Image Cover Sheet

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TITLE

BEAM SPLITTER LAYER EMISSION IN FOURIER-TRANSFORM INFRARED INTERFEROMETERS

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Beam splitter layer emission in Fourier-transform infrared interferometers

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The self-emission from the splitting layer of a Fourier-transform infrared interferometer is modeled with basic properties of optical thin films. The resulting equation gives explicitly the self-emission contribution in terms of the temperature, the complex refractive index, the reflection coefficient, and the thickness of the beam splitter layer.

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Raw spectra generated by Fourier-transform infrared (FTIR) interferometers contain a significant amount of parasite radiation in addition to radiation of the source under study. This parasite radiation is often referred to as self-emission. It is attributed to graybody emission and scattering from the various optical elements that comprise the interferometer system, e.g., from input and output optics and from the beam splitter itself. For performance optimizations often one must quantify the amount of self-emission in a given design. The self-emission from input and output optics is usually well understood and relatively simple to compute from knowledge of the temperature, of the transmission, and of the reflection of each intervening element. However, the self-emission contribution by the beam splitter itself is more complicated to handle. Only a few papers exist in which the effects and consequences of beam splitter self-emission in FTIR interferometers is discussed and explained. For example, Revercomb *et al.*[1] observed and interpreted the dual phase response of the high-resolution interferometer sounder as an effect of beam splitter emission. More recently, Weddigen *et al.*[2] reported phase anomalies in spectra of the Michelson interferometer for passive atmospheric sounding (MIPAS) instrument, which were corrected by taking into account explicitly the beam splitter self-emission. They also proposed a mathematical description for it.

In this Note a simple model to compute the self-emission of a single thin layer beam splitter is proposed. It is developed with the basic properties of optical thin films. The resulting equation was found to be useful in the study of a double input-beam FTIR interferometer optimized to suppress the self-emission contributions.[3,4] Before the self-emission model is described, it is convenient to review the interferometer modulation characteristics when a thin layer beam splitter is used. These characteristics are presented in the context of a double-beam FTIR interferometer configuration.

The ray tracings in Figs. 1(a) and 1(b) indicate that the output amplitudes ($\mathbf{A}_1$ and $\mathbf{A}_2$) for beams of unit amplitude incident on input 1 and input 2 are given by

$$\mathbf{A}_1 = \mathbf{rt} + \mathbf{tr}\exp(i\phi), \tag{1}$$

$$\mathbf{A}_2 = \mathbf{rr} + \mathbf{tt}\exp(i\phi), \tag{2}$$

where $\mathbf{r}$ and $\mathbf{t}$ represent the amplitude reflection and transmission, respectively, of the layer and

$$\phi = 2\pi\nu x \tag{3}$$

expresses the phase difference between the two interfering beams in terms of the wave number ν and the optical path difference x. Following Heavens,[5] the amplitude reflection and transmission of the layer can be evaluated explicitly by summation of the contributions from multiple reflections and transmissions inside the layer. For a layer symmetrically bounded by identical media such as defined in Fig. 1(c) the summations yield

$$\mathbf{r} = \frac{r_1[1 - \exp(-2i\delta)]}{1 - r_1^2\exp(-2i\delta)}, \tag{4}$$

$$\mathbf{t} = \frac{(1 - r_1^2)\exp(-i\delta)}{1 - r_1^2\exp(-2i\delta)}, \tag{5}$$

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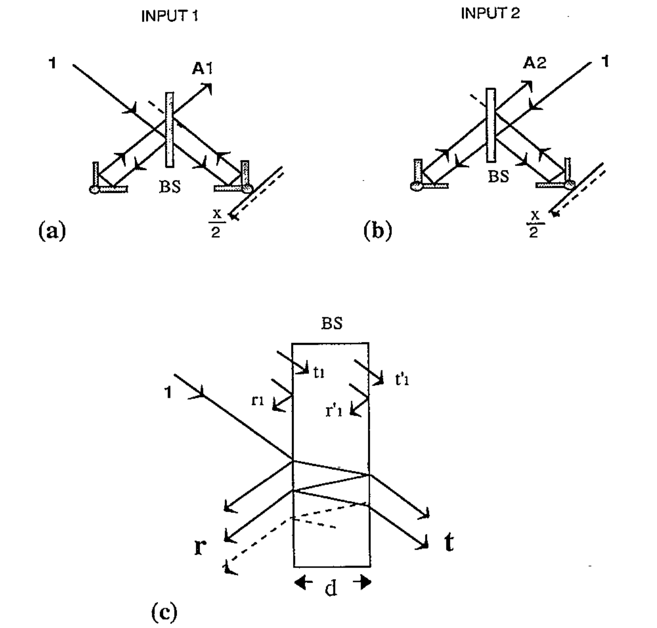

Fig. 1. Ray tracings showing the output amplitudes ($\mathbf{A}_1$ and $\mathbf{A}_2$) for beams of unit amplitude incident on (a) input 1, (b) input 2, (c) the amplitude reflection and transmission coefficients of a thin layer beam splitter.

