Acoustic radiation from an elastic baffled rectangular plate covered by a decoupling coating and immersed in a heavy acoustic fluid

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The vibroacoustic behavior of an elastic, simply supported rectangular plate covered by a locally reacting decoupling layer supporting thickness deformation is presented. The model simulates the vibration and acoustic response of the system immersed in water and subjected to a point force disturbance. A simplified version of the theory is derived in the limiting case of a large decoupling (low mechanical impedance of the layer/high frequency). An appropriate vibratory indicator, representative of the acoustic attenuation provided by the decoupling treatment, and independent of the structure dimensions, is also investigated from the perspective of small-scale laboratory characterization © *2000 Acoustical Society of America.* [S0001-4966(00)00405-7]

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INTRODUCTION

The addition of a decoupling (or compliant) coating on a resonant structure is a common way to reduce the sound radiated by a structure immersed in water. The aim of this treatment is to isolate the fluid from the vibrations of the resonant structure. The decoupling material should have a low stiffness in comparison to the elastic material forming the base structure to support thickness deformation induced by the action of the dynamic pressure in the fluid. This deformation leads to the decoupling of the motions of the base structure and of the fluid in contact with the decoupling layer. The decoupling effect is small if the structure is immersed in a light fluid because the acoustic pressure exerted by a light fluid on the decoupling layer is negligible.

The objective of this article is to propose a theoretical model to describe the vibroacoustic behavior of a rectangular, simply supported elastic plate covered by a decoupling layer and immersed in water in order to quantify the sound attenuation provided by such a system. The rectangular plate is inserted in an infinite rigid baffle that separates vacuum on one side from water on the other side. The vibration of the base plate, the vibration of the outer surface of the decoupling layer, and the sound radiated in the fluid half-space are investigated when the excitation is a point force applied on the base structure.

In order to accurately model the dynamics of the decoupling layer, the three-dimensional theory of elasticity is usually required. This theory ensures that thickness deformation as well as bending, shear, and extensional deformations of the decoupling layer are properly accounted for (Noor *et al.*, 1989; Garrison *et al.*, 1994; Laulagnet *et al.*, 1994). Such models devoted to finite structures lead to very complicated formulations that usually prevent parametric study and high-frequency investigations. In order to obtain simpler formulation, Ko (1997) and Keltie (1998) considered the three-dimensional theory of elasticity but with infinite structures. A finite structure is under consideration in this paper, so that the three-dimensional elasticity theory is not used to model the decoupling layer.

Another category of models to describe the behavior of compliant coating is the locally reacting model, which assumes that the decoupling material behaves as evenly distributed massless springs on the plate. This model is able to account for the transverse strain of the decoupling layer. It is called "locally reacting" because the thickness deformation of the layer at a given position depends on the surface acoustic pressure at this position only. It also neglects the mass of the decoupling layer.

Numerous works have used the locally reacting model to describe the vibroacoustics of a base structure covered by a compliant material and immersed in water. Crighton (1979) investigated the sound reflected from an infinite elastic panel covered by a decoupling material under fluid loading. He used the locally reacting model and examined the plane wave reflection coefficient when the excitation is a plane acoustic wave propagating in the fluid under grazing incidence. Maidanik *et al.* (1968, 1974, 1984) studied the acoustic behavior of multilayered infinite panels immersed in fluid media. They defined two types of layers: the compliant layer, that can support transverse strains, modeled by the locally reacting model, and the blanket that has dynamical properties that are fluidlike, so that the propagation within the blanket can be specified by a density and a speed of sound. Sylwan (1987) investigated the *in vacuo* vibrational behavior of an infinite sandwich beam when the core is modeled by evenly distributed springs in the transverse direction (to take into account transverse strain) and in the longitudinal direction (to take into account shear strain). He compared both effects as a function of the frequency and obtained their relative contribution in the global loss factor of the structure. House (1991) studied the transmission loss of an infinite panel covered by a compliant coating immersed in water. He found good agreement between experimental results in a 1/10th scale section of a ship hull and a simple theory where the hull and the coating are assumed to behave as a rigid wall with a given impedance. Laulagnet *et al.* (1991, 1994) used the locally reacting model to calculate the sound radiated from a thin, simply supported cylindrical shell covered by a compliant coating. This method was compared to a rigorous model that involves the three-dimensional theory of elasticity for the compliant layer motion, and it was shown that the locally reacting model appropriately describes the vibroacoustic behavior of thin shells covered by a decoupling layer. Recently Sandman *et al.* (1995) used the locally reacting model when the base structure is a rigid piston. In order to obtain a very simple model, only limiting cases were considered by these authors for the calculation of the radiation impedance of the rigid piston.

As seen in the literature review, the sound radiation under mechanical excitation of a finite elastic plate, covered by a decoupling coating, is not yet fully covered. It is the purpose of this article to propose a model where the base plate is a rectangular elastic plate and the coating is assumed to conform to the locally reacting model. In particular, an objective of the proposed model is to derive a global vibratory indicator appropriate to represent the acoustic efficiency of a decoupling treatment, independently of the substrate (base plate) dimensions. Such an indicator would be useful to predict the efficiency of a decoupling treatment on a large structure from experiment on small-scale test specimens.

The acoustic behavior of the elastic plate-compliant coating system inserted in an infinite baffle involves the evaluation of the surface acoustic pressure. The modal basis, used for the displacement of the simply supported plate, cannot be used for the surface pressure because on the baffle/plate boundary the displacement is zero, but not the acoustic pressure. Several authors (Ginsberg *et al.*, 1995, 1991) used the surface variational principle (SVP, Pierce, 1987) to evaluate the surface acoustic pressure. These authors define a distance r—from the boundary of the plate—beyond which the surface pressure is assumed to vanish. The surface acoustic pressure is then defined by a set of trigonometric functions that vanish at the distance r. In this paper, the surface acoustic pressure is expanded on a complete Fourier series that allows the nonzero condition of the surface acoustic pressure on the baffle to be satisfied. Furthermore, it avoids the need to define the distance r.

The theoretical formulation of the proposed model is developed in the next section. The locally reacting model and the basic equations that describe the vibroacoustic behavior of the plate-coating-water system are presented. The governing equations are solved to obtain global structural and acoustic indicators of the problem. A simple theory is derived from the proposed model to treat the case of a large decoupling (low stiffness decoupling material/high frequency). The results obtained by the proposed model are analyzed in the numerical results section where the vibration and acoustic isolation effect provided by a compliant layer are presented.

