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THESEUS NAVIGATION ERROR MODEL

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TECHNICAL MEMORANDUM

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Abstract

The Spinnaker Project used an unmanned autonomous underwater vehicle to lay a cable on the bottom of the Arctic Ocean out to a distance of 180 km from shore. The journey was entirely under heavy polar pack ice, and navigation was a serious concern. This paper discusses the navigational error model that was used; it gives a derivation of the model, and it makes a comparison of the predictions with actual measurements.

RÉSUMÉ

Dans le projet Spinnaker on a utilisé un véhicule sous-marin autonome inhabité pour poser un câble sur le fond de l'océan Arctique jusqu'à 180 km des côtes. Le voyage s'est entièrement fait sous une épaisse banquise et la navigation était une préoccupation majeure. Cet article traite du modèle d'erreur de navigation qui a été utilisé; il décrit une variante du modèle, et compare les prévisions et les mesures véritables.

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THESEUS NAVIGATION ERROR MODEL

By

Ronald I. Verrall

EXECUTIVE SUMMARY

INTRODUCTION:

The Spinnaker Project used an unmanned underwater vehicle to lay cable from near-shore out into the Arctic Ocean. The 180-km track was completely covered with polar pack-ice, and so the navigation of the autonomous vehicle was of crucial importance. An initial study of the navigation problem by Rockwell International suggested the use of Doppler sonar to measure velocity and an Inertial Navigation Unit (INU) to determine the vehicle's heading and to do the necessary navigational calculations.

As part of the contract, Rockwell provided a navigation error model, which predicts the build-up of position offsets caused by various sources of error in the INU and the Doppler Sonar. This error model was not a large part of the report, and no details were given as to the mathematical derivations. Later, after the navigation equipment had been purchased and better estimates of the error parameters had been made, there was a requirement to understand and extend this model. The contract was long over, and a few telephone calls did not suggest a quick resolution to the problem, and so the subsequent modelling was done in-house.

SUMMARY OF DISCUSSION:

This paper gives a detailed mathematical derivation of the error model, showing how the various random terms contribute to the final expression for the mean-square position-offset. Our expression for the errors in the outward-bound section of the voyage agrees with that given by Rockwell; however, the Rockwell report appears to have minor mistakes in the expression for the error upon reaching 'home'. Moreover, we derive an expression for the error growth on the return (homeward-bound) half of the voyage. (Rockwell did not show how the error grew with range or time; they gave only the error upon reaching the destination.)

We then give numerical results using our best estimates of the parameters and compare them to that measured at the Nanoose underwater range. The agreement is quite favourable, which indicates that the error model would be a good tool in any future AUV design.

SIGNIFICANCE OF RESULTS:

Unmanned, autonomous underwater vehicles are being used more and more in today's world – both by civilians and by the military. An important component of these vehicles is their navigation system. The navigation must, of course, be sufficiently accurate to do the

intended job. However, very accurate systems are expensive, and a compromise must be reached between accuracy and cost. Consequently, the ability to understand and model the navigation errors expected on a mission is very important. This paper presents the modelling that was carried out for the Autonomous Underwater Vehicle (AUV) Theseus, and it compares the results of the model with mission-length tests. The techniques used in developing the model are explained in some detail so that they can be extended to other navigation problems.

FUTURE PLANS:
There are no plans to use Theseus in the near future, and there are no plans to build other autonomous underwater vehicles for the Canadian military.

THESEUS NAVIGATION ERROR MODEL

Introduction:

The Spinnaker project involved the laying of an underwater fiber-optic cable from the shoreline near CFS Alert on Ellesmere Island out into the Arctic Ocean where it connected with an underwater array 180 km offshore[1,2,3]. The cable was laid by the autonomous underwater vehicle (AUV) Theseus, which had been built by International Submarine Engineering Research (ISER) under contract to the Defence Research Establishment Pacific (DREP).