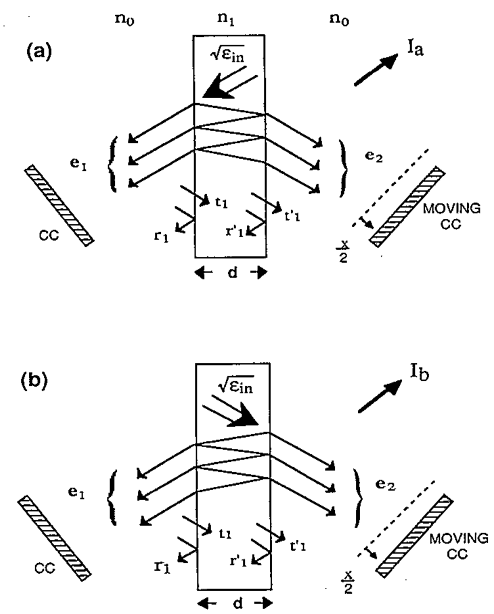

Fig. 2. Ray tracings for the modelization of the self-emission from the splitting layer. Graybody radiation and stray light can emanate from the layer and interfere with itself to create a significant contribution to the raw spectrum (see text).

with

$$\delta = 2\pi \nu n d \cos(\theta), \tag{6}$$

where δ is the phase shift that is due to a layer of optical thickness nd, n is the refractive index of the layer, and θ is the angle of refraction inside the material. Equations (4) and (5) are obtained with the following identities between the Fresnel coefficients (r_1, r_1' and t_1, t_1') associated with the interfaces of the layer (see Ref. 5, pp. 51–57):

$$r_1' = -r_1, \tag{7}$$

$$t_1' t_1 = (1 - r_1^2). \tag{8}$$

Equating the denominators of Eqs. (4) and (5) leads to a simple expression that connects the amplitude reflection and transmission such that

$$\mathbf{t} = \mathbf{r} H \exp\left(-i \frac{\pi}{2}\right), \tag{9}$$

where H is an intermediate variable defined as

$$H = \frac{(1 - r_1^2)}{2 r_1 \sin(\delta)}. \tag{10}$$

Equation (9) shows a 90° phase difference between the amplitude reflection and the transmission of the layer. This is a general property that explains (as seen below) the 180° phase difference between the two interferograms associated with the input and output ports (or between the two inputs for a double-beam inteferometer). In addition, Eqs. (9) and (10) are useful relations that simplify the analysis for obtaining the characteristics of the interferometer.

Using Eqs. (1) and (2) and multiplying the two output amplitudes by their complex conjugates ($\mathbf{A}_1^*$ and $\mathbf{A}_2^*$) yields the two output intensities I_1 and I_2 associated with each input port; i.e.,

$$I_1 = \mathbf{A}_1\mathbf{A}_1^* = [\mathbf{rt} + \mathbf{tr} \exp(i\phi)][\mathbf{r}^*\mathbf{t}^* + \mathbf{t}^*\mathbf{r}^* \exp(-i\phi)], \tag{11}$$

$$I_2 = \mathbf{A}_2\mathbf{A}_2^* = [\mathbf{r}^2 + \mathbf{t}^2 \exp(i\phi)][\mathbf{r}^{*2} + \mathbf{t}^{*2} \exp(-i\phi)]. \tag{12}$$

Taking the coefficient of reflection in intensity of the layer $R = \mathbf{rr}^*$ and inserting Eq. (9) into the two preceding equations yields for the two output intensities

$$I_1 = [2R^2H^2 + 2R^2H^2 \cos(\phi)], \tag{13}$$

$$I_2 = [R^2 + R^2H^2 + 2R^2H^2 \cos(\phi - \pi)]. \tag{14}$$

In Eqs. (13) and (14) the modulated components of intensities represent the interferograms associated with each input port. Note the 180° phase difference between the two interferograms, which yields a perfectly balanced double-beam interferometer.

As discussed above, graybody radiation and stray light can emanate from the splitting layer and interfere with itself. Let us assume that the splitting layer absorbs with a complex refractive index $\mathbf{n}_1 = n_1 - ik_1$ and a thickness d and is bounded symmetrically by identical transparent media of index n_0. Figure 2 represents the layer together with parameters used to model the emission phenomena. The bulk material that composes the layer generates isotropic IR radiation (emission and stray light). Only the radiation inside the field of view of the detector will contribute to the interferogram. The only two components that can contribute to the interferogram are indicated in Fig. 2 where ε_{in} is defined as the emitted intensity (radiance units) of the bulk material. For each direction this emitted intensity generates two beams of amplitudes $\mathbf{e}_1$ and $\mathbf{e}_2$ as a result of the multiple reflections inside the layer. After reflections on the two corner reflectors such as for conventional interferometer ray tracings [see Figs. 1(a) and 1(b)], the two beams are recombined for interference at the detector level.

