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SHIP CAPSIZE RISK IN A SEAWAY USING TIME DOMAIN SIMULATIONS AND FITTED GUMBEL DISTRIBUTIONS

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ABSTRACT

Risk analysis provides the most rational approach for designing and operating a ship to prevent capsize. A capsize risk analysis procedure must adequately consider the dynamics of ship capsize in waves. This paper presents an efficient method for determining capsize risk for a given seaway and ship operational condition using time domain simulations. The risk of capsize during one hour exposure to a given seaway is dependent on the wave process realization, which is determined by a seed number and resulting random wave phases. The dependence of maximum roll angle on wave process is modelled by fitting a Gumbel distribution to maximum roll angles from a moderate number of simulations. Sample computations for a naval frigate demonstrate that a Gumbel distribution provides a very good fit to maximum roll angles from different wave realizations. The sample computations provide guidance regarding the number of wave realizations and simulation durations that should be used for fitting Gumbel distributions.

INTRODUCTION

Risk analysis offers significant advantages over traditional rule-based methods for design and operation of ships. Rule-based methods are typically derived from a limited range of experience, and are not suitable for extrapolation to new design types or operational conditions. In contrast, risk analysis can yield consistent levels of safety among existing and innovative new designs. Another useful aspect of risk analysis is that it can identify which operational conditions are most dangerous for a vessel. For example, de Kat et al. (1994) indicate that a naval frigate should avoid travelling at high speed in severe stern quartering seas. The benefits of risk analysis are leading to its adoption by navies and the shipping industry in areas such as structural design (Mansour et al. 1997).

Existing stability standards for naval ships from several countries are based on the work of Sarchin and Goldberg (1962). Their design criteria were developed by examining capsizes of naval vessels during World War II. Since the development of the existing stability standards, there has been a significant move from narrow sterns to wide transom sterns on modern warships. This design evolution has made modern warships vulnerable to loss of stability in following seas, which can occur when the transom comes out of the water, resulting in large loss of waterplane area.

The Cooperative Research Navies Dynamic Stability Project (de Kat et al., 1994) has been developing new frigate stability criteria which adequately consider the dynamics of ship capsize. In conjunction with this project, the Canadian Department of National Defence has been developing risk analysis methods for ship capsize. A key aspect of the risk analysis is to determine the capsize risk for a ship in given conditions, which are defined by ship speed, heading, significant wave height, and peak wave period. This paper describes the prediction of capsize risk for given conditions using time domain simulation. Of particular importance is the sensitivity of ship capsize to wave process realization.

TIME DOMAIN SIMULATION OF SHIP CAPSIZE

Ship capsize is a complex nonlinear phenomenon for which time domain simulation appears to be required for accurate modelling. The Cooperative Research Navies Dynamic Stability Project has been developing and using the program Fredyn (de Kat, 1993, Hooft and Pieffers, 1989) for simulating capsize of naval frigates in regular and irregular wave conditions. Fredyn computes ship response for six degrees of freedom to forces arising from waves, wind, and maneuvering. The time domain approach allows force nonlinearities to be easily incorporated into motion predictions. Added mass, retardation, and wave diffraction forces are predicted using strip theory, which appears to be sufficiently accurate for naval frigates. The strip theory implementation is robust and fast, with the program running approximately 15 times faster than real time on a personal computer. This high computational speed is essential for applications such as risk analysis, which require large numbers of simulations.

Fredyn has been extensively validated with experimental data for naval frigates, and can model key capsize modes. For a frigate in following seas, these modes include loss of static stability while riding on a wave crest, parametric excitation, and broaching. Recently, de Kat and Thomas (1998) conducted dedicated model tests of frigate capsize to be used for additional validation of Fredyn. Preliminary results confirm that Fredyn is very good at predicting when frigate capsize will occur. To date only limited validation with Fredyn has been done for vessels with low length to beam ratios such as fishing boats; thus, it is uncertain whether the strip theory approach provides sufficient accuracy for predicting capsize of non-slender ships.

CAPSIZE RISK FOR ALL CONDITIONS

Using the program Fredyn described above, a new risk analysis procedure is being developed for ship capsize. While previous work (McTaggart 1993) focussed on the risk of capsize in the extreme annual storm based on significant wave height, the present work considers all combinations of seaways because the most dangerous conditions for capsize won't necessarily be associated with the highest significant wave height. For example, smaller but steeper waves could be more dangerous than the highest waves encountered.

