Quantification of Correlation of Predicted and Measured Transfer Functions for Ship Motions and Wave Loads

Samon Ando
Defence Research Establishment Atlantic, P.O. Box 1012, Dartmouth, NS, Canada B2Y 3Z7

SUMMARY

This paper introduces the total factor error (TFE), an index of correlation of predicted and measured transfer functions for ship motions and wave loads. TFE is defined as the ratio, expressed as percent, of the sum of weighted squares of the difference between the magnitudes (gain factors) of predicted and measured transfer functions to the sum of the weighted squares of the measured value. The wave spectrum of a given sea state is used as the weighting factor. Thus the error of predicted transfer function at a particular wave frequency is weighted by a measure of its importance. Eliminating the visual judgement and the use of words from experimental validation of computer-predicted transfer functions, TFE is an efficient, as well as objective, means of ensuring the reliability of the computer-predicted transfer functions for ship responses.

1. INTRODUCTION

The last twenty years or so have seen ever-increasing emphasis on rational approaches to naval architectural problems. One example is the steady shift of paradigm in ship structural design: instead of relying on arbitrary large factors of safety, the modern approach is based on the idea that loads ("demand") on, and the load-carrying ability ("capability") of, the hull structure should be expressed in terms of probabilities, and the successful design is the one that reduces the probability of failure to an acceptable value. To minimize the uncertainty in the calculated probability of failure, it is necessary, among other things, that all loads be completely determined on the basis of scientific, as opposed to empirical, procedures. Now, a large component of the hull loads at sea comes from the time-varying loads induced by encountering waves and by the ship motions caused by waves, and a considerable stride has been made in advancing the mathematical models for their predictions. But how confident can the naval architect be of the computer predictions? How can he/she gauge their accuracy objectively so as to ensure the reliability of the final results? This paper will try to answer these questions, focusing on the transfer functions for ship motions and wave loads.

Ocean waves can be regarded as a narrow-band, stationary, ergodic Gaussian random process with a zero mean. So too can the wave-induced response of a ship if it is linear, *i.e.,* additive and homogeneous. Since any zero-mean Gaussian process can be completely determined in the statistical sense by the power spectrum, the latter is an extremely useful parameter for the study of a ship's responses to ocean waves such as the peak value, mean, variance, average of the highest $1/n$ values, average number of peaks, and average number of upward or downward crossings of a particular level. The power spectrum is also essential in the study of probability density functions governing such statistics. For instance, the envelope of wave heights and of the ship's response during a short period of time (less than an hour) can be assumed to follow the Rayleigh probability distribution, and the latter's parameter can be estimated from the moments of the power spectral density (PSD) function.

By means of a simple relationship between the PSD of the input and that of the output in the frequency domain (see equation (1) below), the PSD of the ship's response to the given PSD of the sea state can be calculated easily if the transfer function is known. The immediate question is how best to assess the accuracy of the analytically predicted transfer function.

As in other areas of scientific endeavor, experiment is the final arbiter in this regard. Conventionally, experimental validation of the computer predictions is by visual inspection of plots like Figs. 1(a), 2(a), and 5(a) to be discussed in the examples below. The degree of correlation of prediction and experimental data is expressed qualitatively by such terms as "excellent", "good", "fair", "middling", or "poor". Since one does not have precise criteria for these terms, the result is apt to be subjective and inconsistent. It can also be erroneous. To see this, consider Fig. 3. The curve in the figure may represent theory and dots experiment. The vertical distances between the dots and the curve are the same at all values of X, but for some, they look much closer together when the slope is steep (see, e.g. [1]).

If we are to put the experimental validation of predicted transfer functions on a more solid ground, then we will need to invent a scheme that does not depend on either visual judgement or description by words. In short, we need to quantify the degree of correlation of prediction and experiment. Dalzell [2] appears to be the first to articulate this need in the literature. He proposed an index of correlation, which is the ratio expressed as percent of the maximum magnitude of the deviation between theory and experiment to the largest response magnitude observed in the experiment. Its concept is akin to the notion of the fractional uncertainty, or the precision, in error analysis. As a guideline, he suggests that the correlation is acceptable when this index is no more than 10 percent, but not good when it is greater than 30 percent. Another type of index of correlation is proposed by Guedes Soares [3]. Called model error, this index is the slope of the regression line of experimental values on predicted values which is constrained to pass through the origin (i.e. a zero intercept). The closer the model error is to 1, the better the agreement between theory and experiment is regarded to be.