One of the very important considerations in the design of Theseus was that of navigation. In particular, how could Theseus make its way autonomously all the way out to the underwater array and back again? In 1990, ISER and Rockwell International did a study to define a navigation system and strategy for under-ice cable-laying missions[4]. The study recommended that Theseus dead-reckon its position by a combination of an inertial navigation unit (INU) and a Doppler sonar. The Doppler sonar, which bounced acoustic pulses off the ocean bottom, determined Theseus's velocity over the ground, both the fore-and-aft component and the transverse component. The purpose of the INU was to determine the vehicle's attitude and its true heading and to do all the navigational computations.

An error model of this navigation system and an analysis of the expected error was part of Rockwell's report. It gave their assumptions as to the source and magnitude of the various errors, and it calculated the mean square cross-track error as a function of trip-time. However, the report gave no indication of how they did the calculation, and the contract was over long before we really needed to understand their model and make it a tool that we could use. Thus, in order to reproduce their results, to check them for accuracy and to redo the calculations with other initial assumptions, we had to do a fair bit of detective work. This memorandum gives the result of this investigation. It shows (in some detail) how to reproduce the calculations that go from Rockwell's initial assumptions to their final results. Moreover, it shows that some of Rockwell's analysis was in error.

The Error Model:

Description of the Rockwell Model

The error model concentrated on the cross-track error. This error is more important than the along-track error since the primary aim of the navigation is to bring Theseus close to an acoustic beacon, where it can do a position update. This goal is only minimally jeopardized by an along-track error.

The model's first and simplest source of error is the uncertainty of the starting location, although with modern P-code GPS these errors will be very small (10 m, or so). The position error for the return trip will be somewhat larger because the Doppler sonar does not start out with bottom contact. This is discussed later in more detail.

The main sources of error are assumed to be the error in the Doppler sonar velocity and the error in the INU heading determinations. The sonar cross-track velocity error δV_o is assumed to be proportional to the forward speed. This means that the cross-track error due to this inaccuracy will be proportional to the distance travelled. The error is equivalent to an error in heading. The Rockwell navigation study used an estimate of 1% of the AUV speed, which, if it were the only error source, would result in a 2 km error in a 200 km mission.

The modelling of the INU heading error, ϕ, was much more difficult to understand and was the motivation for the work described in this paper. The heading error of the INU is approximated as the sum of several parts:

$$\phi(t) = \phi_0 + \mu_a(t) + \int_0^t [\varepsilon_o + \mu_d(t')]dt' \quad (1)$$

Where ϕ_0 is the initial (constant) heading error, $\mu_a(t)$ is a random variable with the distribution of a 'random walk', and the two terms inside the integral represent the rate of change of the heading error. The term ε_0 is the initial (constant) rate of change of the heading error and $\mu_d(t)$ is another random-walk variable. A random-walk variable (sometimes called a drunkard's walk) has the property that the expectation value of its square grows linearly with time. Thus $E[(\mu_a(T))^2] = T\mu_a^2$ where the 'E' represents an expectation operator, and μ_a^2 is a constant. From the context of equation (1) μ_a^2 has the units degrees squared per hour; equivalently, μ_a has the units degrees per root hour. Similarly, μ_d has the units degrees per hour per root hour.

To summarize, the error terms are:

δR_0	Initial cross - path position error
δR_d	Destination cross - path position error
δV_0	Doppler cross track velocity error
ϕ_0	Initial heading error
ε_0	Initial heading rate error
μ_a(t)	Heading error random walk
μ_d(t)	Heading rate error random walk

Their numerical values will be discussed later.

Predictions of the Model – Outbound Track

For simplicity we consider a straight-line path with the x-axis along the track and the y-axis across the track. The AUV speed, V_0, and the Doppler cross-track error, δV_0, are taken to be constants. The rate of change of the cross-track coordinate, δY, is given by the sum of the initial error, the cross-track velocity error and the error due to the error in heading:

$$\delta Y_{out}(T) = \delta R_0 + T\delta V_0 + \int_0^T V_0\phi(t)\,\mathrm{dt}\,. \qquad (2)$$

We wish to obtain an expression for the root-mean-square of the cross-track error at time *T*. To do this, we first express the cross-track error in terms of the random variables; then we square the result and take the expectation value of the squared random variables. Working with random-walk variables is made much easier if they are expressed in terms of white-noise random variables. The relationship is: $\frac{d\mu(t)}{dt} = w(t)$, where $w(t)$ is a white-noise zero-mean random variable which has the property: $E[w_a(t)\, w_a(t')] = \mu_a^2\, \delta(t-t')$, with $\delta(t-t')$ being the usual delta function. (See, for example, Gelb[5], page 79). The variable $\mu_d(t)$ has similar properties. The delta functions that result from taking expectations greatly simplify the subsequent algebra.

