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AMAD - A FORTRAN PROGRAM TO CALCULATE ADDED MASS AND DAMPING OF TWO-DIMENSIONAL FLOATING BODIES BY THE BOUNDARY INTEGRAL EQUATION METHOD

Raymond A. Comeau - Samon Ando

Defence Research Establishment Atlantic

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ABSTRACT

This Technical Communication describes the underlying theory and use of the FORTRAN program AMAD (Added Mass And Damping), which calculates the hydrodynamic pressure distribution on the body surface, the added mass, and damping of a two-dimensional, laterally symmetric body heaving in the free surface of an unbounded fluid. Because AMAD uses the simple, free-space Green function $G = \log(1/r)$, "irregular frequencies" are not likely to occur.

RÉSUMÉ

Cet exposé technique décrit les principes de base et l'utilisation de AMAD (acronyme de "Added Mass and Damping", ce qui signifie "masse d'eau ajoutée et amortissement") un logiciel en FORTRAN, qui calcule la distribution de pression hydrodynamique sur la surface d'un corps, la masse d'eau ajoutée, et l'amortissement d'un corps bidimensionnel à symétrie latérale en mouvement de pilonnement sur la surface libre d'un fluide illimité. Puisque AMAD utilise la fonction simple de Green en espace libre G = log (l/r), il est peu probable qu'apparaîtront des "fréquences irrégulières".

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NOTATION

a	Complex displacement of the body
A_{ij}	Complex matrix to solve for ϕ
b	Damping
$\bar{b}$	Dimensionless damping
B	Breadth of body
C	Amplitude of body displacement
F	Body surface contour
g	Gravitational acceleration
G	Green function
H	Fluid depth
i	$= \sqrt{-1}$
K_i	Complex vector to solve for ϕ
m	Added mass
$\bar{m}$	Dimensionless added mass
m_o	Wavenumber
$\mathbf{n}$	Unit normal vector to S pointing out of the fluid
n_x, n_y	x and y components of $\mathbf{n}$
N_S	Offset point on the body
N_2	Offset point on the free surface
N_3	Offset point on the radiation boundary
N_T	Total number of offsets
P	Pressure
$\|P\|$	Hydrodynamic pressure amplitude
$\bar{P}$	Nondimensionalized hydrodynamic pressure
P_{ij}	Integral of $\log r$ along a segment
Q_{ij}	Integral of $\frac{\partial}{\partial n} \log r$ along a segment
r	Distance between the field and source points
R	Fluid region
Re	Real part of a complex variable
S	Fluid boundary
S_O	Body surface boundary
S_O^+	Body surface boundary in the plane $x \geq 0$
S_F	Free surface boundary
S_F^+	Free surface boundary in the plane $x \geq 0$
S_B	Sea bottom boundary
S_B^+	Sea bottom boundary in the plane $x \geq 0$
S_L	Left radiation boundary
S_R	Right radiation boundary
$\mathbf{V}$	Body surface velocity
x, y	Coordinates of the field point

x_L, x_R	Left and right radiation boundary positions, respectively
Y	Free surface elevation
α	Vertical displacement of the body
δ_{ij}	Kronecker delta function
ϵ	Hydrodynamic pressure phase angle
η	Offset y-coordinate
θ_j	Segment angle with the horizontal
λ	Wavelength
ν	$=\omega^2/g$
ξ, η	Coordinates of the source point
ρ	Fluid mass density
ϕ	Space component of Φ
ω	Circular frequency of oscillation of the body
Φ	Time dependent velocity potential
∇	Gradient operator

Subscripts

i, j	Indicies for contour segments

1. INTRODUCTION

The purpose of this Technical Communication is to describe the FORTRAN program AMAD (Added Mass And Damping), which calculates the added mass, damping, and the hydrodynamic pressure distribution of a two-dimensional body heaving sinusoidally in the free-surface of water.

Added mass is an effective mass of the fluid which accelerates with a body. Damping is the energy loss due to wave making from the fluid reactions in phase with the velocity of the body. For a given frequency of oscillation the added mass and damping are functions of the body form and the density of the fluid. Under some simplifying assumptions, the added mass and damping may be calculated from the velocity potential on the body in a straightforward manner.

In AMAD, the boundary-value problem for the velocity potential is solved by the boundary-integral equation method using the free-space Green function for two-dimensional Laplace's equation. The radiation boundaries are artificially set at a finite distance from the body for computational purposes. Although AMAD treats only symmetric bodies and a flat sea bottom, the solution method can be modified for any shape of the body and seabed; see, for example, Yeung[1]. As well, extension of the present method to swaying and rolling motions is straightforward because only the boundary condition on the body surface needs to be modified.

The mathematical formulation of the linearized problem is outlined in Section 2. Section 3 describes the solution method, which can be used for both symmetrical and asymmetrical body contours. Hydrodynamic pressure, added mass, and damping are derived in Section 4. In section 5, the effects of offset spacing and radiation boundary position on the added mass and damping are given. Descriptions of the input and output to AMAD are described in Section 6 and 7, respectively. Instruction on the execution of AMAD is described in Section 8. Finally, concluding remarks are given in Section 9.

2. FORMULATION OF THE PROBLEM

Figure 1 shows the coordinate system. The body executes a sinusoidal oscillation in the vertical direction ("heaving motion") about the equilibrium point o in the undisturbed free surface. The amplitudes of motions and velocities of the body and the fluid are assumed small. The undisturbed free surface corresponds to the plane $y = 0$, and the positive $y-$axis points vertically upwards. The body contour is described by $F(x, y, t) = 0$.

It is assumed that the fluid is incompressible, inviscid, and its motion irrotational. The last assumption guarantees the existence of the velocity potential. To find the quantities of interest (hydrodynamic pressure, added mass, and damping), it is first necessary to calculate the velocity potential.

The velocity potential Φ satisfies Laplace's equation

$$\nabla^2\Phi(x,y,t) = \frac{\partial^2\Phi(x,y,t)}{\partial x^2} + \frac{\partial^2\Phi(x,y,t)}{\partial y^2} = 0 \tag{1}$$

in the fluid domain.