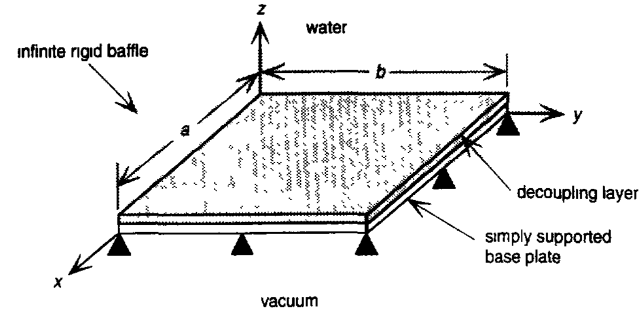

FIG 1 Schematic representation of the baffled plate covered with a decoupling layer and immersed in water

I. THEORETICAL FORMULATION

A. Statement of the problem

The structure under study is assumed to be a rectangular, baffled plate, simply supported on the four edges. The plate-baffle system separates a heavy fluid ($z>0$) from vacuum ($z<0$) (Fig. 1). The structure itself consists of a base flexural plate covered by a compliant layer allowing for thickness deformation. The position of the rigid baffle coincides with the outer surface of the decoupling layer. The base plate is assumed to conform to the Love–Kirchhoff theory that considers only bending deformation, while the coating layer is described with the locally reacting model. The purpose of the following model is to calculate the transverse vibration of the base plate, the transverse vibration at the outer surface of the coating, and the resulting sound radiation in the semi-infinite heavy fluid when the base plate is excited by an harmonic point force.

The output parameters are the mean-square velocity of the plate and of the outer surface of the decoupling layer, the radiation efficiency, and the radiated sound power in the heavy fluid.

B. Equation of motion of the structure

The thickness deformation of the decoupling layer implies that the base plate and the outer surface of the decoupling layer (the surface in contact with the fluid) have distinct transverse displacements (Fig. 2). Thus $\tilde{w}_1$ is defined to be the transverse displacement of the base plate and $\tilde{w}_2$ the transverse displacement of the outer surface of the decou-

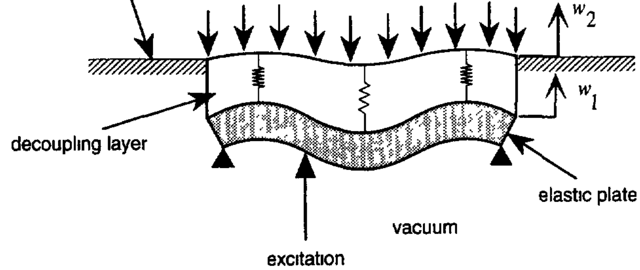

FIG. 2. Schematic representation of the locally reacting model.

pling layer. Throughout this document, tildes represent complex values expressed in the frequency domain.

The equation of motion of the base plate, expressed in the frequency domain, is

$$\tilde{D}\nabla^4\tilde{w}_1(Q)-\rho h\omega^2\tilde{w}_1(Q)=\tilde{f}(Q)-\tilde{\sigma}(Q), \tag{1}$$

where ω is the angular frequency, Q is a point on the plate, $\tilde{D}$ is the complex bending stiffness of the plate: $\tilde{D}=\tilde{E}h^3/12(1-v^2)$, ρ, h and v are, respectively, the density, the thickness, and the Poisson's ratio of the plate, $\tilde{E}$ is the complex Young's modulus, defined by $\tilde{E}=E(1+j\eta)$, E is the real art of the Young's modulus, and η the structural loss factor, ∇^4 is a partial differential operator: $\nabla^4=(\partial^4/\partial x^4)+2(\partial^4/\partial x^2\partial y^2)+\partial^4/\partial y^4$, $\tilde{f}(Q)$ is the external transverse force per unit area applied to the base plate and $\tilde{\sigma}(Q)$ is the normal stress in the z-direction applied by the coating on the base plate.

Assuming that the coating behaves like a massless stiffness, the surface acoustic pressure $\tilde{P}(Q)$ exerted by the fluid on the coating is the same as the normal stress $\tilde{\sigma}(Q)$ exerted by the coating on the structure. Thus, $\tilde{\sigma}(Q)=\tilde{P}(Q)$ in Eq. (1). In the following, the unknowns of this equation are the transverse displacement of the plate $\tilde{w}_1$ and the acoustic pressure $\tilde{P}$. According to previous work (Laulagnet *et al.*, 1994), the zero mass assumption for the coating is acceptable in situations where the mass per unit area of the coating is smaller than the mass per unit area of the substrate.

The simply supported boundary conditions assumes that the transverse displacement and the bending moment vanish on the plate boundaries,

$$\tilde{w}_1(x,y)=0 \text{ and } \frac{\partial^2\tilde{w}_1(x,y)}{\partial x^2}=0 \text{ for } x=0 \text{ and } x=a, \tag{2}$$

$$\tilde{w}_1(x,y)=0 \text{ and } \frac{\partial^2\tilde{w}_1(x,y)}{\partial y^2}=0 \text{ for } y=0 \text{ and } y=b, \tag{3}$$

where a and b are, respectively, the x-dimension and the y-dimension of the plate.

Considering a locally reacting decoupling material allows the following relation to be written for the motion of the decoupling layer under the action of the surface acoustic pressure $\tilde{P}(Q)$:

$$\tilde{P}(Q)=Z_c(\tilde{w}_2(Q)-\tilde{w}_1(Q)). \tag{4}$$

Equation (4) assumes that the thickness deformation of the decoupling layer at point Q depends only on the surface acoustic pressure at point Q. This equation ignores extended reaction of the layer, and thus conforms to a locally reacting assumption. Z_c is the impedance of the decoupling material, which can be written in terms of the elastic properties of the materials,

$$Z_c=\frac{\tilde{B}_c}{h_c}. \tag{5}$$

In Eq. (5), $\tilde{B}_c$ is the complex modulus of the decoupling layer: $\tilde{B}_c=B_c(1+j\eta_c)$, and h_c is the thickness of this material. Recent experiments (Hamm, 1996) have suggested that the *bulk* modulus is the best parameter to characterize the decoupling material when the locally reacting model is used, although a more thorough experimental validation using solid polymers and foams with known elastic properties still needs to be carried out. The decoupling material is thus characterized only by its bulk modulus and its thickness.

The acoustic pressure in the fluid is governed by the Helmholtz equation,

$$\Delta\tilde{P}(M)+k_0^2\tilde{P}(M)=0, \tag{6}$$

where M is a point in the fluid space, k_0 is the acoustic wave number defined by $k_0=\omega/c_0$, and c_0 is the velocity of sound in the fluid.