As a consequence of the above discussion, equation (1) can be rewritten as:

$$\phi(t) = \phi_0 + \int_0^t [w_a(t') + \varepsilon_0 + \mu_d(t')]dt'\,. \qquad (3)$$

from which we obtain

$$\frac{d\phi(t)}{dt} = w_a(t) + \varepsilon_0 + \mu_d(t)\,. \qquad (4)$$

Also, $\frac{d\mu_a(t)}{dt} = w_a(t)$ and $\frac{d\mu_d(t)}{dt} = w_d(t)$.

Let us first consider the outbound track, which is the simpler of the two cases. The evaluation of equation (2) requires the integration of $\phi(t)$. Rather than trying to integrate equation (3) with respect to t, it is easier to use equation (4) and integrate by parts.

$$\int_0^T \phi\,\mathrm{dt} = t\phi(t)\big|_0^T - \int_0^T t\frac{d\phi}{dt}dt$$

$$= T\phi(T) - \int_0^T t[w_a(t) + \varepsilon_0 + \mu_d(t)]\,dt\,, \text{ and using eq (3) for } \phi(T)$$

$$\int_0^T \phi\,\mathrm{dt} = T\phi_0 + T\int_0^T [w_a(t) + \varepsilon_0 + \mu_d(t)]\,dt - \int_0^T t\,w_a(t)\,dt - \frac{\varepsilon_0 T^2}{2} - \int_0^T t\mu_d(t)\,dt$$

$$= T\phi_0 + \int_0^T (T-t)\,w_a(t)\,dt + \varepsilon_0 T^2 + \int_0^T (T-t)\mu_d(t)\,dt - \frac{\varepsilon_0 T^2}{2}$$

$$= T\phi_0 + \int_0^T (T-t)\,w_a(t)\,dt + \frac{\varepsilon_0 T^2}{2} - [0.5(T-t)^2\,\mu_d(t)]\big|_0^T + \int_0^T \frac{(T-t)^2}{2} w_d(t)\,dt$$

and, since $\mu_d(0) = 0$, the third term vanishes,

$$\int_0^T \phi\,\mathrm{dt} = \int_0^T (T-t)\,w_a(t)\,dt + \frac{\varepsilon_0 T^2}{2} + \int_0^T \frac{(T-t)^2}{2} w_d(t)\,dt\,. \qquad (5)$$

Putting equation (5) into (2) we obtain the required expression for the cross-track error,

$$dY_{out} = \delta R_0 + T\delta V_0 + V_0 T\phi_0 + \frac{V_0 T^2 \varepsilon_0}{2} + V_0 \int_0^T (T - t) w_a(t)\,dt + V_0 \int_0^T \frac{(T - t)^2}{2} w_d(t)\,dt\,. \quad (6)$$

The cross-track error is now in terms of the given parameters and the time. The first four terms are probably not independent random variables; they are fixed at the beginning of the run and are not the same as the random variables μ (or w). Rockwell, however, treated them as being independent, and, for consistency, we shall do the same. The last two terms are truly random, and are considered to be independent of each other as well as of the first three terms. We now square δY_{out} and take expectations. We assume that the order of doing the integrals and taking the expectations can be interchanged. Also, because of the assumed statistical independence of the variables, the expectation value of all the cross terms are zero.