Useful as these indices are as measures of correlation between predicted and measured ship responses, their reliability tends to be affected by discrepancies in trends or the location of peaks of theory and experiment. For instance, owing to the assumption of a zero intercept, the systematic (as opposed to random) differences between predicted and measured values are not properly reflected in Guedes Soares's model error. As well, if the maximum value of a transfer function should occur outside the experimentally feasible frequency ranges, then it would be inaccurate to use the maximum of the measured values as the denominator of Dalzell's index. (These instances are illustrated in the examples below.) Although the assumption underlying these indices is that the error in the predicted transfer function is of equal importance for all frequencies, some frequency ranges are more important than others when it comes to calculating power spectra of ship responses in realistic random seas. For example, a 100-percent error in the predicted transfer function can be insignificant if it occurs at very high or very low frequency ranges where the energy content of the waves is relatively small, but it would completely invalidate the results if it occurs at frequencies around the modal frequency of the wave spectrum. An instance of this can also be seen in an example below. (The term "error" needs to be qualified here. In this paper, it is used to mean the deviation of the predicted transfer function from the measured value. We temporarily neglect the inevitable experimental uncertainties that attend all physical measurements, and consider experimental values as reference for the purpose of the present discussion.)

What is needed, therefore, is an index of correlation that takes into account the varying impact of the error in the predicted transfer functions on the final results. To this end, this paper introduces the total factor error (TFE), and demonstrates its main features through simple examples. This paper may be viewed as an extension of Dalzell's work [2].

2. THEORETICAL FORMULATION

Under the assumption of linear relationship between ocean waves and the ship's response, their PSD's are related by

$$S_{resp}(\omega) = |H(\omega)|^2 S(\omega) \tag{1}$$

where $S(\omega)$ and $S_{resp}(\omega)$ are the PSD's of the wave and the ship's response, respectively, and $H(\omega)$ is the transfer function for the ship's response. The absolute value (gain factor) of the transfer function, $|H(\omega)|$, is simply the ratio of amplitude of ship response to that of wave, and $|H(\omega)|^2$ is often called the response amplitude operator (RAO). The wave spectral density $S(\omega)$ can be either a mathematical model (the Pierson-Moskowitz, ITTC, JONSWAP, Ochi-Hubble, etc.) or an actual ocean spectrum.

As the first step in defining the total factor error, the following expression is useful:

$$\int_0^\infty \left\{ |\hat{H}(\omega)| - |H(\omega)| \right\}^2 S(\omega) d\omega, \tag{2}$$

where $\hat{H}(\omega)$ is the predicted value of the transfer function and the $H(\omega)$ the experimental value. The expression (2) may be regarded as the mean square error (MSE) of the predicted transfer function relative to the wave spectrum $S(\omega)$. It gives the weighted deviation between prediction and experiment. The amount of difference is weighted by a measure of its importance in terms of the harmonic content of the wave spectrum $S(\omega)$.

For a prescribed wave spectrum and a particular set of experimental data, the values of (2) can be used to rate the relative merits of two or more sets of predictions. But since the area under the power spectrum is an important parameter in spectral analysis, the ratio of the MSE to the area under the reference response spectrum,

$$\frac{\int_0^\infty \left\{ |\hat{H}(\omega)| - |H(\omega)| \right\}^2 S(\omega) d\omega}{\int_0^\infty |H(\omega)|^2 S(\omega) d\omega}, \tag{3}$$

provides a more meaningful measure of importance of the error associated with the predicted transfer function. As well, MSE (2) is dimensional but the ratio defined by (3) is nondimensional.

In (3), ω ranges from 0 to infinity, but the experimental values of ω are bounded owing to physical constraints: its lowest value is usually limited by the water depth of the tank and the highest by the power of the wavemaker. Moreover, only a finite set of discrete values of the measured transfer function $\{H(\omega_i)\}$ for $\omega \in \{\omega_i\}$, $i = 1, 2, \ldots, N$, say, is available in practice. We therefore define the total factor error (TFE) by the square root ε of the following expression,

$$\varepsilon^2 \equiv \frac{\sum_{i=1}^{N} \left\{ |\hat{H}(\omega_i)| - |H(\omega_i)| \right\}^2 S(\omega_i)}{\sum_{i=1}^{N} |H(\omega_i)|^2 S(\omega_i)}, \tag{4}$$

as a measure of error of the predicted transfer function. (The right-hand side of (4) is numerically equivalent to (3) when ω_i ranges between a sufficiently small value ω_1 such that $H(\omega_1) \approx H(0)$ and a sufficiently large value ω_N such that $H(\omega_N) \approx H(\infty)$, and $|\omega_{i+1} - \omega_i|$ is constant for all i. This, however, does not bear critically on the definition of TFE.) Since ε is usually a small number, it is convenient to multiply it by 100 and express TFE as percent.

Obviously, the value of ε is relative to a particular data set; *i.e.*, it depends on the number and the composition of data points. This makes it practically impossible to rate different theories by their TFE's unless they are calculated using the same data set. In some circumstances, it may be desirable to multiply $H(\omega_j)$ for each i with some weighting factor to account for experimental error. One such instance would be when the data include obvious outliers.