The kinematic condition on the body surface requires that the fluid particles next to the body have the same normal velocity as the body. This condition is expressed by

$$\left.\frac{\partial\Phi}{\partial n}\right|_{F(x,y,t)=0} = \mathbf{V}\cdot\mathbf{n} \tag{3}$$

where $\mathbf{n}=(n_x, n_y) = \mathbf{n} = \nabla F/|\nabla F|$ is the normal vector pointing out of the fluid, ∇ is the gradient operator, and $\partial/\partial n = \mathbf{n}\cdot\nabla$. The velocity of the body surface is

$$\mathbf{V} = \dot{\alpha}(t)\mathbf{j} \tag{4}$$

with $\alpha(t)$ denoting the body displacement in the vertical direction. Thus, the boundary condition on the body surface can be expressed as

$$\left.\frac{\partial\Phi}{\partial n}\right|_{S_O} = n_y\dot{\alpha}(t) \tag{5}$$

where n_y is the y component of the unit vector normal to the body surface S_O.

On the free surface, Φ must satisfy two conditions. One is the kinematic condition that that the fluid particles on the free surface remain on the free surface. Let the free surface elevation be described by $y = Y(x,t)$, then the kinematic condition is given by

$$\left.\frac{\partial\Phi}{\partial n}\right|_{y=Y(x,t)} = \frac{Y_t}{\sqrt{1+Y_x{}^2}} \quad \text{for} \quad \mathbf{n} = \frac{\nabla(y-Y)}{|\nabla(y-Y)|}$$

or,

$$-\Phi_x(x,Y,t)Y_x + \Phi_y = Y_t. \tag{6}$$

The other is the dynamic condition that the pressure on the free surface be equal to the atmospheric pressure (that is, zero gauge pressure). Applying Bernoulli's equation at the free surface gives

$$\Phi_t(x,Y,t) + gY + \frac{1}{2}|\nabla\Phi|^2 = 0. \tag{7}$$

Assuming small body displacement and wave amplitudes, the second order terms of (6) and (7) can be neglected. The resulting linearized conditions

$$\Phi_y - Y_t = 0 \tag{8}$$

and

$$\Phi_t + gY = 0 \tag{9}$$

are applied on the undisturbed free surface $y = 0$. Equations (8) and (9) can be combined to eliminate the unknown Y,

$$\Phi_{tt} + g\Phi_y = 0 \quad \text{on} \quad y = 0. \tag{10}$$

Equation (10) is the condition to be applied on the free surface in the formulation for AMAD.

On the sea bottom, no flow may cross the boundary; so the sea-bottom condition is

$$\left.\frac{\partial\Phi}{\partial n}\right|_{y=-H} = 0 \tag{11}$$

where H is the depth of water.

In addition, the radiation condition at infinity needs to be satisfied to make the solution unique.

2.1 TIME-HARMONIC PROBLEMS

The body is assumed to be heaving sinusoidally at an angular frequency ω with an amplitude C (forced motion). Then, the displacement of the body takes the form

$$\alpha(t) = \text{Re}[a(t)] = \text{Re}[Ce^{-i\omega t}] = C\cos\omega t \tag{12}$$

with $i = \sqrt{-1}$.

The body boundary condition (5) suggests that the velocity potential Φ be expressed as

$$\Phi(x,y,t) = \text{Re}[\phi(x,y)\dot{a}(t)] = \phi_1\dot{\alpha}(t) + \omega\phi_2\alpha(t) \tag{13}$$

where $\phi = \phi_1 + i\phi_2$ represents the velocity potential due to the body motion with unit velocity, and is dependent only upon the space coordinates and has dimensions of length.

Now substituting (13) into the governing equations (1),(10),(5), and (11) gives the following boundary-value problem for ϕ:

$$\nabla^2\phi = \phi_{xx} + \phi_{yy} = 0 \quad \text{for} \quad y < 0 \tag{14}$$

$$\phi_y - \nu\phi = 0 \quad \text{on} \quad y = 0, \tag{15}$$

$$\left.\frac{\partial\phi}{\partial n}\right|_{S_o} = n_y \tag{16}$$

$$\left.\frac{\partial\phi}{\partial n}\right|_{S_B} = 0 \tag{17}$$

where $\nu = \omega^2/g$, S_O is the body contour below $y = 0$, and S_B is the sea bed. In addition, a condition at infinity is needed to make the solution unique. Here the following Sommerfeld radiation condition is adopted:

$$\lim_{x\to\pm\infty}\sqrt{x}\left(\frac{\partial\phi}{\partial x} \mp im_o\phi\right) = 0 \tag{18}$$

where m_o is the wave number, and is the root of the following transcendental equation

$$m_o \tanh m_o H = \nu. \tag{19}$$

Note that $m_o \to \nu$ as $H \to \infty$. For computational purposes, the radiation condition is set at a finite distance from the body. This condition will be valid if, where the condition is set, the difference between the total potential and the propagating wave potential is negligible. Radiation conditions now take the form

$$\frac{\partial\phi}{\partial x}(x_R, y) = im_o\phi \quad , \quad x_R \gg 0 \tag{20}$$

$$\frac{\partial\phi}{\partial x}(x_L, y) = -im_o\phi \quad , \quad x_L \ll 0. \tag{21}$$

3. SOLUTION METHOD

The present method for solving the boundary-value problem for ϕ expressed by (14)–(17), (20), and (21) is based on the simple Green's function approach. That is, the Green's function G is the simple source function

$$G(x,y;\xi,\eta) = \log\frac{1}{r} \tag{22}$$

where (x,y) and (ξ,η) are two points in the fluid domain, and r is the distance between these points, i.e., $r = \sqrt{(x-\xi)^2+(y-\eta)^2}$. A more traditional Green's function (see, for example, Frank[2]) satisfies the boundary conditions on the free surface and a flat bottom of an infinitely deep fluid, and the radiation condition at infinity. Using such a Green function has the advantage over the present method in that only the body boundary needs to be discretized; however, while the present method can be modified to accomodate an arbitrary sea bottom contour at finite depth, Frank's method cannot. Also, Frank's method breaks down at certain wave frequencies, called irregular frequencies. The method used here is unlikely to suffer from the phenomenon of irregular frequencies because of its simple kernel.