The equation of continuity of the structural and acoustic normal accelerations on the outer surface of the decoupling material is

$$\frac{\partial\tilde{P}(Q)}{\partial z}=\rho_0\omega^2\tilde{w}_2(Q), \tag{7}$$

where ρ_0 is the density of the fluid.

The acoustic pressure on the surface of the coating is evaluated with the Rayleigh integral,

$$\tilde{P}(Q)=-\omega^2\rho_0\int\int_s\tilde{w}_2(M)G(M,Q)ds_M, \tag{8}$$

where s is the surface of the decoupling layer, points Q and M are on the surface of the decoupling layer, and the acoustic Green's function G is defined in the Appendix. The system formed by Eqs. (1), (4), and (8) has three unknowns: $\tilde{w}_1(Q)$, $\tilde{w}_2(Q)$, and $\tilde{P}(Q)$. Equation (4) allows the displacement $\tilde{w}_2(Q)$ to be expressed in terms of $\tilde{w}_1(Q)$ and $\tilde{P}(Q)$, so that the two remaining unknowns are $\tilde{w}_1(Q)$ and $\tilde{P}(Q)$. Using Eq. (4), Eq. (8) becomes

$$\tilde{P}(Q)=-\omega^2\rho_0\int\int_s\left(\tilde{w}_1(M)+\frac{\tilde{P}(M)}{Z_c}\right)G(m,Q)ds_M. \tag{9}$$

The displacement $\tilde{w}_1(Q)$ is expanded over the orthogonal *in vacuo* modes of the plate, while the acoustic pressure $\tilde{P}(Q)$ is expanded over a complete orthogonal Fourier series,

$$\tilde{w}_1(Q)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\tilde{a}_{mn}w_{mn}(Q), \tag{10a}$$

$$\tilde{P}(Q)=\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\tilde{b}_{pq}\psi_{pq}(Q), \tag{10b}$$

where $w_{mn}(Q)=\sin(m\pi x/a)\sin(n\pi y/b)$, and $\psi_{pq}(Q)=e^{j2\pi(px/a+qy/b)}$.

Using Eq. (10), Eqs. (1) and (9) lead to two linear systems of coupled equations written in terms of the unknowns of the problem, i.e., the modal displacements $\tilde{a}_{mn}$ and the Fourier coefficients $\tilde{b}_{pq}$ for the surface pressure,

$$\rho h \frac{ab}{4} \tilde{a}_{mn}(\omega_{mn}^2(1+j\eta)-\omega^2)$$

$$= \tilde{f}_{mn} - \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} b_{ij} S_{mnij}, \tag{11a}$$

$$ab \cdot \tilde{b}_{pq} = -\omega^2 \rho_0 \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \tilde{a}_{kl} H_{klpq}$$

$$- \omega^2 \rho_0 \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \frac{\tilde{b}_{ij} L_{ijpq}}{Z_c}, \tag{11b}$$

where ω_{mn} are the *in vacuo* natural angular frequencies of the plate,

$$\omega_{mn} = \sqrt{\frac{D}{\rho h}} \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right], \tag{12}$$

and D is the real part of $\tilde{D}$.

The vector of the generalized forces $\tilde{f}_{mn}$ in the case of a point force is

$$\tilde{f}_{mn} = F_0 \sin\left(\frac{m\pi x_0}{a} \right) \sin\left(\frac{n\pi y_0}{b} \right), \tag{13}$$

where F_0 is the magnitude of the force and x_0 and y_0 are the coordinates of the excitation point. In Eqs. (11a) and (11b), the three terms S_{mnij}, H_{klpq}, and L_{ijpq} are defined by the following integrals:

$$S_{mnij} = \int_s \int w_{mn}(Q) \psi_{ij}(Q) ds,$$

$$H_{klpq} = \int_s \int \int_s \int w_{kl}(Q) G(Q,M) \psi_{pq}^*(M) ds_Q ds_M, \tag{14}$$

$$L_{ijpq} = \int_s \int \int_s \int \psi_{ij}(Q) G(Q,M) \psi_{pq}^*(M) ds_Q ds_M.$$

The calculation of S_{mnij} is trivial. The integrals H_{klpq} and L_{ijpq} need to be calculated numerically; this calculation is done following the idea of Sandman (1975) and Nelisse (1996) that allows the quadruple integral to be analytically reduced into a double integral using an appropriate change of integration variables. The resulting double integral is then calculated with standard Gauss integration (see Appendix). The method to avoid the singularity problem of these integrals is also described in the Appendix.

The equation (11) can be written in matrix form,

$$\begin{bmatrix} [A][B] \\ [C][D] \end{bmatrix} \begin{Bmatrix} \tilde{a}_{mn} \\ \tilde{b}_{pq} \end{Bmatrix} = \begin{Bmatrix} \tilde{f}_{mn} \\ 0 \end{Bmatrix}, \tag{15}$$

where

$$[A] = \left[\quad \rho \frac{ab}{4} (\omega_{mn}^2 (1+j\eta) - \omega^2) \quad . \right] \text{ (diagonal matrix)},$$

$$[B] = [S_{mnij}] \text{ (full matrix)},$$

$$[C] = [\omega^2 \rho_0 H_{klpq}] \text{ (full matrix)}, \tag{16}$$

$$[D] = \left[\omega^2 \frac{\rho_0}{Z_c} L_{ijpq} \right] + [\ ab\ .\ ] \text{ (full matrix)}.$$

The infinite series on *m, n, p, q, i, j, k, l* are now truncated such that *m, n, k, l* vary between 1 and N for the displacement and *p, q, i, j* vary between $-N_p$ and N_p for the acoustic pressure. The system can be written in the form

$$\{\tilde{a}_{mn}\} = [A]^{-1}(\{\tilde{f}_{mn}\} - [B]\{\tilde{b}_{pq}\}), \tag{17}$$

$$([D] - [C][A]^{-1}[B])\{\tilde{b}_{pq}\} = [C][A]^{-1}\{\tilde{f}_{mn}\}. \tag{18}$$

To solve the problem, Eq. (18) has to be first solved for the magnitude of the surface acoustic pressures $\tilde{b}_{pq}$ and then Eq. (17) can be used to calculate the modal displacements $\tilde{a}_{mn}$. Due to the diagonal structure of $[A]$, $[A]^{-1}$ is trivial.

C. Vibroacoustic indicators

Several vibroacoustic indicators can be deduced from the modal displacements $\tilde{a}_{mn}$ and the magnitude of the surface pressure $\tilde{b}_{pq}$. The indicators used in this paper are the mean-square transverse velocity of the base plate, the mean-square transverse velocity of the outer surface of the decoupling layer, the radiated sound power into the fluid, and the radiation efficiency of the system.