For the emitted intensity contribution in the direction of the fixed corner reflector [Fig. 2(a)] the amplitude $\mathbf{e}_1$ of the first beam can be evaluated by the addition of multiple reflections that intervene inside the layer. Following Heavens[5] and according to Fresnel coefficients defined in Fig. 2, this amplitude is given by

$$\mathbf{e}_1 = \sqrt{\varepsilon_{in}}\,[t_1' + r_1'^2 t_1' \exp(-2\alpha)\exp(-2i\Delta) + r_1'^4 t_1' \exp(-4\alpha)\exp(-4i\Delta) + \ldots], \quad (15)$$

where

$$\Delta = 2\pi\nu n_1^{\text{eff}} d \cos(\theta_1), \quad (16)$$

$$\alpha = 2\pi\nu k_1^{\text{eff}} d \cos(\theta_1), \quad (17)$$

where n_1^{eff}, k_1^{eff}, and θ_1 are the effective refractive index, absorption coefficient, and refractive angle, respectively, inside the layer. These effective quantities can be derived by application of Snell's law for absorbing medium. Knowing that $r_1' = -r_1$ and adding terms in Eq. (15) yields

$$\mathbf{e}_1 = \sqrt{\varepsilon_{in}}\,\frac{t_1' \exp(-\alpha)\exp(-i\Delta)}{[1 - r_1^2 \exp(-2\alpha)\exp(-2i\Delta)]}, \quad (18)$$

which can be expressed in a more compact form with the amplitude transmittance ($\mathbf{t}$) expression of the layer defined by Eqs. (5) and (8); i.e.,

$$\mathbf{e}_1 = \sqrt{\varepsilon_{in}}\,\frac{\mathbf{t}}{t_1} \exp(\alpha)\exp(i\Delta). \quad (19)$$

From similar considerations the amplitude $\mathbf{e}_2$ of the second beam [Fig. 2(a)] is found to be

$$\mathbf{e}_2 = \sqrt{\varepsilon_{in}}\left(\frac{-r_1}{t_1}\right)\mathbf{t}. \quad (20)$$

Consequently, the output amplitudes that correspond to the two interfering beams associated with this self-emission component [Fig. 2(a)] are

$$\mathbf{A}_1 = \mathbf{e}_1\mathbf{t} = \sqrt{\varepsilon_{in}}\,\frac{\mathbf{t}^2}{t_1} \exp(\alpha)\exp(i\Delta), \quad (21)$$

$$\mathbf{A}_2 = \mathbf{e}_2\mathbf{r} \exp(i\phi) = \sqrt{\varepsilon_{in}}\,\frac{-r_1}{t_1}\,\mathbf{tr} \exp(i\phi), \quad (22)$$

which yields

$$\mathbf{A}_1 + \mathbf{A}_2 = \sqrt{\varepsilon_{in}}\,\frac{\mathbf{t}}{t_1}\,[\mathbf{t} \exp(\alpha)\exp(i\Delta) - r_1\mathbf{r} \exp(i\phi)]. \quad (23)$$

The associated intensity I_a is found by multiplication of Eq. (23) by its complex conjugate yielding

$$I_a = \varepsilon_{in}\,\frac{T}{T_1}\,(T \exp(2\alpha) + RR_1 - r_1 \exp(\alpha) \times \{\mathbf{rt}^* \exp[i(\phi - \Delta)] + \mathbf{r}^*\mathbf{t} \exp[-i(\phi - \Delta)]\}), \quad (24)$$

where the uppercase letters are the intensities defined as $T = \mathbf{tt}^*$, $R = \mathbf{rr}^*$, and $R_1 = r_1 r_1$.

When the emitted intensity contribution is in the direction of the moving corner reflector [Fig. 2(b)], the two amplitudes $\mathbf{e}_1$ and $\mathbf{e}_2$ are simply the reverse situation of Fig. 2(a). In this case we have

$$\mathbf{e}_1 = \sqrt{\varepsilon_{in}}\,\frac{-r_1}{t_1}\,\mathbf{t}, \quad (25)$$

$$\mathbf{e}_2 = \sqrt{\varepsilon_{in}}\,\frac{\mathbf{t}}{t_1} \exp(\alpha)\exp(i\Delta), \quad (26)$$

which for the corresponding output intensity I_b [Fig. 2(b)] incident on the detector yields

$$I_b = \varepsilon_{in}\,\frac{T}{T_1}\,(R \exp(2\alpha) + TR_1 - r_1 \exp(\alpha) \times \{\mathbf{rt}^* \exp[i(\phi + \Delta)] + \mathbf{r}^*\mathbf{t} \exp[-i(\phi + \Delta)]\}). \quad (27)$$