$$E\left[(\delta Y_{out})^2\right] = (\delta R_0)^2 + (T\delta V_0)^2 + (V_0 T\phi_0)^2 + (\frac{V_0 T^2}{2}\varepsilon_0)^2$$

$$+ V_0^2 \int_0^T\int_0^T (T - t)(T - t')\mathrm{E}\left[w_a(t)w_a(t')\right] dt\,dt' + V_0^2 \int_0^T\int_0^T \frac{(T - t)^2}{2}\frac{(T - t')^2}{2}\mathrm{E}\left[w_d(t)w_d(t')\right] dt\,dt'$$

$$= (\delta R_0)^2 + (T\delta V_0)^2 + (V_0 T\phi_0)^2 + (\frac{V_0 T^2}{2}\varepsilon_0)^2 + V_0^2 \mu_a^2 \int_0^T (T - t)^2\,dt + \mu_d^2 \int_0^T \frac{(T - t)^4}{4} dt$$

$$E\left[(\delta Y_{out})^2\right] = (\delta R_0)^2 + (T\delta V_0)^2 + (V_0 T\phi_0)^2 + \frac{V_0^2 T^4}{4}\varepsilon_0^2 + V_0^2 \frac{T^3}{3}\mu_a^2 + V_0^2 \frac{T^5}{20}\mu_d^2 \quad (7)$$

where μ_a^2 and μ_d^2 are constants.

In the above analysis we have used the fact that the expectation E[$w(t)w(t')$] is equal to a delta function times a constant ($\mu^2\,\delta(t\text{-}t')$). An integration over t' is then trivial.

Equation (7) agrees with the expression given by the Rockwell report for the outbound trip, which gives us confidence that we are using the same approach. The time, T, in equation (7) can be used as a variable to examine how the error grows with time or with distance.

Predictions of the Model - Inbound Track

Now let us consider the return (homeward-bound) track. We will assume, as Rockwell did, that at the end of the outbound trip we know the outbound error, δY_{out}, and that we will blame it entirely on a heading error. This reasoning gives a heading error of $\phi_c = \delta Y_{out} / V_0 T$. For the return trip we will subtract this amount from the heading that would otherwise be computed.

As before, the approach is to calculate the off-track error, δY_{in}, for the return trip, taking account of the correction. Once we have the error δY_{in} expressed in terms of our initial variables we can find the expected value of its square. We assume that $\phi(t)$ and $\varepsilon(t)$ start out where they were at the end of the outbound track (i.e., as $\phi(T)$ and $\varepsilon(T)$). Our time now runs from T to $2T$. With these assumptions, and with the addition of the heading-correction term, equation (2) becomes

$$\delta Y_{in}(2T) = \delta R_d + T\delta V_0 + \int_T^{2T} V_0[\phi(t) - \phi_c]dt$$

$$= \delta R_d + T\delta V_0 + V_0 \int_T^{2T} \phi(t)dt - \delta Y_{out} \,. \qquad (8)$$

Where δR_d is the cross-track error on departure from the Ice Camp en route home. Using equation (2) for δY_{out} this becomes

$$\delta Y_{in} = \delta R_d - \delta R_0 + V_0 \int_T^{2T} \phi(t)dt - V_0 \int_0^T \phi(t)dt$$

$$= \delta R_d - \delta R_0 + V_0 \left(\int_0^{2T} \phi(t)dt - 2\int_0^T \phi(t)dt \right) .$$

To evaluate the integrals we use equation (5), evaluating it at '$2T$' for the first integral and at 'T' for the second. This gives us:

$$dY_{in} = \delta R_d - \delta R_0 + V_0 \left(\int_0^{2T} (2T - t) w_a(t)\, dt + \int_0^{2T} \frac{(2T - t)^2}{2} w_d(t)\, dt + 2\varepsilon_0 T^2 \right.$$

$$\left. - 2\int_0^T (T - t) w_a(t)\, dt - 2\int_0^T \frac{(T - t)^2}{2} w_d(t)\, dt - 2\frac{\varepsilon_0 T^2}{2} \right) . \qquad (9)$$

As before, the procedure is to square, take expectations and integrate. Again, the cross terms involving different statistical variables vanish. There are, however, non-zero cross terms involving integrals that have different upper limits. An example is the cross term involving w_a:

$$\int_0^{2T} (2T - t) w_a(t)\, dt \int_0^T (T - t) w_a(t)\, dt = \int_0^{2T} \int_0^T (2T - t)(T - t') w_a(t')\, w_a(t) dt'\, dt$$