The mean-square velocity of the base plate is defined by

$$\langle V \rangle^2 = \frac{\omega^2}{2s} \int \int_s \tilde{w}_1(Q) \tilde{w}_1^*(Q) ds_Q, \tag{19}$$

where * designates the complex conjugate. Using the expansion of $\tilde{w}_1$ over the orthogonal *in vacuo* modes of the plate, one gets

$$\langle V_1 \rangle^2 = \frac{\omega^2}{8} \sum_{m=1}^{N} \sum_{n=1}^{N} |\tilde{a}_{mn}|^2. \tag{20}$$

Similarly, the mean-square velocity of the outer surface of the decoupling layer is

$$\langle V_2 \rangle^2 = \frac{\omega^2}{2s} \int \int_s \tilde{w}_2(Q) \tilde{w}_2^*(Q) ds_Q$$

$$= \frac{\omega^2}{8} \sum_{m=1}^{N} \sum_{n=1}^{N} |\tilde{a}_{mn}|^2 + \frac{\omega^2}{2|Z_c|^2} \sum_{p=-N_p}^{N_p} \sum_{q=-N_p}^{N_p} |\tilde{b}_{pq}|^2$$

$$+ \frac{\omega^2}{ab} \mathrm{Re} \left[\sum_{k=1}^{N} \sum_{l=1}^{N} \sum_{i=-N_p}^{N_p} \sum_{j=-N_p}^{N_p} \frac{a_{kl}^* b_{ij} S_{kloj}}{Z_c} \right], \tag{21}$$

where use has been made of the orthogonal properties of the $\psi_{pq}(Q)$, and Re denotes the real part.

The radiated sound power in the fluid is the integration of the active acoustic intensity normal to the decoupling layer,

$$W = \frac{1}{2} \mathrm{Re} \left[\int \int_s \tilde{P}(Q)(-j\omega) \tilde{w}_2^*(Q) ds_Q \right], \tag{22}$$

which gives

$$W=\frac{1}{2}\text{Re}\left[(-j\omega)\left[ab\sum_{p=-N_p}^{N_p}\sum_{q=-N_p}^{N_p}\frac{|b_{pq}|^2}{Z_c^*}\right.\right.$$
$$\left.\left.+\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{p=-N_p}^{N_p}\sum_{q=-N_p}^{N_p}a_{mn}^*S_{mnpq}b_{pq}\right]\right]. \quad (23)$$

Finally, the radiation efficiency in the heavy fluid is defined by

$$\sigma=\frac{W}{\rho_0 c_0 s\langle V_2\rangle^2}. \quad (24)$$

In the following, a simplified version of the above formulation is derived for the case of a low stiffness decoupling material.

D. Case of large decoupling

The motion of the base plate and the decoupling layer is largely decouple when the decoupling layer has a low stiffness (Z_c small). In this case the equation of the problem [Eqs. (11a), (11b)] can be rewritten in a simplified form. Two assumptions may be done in the case of high decoupling.

First, if the motion of the base plate and the decoupling layer are uncoupled, the fluid loading exerted on the base plate can be neglected. Thus, the equation of motion of the base plate is

$$\rho h\frac{ab}{4}\tilde{a}_{mn}(\omega_{mn}^2(1+j\eta)-\omega^2)=\tilde{f}_{mn}. \quad (25)$$

It is the *in vacuo* equation of motion of the base plate which can be readily solved for the modal displacement $\tilde{a}_{mn}$,

$$\tilde{a}_{mn}=\frac{4\tilde{f}_{mn}}{\rho hab(\omega_{mn}^2(1+j\eta)-\omega^2)}. \quad (26)$$

Let us now consider the transverse displacement of the outer surface of the decoupling layer $\tilde{w}_2(Q)$, and expand this displacement over a complete Fourier series,

$$\tilde{w}_2(Q)=\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\tilde{c}_{pq}\psi_{pq}(Q), \quad (27)$$

where ψ_{pq} is defined in Eq. (10b). Using Eq. (4), the Rayleigh integral [Eq. (8)] can be written

$$w_2(Q)-w_1(Q)=-\omega^2\frac{\rho_0}{Z_c}\int\int_s\tilde{w}_2(M)G(M,Q)ds_M. \quad (28)$$

The assumption of a large decoupling implies that the motion of the outer surface of the decoupling layer can be neglected in comparison to the motion of the base plate,

$$w_2(Q)\ll w_1(Q). \quad (29)$$

Thus, Eq. (28) simplifies into

$$w_1(Q)=\omega^2\frac{\rho_0}{Z_c}\int\int_s\tilde{w}_2(M)G(M,Q)ds_M. \quad (30)$$

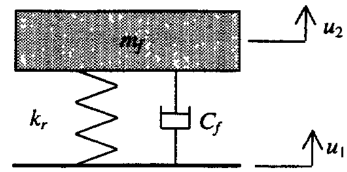

FIG 3 Schematic representation of the lumped parameter system.

Integrating this equation over the surface of the plate and using orthogonal properties in *in vacuo* modes of the plate, leads to

$$\frac{ab}{4}a_{mn}=\frac{\omega^2\rho_0}{Z_c}\sum_{r=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}\tilde{c}_{rs}H_{mnrs}. \quad (31)$$

Equation (30) allows the unknown coefficients of the transverse displacement of the outer surface of the decoupling layer $\tilde{c}_{rs}$ to be calculated,

$$\{\tilde{c}_{rs}\}=\frac{abZ_c}{4\omega^2\rho_0}[H_{mnrs}]^{-1}\{\tilde{a}_{mn}\}. \quad (32)$$

This approximate equation relates the motion of the base plate and the motion of the surface of the decoupling layer, and hence reflects the amount of decoupling provided by the layer. The mean-square velocity of the outer surface of the decoupling layer may then be expressed as

$$\langle V_2\rangle^2=\frac{\omega^2}{2}\sum_{r=-N_p}^{N_p}\sum_{s=-N_p}^{N_p}|\tilde{c}_{rs}|^2. \quad (33)$$

Equation (32) shows that the Fourier coefficients for the displacement of the outer surface of the decoupling layer are proportional to the impedance of the decoupling material Z_c. This amount of the decoupling provided by the coating is thus simply related to the coating stiffness. This result is similar to classical vibration isolation using compliant mount. On the other hand, the frequency dependence of the decoupling cannot be simply drawn from Eq. (32) since the matrix H_{mnrs} has a complex dependence on the frequency.