If desired, the expression (4) can be extended to include more than one wave spectrum. For example, let $S_j(\omega)$ denote a wave spectrum for the jth of a total of M spectra, say, with its particular set of parameters, and ε_j the corresponding TFE. Then (4) can be calculated for each wave spectrum and the results added with an appropriate weighting factor:

$$\varepsilon^2 = \sum_{j=1}^{M} p_j \varepsilon_j^2, \tag{5}$$

where $\{ p_j \}$ are the user-defined weighting factors for the jth wave spectrum such that

$$\sum_{j=1}^{M} p_j = 1. \tag{6}$$

Using the notation used in this paper, Dalzell's correlation index and Guedes Soares's model error can be expressed as follows:

$$\text{Dalzell's correlation index} = \frac{\max\left|\left\{\left|\hat{H}(\omega_i)\right| - \left|H(\omega_i)\right|\right\}\right|}{\max\left\{\left|H(\omega_i)\right|\right\}}, \quad i = 1, 2, \ldots, N, \tag{7}$$

and

$$\text{Guedes Soares's model error} = \frac{\sum_i \left|\hat{H}(\omega_i)\right|\left|H(\omega_i)\right|}{\sum_i \left|H(\omega_i)\right|^2}. \tag{8}$$

3. EXAMPLES

Use of the TFE is illustrated in the following two examples. The first involves two hypothetical theories A and B. Figure 1(a) shows the correlation of the nondimensional midship horizontal shear-force coefficient $C_{SF} = V_2 / \rho gBLa$ predicted by theory A and measurements with a model advancing at Froude number $F_n =$ 0.15 in the direction 150 deg relative to the wave direction (180 deg is head waves) in regular waves. Here,

V_2 is the midship horizontal shear force (HSF), ρ the mass density of water, g the gravitational acceleration, B the beam, L the length between perpendiculars, and a the amplitude of the incident wave. The ordinate is the nondimensional wave frequency, $\overline{\omega} = \omega\sqrt{L/g}$, where ω is the angular wave frequency. The large difference between the predicted and measured C_{SF}'s at higher frequencies in Fig. 1(a) may look artificially large, but the figure is identical to Fig. 12 of Salvesen *et al.* [4], which compares their strip theory and experiment; the only difference here is that the wave direction is assigned 150 deg for the purpose of this example instead of 50 deg in [4]. Apart from the high frequency range, the theory agrees well with experiment. In Fig. 1(b), the scatter diagram of the predicted and measured C_{SF}'s is shown, along with the regression line of the measure C_{SF} on the predicted C_{SF} and the fitted regression line passing through the origin. The slope (0.73) of the latter, equation (8), is the model error of Guedes Soares for theory A. The large overestimate at high frequencies is the major cause of reducing the slopes of the regression lines. From (7), Dalzell's correlation coefficient for this case is 57 per cent, which is bad.

In Fig. 2(a), the experimental data in Fig. 1 are compared against theory B. There are some obvious differences between Figs. 1 and 2. First, the pattern of correlation between theory and experiment in Fig. 2(a) is nearly opposite to that in Fig. 1(a) in that over the low frequency range where agreement between theory A and experiment is good, theory B fares poorly, and over the high frequency range where theory A shows large deviation from experiment, theory B is in good agreement with experiment. Second, while deviations of theory A from the experimental data points over much of the frequency range in Fig. 1(a) seem random, the difference between theory B and experiment in Fig. 2(a) appears systematic in that the predicted values are consistently greater than the measured values. This contributes to the fact that the points in Fig. 2(b) are more widely dispersed about the fitted regression line through the origin than that in Fig. 1(b). Thus the coefficient of determination, a common measure of how well a regression model describes the data, for the fitted regression line in Fig. 2(b) is 0.36, which is less than 2/5 of that (0.93) for the fitted regression line in Fig. 1(b). (The closer the coefficient of determination to 1, the stronger the association between the independent and dependent variables, $\hat{H}(\omega)$ and $H(\omega)$.) This clearly shows that the fitted regression line in Fig. 2(b) is less reliable than that in Fig. 1(b), even though they possess the same slope. These differences notwithstanding, both Dalzell's correlation index (57 percent) and Guedes Soares's model (0.73) are the same for both theories A and B, suggesting that the two theories are of equal accuracy.

We now calculate the TFE for theories A and B from Figs. 1(a) and 2(a). To this end, a wave spectrum needs to be specified. For this example, we take the ITTC wave spectrum,

$$S(\omega) = \frac{5}{16}\frac{H_s^2\omega_0^4}{\omega^5}\exp\left(-\frac{5}{4}\frac{\omega_0^4}{\omega^4}\right), \tag{9}$$

and set the significant wave height $H_s = 5.0$ m and the modal angular frequency $\omega_0 = 0.507$ rad/s (sea state 6). For this wave spectrum, the value of ε is independent of H_s. In Fig. 4(a), the predicted and measured RAO's are shown, along with the wave spectrum for $H_s = 5.0$ m and $\omega_0 = 0.507$ rad/s. As a function of the encounter frequency, the wave spectrum is expressed by

$$S(\omega_e) = S(\omega) / \sqrt{1-(4\omega_e V/g)\cos\beta},$$

where $\omega_e = \omega - (\omega^2 / g)U \cos\beta$ is the encounter frequency, and β the wave direction. In Table 1, the values of TFE for theories A and B are compared with Dalzell's correlation indices and Guedes Soares's model errors for the two theories.