The present approach evaluates the capsize risk in a seaway of duration D as follows:

$$P(C_D) = \sum_{i=1}^{N_{V_s}} \sum_{j=1}^{N_\beta} \sum_{k=1}^{N_{H_s}} \sum_{l=1}^{N_{T_p}} p_{V_s}(V_{s-i})\, p_\beta(\beta_j) \times p_{H_s,T_p}(H_{s-k}, T_{p-l}) \times P(C_D|V_s, \beta, H_s, T_p) \quad (1)$$

where N_X is the number of discretized values of variable X, $p_X(X)$ is the probability mass function of X, V_s is calm water ship speed, β is the ship heading relative to the waves, H_s is significant wave height, T_p is peak wave period, and $P(C_D|V_s, \beta, H_s, T_p)$ is the conditional probability of capsize for the given conditions. Dahle and Myrhaug (1994,1996) and Kobylinski (1997) give similar equations for prediction of capsize risk.

Once the probability of capsize for seaway duration D has been computed using Equation (1), the associated annual probability of capsize can be computed as follows:

$$P(C_{annual}) = 1 - [1 - P(C_D)]^{1\ year/D} \quad (2)$$

A similar approach can be used to determine capsize probability during the life of the ship.

When evaluating ship motions in waves, it is conservatively assumed that waves are unidirectional. This assumption is quite reasonable because directional spreading tends to decrease with higher wave conditions, which are most relevant to ship capsize.

Joint distributions of wave height and wave period are available from Bales (1984) and from BMT Global Wave Statistics (1986). The data from Bales are used here because BMT Global Wave Statistics appears to have some unrealistically large wave steepnesses, which lead to simulation instabilities and unrealistically large capsize risks. The validity of large wave steepnesses from BMT Global Wave Statistics will be further investigated in future work.

An important assumption of Equation (1) is that desired ship speed and heading are independent of wave conditions. This assumption is conservative because ship operators will alter speed and course to reduce capsize risk. The Canadian Navy and other organizations are analysing ship operational data to examine how ship operators alter course and heading in response to wave conditions. The influence of operator response to wave conditions could likely be included in evaluating capsize risk as follows:

$$P(C_D) = \sum_{i=1}^{N_{V_s}} \sum_{j=1}^{N_\beta} \sum_{k=1}^{N_{H_s}} \sum_{l=1}^{N_{T_z}} p_{V_s|H_s}(V_{s-i}|H_{s-k}) \times p_{\beta|H_s}(\beta_j|H_{s-k})\, p_{H_s,T_p}(H_{s-k}, T_{p-l}) \times P(C_D|V_s, \beta, H_s, T_p) \quad (3)$$

The above equation ignores weather routing, but considers ship speed and heading as conditional probabilities of significant wave height.

CAPSIZE RISK FOR GIVEN CONDITIONS USING FITTED GUMBEL DISTRIBUTIONS

The analysis of capsize risk requires evaluation of the conditional capsize probabilities $P(C_D|V_s, \beta, H_s, T_p)$ for relevant possible combinations. The occurrence of capsize in an irregular seaway will depend on the wave process realization, as discussed in detail by Belenky et al. (1998). For an irregular seaway generated by the program Fredyn, an input integer seed number is provided by the user to randomly generate phase angles of wave components. Using N_s different seed numbers and resulting simulations, an estimate of conditional capsize probability can be obtained as follows:

$$P(C_D|V_s, \beta, H_s, T_z) = \frac{N_C}{N_s} \quad (4)$$

where N_C is the number of simulations for which capsize occurs. The main problem with the above approach is that a large number of simulations can be required to determine $P(C_D|V_s, \beta, H_s, T_z)$ to sufficient accuracy, particularly if the conditional capsize risk is small. For risk analysis, there is also a question regarding what seaway duration D should be used.

