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Spatial Deductive Reasoning

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Three-dimensional spatial deductive reasoning is essential for Air-to-Air combat maneuvers. Yet, no study has determined the processes that underlie this type of deductive reasoning. This study addresses this issue by pitting two opponent theories of deductive reasoning: the Formal Rules theory and the Mental Models theory. Method. Twenty-six subjects resolved 144 spatial deductive problems. These problems described one-dimensional layouts (1D), two-dimensional layouts (2D), and three-dimensional layouts (3D). Each of these dimensional conditions included three problem types which varied by the number of mental models and/or the length of the formal derivation required for their solution. Each problem type included eight sets of premises, each set being presented once for each question type (question EB and question ED). Formal derivations were identical for all dimensional conditions (question EB), and identical for dimensional conditions 2D and 3D (question ED). Results. The difficulty of the problems increased significantly with the number of spatial dimensions despite the identical length of their formal derivations. For dimensional conditions 2D and 3D, problems based on two models were significantly more difficult to resolve than problems based on one model despite the fact that the former required shorter formal derivations than the latter. The results corroborated the Mental Models theory of deductive reasoning.

Introduction

Combat pilots must represent aircraft positions in various stages of flight. They must also deduce the location of aircraft from others which position may be determinate, indeterminate, or unknown. Their deductions can be derived from perceptual observations or linguistic sentences (i.e., premises). Consider the following premises in which the objects A, B, and C are collinear horizontally and in a fronto-parallel plane:

1. The bomber (A) is on the right of the fighter (B)
2. The tanker (C) is on the left of the fighter (B)

One can infer the valid conclusion: The tanker is on the left of the bomber.

How do humans make these deductions? The orthodox answer, provided by Formal Rules theorists, is to assume that humans use formal rules of inference to derive conclusions from the logical form of the premises (Braine, 1978; Braine & O'Brien, 1991; Hagert, 1985; Piaget, 1972; Rips, 1983). If a formal rule of inference matches the logical form of the premises, it can then be applied to these premises to derive a conclusion. For example, a formal rule of inference is the rule of logical transitivity. It has been formalised using a branch of logic called first order predicate calculus:

If Right (A, B) *and* Left (C, B) *then* Left (left (C, B)) A)

When applied to the above premises, one can derive the conclusion: C is to the left of B and these two are to the left of A.

A radical alternative theory (Johnson-Laird & Byrne, 1991, 1993) repudiates this view completely. Logical competence is simply not based on formal rules of inferences. Rather, it is achieved through semantic procedures that construct and validate mental models of the premises. For example, given the above two premises, subjects would construct the following one-dimensional model:

C B A

The model yields the putative conclusion: A is on the right of C. No alternative model can be constructed to falsify it. Therefore, it must be true given that the premises are true.

However, the following set of premises:

A is on the left of B; B is on the right of C

yields at least two models or solutions consistent with their truth conditions:

1. A C B and 2. C A B

An alternative model can be constructed and it falsifies the putative conclusion that A is on the left of C. Hence, the conclusion is not logically valid.

Because Johnson-Laird's (Johnson-Laird & Byrne, 1991, 1993) Mental Models theory is based on logics (i.e., the model-theoretic approach) which do not require the use of formal rules of inference, it is viewed quite incompatible with the Formal Rules theories account of deductive competence (i.e., based on the proof-theoretic approach in logic). Also, the debate between both sorts of theories has been quite intense in the last few years (see Evans, 1991; Evans, Newstead, & Byrne, 1993; Roberts, 1993). The question concerning the processes which underlie deductive reasoning is thus a crucial and unresolved one.

Objectives and Hypotheses

The principle objective of this study is to determine the processes by which humans reason about three-dimensional spatial layouts, a type of deductive reasoning that has not yet been investigated. The difficulty of these spatial deductions will be compared to those based on one-dimensional layouts and two-dimensional layouts. The experiment is designed to pit two opponent theories of spatial deductive competence: Hagert's Formal Rules theory (Hagert, 1985; Hagert & Hansson, 1983, 1984), and Johnson-Laird's Mental Models theory (Johnson-Laird & Byrne, 1991, 1993).