As illustrated in Figs. 1 and 2, the geometry of the problem is described as follows. Let R be a closed regular region in two dimensional space, whose boundary is S. If we restrict the point (x,y) to lie on S, then by applying Green's second identity, we get (see, for example, Newman[3])

$$\pi\phi(x,y) = \oint_S \left(\frac{\partial\phi}{\partial n}G - \phi\frac{\partial G}{\partial n}\right) dS. \tag{23}$$

Referring to Figure 2, let S consist of the body contour S_O, the free surface S_F, two radiation surfaces S_R and S_L, and the bottom S_B. Then we obtain from equation (23) together with equations (14)–(17), (20), (21), the following integral equation for the unknown values of ϕ on S:

$$\begin{aligned}\pi\phi(x,y) = &\int_{S_O} \phi\frac{\partial}{\partial n}(\log r)dS + \int_{S_F} \phi[\frac{\partial}{\partial n}\log r - \nu\log r]dS + \int_{S_B} \phi\frac{\partial}{\partial n}(\log r)dS \\ &+ \int_{S_R} \phi[\frac{\partial}{\partial n}\log r - im_o\log r]dS + \int_{S_L} \phi[\frac{\partial}{\partial n}\log r - im_o\log r]dS - \int_{S_O} n_y\log r dS.\end{aligned} \tag{24}$$

3.1 HEAVING MOTION OF SYMMETRIC BODIES

For heaving motion of laterally symmetric bodies, the potential ϕ is symmetric about $x = 0$,

$$\phi(-x,y) = \phi(x,y). \tag{25}$$

The integral equation (24) can then be written as

$$\begin{aligned}\int_{S_O^+} n_y[\log r^+ + \log r^-]dS = &-\pi\phi(x,y) + \int_{S_O^+} \phi[\frac{\partial}{\partial n^+}\log r^+ + \frac{\partial}{\partial n^-}\log r^-]dS \\ &+ \int_{S_F^+} \phi[(\frac{\partial}{\partial n^+}\log r^+ - \nu\log r^+) + (\frac{\partial}{\partial n^-}\log r^- - \nu\log r^-)]dS\end{aligned}$$

$$+\int_{S_R} \phi[(\frac{\partial}{\partial n^+}\log r^+ - im_o \log r^+) + (\frac{\partial}{\partial n^-}\log r^- - im_o \log r^-)]dS$$

$$+\int_{S_B^+} \phi[\frac{\partial}{\partial n^+}\log r^+ + \frac{\partial}{\partial n^-}\log r^-]dS \tag{26}$$

where S_O^+, S_F^+ and S_B^+ indicate respectively the segments of S_O, S_F, and S_B that are in the half plane, $x \geq 0$, and

$$\frac{\partial}{\partial n^+} = \frac{\partial}{\partial \xi}n_x + \frac{\partial}{\partial \eta}n_y \tag{27}$$

$$\frac{\partial}{\partial n^-} = -\frac{\partial}{\partial \xi}n_x + \frac{\partial}{\partial \eta}n_y$$

$$r^+ = [(x-\xi)^2 + (y-\eta)^2]^{\frac{1}{2}} \tag{28}$$

$$r^- = [(x+\xi)^2 + (y-\eta)^2]^{\frac{1}{2}}.$$

3.2 DISCRETIZATION OF THE INTEGRAL EQUATION

After discretization of the boundary, the integral equation (25) is reduced to a set of linearly independent equations for ϕ. The solution gives the values of ϕ along the boundary. Then, as shown in Section 4 below, hydrodynamic pressure on the body, added mass, and damping can be calculated using the potential along the body.

The entire fluid boundary is divided into a series of straight line segments running from offset (ξ_j, η_j) to (ξ_{j+1}, η_{j+1}); see Figure 3. The points $j = 1$ and $j = N_T$ are on the line of symmetry. Each segment makes an angle θ_j with the horizontal, defined by

$$\theta_j = \arctan\frac{\eta_{j+1} - \eta_j}{\xi_{j+1} - \xi_j} \tag{29}$$

and the normal vector's y component is

$$n_{y_j} = \cos\theta_j. \tag{30}$$

The distribution of ϕ along each segment is assumed to be constant with $\phi_j = \phi(\frac{\xi_j+\xi_{j+1}}{2}, \frac{\eta_j+\eta_{j+1}}{2})$ for $j = 1, 2, \cdots, N_T - 1$. The field point (x_i, y_i) is taken at the midpoint of each line segment.

To simplify the ensuing mathematical expressions, the following notation is introduced. (For details of (31)–(34), see, for example, Frank[2].) Let

$$P_{ij}^R = \int_{(\xi_j,\eta_j)}^{(\xi_{j+1},\eta_{j+1})} \log[(x_i-\xi)^2 + (y_i-\eta)^2]^{\frac{1}{2}} dS$$

$$= \cos\theta_j\{(\xi_j - \xi_{j+1}) - \frac{1}{2}(x_i - \xi_{j+1})\log[(x_i-\xi_{j+1})^2 + (y_i-\eta_{j+1})^2] + \frac{1}{2}(x_i-\xi_j)\log[(x_i-\xi_j)^2$$

$$+ (y_i-\eta_j)^2]\} + \sin\theta_j\{(\eta_j - \eta_{j+1}) - \frac{1}{2}(y_i-\eta_{j+1})\log[(x_i-\xi_{j+1})^2 + (y_i-\eta_{j+1})^2]$$

$$+ \frac{1}{2}(y_i-\eta_j)\log[(x_i-\xi_j)^2 + (y_i-\eta_j)^2]\} - Q_{ij}^R[(y_i-\eta_{j+1})\cos\theta_j - (x_i-\xi_{j+1})\sin\theta_j] \tag{31}$$

$$P_{ij}^L = P_{ij}^R(-\xi_j, \eta_j; -\xi_{j+1}, \eta_{j+1}) \tag{32}$$

and

$$Q_{ij}^{R} = \int_{(\xi_j,\eta_j)}^{(\xi_{j+1},\eta_{j+1})} \frac{\partial}{\partial n^{+}} \log[(x_i-\xi)^2+(y_i-\eta)^2]^{\frac{1}{2}} dS$$

$$= \arctan(\frac{y_i-\eta_j}{x_i-\xi_j}) - \arctan(\frac{y_i-\eta_{j+1}}{x_i-\xi_{j+1}}) \qquad (33)$$

$$Q_{ij}^{L} = -Q_{ij}^{R}(-\xi_j,\eta_j;-\xi_{j+1},\eta_{j+1}). \qquad (34)$$

Thus, $Q_{ij}^{R}=0$ when $i=j$.