The current study introduces an alternative method for predicting the probability of capsize in a given seaway. Madsen et al. (1986) and Thoft-Christensen and Baker (1982) indicate that the maximum value of a stationary random process during a specified time interval can be modelled using a Gumbel distribution; thus, a Gumbel distribution could be appropriate for modelling the distribution of $\phi_{max,D}|V_s, \beta, H_s, T_p$, the maximum roll angle for given conditions. The cumulative distribution function (CDF) of the Gumbel distribution is as follows:

$$F_X(X) = \exp\{-\exp[\alpha_X(X - u_X)]\} \tag{5}$$

where u_X and α_X are distribution parameters. The mean μ_X and standard deviation σ_X of the Gumbel distribution are related to the distribution parameters as follows:

$$\mu_X = u_X + \frac{\gamma}{\alpha_X} \tag{6}$$

$$\sigma_X = \frac{\pi}{\alpha_X \sqrt{6}} \tag{7}$$

where γ is Euler's constant (0.5772...).

Two methods are available for determining fitted Gumbel distribution parameters. The method of moments relates the distribution parameters to the computed mean and standard deviation for the variable X. The second method uses a least squares linear fit between the variable X or its transformation (e.g. $\ln X$) and cumulative distribution function $F_X(X)$ or its transformation (e.g. $\ln F_X(X)$). A significant advantage of the least squares method is that it can be applied to a limited variable range of greatest interest. When simulating ship motions using the program Fredyn, a simulation will terminate when the ship roll angle exceeds 90 degrees; thus, a Gumbel least squares fit provides a suitable approach for handling the limited range of valid maximum roll angles.

The equation for determining a least squares fit of Gumbel parameters is as follows:

$$X = \frac{1}{\alpha_X} \ln[-\ln(F_X(X))] + u_X \tag{8}$$

Placing the variable X on the left-hand side of the above equation minimizes the error in X for a given value of $\ln[-\ln(F_X(X))]$. The linear regression fit has a slope of $1/\alpha_X$ and an intercept of u_X. When determining the least squares fit parameters from N_s simulations in a given seaway, the simulated maximum roll angles can be ranked in ascending order to assign cumulative distribution values as follows:

$$F(\phi_{max,i}) = \frac{i}{N_s + 1} \tag{9}$$

where $\phi_{max,i}$ is the sorted maximum roll angle with rank i. Once the Gumbel parameters have been determined using a least squares fit for a given seaway, the associated capsize probability can be easily estimated using Equation (5).

If a maximum value from simulations of duration D_s has a Gumbel distribution, then the maximum value for another duration D will have the following properties:

$$\mu_X(D) = \mu_X(D_s) + \sigma_X(D_s)\frac{\sqrt{6}}{\pi} \ln\left(\frac{D}{D_s}\right) \tag{10}$$

$$\sigma_X(D) = \sigma_X(D_s) \tag{11}$$

The above equations imply that a shorter simulation duration D_s can be used for determining Gumbel distribution parameters for a longer duration D.

The fitting of a Gumbel distribution for a given seaway provides an estimate of the following conditional roll exceedence distribution:

$$Q_{\phi_{max,D}}(\phi_{max,D}|V_s, \beta, H_s, T_p) = 1 - F_X(X) \tag{12}$$

An additional benefit of using fitted Gumbel distributions is that the distribution of maximum roll for all seaways can then be obtained as follows:

$$Q_{\phi_{max,D}}(\phi_{max,D}) = \sum_{i=1}^{N_{V_s}} \sum_{j=1}^{N_\beta} \sum_{k=1}^{N_{H_s}} \sum_{l=1}^{N_{T_z}} p_{V_s}(V_{s-i}) \times p_\beta(\beta_j)\, p_{H_s,T_p}(H_{s-k}, T_{p-l}) \times Q_{\phi_{max,D}|V_s,\beta,H_s,T_p}(\phi_{max,D}|V_s, \beta, H_s, T_p) \tag{13}$$

SIMULATIONS FOR A NAVAL FRIGATE

Parametric simulations have been conducted to examine the applicability of the Gumbel distribution to maximum roll angles from simulations. The Canadian Patrol Frigate was used for the parametric simulations, with properties as given in Table 1. Table 2 shows the base parameters used for the simulations. The ship heading of 60 degrees (stern quartering seas) was selected because it is among the most likely to cause capsize for a naval frigate.