Under the assumption that these two theories propose a universal account of deductive competence, then the predictions made by either theory should generalise across dimensional conditions (i.e., 1D, 2D, and 3D). For each dimensional condition, we thus created three problem types which differed by the number of mental models and/ or the length of the formal derivation required for their solution. The length of the formal derivations were held constant across dimensional conditions.

The Formal Rules theory predicts that problems based on identical formal derivations should be equally difficult regardless of the number of spatial dimensions described among objects. Within each dimensional condition, the difficulty of the problems should increase with the length of the formal derivations underlying their solution. In contrast, the Mental Models theory predicts that the difficulty of the problems should increase with the number of spatial dimensions because each dimension increases the mental workload required to construct an integrated mental model of a spatial layout. Within each dimensional condition, the difficulty of the problems should increase with the number of mental models required for their solution.

Method

Subjects

A total of 26 subjects (20 males, 6 females; 27 to 47 years old) of various ranks (private to commander; clerk to scientist) and educational levels (high school to post-graduate degree) completed the experiment. Each subject received a stress allowance in addition to their regular duty wage.

Materials

Spatial Deductive Problems

Each spatial deductive problem consisted of four premises and a question type. Each premise described the location of two objects. Together, these four premises formed a set which described the layout of five objects. These objects were the following one syllable words: a bomb, a ship, a tank, a gun, and a mine (i.e., a minefield). The subject's task was to determine whether a relation (e.g., is E left of D?) presented in a question type did or did not necessarily follow from a set of premises.

We constructed a total of 72 sets of premises. These sets of premises described one-dimensional layouts (24 sets), two-dimensional layouts (24 sets), or three-dimensional layouts (24 sets). Each of these dimensional conditions included three problem types. For each dimensional condition, problems type 1 and problems type 3 required one model for their solution, while problems type 2 were based on two models. Problems type 2 were created by introducing an indeterminate relation between two of the objects. For dimensional conditions 2D and 3D, problems type 1 and problems type 2 required identical formal derivations, while problems type 3 required an additional step in the formal derivation. Each problem type included 8 sets of premises. Each set was presented twice once for each question type (question ED and question EB). Up to three relations, Q1, Q2, and Q3, were asked in a question type. The experiment thus included a total of 144 problems. Table 1 presents examples of problems[1] (type 2) constructed for each dimensional condition. The structure of the premises was the same for all subjects, but the lexical tokens (e.g., ship, bomb) were assigned randomly to the five terms (A, B, C, D, E).

Problem Structure	**Dimensional condition**		
Premises	**One dimension**	**Two dimensions**	**Three dimensions**
E Relation C	Tank directly below mine	Mine directly right of tank	Ship directly right of bomb
B Relation D	Bomb directly above ship	Ship [2]directly left of tank	Mine directly in front of tank
C Relation B	Mine below bomb	Tank below ship	Bomb below mine
B Relation A	Bomb above Gun	Ship above gun	Mine above gun
Question ED: Q1	Tank (E) below ship (D)?	Mine (E) below tank (D)?	Ship (E) below tank (D)?
Q2			Ship (E) right of tank (D)?
Q3			Ship (E) in front of tank (D)?
Question EB: Q1	Tank (E) below bomb (B)?	Mine (E) right of ship (B)?	Ship (E) right of mine (B)?
Q2		Mine (E) below ship (B)?	Ship (E) below mine (B)?

Table 1. Examples of Spatial Deductive Reasoning Problems

Apparatus

Subjects participated independently in sound-attenuated rooms each equipped with a 386 MS-DOS personal computer and a two-button mouse for response collection. All problems were generated and controlled by the personal computer.

Procedure

Subjects were given written instructions describing the meaning of the directions indicated in the premises. They were to assume that the objects A, B, C were: (1) collinear horizontally when related by the directions left or right, or (2) collinear vertically when related by the directions above or below. All objects were to be located in the same fronto-parallel plane with respect to them. They then received six practice problems followed briefly by the experimental problems.