Table 1. Values of three types of indices of correlation for hypothetical theories A and B shown in Figs. 1 and 2; TFE is based on ITTC spectrum with H_s = 5.0 m and ω_0 = 0.507 rad/s.

	Correlation index (Dalzell)	Model error (Guedes Soares)	TEF $\omega_0 = 0.507$
Theory A	57%	0.73	36%
Theory B	57%	0.73	100%

It can be seen in Fig. 4(a) that in the frequency band where most of the energy content of the wave spectrum lies, theory A is unquestionably more accurate (*i.e.*, closer to experimental data) than theory B. In fact, the values of ε in Table 1 suggest that theory A is about three times more accurate than theory B in this particular combination of wave spectrum, ship speed, and wave direction. On the other hand, the indices of correlation of Dalzell and of Guedes Soares suggest that the two theories A and B are equally accurate. This apparent contradiction arises because in the calculation of these indices, the predictions at all wave frequencies are assigned equal importance.

Figure 4(b) shows the response spectra for the case shown in Fig. 4(a). In terms of the variance (the zeroth moment of response spectrum) of V_2, the difference in the values of ε for the two theories translates to an overestimation by some 179 percent by theory B compared with the value predicted by theory A. As well, there is a wide difference in the modal encounter frequencies: 0.85 rad/s (theory B) and 1.55 rad/s (theory A).

As the second example, the accuracy of the heave amplitudes predicted for a destroyer model at F_n = 0.194 in head waves by three computer programs shown in Fig. 5(a) are compared in terms of TFE for ITTC spectra shown in the figure. The TFE's are calculated based on the 13 data points in the figure and the ITTC wave spectrum (9) for two significant wave heights and three differing modal frequencies each. The results are shown in Table 2. Since the values of ε are independent of H_s, they are plotted as a function of ω_0 in Fig. 5(b).

Table 2. Values of TFE for computer predictions shown in Fig. 5 based on ITTC spectrum and Guedes Soares's model error [3].

	Total factor error ε (%)						Model error (Guedes Soares)
	H_s = 3.25 m (Sea State 5)			H_s = 7.5 m (Sea State 7)			
ω_0	0.405 (rad/s)	0.648 (rad/s)	0.757 (rad/s)	0.340 (rad/s)	0.419 (rad/s)	0.532 (rad/s)	
Program S1	10.6	19.2	26.4	9.9	10.8	13.7	0. 93
Program S2	9.0	17.4	24.8	8.5	9.2	12.0	0.99
Program S3	8.7	15.5	21.5	8.2	8.8	11.1	1.01

At least for these sea states, the TFE suggests that program S3 is superior to the other two codes, although the differences between programs S2 and S3 are not very large. According to Guedes Soares's model error, programs S2 and S3 are equally accurate. Dalzell's correlation index could not be calculated for this case because the maximum of experimental values, $\max|H(\omega)|$ in (7), is likely to be the asymptotic value as $\overline{\omega} \to 0$ (wavelength $\to \infty$), which is obviously outside the experimentally feasible range.

4. CONCLUDING REMARKS

No new software should be used without first being subjected to thorough and rigorous tests. Thus, experimental validation is an essential part of code development. But the conventional method of code validation, based as it is on the visual judgement and the use of words, is subjective and apt to be inconsistent owing to lack of precise criteria. It is also inefficient. To give precision and objectivity to the code validation, the total factor error (TFE) was introduced in this paper.

The TFE is an index of correlation between predicted and measured transfer functions for ship motions and wave loads. What differentiates TFE from other indices that have been devised for the same purpose is that it accounts for the varying impact of the error in the predicted transfer function on the final result. Because of this, TFE can rate the candidate prediction methods' accuracy uniquely even when other indices fail to do so. Calculation of TFE can be programmed easily. So by checking the values of TFE, it is possible to ensure the reliability of the computer prediction of the transfer functions for ship motions and wave loads as a matter of routine. The objective of this paper is to outline the concept of TFE and show its principal features through simple examples. More work is needed to develop the concept fully and to calibrate TFE. Only then will it be possible to answer such a question as "How accurate is the predicted transfer function if its TFE is 10 percent?". In fact, part of Guedes Soares's work [3] is along these lines.