Using (31)–(34), discretization of (26) can be expressed as

$$-\pi\phi(x_i,y_i) + \sum_{j=1}^{N_s}\phi_j[Q_{ij}^{R}+Q_{ij}^{L}] + \sum_{j=N_S+1}^{N_2}\phi_j[Q_{ij}^{R}-\nu P_{ij}^{R}+Q_{ij}^{L}-\nu P_{ij}^{L}]$$

$$+\sum_{j=N_2+1}^{N_3}\phi_j[Q_{ij}^{R}-im_oP_{ij}^{R}+Q_{ij}^{L}-im_oP_{ij}^{L}] + \sum_{j=N_3+1}^{N_T-1}\phi_j[Q_{ij}^{R}+Q_{ij}^{L}] = \sum_{j=1}^{N_S} n_{y_j}[P_{ij}^{R}+P_{ij}^{L}] \qquad (35)$$

for $i=1,2,3,\cdots N_T-1$.

Equation (35) can be simplified as

$$\sum_{j=1}^{N_T-1} A_{ij}\phi_j = K_i \quad i=1,2,3,\cdots,N_T-1 \qquad (36)$$

$$A_{ij} = \begin{cases} -\pi\delta_{ij}+Qij^{R}+Q_{ij}^{L}, & \text{for } j=1,\cdots,N_S \quad j=N_3+1,\cdots,N_T-1 \\ -\pi\delta_{ij}+(Q_{ij}^{R}-\nu P_{ij}^{R})+(Q_{ij}^{L}-\nu P_{ij}^{L}) & j=N_S+1,\cdots,N_2 \\ -\pi\delta_{ij}+(Q_{ij}^{R}-im_0P_{ij}^{R})+(Q_{ij}^{L}-im_0P_{ij}^{L}) & j=N_2+1,\cdots,N_3 \end{cases}$$

$$K_i = \sum_{j=1}^{N_S} n_{y_j}[P_{ij}^{R}+P_{ij}^{L}]$$

where the Kronecker delta function δ_{ij} is defined by

$$\delta_{ij}=1, \quad \text{if} \quad i=j, \quad \text{and} \quad \delta_{ij}=0, \quad \text{if} \quad i\neq j. \qquad (37)$$

Solving equation (36) will give the potential along all boundaries.

4. HYDRODYNAMIC PRESSURE, ADDED MASS, AND DAMPING

Define

$$m+i\frac{1}{\omega}b = \rho\int_{S_O}\phi n_y dS \qquad (38)$$

where S_O includes the left hand side of the body, m is the added mass, b is the damping, and ρ is the density of the fluid.

Discretization of (38) gives

$$m + i\frac{1}{\omega}b = 2\sum_{j=1}^{N_S} \rho\phi_j n_{y_j}[(\xi_{j+1} - \xi_j)^2 + (\eta_{j+1} - \eta_j)^2]^{\frac{1}{2}}. \quad (39)$$

Introduce dimensionless added-mass and damping coefficients as follows:

$$\bar{m} = \frac{m}{\frac{1}{2}\pi\rho(\frac{B}{2})^2} \quad (40)$$

$$\bar{b} = \frac{b}{\frac{1}{2}\pi\rho\omega(\frac{B}{2})^2} \quad (41)$$

where B is the breadth of the body at $y = 0$. Then (39) becomes

$$\bar{m} + i\bar{b} = \frac{16\sum_{j=1}^{N_S} \phi_j n_y[(\xi_{j+1} - \xi_j)^2 + (\eta_{j+1} - \eta_j)^2]^{\frac{1}{2}}}{\pi B^2}. \quad (42)$$

Values for added mass and damping for an infinitely deep fluid are calculated by Frank[2] using a more conventional Green function. Figure 4 plots values of added mass and damping calculated by AMAD and Frank, against frequency. These values agree quite well with each other at high frequencies, since a depth of a third of a wavelength can be considered infinitely deep. At low frequencies, the values diverge. This is expected, since the effects of finite depth are more noticeable.

Hydrodynamic pressure P on the body surface is found from the linearized Bernoulli's equation

$$\frac{P}{\rho} = -\frac{\partial\Phi}{\partial t}. \quad (43)$$

Substituting (13) into (43),

$$P = \omega^2\rho C[\phi_1 \cos\omega t + \phi_2 \sin\omega t]. \quad (44)$$

The amplitude of the hydrodynamic pressure is

$$|P| = \rho\omega^2 C\sqrt{\phi_1^2 + \phi_2^2}. \quad (45)$$

and the phase angle ϵ

$$\epsilon = \arctan\left(\frac{\phi_2}{\phi_1}\right). \quad (46)$$

If ϵ is positive, hydrodynamic pressure is lagging the displacement.

The hydrodynamic pressure amplitude is nondimensionalized as follows:

$$\bar{P} = \frac{|P|}{\rho g C} = \nu\sqrt{\phi_1^2 + \phi_2^2}. \quad (47)$$

5. BOUNDARY DISCRETIZATION PARAMETERS

A scheme for determining the radiation boundary position, and the offset spacing for the free surface, radiation, and bottom boundaries is given here.

In general, the dimensionless frequencies of interest in ship motion are

$$0.2 < \frac{\omega^2 B}{2g} < 1.5. \tag{48}$$

Therefore, test calculations were made at low, middle, and high frequencies, ($\omega^2 B/2g = 0.2, 0.9$, and 1.5) with two different values of $(2H/B)$.

5.1 RADIATION BOUNDARY POSITION

Figures 5(a)–5(c) show the effect of radiation boundary position on the added mass and damping. For the range of the frequency parameter indicated in (48), the effect of variations of the values of $(x_R - B/2)/H$ on the added mass and damping is seen to be small. These values are calculated with a free surface spacing, $(\xi_{j+1} - \xi_j)/\lambda = 0.05$, and 6 points on the radiation boundary to a depth of a third of a wavelength.

5.2 FREE SURFACE SPACING

Offset spacing on the free surface is uniform, that is, $\xi_{j+1} - \xi_j = l_F =$ constant, for $N_S + 1 \leq j \leq N_2$. Figures 6(a)–6(c) are calculated using the value $(x_R - B/2)/H$ of 0.8, and 6 points on the radiation boundary to a depth of a third of a wavelength. It is seen that the effect of the value of l_F/λ on the calculated added mass and damping is small.