Figures 1 and 2 show the mean and standard deviation of maximum roll angle versus significant wave height. These figures also show predicted values based on RMS roll and linear theory, for which Madsen et al. (1986) give the following equations for a narrow-banded Gaussian process:

$$E(\phi_{max}) = \sigma_\phi \left[\sqrt{2 \ln N_{cycle}} + \frac{\gamma}{\sqrt{2 \ln N_{cycle}}}\right] \tag{14}$$

$$\sigma(\phi_{max}) = \sigma_\phi \frac{\pi}{\sqrt{6}} \frac{1}{\sqrt{2 \ln N_{cycle}}} \tag{15}$$

Table 1: Main Particulars for Canadian Patrol Frigate

Length, L	124.5 m
Beam, B	14.7 m
Midships draft, T	4.64 m
Trim by stern, t_s	0.0 m
Displacement, $\triangle$	4077 tonnes
Vertical centre of gravity, $\overline{KG}$	6.44 m
Corrected metacentric height, GM_{fluid}	1.224 m

Table 2: Base Parameters for Parametric Simulations

Number of seed numbers, N_s	10
Simulation duration, D_s	10 minutes
Desired heading, β	60 degrees
Calm water ship speed, V_s	10 knots
Zero-crossing wave period, T_p	9.7 seconds

where σ_ϕ is the RMS roll angle and N_{cycle} is the number of roll cycles.

Figure 1 indicates that the maximum roll angle increases approximately linearly with wave height up to a wave height of 8.5 m, with a corresponding expected maximum roll of 28 degrees. For larger wave heights, maximum roll angle increases sharply with wave height. The maximum roll angles predicted using Equation (14) show good agreement with the simulated values when the wave height is below 8.5 m.

In Figure 2, the standard deviation of maximum roll angle from simulations is a non-smooth function of significant wave height. The large standard deviations at higher wave heights indicate that maximum roll angle becomes more dependent on the input wave phase seed number as wave height increases. A larger number of simulations would likely give a smoother function for standard deviation versus wave height. Equation (15) severely underpredicts the standard deviation of maximum roll at higher wave heights.

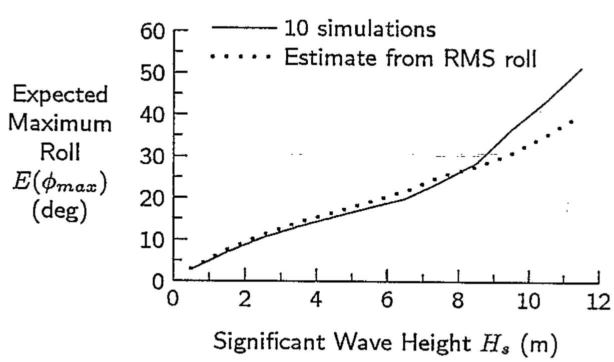

Figure 1: Expected Value of Maximum Roll Versus Significant Wave Height

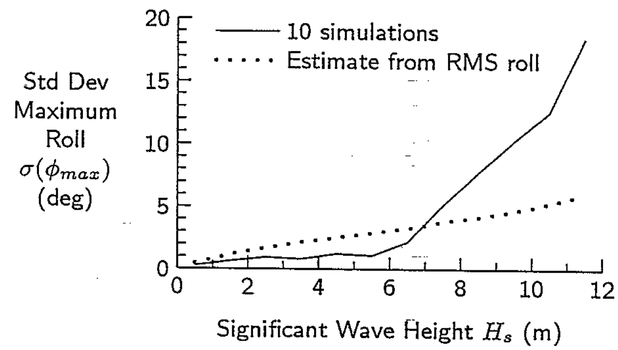

Figure 2: Standard Deviation of Maximum Roll Versus Significant Wave Height

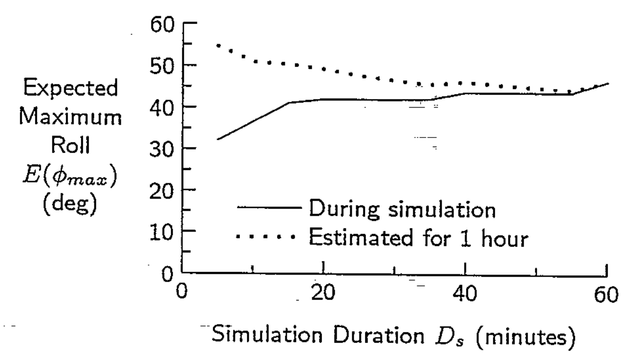

Figure 3: Expected Value of Maximum Roll Versus Simulation Duration, $H_s = 9.5$ m