E. Lumped parameter system

This section presents an alternative and simplified model of a base structure covered by a decoupling layer, by instead considering a lumped parameter system (Fig. 3). This approach is closely related to a previous work by Sandman *et al.* (1995). In this case, the substrate plate reduces to a rigid piston, and the decoupling coating still behaves as a massless stiffness. For such a system the displacement u_2 of the outer surface of the coating is expressed as a function of the displacement of the base u_1 by the following equation:

$$u_2=\frac{k_r}{k_r-\omega^2(m_f+jC_f)}u_1. \quad (34)$$

In this analogy, u_1 represents the displacement of the base plate, u_2 the displacement of the outer surface of the decoupling layer, k_r the stiffness of the decoupling layer, m_f represents the added mass due to the fluid, and C_f the radiation damping due to the fluid. In high frequency, when $\omega^2 m_f \gg k_r$, the displacement u_2 is proportional to k_r, as in Eq.

TABLE I. Characteristics of the steel base plate used in the numerical simulations.

x-dimension (m) a	y-dimension (m) b	Thickness (mm) h	Density (kg/m³) ρ	Young's modulus (Pa) E	Loss factor η	Poisson's ratio v
0.6	0.6	9	7850	2.1×10^{11}	0.005	0.3

(32). In the case of a rigid circular piston, the fluid loading term m_f+jC_f is given by

$$m_f+jC_f=\pi(r_p)^2\rho_0c_0\frac{Z_f}{j\omega}, \quad (35)$$

where r_p is the piston radius, the radiation impedance Z_f is given by

$$Z_f=R+jX,$$

$$\text{with } R=1-\frac{2J_1(2k_0r_p)}{2k_0r_p} \text{ and } X=\frac{2H_1(2k_0r_p)}{2k_0r_p}. \quad (36)$$

J_1 is the first-order Bessel function defined by

$$J_1(x)=\frac{(x/2)}{(1!)^2}-2\frac{(x/2)^3}{(2!)^2}+3\frac{(x/2)^5}{(3!)^2}-\dots . \quad (37)$$

H_1 is the first-order Struve function,

$$H_1(x)=\frac{2}{\pi}\left(\frac{x^2}{1^2\cdot3}-\frac{x^4}{1^2\cdot3^2\cdot5}+\frac{x^6}{1^2\cdot3^2\cdot5}-\dots\right). \quad (38)$$

When $2k_0r_p\gg1$ Pierce (1981, p. 234, Eqs. 5-4.12a and 5-4.12b) uses an asymptotic expression,

$$J_1(x)\approx\sqrt{\frac{2}{\pi x}}\cos\left(x-\frac{3\pi}{4}\right), \quad (39)$$

$$H_1(x)\approx\frac{2}{\pi}+\sqrt{\frac{2}{\pi x}}\sin\left(x-\frac{3\pi}{4}\right). \quad (40)$$

Results given by this simple one degree of freedom system are compared to the proposed model of this paper in Sec. II.

II. NUMERICAL RESULTS

This section presents numerical results of the vibration reduction or sound attenuation that can be achieved by adding a compliant layer or an elastic base plate immersed in water. The vibroacoustic indicators calculated are the mean-square velocity of the base plate, the mean-square velocity of the outer surface of the decoupling layer, the radiation efficiency of the structure, and the radiated sound power in water. In order to characterize the decoupling efficiency, the ratio of the mean-square velocity of the base plate to the mean-square velocity of the outer surface of the coating is also presented. One underlying motivation of this numerical study is to derive a vibratory indicator representative of the acoustic performance of the decoupling treatment, and independent of the base plate dimensions.

The characteristics of the steel base plate used for all the numerical simulations are described in Table I. The excitation is a transverse point force applied at $x=y=0.06$ m from a corner of the plate. In the locally reacting model the decoupling material is characterized by its impedance $Z_c=B_c(1+j\eta_c)/h_c$; thus, increasing the modulus B_c is equivalent to decreasing the thickness h_c. In this study the thickness of the decoupling layer is fixed to 1 cm, whereas its modulus varies from 10^8 to 10^5 Pa. The loss factor of the decoupling layer is set to zero in order to study the decoupling effect without damping. It all cases the structure is immersed in water (density $\rho_0=1000$ kg/m³, sound speed $c_0=1460$ m/s. The expansion order used for this displacement and the acoustic pressure is $N=N_p=15$, and the number of Gauss points used for the numerical calculation of H_{klpq} and L_{ijpq} is $N_g=20$.

A. Vibration isolation results

Figures 4 and 5 show the mean-square velocity of the base plate $\langle V_1\rangle^2$ [Eq. (20)] and the mean-square velocity of the outer surface of the decoupling layer $\langle V_2\rangle^2$ [Eq. (21)] when $B_c=10^8$ and $B_c=10^6$ Pa. When $B_c=10^8$ Pa, the velocity of the layer outer surface is not significantly reduced as compared to the velocity of the base plate; the decoupling provided by the layer is low in this case. When $B_c=10^6$ Pa, the decoupling is much larger. It is seen from Figs. 4 and 5 that for a given value of B_c the frequency behavior of $\langle V_1\rangle^2$ and $\langle V_2\rangle^2$ shows the same peaks, corresponding to the resonance frequencies of the fluid-loaded base plate with the decoupling layer system. Moreover, when B_c is decreased, these resonance frequencies are shifted to higher frequencies, corresponding in the limiting case of a very compliant coating to the natural frequencies of the *in vacuo* base plate. In such a case, the base plate is isolated from the fluid loading by the compliant coating, and thus vibrates like an *in vacuo* plate. The approximate equation (33) would also predict that

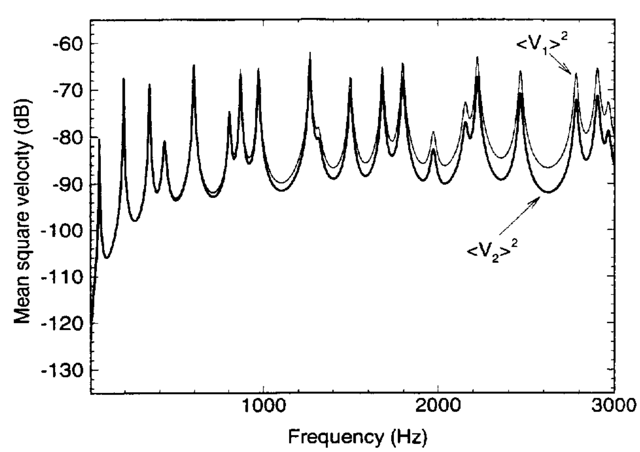

FIG 4 Mean-square velocity of the base plate $\langle V_1\rangle^2$ and mean-square velocity of the surface of the decoupling layer $\langle V_2\rangle^2$ when $B_c=10^8$ Pa, $\eta_c=0$, $h_c=1$ cm