All problems were presented in a visual sequential mode where each premise and each relation of a question type is presented individually according to predefined temporal parameters. A pilot study lead to the following temporal parameters (Table 2):

Parameters	Time
Display time of a premise	4 seconds
Interpremise delay time	5 seconds
Display time of a relation	Indefinite
Interrelation delay time	400 milliseconds
Interproblem delay time	10 seconds

Table 2. Temporal Parameters

Each premise remained visible on the computer screen for 4 seconds. A blank screen of 5 seconds separated each premise, and the last premise and the first (or only) relation of a

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[1] The letters in parenthesis were not presented to the subjects.

[2] Formal logical reasoning should apply to the logical structure of the premises regardless of their content.

question type. Subjects were then shown up to three consecutive relations, as illustrated in Table 1, that required them to determine whether the location of a pair of objects (i.e., ED or EB) was true or false. The correct or incorrect conclusion presented in a relation was varied according to a different random order for each subject.

Each relation (or the relation) remained visible on the computer screen until the subject provided an answer. The subject was to answer by selecting either the left mouse button (yes: the conclusion is correct), or the right mouse button (no: the conclusion is incorrect). If they did not know the answer, they were to press any key on the keyboard. These answers were treated as incorrect answers. Subjects were instructed to solve each problem as quickly and as accurately as they could. Response times were measured to the nearest millisecond from the onset of a relation on the computer screen. A blank screen of 400 milliseconds then separated the subject's answer to one relation and the presentation of the next relation. The interrelation delay time obviously occurred only when a question type involved more than one relation. A blank interval of 10 seconds separated each problem.

The experimental problems were presented in eight sessions of approximately 15-20 minutes each. These sessions were presented in two days, each of which included four sessions separated by a 3-5 minute pause. Each session included 18 problems, half of which involved question ED and the other half, question EB. The problems, the question types of a problem, and the relations of a question type were presented in a different random order for each subject. After completing 72 problems, subjects were to explain their strategies.

Design

Each subject was assigned to 18 completely crossed experimental conditions: 3 dimensional conditions x 3 problem types x 2 question types. The number of relations asked in each question type depended on the dimensional condition.

Results and Discussion

Statistical Analysis

Univariate analyses of variance for repeated measures were performed respectively on the mean numbers of correct responses (CR) and on the response times (RT) obtained for those responses. We performed an arcsine (or angular) transformation on the mean number of CR obtained for each subject and for each experimental condition. With one exception, the results of the analyses of variance obtained after this transformation were the same as those obtained before it. We will retain the first of these analyses of variance. RT were normally distributed. Consequently, the analyses of variance were carried out directly on that dependent measure. Geiser-Greenhouse epsilon corrections were performed to adjust the degrees of freedom of each effect. Contrasts between pairs of means were also calculated for the levels of the significant effects. In this paper, we will present the respective effects of dimensional condition and problem type. The experiment is fully described in Boudreau, Pigeau, & McCann (1995).

Effect of Dimensional Condition

The ease with which subjects resolved the problems decreased monotonically with the increasing number of dimensions. For relation Q1, dimensional condition had a significant main effect on both the RT [$F(2, 50) = 29.858, p < .0001$] and the CR [$F(2, 50) = 12.702, p < .0001$]. With one exceptions , all three dimensional conditions differed significantly from one another both in terms of the speed ($p < .001$) and the accuracy ($p < .05$) with which subjects resolved the problems. For relation Q2, RT also increased significantly from dimensional condition 2D to dimensional condition 3D [$F(1, 25) = 7.811, p < .0098$].

The length of the formal derivations were identical for all dimensional conditions (question EB), and identical for dimensional conditions 2D and 3D (question ED). Except for dimensional condition 1D (question ED), the Formal Rules theory cannot account for these results (Hagert, 1985, Hagert & Hansson, 1983, 1984). In contrast, the Mental Models theory (Johnson-Laird & Byrne, 1991, 1993) can explain them as each additional dimension increases the mental workload required to construct an integrated mental model. These results thus corroborate Johnson-Laird's Mental Models theory of spatial deductive competence.