Figures 3 and 4 show the expected value and standard deviation of maximum roll angle versus simulation duration for a significant wave height of 9.5 m. These figures also give predicted values for one hour duration based on Equations (10) and (11). As expected, Figure 3 shows that the mean value for maximum roll angle increases with simulation duration. Equation (10) tends to overpredict the hourly maximum roll angle, with the degree of overprediction decreasing as simulation duration increases. Although Equation (11) indicates that the standard deviation of maximum roll will be independent of simulation duration, Figure 4 shows that the standard deviation varies with simulation duration. For longer simulation durations, the standard deviation appears to converge.

Figures 5 and 6 show the influence of the number of simulations on statistics for maximum roll angle. Both the mean and standard deviation converge as the number of simulations increases. Twenty simulations appear to provide a reasonable compromise between accuracy and computational time.

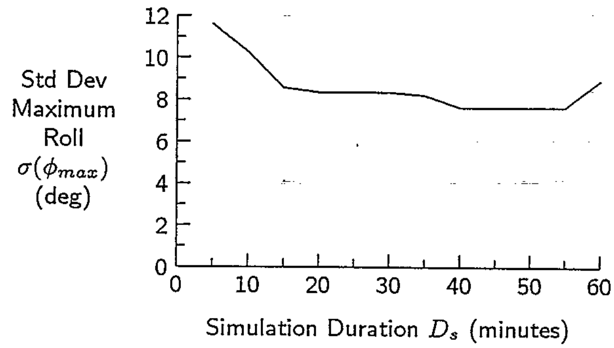

Figure 4: Standard Deviation of Maximum Roll Versus Simulation Duration, $H_s = 9.5$ m

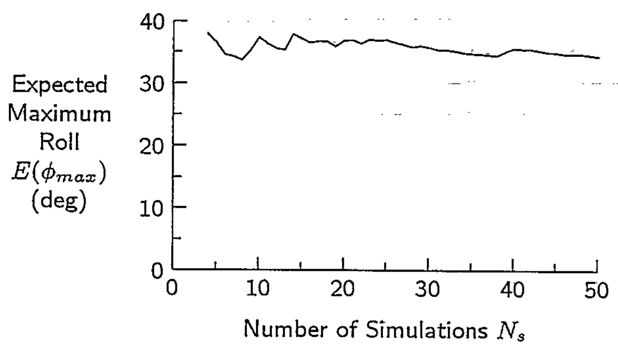

Figure 5: Expected Value of Maximum Roll Versus Number of Simulations, $H_s = 9.5$ m

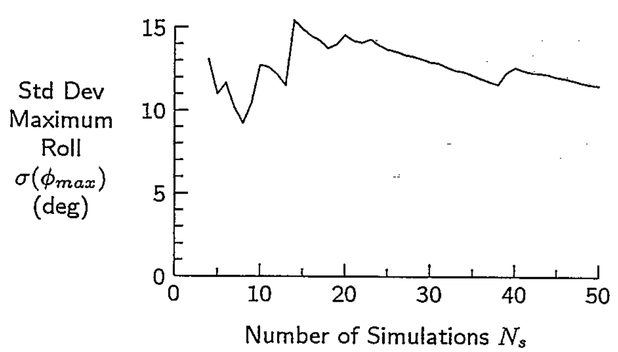

Figure 6: Standard Deviation of Maximum Roll Versus Number of Simulations, $H_s = 9.5$ m

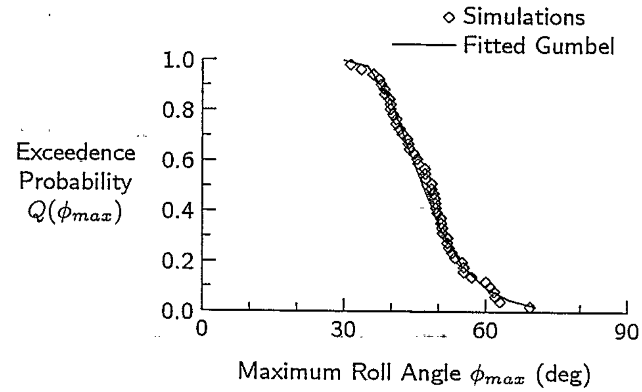

Figure 7: Roll Exceedence Probability Versus Roll Angle for Different Distributions, $H_s = 9.5$ m