5.3 RADIATION BOUNDARY SPACING

The effects of waves on fluid motion rapidly diminish with depth, so that offset spacing may be increased with increasing depth. Also, virtually no fluid motion occurs beyond a depth of approximately a third of a wavelength. In the light of these facts, offset spacing on the radiation boundary in the formulation of AMAD is increased with increasing depth according to the cosine distribution from the free surface down to a depth of a third of a wavelength: that is,

$$\eta_j = -\frac{\lambda}{3}\left[1 - \cos\left(\frac{\pi}{2}\frac{j - N_2 + 1}{N}\right)\right], \quad N_2 + 1 \leq j \leq N + N_2 + 1.$$

The value of N is set equal to 4 in AMAD. For the rest of the radiation boundary and the sea bottom, offset spacing is made equal to the final spacing (i.e. spacing at depth one third the wavelength). Figures 7(a)–7(c) show the effect of the number of offsets on the added mass and damping, where NRAD is the number of offsets on the radiation boundary to a depth of $\lambda/3$. The number of offsets to this depth should be 5 or greater. These values are calculated with a free surface spacing, $(\xi_{j+1} - \xi_j)/\lambda = 0.05$, and radiation boundary position of $(x_R - B/2)/H$ of 0.8 .

6. DESCRIPTION OF INPUT

Data Set 1	(72 characters or less - CHARACTER FORMAT)
TITLE	Used for identification of the body section.

Data Set 2 NP	(1 Integer - FREE FORMAT) The number of offsets which makes up the body.
Data Set 3 (XI(I),I=1,NP)	(NP Real numbers - FREE FORMAT) The X-coordinates of the body points. The first point must be on the y-axis (XI(1)=0), and all points must be positive.
Data Set 4 (ETA(I),I=1,NP)	(NP Real numbers - FREE FORMAT) The Y-coordinates of the body points. The last point must be on the x-axis (ETA(NP)=0), and all points must be negative. The last point will be taken as the half breadth of the body (ETA(NP)=B/2)
Data Set 5 XRAD	(1 Real - FREE FORMAT) Dimensionless radiation boundary condition position, $XRAD = \frac{X_R - B/2}{H}$. A value of 0.8 or greater is suggested.
Data Set 6 FS	(1 Real - FREE FORMAT) Dimensionless free surface segment length FS= (OFFSET SPACING)/λ. A value of 0.06 or smaller is suggested.
Data Set 7 NRAD	(1 Integer - FREE FORMAT) The number of offsets on the radiation boundary to a depth of $\lambda/3$ which are spaced by a cosine function. A value of 5 or greater is suggested.
Data Set 8 NOD	(1 Integer - FREE FORMAT) The number of different dimensionless fluid depths
Data Set 9 (DH(I),I=1,NOD)	(NOD Real Numbers - FREE FORMAT) Dimensionless fluid depths ($DH(I)=\frac{2H}{B}$).
Data Set 10 NOF	(1 Integer - FREE FORMAT) Number of dimensionless frequencies of oscillation for the given depths.
Data Set 11 (DOMEGA(I),I=1,NOF)	(NOF Real numbers - FREE FORMAT) Dimensionless frequencies of oscillation. ($DOMEGA(I)=\frac{\omega^2 B}{2g}$).

A sample input is given in Appendix A.

7. DESCRIPTION OF OUTPUT

Appendix B shows the corresponding output from the sample input of Appendix A. Output begins with a printout of the input file name, the title, the body breadth, the offsets constituting the body form, and parameters dealing with boundary offset spacing, and radiation condition position. Then for each combination of dimensionless depth (DH), and frequency of oscillation (DS), values for dimensionless added mass and damping are printed. Hydrodynamic pressure distribution follows, giving the dimensionless pressure amplitude (PBAR), and phase angle (EPSILON), for each segment of the body.

8. EXECUTION OF AMAD

The following is an example of the execution of AMAD

```
RUN AMAD                                                    (SYSTEM)

AMAD - This program will determine the added mass, damping, and hydrodynamic
pressure distribution for a laterally symmetric cylinder heaving in the
free surface of fluid with a flat bottom.

Input filename?  (9 characters)                      (SYSTEM PROMPT)
CIRC1.DAT                                               (USER INPUT)

Output filename?  (9 characters)                     (SYSTEM PROMPT)
ADMAD.DAT                                               (USER INPUT)
```

AMAD will now run and print calculated values into the output file.

9. Concluding Remarks

The boundary-integral equation method using the simple source, $\log(1/r)$, is applied to calculate the hydrodynamic pressure distribution, added mass and damping of a two dimensional body. Although only the case where the body has lateral symmetry and is heaving in the free surface of a fluid with a flat bottom is treated in AMAD, extension to asymmetric bodies as well as to swaying and rolling motions is relatively straightforward.

The present method requires discretization of all boundaries of the fluid domain including radiation boundaries. While this does result in large matrices to solve, this method is advantageous in two respects. First, the phenomenon of "irregular frequencies," at which the more traditional Green function method fails, is not likely to occur. Second, the resulting equations have simple kernels and are easy to program.

On the other hand, compared with the method based on the Green function that satisfies all the boundary conditions except for the body boundary condition, the present method requires considerably more computer time. For example, on a DEC 20 computer, Frank's method takes less than 1 second of CPU time to execute the sample input given in Appendix A, while AMAD takes approximately 5 seconds. (Note that, in Frank's method, the fluid is treated as infinitely deep.) A major portion of the computer time for AMAD is spent on computing the terms of the main matrix. In setting up the boundaries for the problem, it should be borne in mind that the computation time is roughly proportional to the square of the number of line segments.

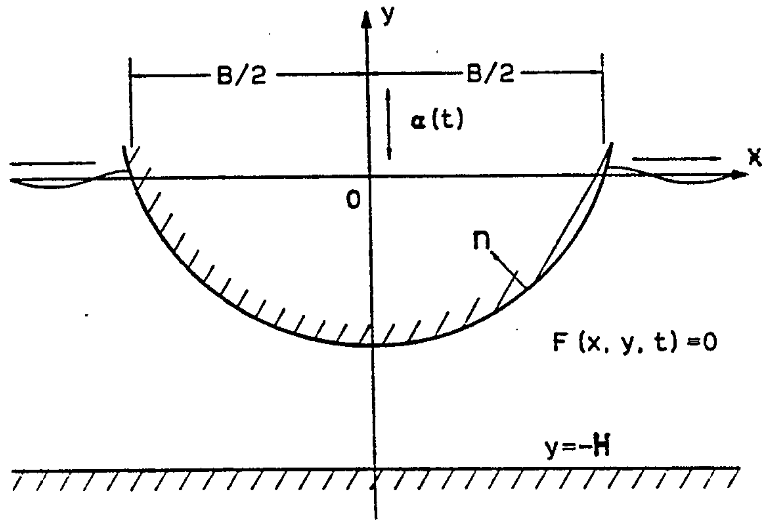

Fig. 1. Geometry of the problem.