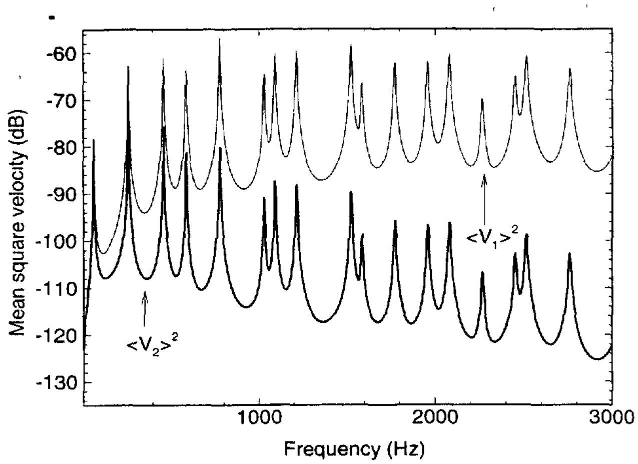

FIG 5 Mean-square velocity of the base plate $\langle V_1\rangle^2$ and mean-square velocity of the surface of the decoupling layer $\langle V_2\rangle^2$ when $B_c = 10^6$ Pa, $\eta_c=0$, $h_c=1$ cm

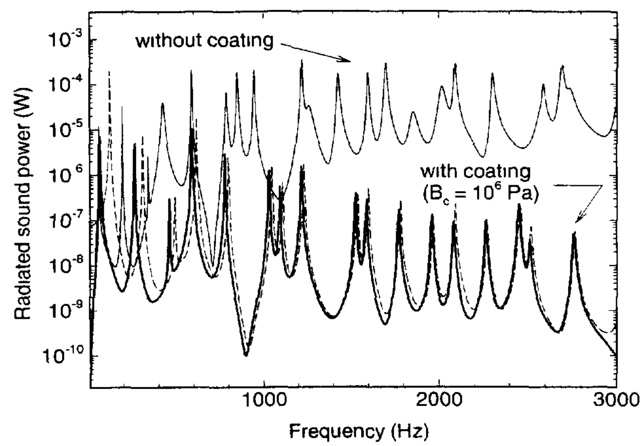

FIG 7 Radiated sound power in water without and with a decoupling layer ($B_c=10^6$ Pa, $\eta_c=0$, $h_c=1$ cm Comparison between the proposed model (—) and the high decoupling approximation (---)

decreasing the layer modulus by a factor 10 in this case still decreases $\langle V_2\rangle^2$ by an amount of 20 dB [$(Z_c)^2$ dependence of $\langle V_2\rangle^2$, Eq. (33)].

B. Sound radiation results

Figure 6 shows the radiation efficiency for the base plate without coating and with coating ($B_c=10^6$ Pa). This result shows that the radiation efficiency of the structure is globally unchanged by the addition of a compliant layer. Figure 7 shows the corresponding sound power radiated into water from the base plate without coating, and from the plate with coating ($B_c=10^6$ Pa). The acoustic attenuation provided by the coating is commensurate with the vibration attenuation provided by the coating, since the radiation efficiency is essentially unchanged by the coating.

Figure 7 shows that the "large decoupling" approximation (Sec. I D) satisfactorily represents the radiated sound power of the structure. This validates the *in vacuo* approximation used to calculate the motion of the base plate as well as the assumption that neglects the motion of the outer surface of the coating in comparison to the motion of the base plate.

C. Lumped parameter system

Results given by the lumped parameter system presented in Sec. I E are compared to the proposed model of this paper in this section. The coating material has the following properties: $B_c=10^6$ Pa, $h_c=10$ mm, $\eta_c=0$. The equivalent spring stiffness is given by: $k_r=\pi(r_p)^2B_c/h_c$, the piston area is chosen to be equal to the late area: $ab=\pi(r_p)^2$. In the numerical results, eight terms were used in expansion of the Bessel function [Eq. (37)] and of the Struve function [Eq. (38)]. This allows an accurate calculation of this piston radiation impedance up to 2 kHz in this case; above this frequency more terms should be used in the series expansions, or the high-frequency approximation given by Pierce should be used. The velocity ratio is the indicator chosen for the comparison between the models. In Fig. 8, the indicator $10\log_{10}(|u_1/u_2|^2)$ defined for the lumped parameter system is compared to the mean-square velocity ratio $10\log_{10}[\langle V_1\rangle^2/\langle V_2\rangle^2]$ defined for the plate-coating system. The two curves show the same tendencies; however, the lumped parameter system overestimates the elastic model by around 10 dB at low frequencies. This deviation is attributed to the difference in the type of motion (rigid body versus elastic) in both models. Above 2 kHz the lumped model

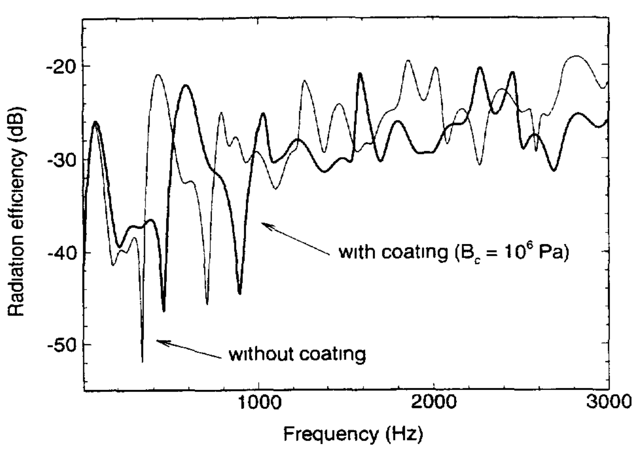

FIG 6. Radiation efficiency of the structure without and with a decoupling layer when $B_c=10^6$ Pa, $\eta_c=0$, $h_c=1$ cm.