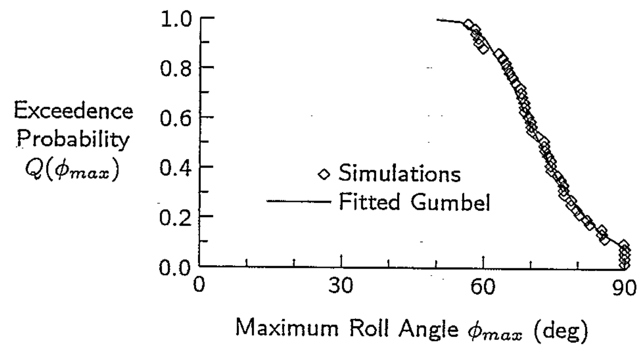

Figure 8: Roll Exceedence Probability Versus Roll Angle for Different Distributions, $H_s = 11.5$ m

Roll exceedence probabilities and fitted distributions are given for significant wave heights of 9.5 m and 11.5 m in Figures 7 and 8. The Gumbel distribution provides a very good fit to the maximum roll angles from simulations. In Figure 8, the concentration of data at a roll angle of 90 degrees is caused by the termination of Fredyn when roll angle exceeds 90 degrees.

Figures 9 and 10 show fitted Gumbel distributions for maximum hourly roll angle derived from simulation durations of 10, 30, and 60 minutes. For maximum roll angles of 70 degrees and greater, which is the range of greatest interest for ship capsize, the 10 and 30 minute simulations lead to some overprediction of exceedence probability for hourly maximum roll angles.

Figures 11 and 12 show fitted distributions based on 10, 20 and 50 simulations of one hour duration. For the first significant wave height of 9.5 m, the fitted distributions from the different numbers of simulations are quite similar. With the second wave height of 11.5 m, roll exceedence probabilities from 10 simulations are significantly less than corresponding values from 20 and 50 simulations,

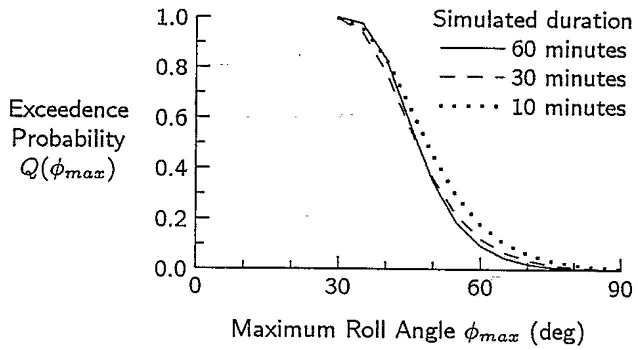

Figure 9: Hourly Roll Exceedence Probability Versus Roll Angle from Different Simulation Durations, 50 Realizations, $H_s = 9.5$ m

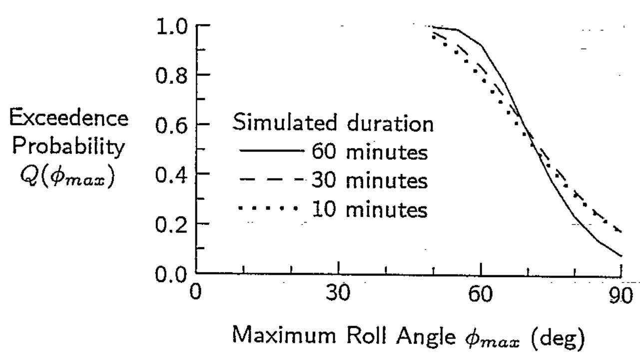

Figure 10: Hourly Roll Exceedence Probability Versus Roll Angle from Different Simulation Durations, 50 Realizations, $H_s = 11.5$ m