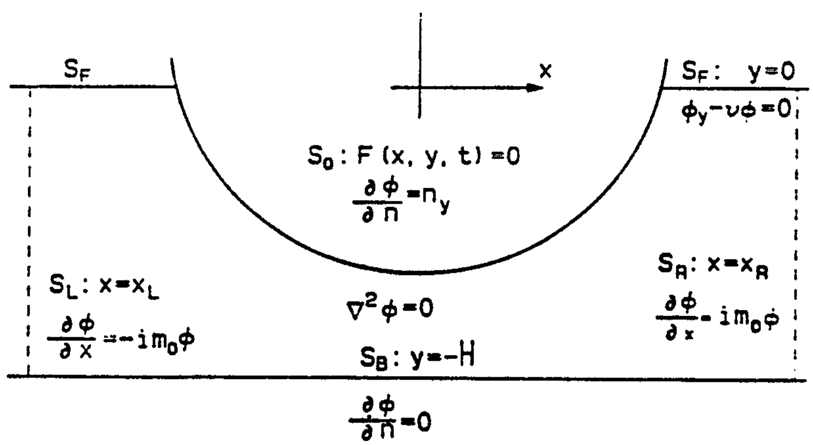

Fig. 2. Boundary value problem for ϕ.

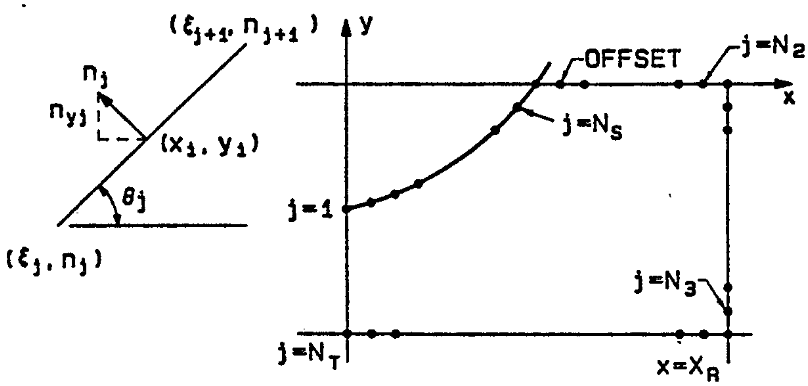

Fig. 3 Discretization of the boundary.

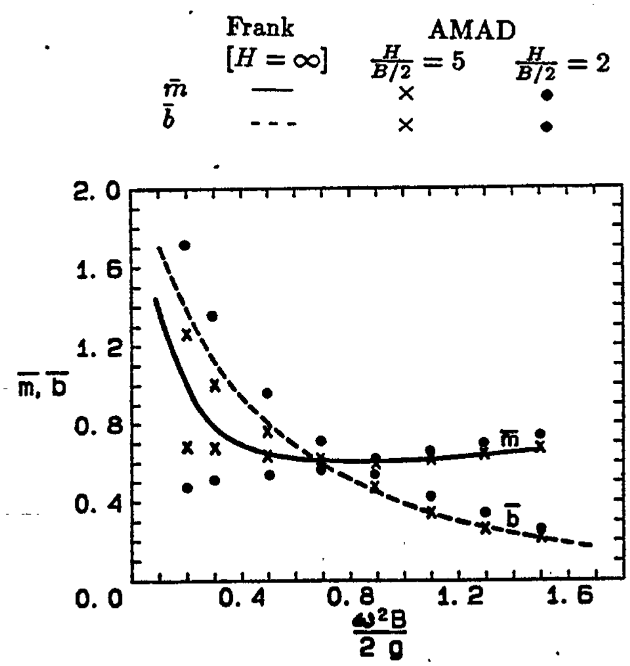

Fig. 4. Dimensionless added mass ($\bar{m}$) and damping ($\bar{b}$) for a heaving circular cylinder (Fig. 1) calculated by Frank's method[2] and by AMAD.

(a)

(b)

(c)