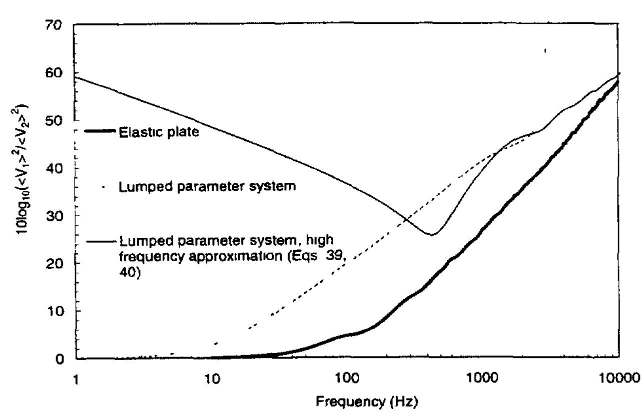

FIG 8 Comparison between the elastic plate + locally reacting model proposed in this paper and a simple one degree of freedom system where the radiation impedance is modeled using a circular rigid piston

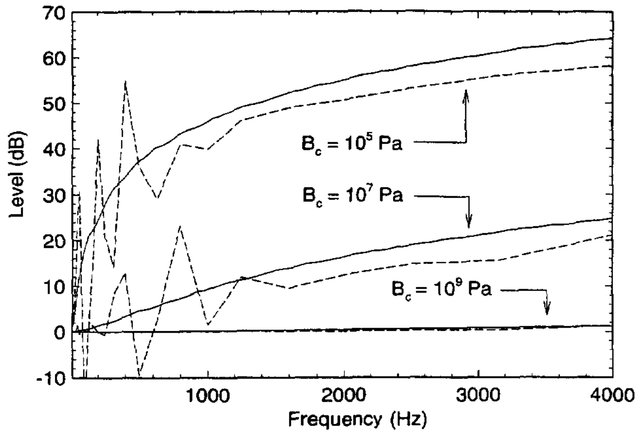

FIG 9 Comparison between the ratio of the mean-square velocity of the base plate to the mean-square velocity of surface of the decoupling layer (—) and the acoustic insertion loss (---), where $B_c=10^9$, $B_c=10^7$, $B_c=10^5$ Pa ($\eta_c=0$, $h_c=1$ cm in all cases)

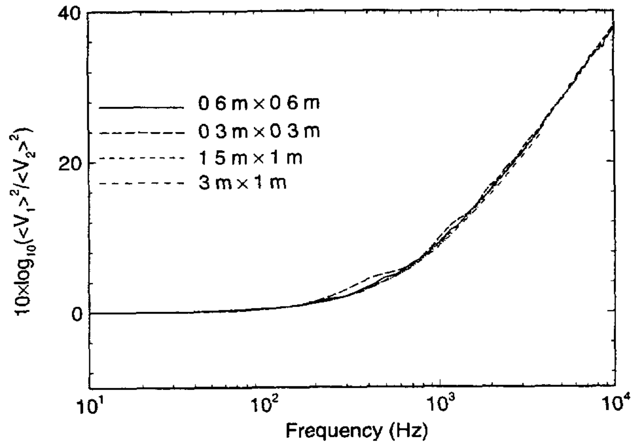

FIG 10 Ratio of the mean-square velocity of the base plate to the mean-square velocity of the surface of the decoupling layer when the plate dimensions are 0.6×0 6, 0.3×0 3, 1.5×1, 3×1 m ($B_c=10^7$ Pa, $\eta_c=0$, $h_c=1$ cm)

diverges because of the insufficient number of terms used to evaluate the piston radiation impedance. The high-frequency approximation of the lumped model (using a high-frequency asymptotic expression of the piston radiation impedance) was plotted on the same graph; it satisfactorily approximates the elastic plate model at high frequencies but diverges at low frequencies. This example shows that a very simple system is able to approximate tendencies in the behavior of the plate-coating system, but it does not allow accurate prediction of the vibration or noise attenuation of an elastic plate covered by a compliant coating and immersed in water.

D. Decoupling indicator

In Fig. 9, the ratio of the mean-square velocity of the base plate to the mean-square velocity of the decoupling layer, $10\log_{10}[\langle V_1\rangle^2/\langle V_2\rangle^2]$,is compared to the acoustic power insertion loss, $10\log_{10}(W_0/W)$, where W_0 is the radiated acoustic power of the structure without coating and W is the radiated acoustic power with coating. These results were obtained by solving Eqs. (17) and (18), without large decoupling approximation. From Fig. 9, it is seen that the mean-square velocity ratio is a nonmodal parameter that shows a smooth increase of the decoupling efficiency as a function of the frequency.

The acoustic power insertion loss characterizes the noise attenuation provided by the treatment. As seen in Fig. 7 there is a shift of the resonance frequencies between the response of the base plate alone and the response of the plate covered by a coating. This shift yields many peaks and dips when the acoustic power is plotted in the narrow-frequency band; the acoustic power insertion loss was thus plotted in 1/3-octave bands in Fig. 9. The differences at low frequencies between the mean-square velocity ratio and the insertion loss are due to the aforementioned frequency shifts. Both indicators show approximately the same frequency dependence but the velocity ratio is about 3 to 5 dB lower than the insertion loss. If the radiation efficiency is assumed to be unchanged when the coating is added (see Fig. 6), one can explain this 3 to 5 dB as the difference between the velocity of the base plate with and without coating.

E. Variation of the base structure dimensions

The effect of changing the base plate dimensions on the vibration isolation is investigated in this section. This point addresses the question of characterizing the vibroacoustic behavior of a decoupling layer on a test structure of small size, and transferring the result on a full scale, large structure. Figure 10 compares the mean-square velocity ratio for four different dimensions of the base plate (0.6×0.6, 0.3×0.3, 1.5×1, and 3×1 m). For all cases the coating characteristics are: $B_c=10^7$ Pa, $h_c=1$ cm, $\eta_c=0$. Clearly, this ratio is independent of the substrate size; similar results were obtained for all plate dimensions.

III. CONCLUSION

The vibroacoustic behavior of an elastic simply supported, rectangular baffled plate covered by a locally reacting decoupling layer supporting thickness deformation was presented in this paper. The model simulates the vibration and acoustic response of the system immersed in water, and subjected to a point force disturbance. A simplified version of the theory was derived in the limiting case of a large decoupling (low impedance of the layer/high frequency). On the other hand, the proposed model shows that the radiation efficiency of the structure is marginally affected by the addition of a locally reacting decoupling layer; the vibration attenuation provided by the decoupling layer is thus representative of the acoustic attenuation. Finally, the ratio of the mean-square velocity of the substrate to the mean-square velocity of the layer's surface is a smooth, nonmodal indicator approximately independent of the system x- and y-dimensions, and representative of the global acoustic performance of the decoupling treatment.