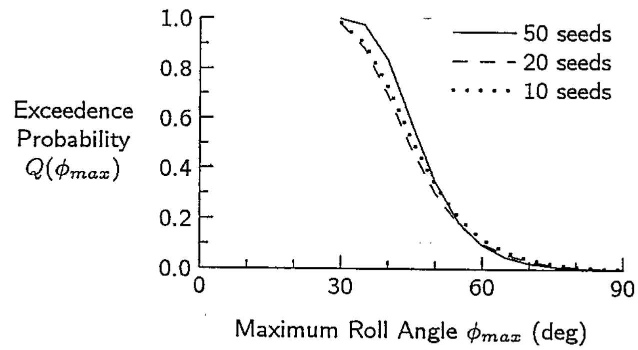

Figure 11: Roll Exceedence Probability Versus Roll Angle for Different Numbers of Seeds, 1 Hour Simulations, $H_s =$ 9.5 m

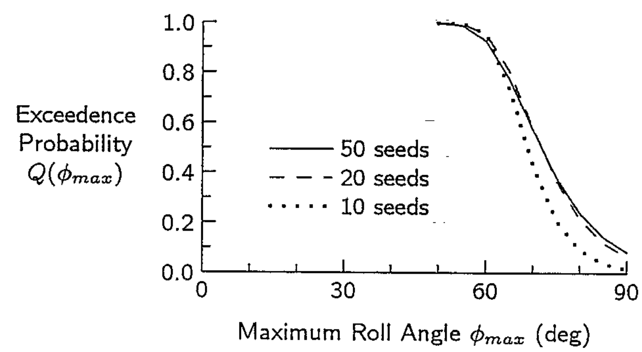

Figure 12: Roll Exceedence Probability Versus Roll Angle for Different Numbers of Seeds, 1 Hour Simulations, $H_s =$ 11.5 m

particularly for larger roll angles.

Figures 9 to 12 indicate that predicted hourly roll exceedence probabilities can vary significantly with simulation duration and number of realizations, particularly for higher roll angles with low exceedence probabilities. However, the simulation results indicate that the maximum roll angle associated with a given exceedence probability is relatively insensitive to the simulation duration and number of realizations. For capsize risk analysis using small numbers of realizations and short simulation durations, a safety margin of approximately 10 degrees in the capsize roll angle could be applied to ensure that the resulting predicted capsize risk was conservative.

CONCLUSIONS

For a ship operating in a given irregular seaway, the maximum roll angle is very dependent on the wave realization. Simulations for a naval frigate suggest that the expected maximum roll angle will show good agreement with the predicted value from linear theory for maximum roll angles up to 28 degrees. For higher maximum roll angles, linear theory significantly underpredicts the expected value of maximum roll angle and its standard deviation.

For given conditions, a fitted Gumbel distribution offers a very efficient approach for modelling the dependence of maximum roll angle on wave realization. A least squares linear fit procedure can be applied to the range of roll angles of greatest interest, and can manage the practical problem of simulations terminating when the roll angle exceeds 90 degrees. Gumbel distributions provide very good fits to maximum roll angles from large numbers of simulations.

A significant benefit of the Gumbel fit procedure is that a relatively small number of simulations can be used to estimate small capsize probabilities. For example, 50 simulations can likely give reliable estimates for a capsize risk level of 0.01. Sample computations for a naval frigate indicate a minimum of approximately 20 wave realizations should be used for accurately determining fitted Gumbel parameters. Another benefit of the Gumbel procedure is that simulations of shorter duration can be used to estimate maximum roll properties for longer durations. Sample computations for a naval frigate indicate that simulations should have a minimum duration of 10 minutes for estimating maximum roll properties for one hour. Estimates of maximum hourly roll properties will improve as the number of simulations and simulation duration increase; however, increased accuracy requires increased total simulation time. If small numbers of realizations and short simulation durations are used, then a safety margin should likely be applied to the capsize roll angle to ensure that the resulting predicted capsize risk is conservative.

Ongoing work is examining capsize risk for all conditions by combining results from Gumbel fits for each possible seaway. Results from the present study help to minimize the required computation time when considering all possible conditions. Future work will consider the influence of capsize avoidance action by considering ship speed and heading to be conditionally dependent on wave conditions.

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