This paper brought into focus the need for some standard benchmark model-test data for ship motions and wave loads. They are necessary not only to enable the naval architect to calibrate the accuracy of the computer-predicted transfer functions at his/her disposal, but also to fully realize the usefulness of an index of correlation as a universal figure of merit of accuracy of predicted transfer functions. Once established, such benchmark data will be revised as more and more precise measurements become available. This consideration undoubtedly constitutes a large part of the motivation for the experimental work [5] by Lloyd and his colleagues. Though an extremely valuable step forward, it has not been followed up, however. This of course has a great deal to do with the large expense such elaborate tests will incur. So the initiatives and coordination by RINA, SNAME, and other societies of naval architects as well as international umbrella organizations such as ITTC are essential. In this respect, Murdey [6] pointed out that the need for a set of benchmark data for validation of seakeeping predictions had been recognized by the ITTC for some time and that the Loads and Responses Committee had been assigned the task of reviewing all benchmark data with a view to publication in a standard format for validation in seakeeping in 1999.

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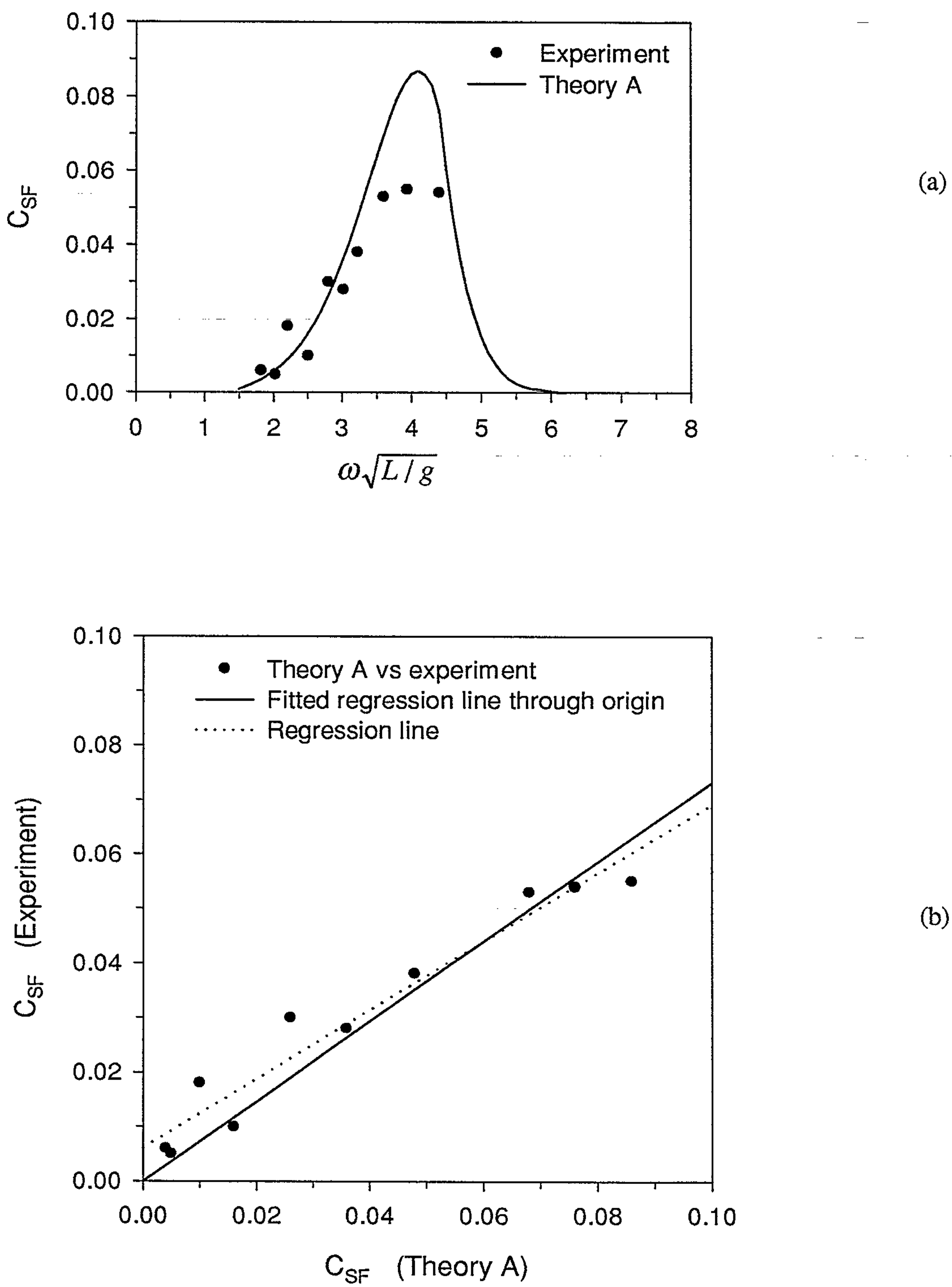
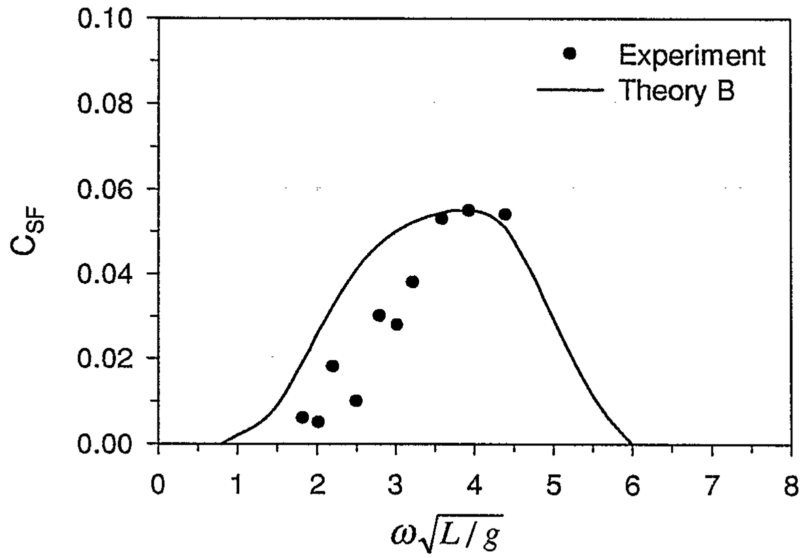