Here we must be careful with the limits after we take expectation values. Values of t lying between T and $2T$ in the above integral have no matching value of t', which runs only from 0 to T. Thus the δ function is zero whenever t is greater than T, Consequently, the final integration runs only from 0 to T. The integral in question works out to be:

$$\mu_a^2 \int_0^T (2T - t)(T - t)dt = \mu_a^2 \frac{5T^3}{6} \,.$$

Once equation (9) is squared and integrated, we obtain the following mean squared value of the cross-track error:

$$E\left[(\delta Y_{in})^2\right] = (\delta R_d)^2 + (\delta R_0)^2 + V^2 T^4 \varepsilon_0{}^2 + \frac{2}{3} V^2 T^3 \mu_a^2 + \frac{46}{60} V^2 T^5 \mu_d^2 \,. \qquad (10)$$

Unlike Rockwell's version, this has no direct dependence on δV_0 nor on ϕ_0. This dependence has vanished because of the heading correction made at the end of the outbound leg and because the terms in δV_0 and ϕ_0 are linear in T. This certainly seems reasonable, and, since Rockwell made the same heading correction, it implies that there may be errors in their analysis.

In equation (10) the coefficient of $V^2T^4\varepsilon_0^2$ is 1, rather than Rockwell's 3/4. The coefficient of $V^2T^3\mu_a^2$ is the same as Rockwell's, and the coefficient of $V^2T^5\mu_d^2$ is 46/60 rather than Rockwell's 11/60.

Equation (10) cannot be used to plot the expected error as a function of time or distance on Theseus's return; the return time (T) in the equation is the same time it took Theseus to go out; it is not an arbitrary time.

In order to examine the growth in error as a function of time on the return voyage we must integrate equation (8) when the final time is T', an arbitrary value ($>T$), rather than the special $2T$. (This dependence was not given by Rockwell.) In this case, equation (8) becomes

$$\delta Y_{in}(T') = dR_d + (T' - T)\delta V_0 + \int_T^{T'} V_0[\phi(t) - \phi_c]dt$$

$$= dR_d + (T' - T)\delta V_0 + V_0\int_T^{T'} \phi(t)dt - \frac{V_0(T' - T)}{V_0 T}\delta Y_{out}$$

$$= dR_d + (T' - T)\delta V_0 + V_0\int_0^{T'} \phi(t)dt - V_0\int_0^{T} \phi(t)dt - \frac{(T' - T)}{T}\left(\delta R_0 + T\delta V_0 + V_0\int_0^{T}\phi(t)dt\right)$$

$$= dR_d - \frac{(T' - T)}{T}\delta R_0 + V_0\left(\int_0^{T'} \phi(t)dt - \frac{T'}{T}\int_0^{T}\phi(t)dt\right). \qquad (11)$$

Again using equation (5) for the integrals, we obtain.

$$dY_{in}(T') = \delta R_d - \frac{(T' - T)}{T}\delta R_0 + V_0\left(\int_0^{T'}(T' - t)w_a(t)\,dt + \int_0^{T'}\frac{(T' - t)^2}{2}w_d(t)\,dt + \frac{\varepsilon_0 T'^2}{2}\right.$$

$$\left. - \frac{T'}{T}\int_0^{T}(T - t)w_a(t)\,dt - \frac{T'}{T}\int_0^{T}\frac{(T - t)^2}{2}w_d(t)\,dt - \frac{T'}{T}\frac{\varepsilon_0 T^2}{2}\right).$$

$$dY_{in}(T') = \delta R_d - \frac{(T' - T)}{T}\delta R_0 + \frac{V_0\varepsilon_0 T'(T' - T)}{2} + V_0\left(\int_0^{T'}(T' - t)w_a(t)\,dt + \int_0^{T'}\frac{(T' - t)^2}{2}w_d(t)\,dt\right.$$

$$\left. - \frac{T'}{T}\int_0^{T}(T - t)w_a(t)\,dt - \frac{T'}{T}\int_0^{T}\frac{(T - t)^2}{2}w_d(t)\,dt\right). \qquad (12))$$