The quantities of interest in Eqs. (24) and (27) are the two modulated components that add together to produce the interferogram that corresponds to the layer self-emission. With this self-emission defined as $\mathbf{E}_{lay}$, it follows that

$$\mathbf{E}_{lay} = \varepsilon_{in}\,\frac{T}{T_1}\,[-r_1 \exp(\alpha) 2 \cos \Delta][\mathbf{rt}^* \exp(i\phi) + \mathbf{r}^*\mathbf{t} \exp(-i\phi)]. \quad (28)$$

To reduce Eq. (28) further, it is convenient to reuse the connecting relation between $\mathbf{t}$ and $\mathbf{r}$ [Eq. (9)] but for an absorbing layer; i.e.,

$$\mathbf{t} = \mathbf{rH} \exp(i\pi/2), \quad (29)$$

where $\mathbf{H}$ is now a complex number given by

$$\mathbf{H} \equiv H \exp(i\theta_H) = \left(\frac{1 - r_1^2}{r_1}\right)\frac{1}{[\exp(\alpha - i\Delta) - \exp(-\alpha + i\Delta)]}. \quad (30)$$

Note that Eq. (30) is obtained directly by insertion of the complex phase associated with an absorbing layer, $\delta = \Delta - i\alpha$, into Eq. (10), where Δ and α are defined by Eqs. (16) and (17), respectively. Taking

Eq. (30) into account and combining Eq. (29) with Eq. (28) yields

$$\mathbf{E}_{\mathrm{lay}} = \varepsilon_{\mathrm{in}} \frac{T}{T_1} (-r_1 \exp(\alpha) 2 \cos \Delta) \left\{ \mathbf{r}\mathbf{r}^* H \exp\left[i\left(\phi - \frac{\pi}{2} - \theta_H\right)\right] + \mathbf{r}^*\mathbf{r} H \exp\left[-i\left(\phi - \frac{\pi}{2} - \theta_H\right)\right]\right\}, \quad (31)$$

or, in a simpler form,

$$\mathbf{E}_{\mathrm{lay}} = \varepsilon_{\mathrm{in}} \frac{T}{T_1} [r_1 \exp(\alpha) 2 \cos \Delta](2RH)\cos\left(\phi + \frac{\pi}{2} - \theta_H\right), \quad (32)$$

where $R = \mathbf{r}\mathbf{r}^*$ and with H and θ_H defined by Eq. (30).

Equation (32) gives an analytical expression for computing the self-emission of a thin layer beam splitter. It must be emphasized that Eq. (32) is not rigorously exact mainly because the phase shift (when the layer is absorbing) associated with the air-layer reflection was neglected in this development. This phase shift becomes important for a strongly absorbing layer. Thus Eq. (32) constitutes a proper approximation for a slightly absorbing layer material, which is usually the case, because this is the condition that maximizes the sensitivity of the interferometer. In addition, for a slightly absorbing layer material, θ_H becomes negligible. In this case the layer self-emission interferogram [Eq. (32)] appears to have an extra phase of 90° over the source interferograms [Eqs. (13) and (14)]. This is in agreement with the result found by Weddigen *et al.*[2]

The driving term in Eq. (32) is the emitted intensity $\varepsilon_{\mathrm{in}}$ defined as the graybody radiation and stray light emanating from the bulk of the material that comprises the layer. For a well-designed beam splitter the scattering inside the splitting layer is in principle negligible, thus eliminating the stray light contribution. Consequently, the only significant contribution to the emitted intensity $\varepsilon_{\mathrm{in}}$ arises from the graybody emission of the layer material, which is defined as

$$\varepsilon_{\mathrm{in}} = [1 - \exp(-2\alpha)]B(T_{\mathrm{BS}}), \quad (33)$$

where $B(T_{\mathrm{BS}})$ is the Planck radiance that corresponds to the temperature of the beam splitter, T_{BS}, and 2α is the optical depth associated with the absorption of the layer material. In the beam splitter configuration selected for the double-beam FTIR interferometer[3,4] the splitting layer consists of a thin film of air. In this case there is no self-emission associated with this air layer, because $\alpha = 0$.

It is important to point out that in the development leading to Eq. (32) the polarization effects from the beam splitter were neglected. Because beam splitters (in an interferometer) are usually utilized at nonnormal incidence, there should be significant polarization effects. In this case Eq. (32) can be used to compute the self-emission for each polarization (s and p), provided that the Fresnel coefficients are defined accordingly. Finally, it is believed that this approach to computing the self-emission for a single layer can be easily extended to more complex multilayer beam splitters.

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