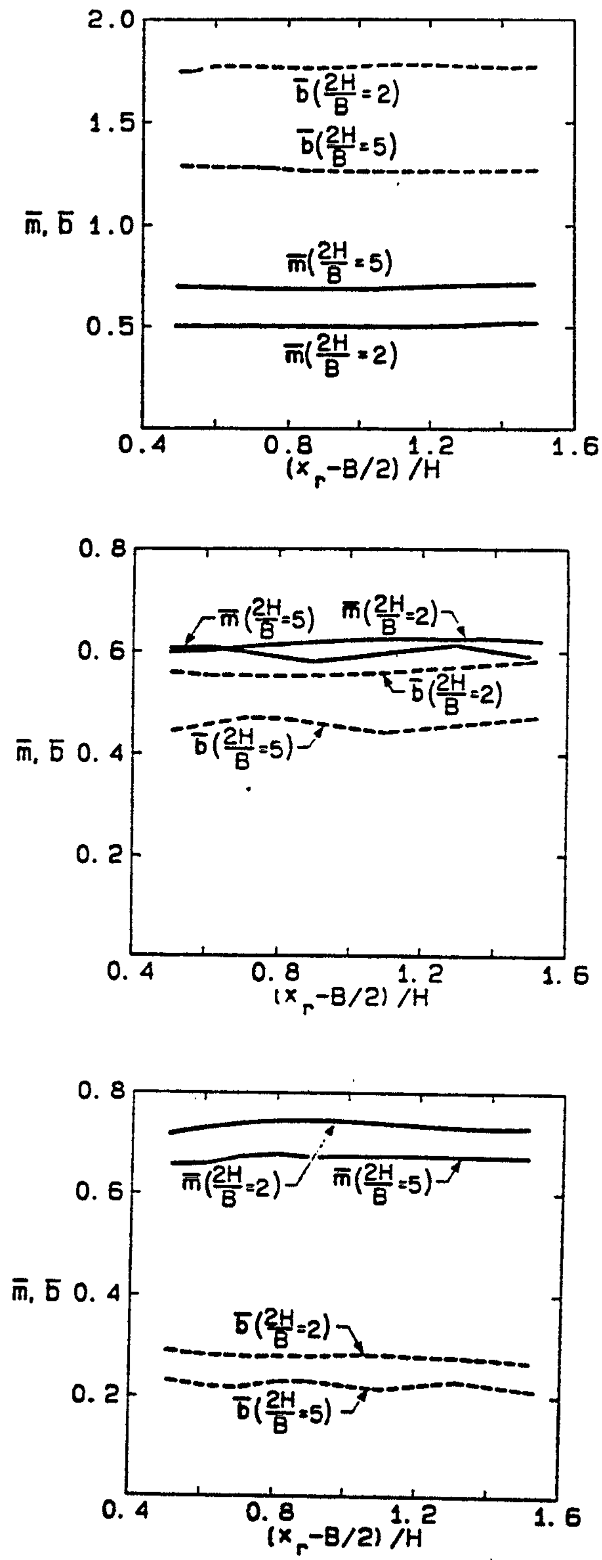

Fig. 5. Effect of the radiation boundary position on the added mass and damping of a heaving circular cylinder in Fig. 1 at $\frac{\sigma^2 B}{2g} = 0.2$ (a), 0.9 (b), and 1.5 (c).

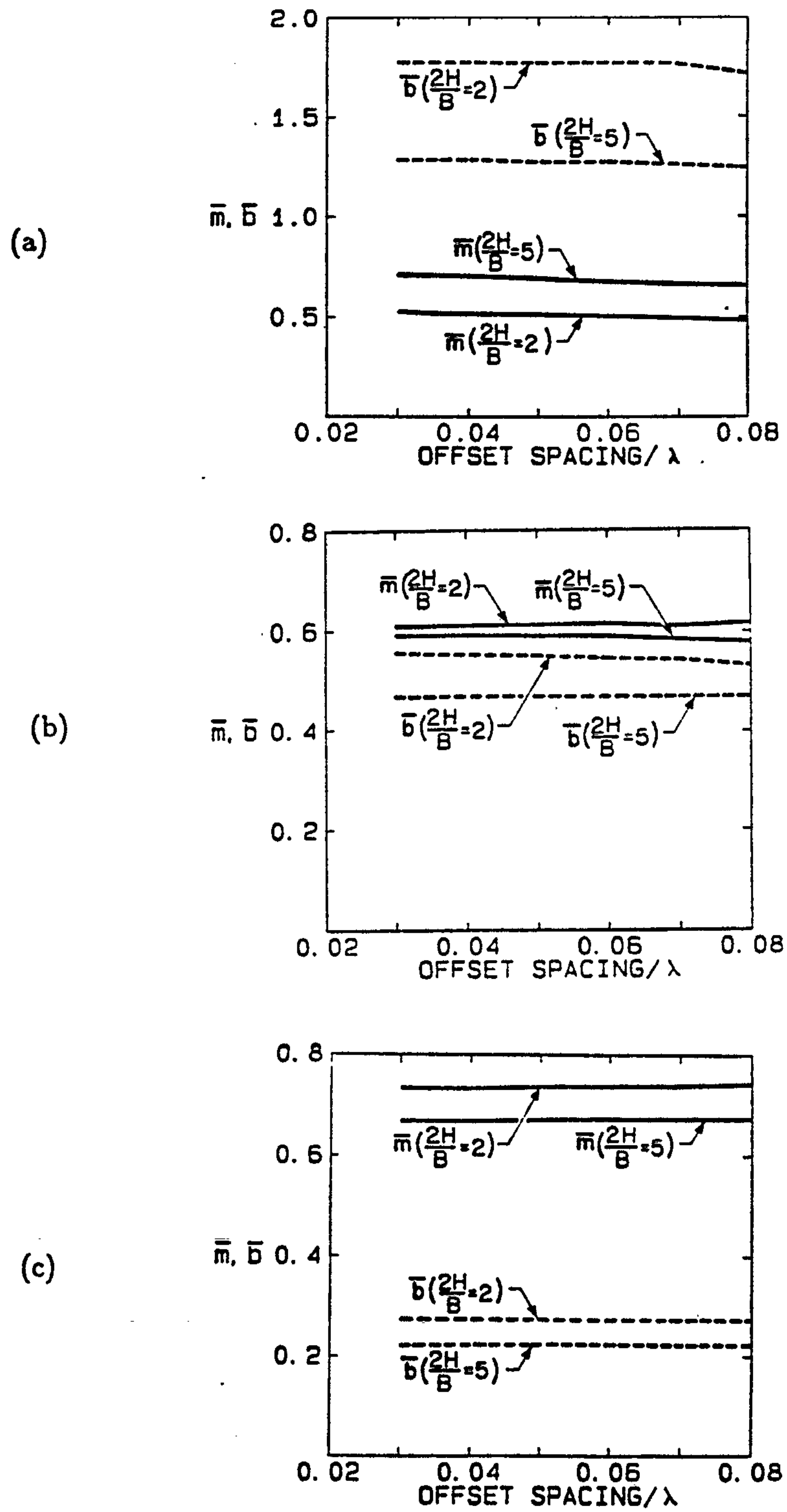

Fig. 6. Effect of the free surface spacing on the added mass and damping of a heaving circular cylinder in Fig. 1 at $\frac{\sigma^2 B}{2g} = 0.2$ (a), 0.9 (b), and 1.5 (c).

