Time Dependant Reliability Have to be Computed

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ALGORITHM FOR INSTANTENOUS RELIABILITY

$$P_f = \int_{\Omega} f(X(t))dX = \int_{\Omega} f(X_1, X_2 \ldots. X_n)dX$$

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ALGORITHM FOR PROGRESSIVE / TIME DEPENENT RELIABILITY

◆ Interested in the Structural Reliabiltiy with Time, $\gamma(t)$

$$\gamma(t) = \exp[-\int_{0}^{t} \lambda(\tau)d\tau]$$

◆ $\lambda(t)$ = Conditions Probability Function

$\lambda(t)$ dt = *Prob*[failure between *t* and *t+dt*|no failure up to *t*]

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EXAMPLE: ESTIMATION OF SERVICE MISSILE STORED AT TWO SITES

Values of Material and Geometric Parameters

PARAMETER	TYPICAL VALUE
b	3 21
a	1
α_p	$85\times10^{-6}/°C$
ν_c	0 33
α_c	$23.6\times10^{-6}/°C$
T_{sfree}	60°C

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EXAMPLE: ESTIMATION OF SERVICE MISSILE STORED AT TWO SITES

Values of Material and Geometric Parameters

PARAMETERS	SEASONAL VARIATION		DIURNAL CYCLE	
	T_y(C)	T_m(C)	T_d(C)	$\sigma_{Td}(C)$
Alert	20 8	-12 88	-0 35	5 46
Valcartier	17 8	5 8	2 41	4 92

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EXAMPLE: ESTIMATION OF SERVICE MISSILE STORED AT TWO SITES

I	G_i(PA)	$\lambda_i(S)$
1	77 4520 x 10^6	3 2967 x 10^9
2	40 0178 x 10^6	2 7109 x 10^8
3	21 2522 x 10^6	2 2291 x 10^7
4	14 8369 x 10^6	1 8330 x 10^6
5	10 9891 x 10^6	1 5073 x 10^5
6	7 9458 x 10^6	1 2394 x 10^4
7	4 8011 x 10^6	1 0192 x 10^3
8	2 5918 x 10^6	8 3804 x 10^3
9	1 3000 x 10^6	6 8912 x 10^2
10	6 6377 x 10^6	5 6667 x 10^2
11	3 2938 x 10^6	4 6596 x 10^2
12	1 9124 x 10^6	3 8315 x 10^1
13	6 4188 x 10^6	3 1506 x 10^2

EXAMPLE: ESTIMATION OF SERVICE MISSILE STORED AT TWO SITES

RANDOM VARIABLE	DISTRIBUTION TYPE	MEAN VALUE	COV
Capacity	Normal	Obtained from $\log Ra_T$	0 1
Outside Diameter (b)	Fixed	3 21	
Inside Diameter (a)	Fixed	1	
Coefficient of Thermal Expansion of Case (α_c)	Fixed	2 36E-05	
Coefficient of Thermal Expansion of Propellant	Fixed	8.5E-05	
Poisson Ratio of Case	Fixed	0 33	
Stress Free Temperature	Fixed	60°C	
Daily Mean Temperature (T_m)	Fixed	-12 88°C (Alert) 6°C (Valcartier)	
Yearly Amplitude of Temperature T_y	Fixed	20 8°C (Alert) 18°C (Valcartier)	
Peak Value of Daily Temperature	Normal	-0 35 (Alert) 2 5 (Valcartier)	2 1

EXAMPLE: ESTIMATION OF SERVICE MISSILE STORED AT TWO SITES

Summary of Ratio of Daily Critical Response
to Daily Capacity at Alert and Valcartier

	VALCARTIER Range of Values	ALERT Range of Values
$\frac{\sigma_{cp}}{\sigma_r}$	8.89 - 3.72	5.53 - 2.6
$\frac{\sigma_p}{\sigma_c}$	4.0 - 1.68	2.5 - 1.14
$\frac{\varepsilon_p}{\varepsilon_r}$	8.77 - 4.09	5.52 - 2.96