FIG. 1. Comparison of nondimensional midship horizontal shear force C_{SF} predicted by hypothetical theory A with experiment. $F_n = 0.15$, wave direction 150 deg. (a) wave frequency vs. C_{SF}; (b) regression line (dotted line: slope 0.63, intercept 0.006) of experimental value on predicted value, for which the sum of squares of residuals is minimum, and the fitted regression line passing through origin (solid line: slope 0.73).



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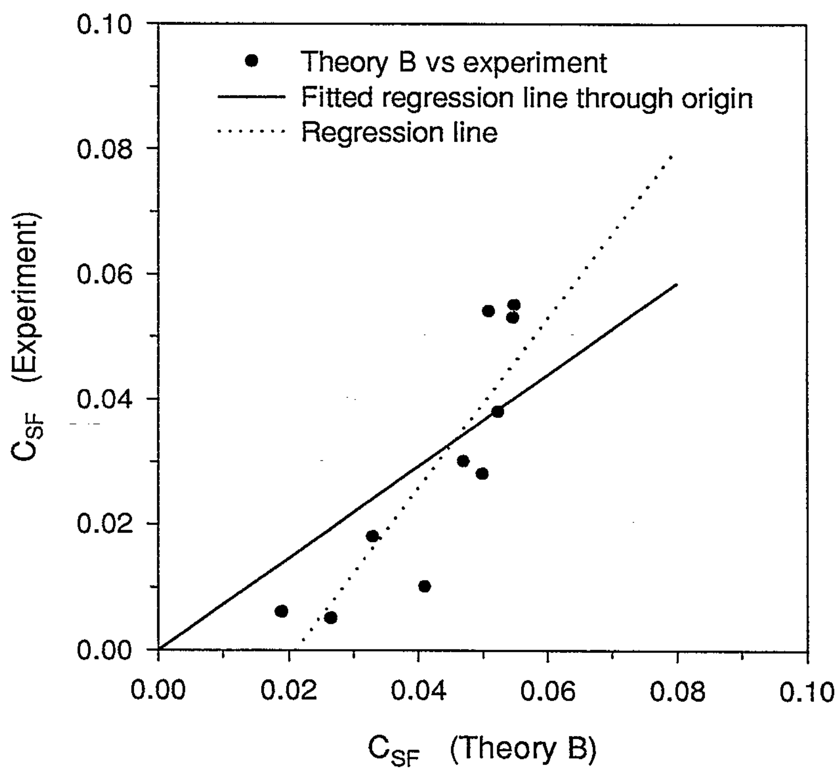


(a)

FIG. 2. Comparison of nondimensional midship horizontal shear force C_{SF} predicted by hypothetical theory B with experiment. $F_n = 0.15$, wave direction 150 deg. (a) wave frequency vs. C_{SF}; (b) regression line (dotted line: slope 1.36, intercept -0.029) of experimental value on predicted value, for which the sum of squares of residuals is minimum, and the fitted regression line passing through origin (solid line: slope 0.73).