ACKNOWLEDGMENT

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APPENDIX: CALCULATION OF MATRICES H_{klpq} and L_{ijpq}

Matrices H_{klpq} and L_{ijpq} are written,

$$H_{klpq}=\int_0^a\int_0^b\int_0^a\int_0^b w_k(x)w_l(y)G(x,y,x_0,y_0)\times\psi_p^*(x_0)\psi_q'(y_0)dxdydx_0dy_0, \quad (A1)$$

$$L_{ijpq}=\int_0^a\int_0^b\int_0^a\int_0^b \psi_i(x)\psi_j(y)G(x,y,x_0,y_0)\times\psi_p^*(x_0)\psi_q'(y_0)dxdydx_0dy_0, \quad (A2)$$

where * denotes the complex conjugate, $\psi_p(x)=e^{j2\pi(px/a)}$, $\psi_q(y)=e^{j2\pi(qy/b)}$, $w_k(x)=\sin(k\pi x/a)$, $w_l(y)=\sin(l\pi(y/b)$, $G=e^{-jk_0R}/2\pi R$ and $R=\sqrt{(x-x_0)^2+(y-y_0)^2}$.

The following change of variables is used:

$$\begin{cases} u=\dfrac{x-x_0}{a} \\ v=\dfrac{x_0}{a} \end{cases} \text{and} \begin{cases} u'=\dfrac{y-y_0}{b} \\ v'=\dfrac{y_0}{b}. \end{cases} \quad (A3)$$

Using this change of variables, H_{klpq} and L_{ijpq} become

$$H_{klpq}=a^2b^2\int_{u'}\int_{v'}\left[\int_{-1}^0\int_{-u}^1 w_k(u,v)w_l(u',v')\times G(u,u')\psi_p^*(v)\psi_q^*(v')dvdu + \int_0^1\int_0^{1-u} w_k(u,v)w_l(u',v')G(u,u')\times\psi_p^*(v)\psi_q^*(v')dvdu\right]du'dv', \quad (A4)$$

$$L_{ijpq}=a^2b^2\int_{u'}\int_{v'}\left[\int_{-1}^0\int_{-u}^1 \psi_i(u,v)\psi_j(u',v')\times G(u,u')\psi_p^*(v)\psi_q^*(v')dvdu + \int_0^1\int_0^{1-u} \psi_i(u,v)\psi_j(u',v')G(u,u')\times\psi_p^*(v)\psi_q^*(v')dvdu\right]du'dv', \quad (A5)$$

In Eqs. (A4) and (A5), the limits of the integration of u' and v' are similar to the limits of integration on u and v, respectively.

The change of variable $u=-\tilde{u}$ is used in the integrals over u in (A4) and (A5). The first term of (A4) then leads to

$$\int_0^1\int_{-u}^1 w_k(-u,v)w_l(u',v')G(u,u')\psi_p^*(v)\psi_q^*(v')dvdu, \quad (A6)$$

and the first term of (A5) leads to

$$\int_0^1\int_{-u}^1 \psi_i(-u,v)\psi_j(u',v')G(u,u')\psi_p^*(v)\psi_q^*(v')dvdu, \quad (A7)$$

One can then analytically calculate the integrals over v in Eqs. (A4) and (A5). Repeating the same process for u' and v' leads to

$$H_{klpq}=a^2b^2\int_0^1\int_0^1 F_{kp}^B(u)F_{lq}^B(u')G(u,u')dudu', \quad (A8)$$

$$L_{ijpq}=a^2b^2\int_0^1\int_0^1 F_{ip}^C(u)F_{jq}^C(u')G(u,u')dudu', \quad (A9)$$

where $G(u,u')=(e^{-jk_0R}/2\pi R)$ and $R=\sqrt{a^2u^2+b^2(u')^2}$.

The function $F_{kp}^B(u)$ is defined by

$$F_{kp}^B(u)=\int_{-u}^1 w_k(-u,v)\psi_p^*(v)dv + \int_0^{1-u} w_k(u,v)\psi_p^*(v)dv. \quad (A10)$$

If k is even,

$$F_{kp}^B(u)=\frac{2j}{\pi(k^2-2p^2)}[2p\sin(\pi ku)-k\sin(2\pi pu)]. \quad (A11)$$

If k is odd,

$$F_{kp}^B(u)=\frac{2k}{\pi(k^2-4p^2)}[\cos(2\pi pu)+\cos(\pi ku)]. \quad (A12)$$

If $k=2p$,

$$F_{kp}^B(u)=j(u-1)\cos(\pi ku) - \frac{j}{\pi(k+2p)}[\sin(\pi ku)+\sin(2\pi pu)]. \quad (A13)$$

If $k=-2p$,

$$F_{kp}^B(u)=j(u-1)\cos(\pi ku) + \frac{j}{\pi(2p-k))}[\sin(\pi pu)-\sin(\pi ku)]. \quad (A14)$$

The function $F_{ip}^C(u)$ is defined by

$$F_{ip}^C(u)=\int_{-u}^1 \psi_i(-u,v)\psi_p^*(v)dv + \int_0^{1-u} \psi_i(u,v)\psi_p^*(v)dv. \quad (A15)$$

If $i\neq p$,

$$F_{ip}^C(u)=-\frac{2}{\pi(i-p)}\cos(\pi(i+p)u)\sin(\pi(i-p)u). \quad (A16)$$

If $i=p$,

$$F_{ip}^C(u)=2(1-u)\cos(2\pi iu). \quad (A17)$$

The function $F_{ip}^C(u)$ satisfies the symmetry relation $F_{ip}^C(u)=F_{pi}^C(u)$, such that: $L_{ijpq}=L_{pjiq}=L_{iqpj}=L_{pqij}$. The integrals (A8) and (A9) are finally solved using Gaussian quadrature formulas,

$$H_{klpq}=a^2b^2\sum_{r=0}^{N_g}\sum_{s=0}^{N_g} p_r p_s F^B_{kp}(u_r)F^B_{lq}(u'_s)G(u_r,u'_s)$$

$$\forall(u_r,u'_s)\in]0,1[, \quad \text{(A18)}$$

$$L_{ijpq}=a^2b^2\sum_{r=0}^{N_g}\sum_{s=0}^{N_g} p_r p_s F^C_{ip}(u_r)F^C_{jq}(u'_s)G(u_r,u'_s)$$

$$\forall(u_r,u'_s)\in]0,1[, \quad \text{(A19)}$$

where p_r and p_s are the weights associated with the Gauss points u_r and u'_s. N_g is the number of Gauss points for the integration. The singularity $u_r=u'_s=0$ is automatically removed because the Gauss points are in the interval]0,1[.

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