As before, we square, take expectations and integrate.

$$E\left[(\delta Y_{in})^2\right] = (\delta R_d)^2 + \frac{(T' - T)^2}{T^2}(\delta R_0)^2 + \frac{T'^2}{4}(T' - T)^2 V_0^2\varepsilon_0^2 + \frac{T'}{3}(T' - T)^2 V_0^2\mu_a^2$$

$$+ \frac{T'}{60}(T' - T)^2(3T'^2 + 6TT' - T^2)V_0^2\mu_d^2. \qquad (13)$$

This equation allows us to monitor the growth of the error along the return track. (As a check, note that this reduces to (10) if T' is set equal to 2T. Note also that the terms in δV_0 and ϕ_0 have vanished because of the linearity of these terms in T.)

Numerical Results:

Rockwell's original study used their best estimates of the error parameters. They are shown in Table 1.

Table 1: Rockwell's Initial Assumptions

	Value	Units
Initial Cross-Track Position Error	10.	metres
Return Cross-Track Position Error	100.	metres
Cross-Track Velocity Error	1.	Percent
Initial Heading Error	0.2	Degree
Initial Heading Error Rate	0.04	Deg/hr
Random Heading Error Rate	0.05	Deg/hr$^{1/2}$
Change in Heading Error Rate	0.005	Deg/hr$^{3/2}$

The expected cross-track error (as a function of distance), which is computed from equations (10) and (13), is plotted in Figure 1. As can be seen, the errors grow quite large, especially on the return trip. Moreover, the plotted values are just the standard deviation of the error distribution; on the assumption of a normal distribution there is approximately a one third chance that the errors will be bigger than those quoted. It is unlikely that the errors would be larger than three standard deviations, but three standard deviations corresponds to an error of about 13 km, which is much farther than Theseus can detect an acoustic beacon. It was for reasons like this that several beacons were installed en route so that Theseus could get periodic updates in its position.

As it turned out, the estimates of the error parameters were quite large. This conservative choice was deliberate; the parameters were estimated at a time when we had not decided what equipment to buy. However, after the Doppler Sonar (EDO 3050) and the INU (Honeywell MAPS INU) had been chosen, purchased and tested, we were able to refine these estimates. The Doppler velocity error, δV_o, was quoted by the manufacturer to be 0.1% (long-term average). The initial heading error, ϕ_o, was left at 0.2 degrees – the same as the Rockwell estimate – although the number was now deemed to include the effects of any misalignment between the Doppler and the INU.

The initial heading rate error, ε_o, is the slope of the heading error curve at the time that the INU is switched from 'Gyrocompass Alignment mode' into 'Nav mode'. During Arctic trials in 1992 and 1993 a number of tests were made of the heading error curve after alignment periods of 120 minutes. It was found that the heading error curve had an initial slope that was essentially zero. Zero, then, is the value that was used in the model. For the random heading error rate, μ_a, and the random change in heading error rate, μ_d, we used the values quoted by Honeywell for this INU – namely, 0.003 degrees per root hour, and 0.001 degrees per hour root hour, respectively.