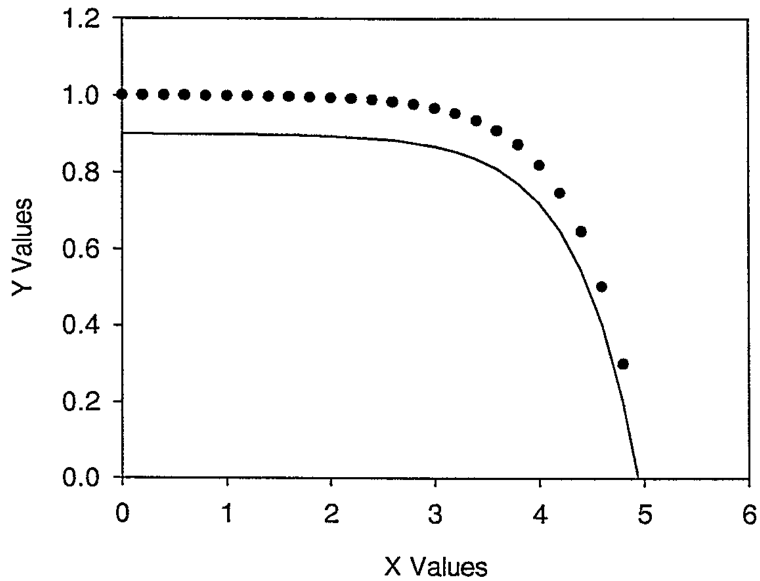
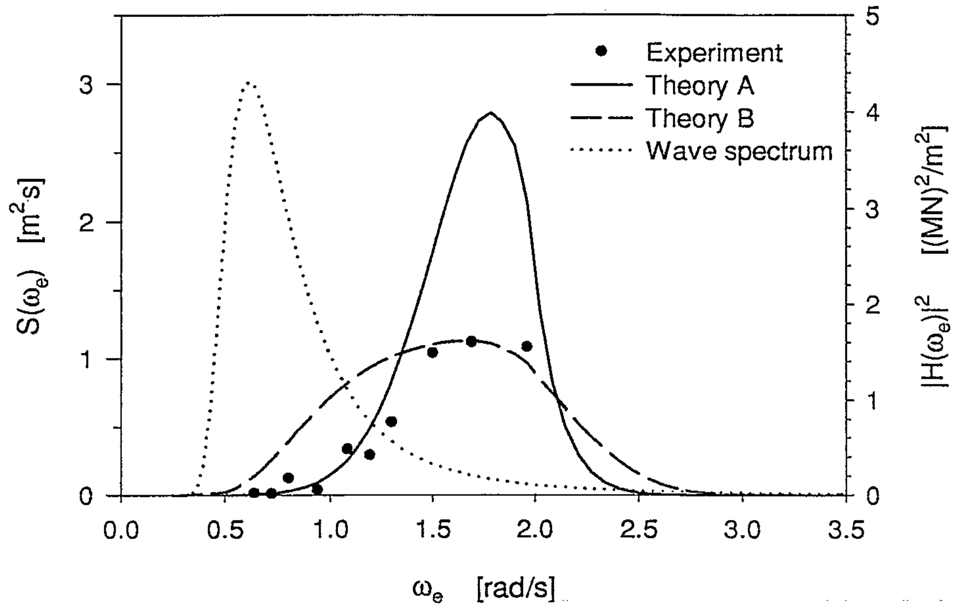


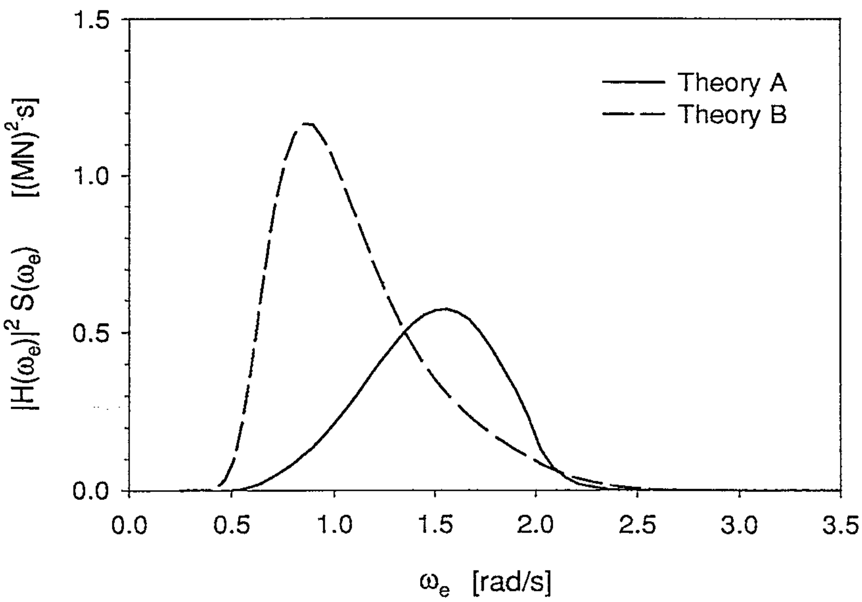
FIG. 3. Hazard of visual judgment. The vertical distances between curve and points are equal at all values of X, but for some, the distance appears to get smaller as the X-value increases; see, e.g. [1].