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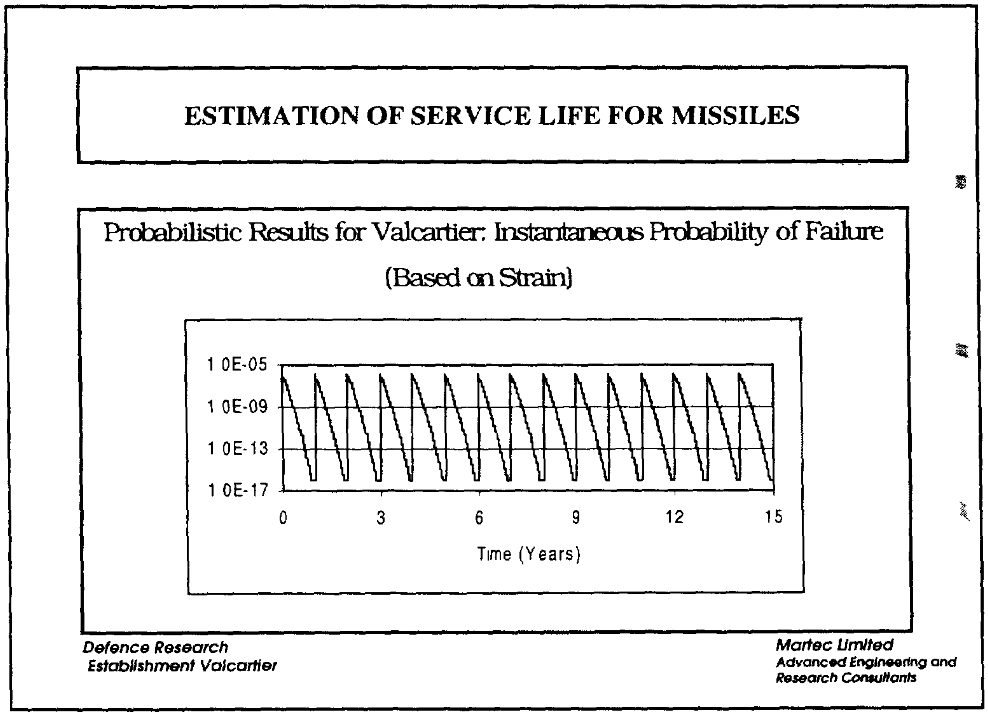

ESTIMATION OF SERVICE LIFE FOR MISSILES

Probabilistic Results for Valcartier: Instantaneous Probability of Failure (Based on Strain)

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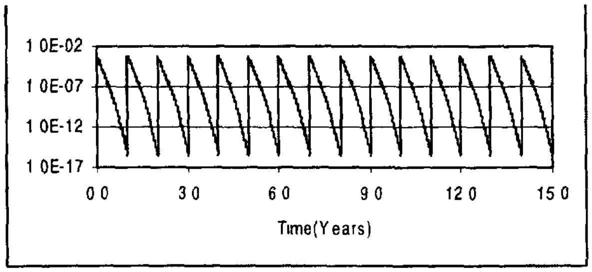
ESTIMATION OF SERVICE LIFE FOR MISSILES

Probabilistic Results for Alert : Instantaneous Probability of Failure

(Based on Strain)



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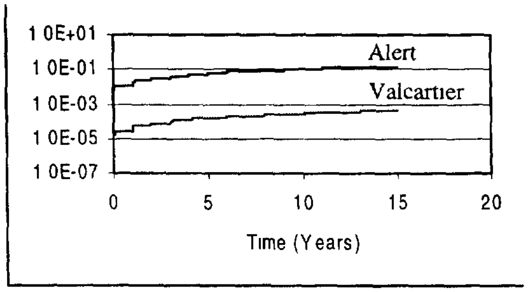
ESTIMATION OF SERVICE LIFE FOR MISSILES

Probabilistic Results :

Comparative Values of Progressive Probability of Failure



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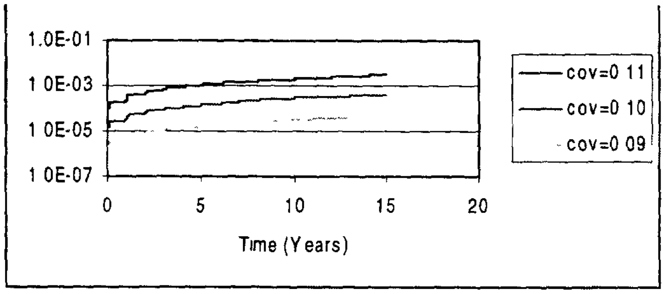
ESTIMATION OF SERVICE LIFE FOR VALCARTIER SITE

Sensitivity to COV of Strain Capacity: Valcartier



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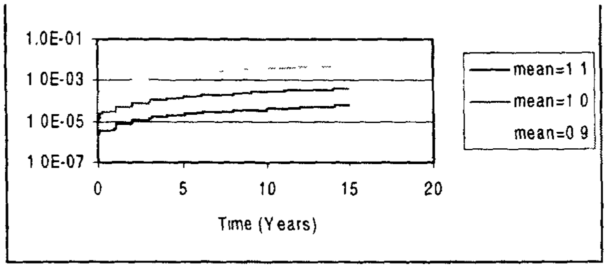
ESTIMATION OF SERVICE LIFE FOR VALCARTIER SITE

Sensitivity to Mean Values of Strain Capacity: Valcartier



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LIMITATION OF RESULTS

- Effect of Cumulative Damage Not Considered
- Probabilistic Distribution for Strain Capacity Assumed and Not Calibrated From Data
- Variables That Affect the Structural Responses Modeled as Fixed Variables

SUMMARY/CONCLUSIONS

- A Time Dependent Probabilistic Strategy For Assessing the Service Life of Missile Structure Has been Developed
- Example Problems That Demonstrate the Approach Have been Demonstrated
- Limitations of Method Have been Highlighted
- Alternative Probabilistic Service Life Assessment Strategies that Circumvent the Limitations are Being Pursued