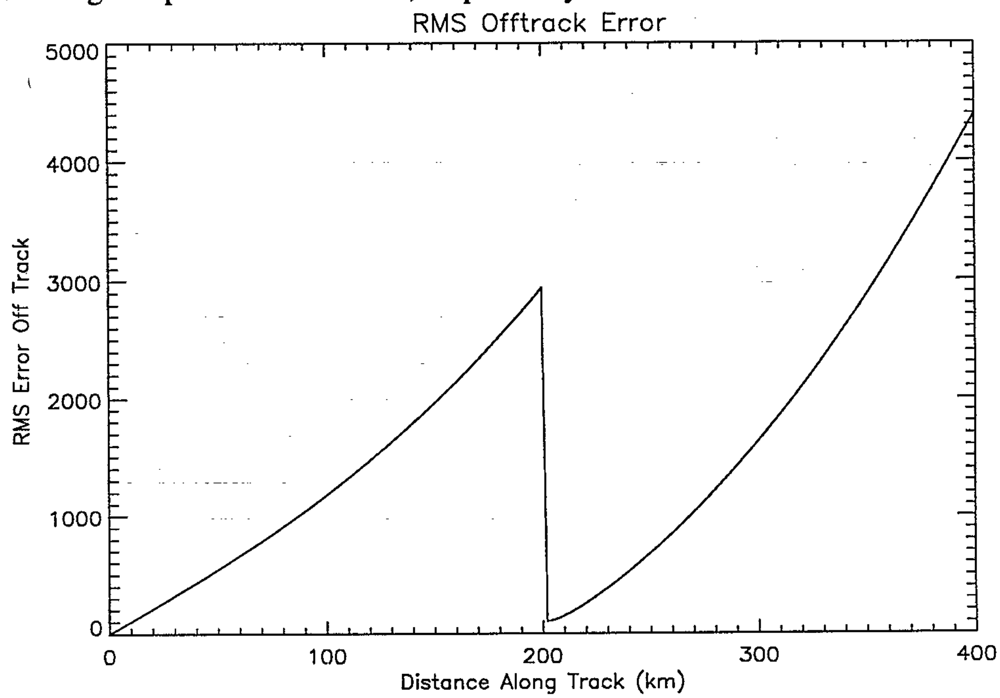

Figure 1: A prediction of Theseus's navigational errors as a function of its distance along the track. At the 200-km point Theseus gets a position update and a heading correction. It then turns around and returns home. This curve uses the error parameters originally estimated by Rockwell.

The initial position error, δR_0, was left at 10 m, which is roughly what one can get from P-code GPS, but a new and somewhat larger estimate of the destination position error, δR_d, was made. For some time after Theseus leaves the ice camp (at the location of the underwater array) its Doppler sonar will not be able to 'see' the bottom. During this time it gets its velocities from reflections within the water column. This is inaccurate to the extent that the water mass may be moving relative to the bottom. It may take Theseus 45 minutes to get close enough to the bottom (at a descent angle of 5°), and during this time the current (estimated at a maximum of 10 cm/s) may have carried it 260 m away from its dead-reckoned position. Since the direction of the current is not known, the average error of the cross track position would be about 130 m. This value has been used in the following computation.

Table 2 gives a summary of these parameters, and Figure 2 shows a plot of the mean square error as a function of the distance travelled. Again, it is to be remembered that a correction to both the location and the heading is applied once Theseus arrives at the ice camp, and this has been taken into account by equation (16). In practice, we effected the heading correction by adjusting (in Theseus's control software) the alignment between the Doppler Sonar and the INU heading.

As can be seen, the cross-track errors are much smaller than the previous estimates. For example, on the return trip the mean-square error increases from an initial value of 130 m (which, itself, is probably conservatively high) to only 450 m. This predicted increase in error of 320 m is only 0.16% of the 200 km distance travelled. Although this seems unbelievably small, controlled measurements made at the Canadian Force's Maritime Experimental and Test Range (CFMETR) at Nanoose Bay on Vancouver Island showed a cross-track error of only 0.05% after the heading correction had been made[2]. Thus, the error predicted in this model on the basis of some measurements and some factory specifications was reasonably accurate. If anything, it was too large.

Table 2: Improved Estimates of the Error Parameters

	Value	Units
Initial Cross-Track Position Error	10.	Metres
Return Cross-Track Position Error	130.	Meters
Off-Track Velocity Error	0.1	Percent
Initial Heading Error	0.2	Degree
Initial Heading Error Rate	0.0	Deg/hr
Random Heading Error Rate	0.003	Deg/hr$^{1/2}$
Change in Heading Error Rate	0.001	Deg/hr$^{3/2}$