	Requirements	*Prediction*
Array Size:	64 x 64	√
Pixel Pitch:	200 μm	√
Instantaneous Dynamic Range:	> 60 dB	√
Max Power:	2 W	0.2
Operating Temperature Range:	15°C - 50°C	√
Bandpass Cutoff Frequency:	> 10 KHz	√
Bandpass Cutoff Frequency:	> 8 MHz	√
Gain States:	10, 100	2, 20, 200
Gain Accuracy:	± 30%	± 10%
Gain Variation: (Pixel to Pixel Standard Deviation)	5%	√
Distortion:	- 60 dB THD w/2V Swing	√
Input Referred Noise at Max Gain:	5 nV/rtHz	6.6nV/rtHz
Sample and Hold Number:	16 Minimum	20
Maximum Sample Frequency:	20 MHz	√
Piezoelectric Detector Minimum Series Capacitance:	0.15 pF over Frequency Range	√

Figure F-3. BUDI ROIC Performance Specification

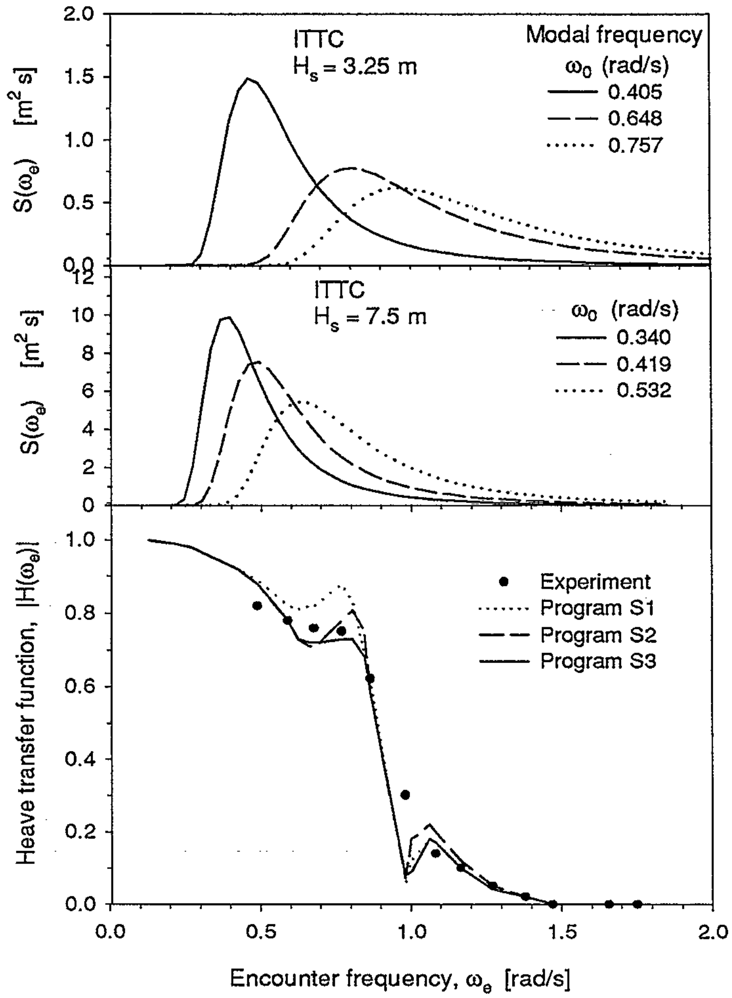


(a)

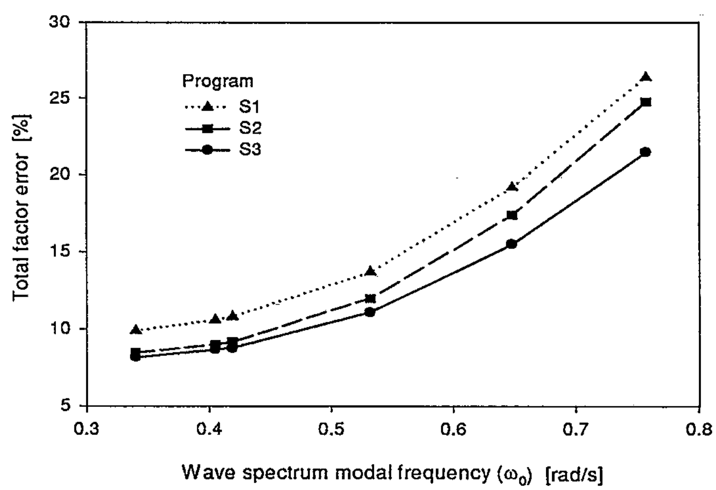


(b)

FIG. 4. Comparison of hypothetical theories A and B in Figs. 1 and 2. $F_n = 0.15$, wave direction 150 deg for a ship of B = 18.75 m, L = 121.9 m. (a) RAO's for horizontal shear force amidships and experimental values, along with ITTC wave spectrum for $H_s = 5.0$ m, $\omega_0 = 0.507$ rad/s; (b) Midship horizontal shear force response spectra obtained from Fig. 4(a).



(a)



(b)

FIG. 5. Comparison of TFE for heave transfer functions predicted for a destroyer model in head seas, F_n = 0.194, by three candidate computer codes. (a) ITTC wave spectra used for calculation of TFE in relation to predicted and measured heave transfer functions and (b) TFE for the three codes as a function of modal frequency of wave spectra.

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