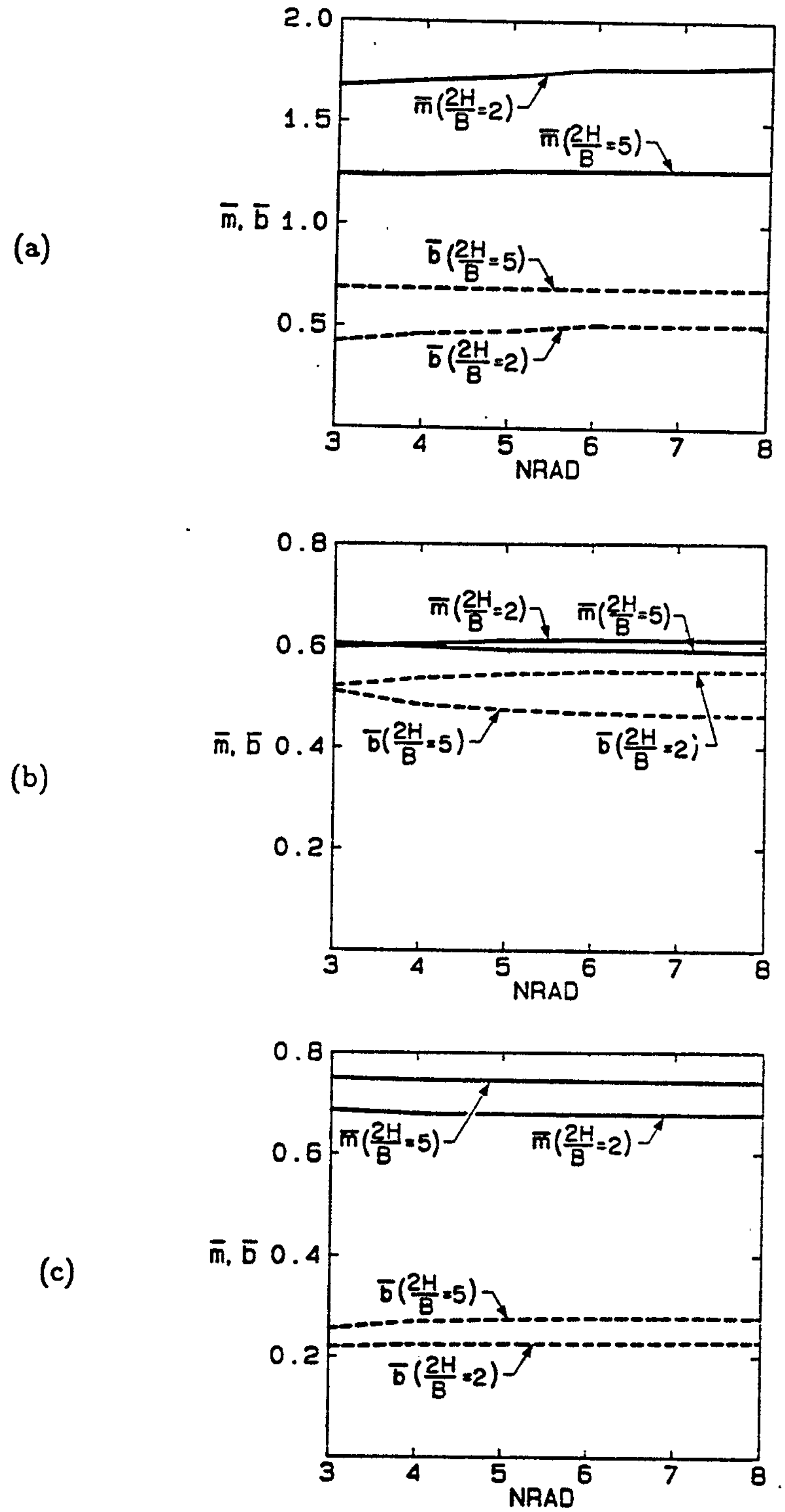

Fig. 7. Effect of NRAD (number of offsets on the radiation boundary over a depth of $\lambda/3$ from the free surface) on the added mass and damping of a heaving circular cylinder in Fig. 1 at $\frac{\sigma^2 B}{2g} = 0.2$ (a), 0.9 (b), and 1.5 (c).

APPENDIX A: SAMPLE INPUT

Data Set 1	CIRCLE OF RADIUS ONE
Data Set 2	11
Data Set 3	0.0 0.1564344 0.3090170 0.4539904 0.5877852 0.7071068 0.8090170 0.8910065 0.9510565 0.9876883 1.0
Data Set 4	-1.0 -0.9876884 -0.9510565 -0.8910066 -0.8090170 -0.7071068 -0.5877852 -0.4539905 -0.3090170 -0.1564345 0.0
Data Set 5	.8
Data Set 6	.05
Data Set 7	5
Data Set 8	1
Data Set 9	5.
Data Set 10	1
Data Set 11	.9

APPENDIX B: SAMPLE OUTPUT

FILEIN=CIRC1.DAT
TITLE=CIRCLE OF RADIUS ONE
BREADTH= 2.00000
SYMMETRIC BODY

OFFSET	XI	ETA
1	0.00000	-1.00000
2	0.15643	-0.98769
3	0.30902	-0.95106
4	0.45399	-0.89101
5	0.58779	-0.80902
6	0.70711	-0.70711
7	0.80902	-0.58779
8	0.89101	-0.45399
9	0.95106	-0.30902
10	0.98769	-0.15643
11	1.00000	0.00000

[Xr-B/2]/H, XRAD= 0.80
(FREE SURFACE SEGMENT LENGTH)/LAMBDA = 0.0500
NUMBER OF OFFSETS TO A DEPTH OF LAMBDA/3 = 4

DH=2H/B= 5.00
DS=(SIGMA^2*B)/(2G)= 0.90
DIMENSIONLESS ADDED MASS= 0.59150
DIMENSIONLESS DAMPING= 0.47304
HYDRODYNAMIC PRESSURE DISTRIBUTION

OFFSET	PBAR (-)	EPSILON (DEGREES)
1	0.6668	23.0
2	0.6475	24.3
3	0.6109	27.3
4	0.5619	32.4
5	0.5096	40.5
6	0.4691	52.6
7	0.4610	67.9
8	0.5005	83.1
9	0.5850	94.4
10	0.6968	100.6

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3. Newman, J.N.: *Marine Hydrodynamics*, MIT Press, Cambridge, Mass., 1977.

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This Technical Communication describes the underlying theory and use of the FORTRAN program AMAD (Added Mass And Damping), which calculates the hydrodynamic pressure distribution on the body surface, the added mass, and damping of a two-dimensional, laterally symmetric body heaving in the free surface of an unbounded fluid. Because AMAD uses the simple, free-space Green function $G = \log (1/r)$, "irregular frequencies" are not likely to occur.

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two-dimensional
radiation problem
computer program
boundary integral equation method
floating bodies
hydrodynamic pressure
potential theory