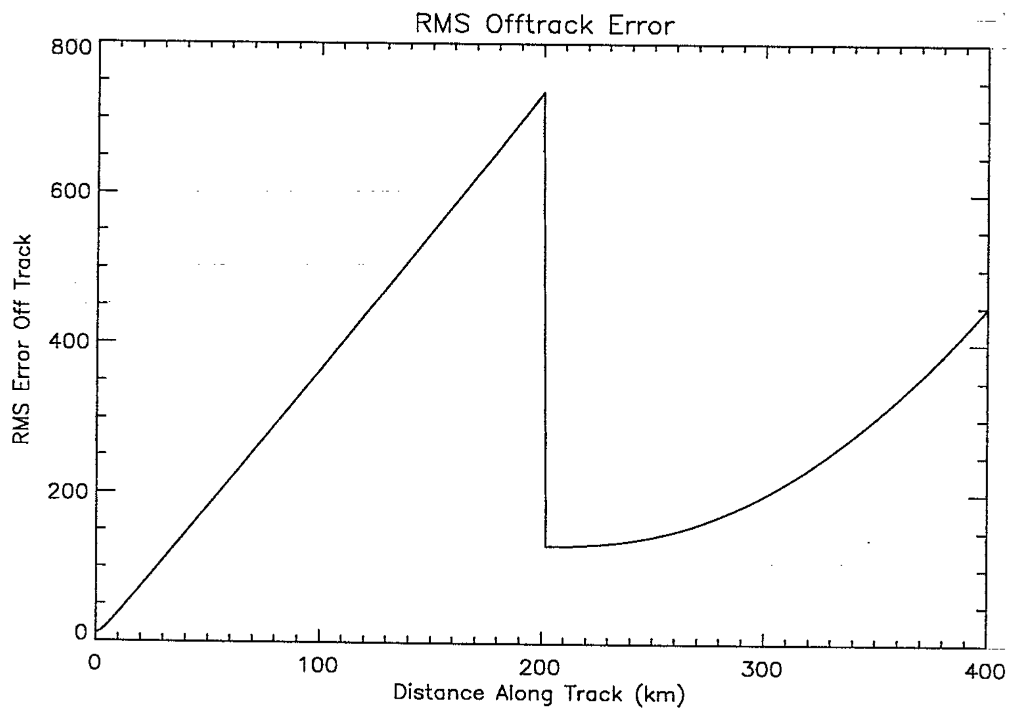

Figure 2: The calculated RMS error as a function of distance travelled. This curve is calculated with the refined error parameters.

Summary and Conclusions:

The Theseus navigational error model was first developed by Rockwell International in collaboration with International Submarine Engineering Research (ISER) on a government contract. However, the report gave no details as to how the calculation was done. This report gives full details of the mathematical development of the error expressions. It agrees with the Rockwell results for the outward-bound track, but it does not agree with their expression for the error expected upon arriving back at the starting point. This report also produces the analysis of the expected error as a function of time (or distance) on the return track, something that was not done in the Rockwell report.

Rockwell used their knowledge of navigational systems to estimate the magnitude of the various parameters in the error model. This was done before the Spinnaker team had decided what navigation equipment to buy; in fact the purpose of the study, in part, was to help decide what was necessary. The numbers they used turned out to be overly large for the equipment that was available to us, and their RMS error predictions for the return track was more than 4 km. Partly as a result of this, and partly out of uncertainty as to what other problems might arise, six acoustic beacons were installed in the ice to help Theseus in its navigation. (Theseus did position updates at each beacon.)

Once the equipment was purchased and tested, refinements could be made to the error model. The error estimates derived from the new parameters were substantially lower than the original values, and these estimates were justified (and, in fact, shown to be conservative) by actual tests. It now seems quite certain that Theseus could have navigated from shore to the neighbourhood of the underwater array with no intervening acoustic beacons, although it would still have needed two beacons to help guide it through the catchment loop at the ice camp. The trip back to shore would have needed one beacon to help guide it through a narrow channel and a second beacon at the recovery hole.

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The Spinnaker Project used an unmanned autonomous underwater vehicle to lay a cable on the bottom of the Arctic Ocean out to a distance of 180 km from shore. The journey was entirely under heavy polar pack ice, and navigation was a serious concern. This paper discusses the navigational error model that was used; it gives a derivation of the mathematics, and it makes a comparison of the predictions with actual measurements.

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