Effect of Problem Type

Table 3 summarizes the Formal Rules theory's and the Mental Models theory's respective hypotheses regarding the order of difficulty of problems type 1, 2, and 3 for dimensional conditions 2D and 3D. The Formal Rules theory's hypotheses are based on the length of the formal derivations underlying a deduction; while the Mental Models theory's hypotheses are based on the number of mental models. Problems type 2 and type 3 are critical for a theoretical verdict since both theories make opposite predictions for these problem types.

Formal Rules theory		Mental Models theory	
Length of the Formal derivations	Order of difficulty	Number of Mental Models	Order of difficulty
1. EC[3], CB => EB; EB, BD => ED	1 = 2	1. one model	1 > 2
2. EC, CB => EB; EB, BD => ED	1 > 3	2. two models	1 = 3
3. EC, CB => EB; EB, BA, AD =>ED	2 > 3	3. one model	3 > 2

Table 3. The Formal Rules theory's and the Mental Models theory's respective Hypotheses regarding the Order of Difficulty of problems type 1, 2, and 3

Problem type had a significant main effect on the RT obtained for each dimensional condition, relation Q1 [$F(2, 50) = 10.191, p < .0002$]. The interaction between problem types and dimensional conditions was also significant [$F(4, 100) = 3.959, p < .0050$].

For dimensional condition 3D, problems type 1 did not differ significantly from problems type 2 [$F(1, 25) = .307, p > .5806$], nor did either of these problem types differ reliably and respectively from problems type 3 [$F(1,25) = .836, p < .3628$; $F(1, 25) = .130, p > .7196$]. Since dimensional condition 3D produced the longest RT, the difficulty of the condition may account for this ceiling effect.

Now, the results of dimensional condition 2D are quite interesting for the Mental Models theory. Problems type 2 based on two models required significantly more time to resolve than problems type 1 based on one model [$F(1, 25) = 12.170, p < .0007$]. Problems type 3 based on one model took no longer to resolve than problems type 1 which are also based on one model [$F(1, 25) = 1.635, p > .2039$]. More importantly, problems type 3 based on one model were systematically easier than problems type 2 based on two models [$F(1, 25) = 4.883, p < .0294$] despite the fact that the former required a shorter formal derivation than the latter (see Table 3). There were no significant interaction between question type and problem type ($p > .01$). With regard to dimensional condition 2D, these results thus confirm each of the Mental Models' hypotheses while refuting each of those of the Formal Rules theory.

Overall, the effects of problem type on the CR were not statistically significant ($p > .01$) perhaps because of a speed-accuracy tradeoff. However, an analysis of variance performed on the arcsin of the mean CR revealed a significant effect of problem type for both dimensional conditions 2D and 3D, relation Q2. Problems type 2 based on two models lead systematically to more errors than problems type 3 which are based one model [$F(1, 25) = 5.112, p < .0281$]. As for the RT obtained for dimensional condition 2D, this result confirms the Mental Models principle hypothesis but counters that of the Formal Rules theory.

Conclusions

The results have shown that the difficulty of spatial deductions increases systematically with the number of spatial dimensions described among objects. With the exception of dimensional condition 1D (question ED), all dimensional conditions required, within question type, identical formal derivations. Apart from the above exception, the Formal Rules theory would not predict any differences in difficulty between the three dimensional

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[3] Each pair of letters denotes a relation between two objects, i.e., a step in a formal derivation. The sign "=>" means implies; the sign "=" denotes the logical identity of the relation of two pairs of objects.

conditions. However, the Mental Models theory would argue that each additional dimension increases the load on working memory. With each incoming premise, subjects must construct a temporary layout of the objects in their respective dimensions and integrate these layouts in a unitary representation. Subjects' explanations indicate that these representations are visual mental models rather than verbal propositions. The effects of dimensional conditions thus corroborate Johnson-Laird's Mental Models theory.

The effects of the problem types on the RT interacted with those of the dimensional conditions. For dimensional condition 2D, the effects of problem type on the subjects' RT supported each of the Mental Models theory's hypotheses, but overturned each of those of the Formal Rules theory. Dimensional condition 3D produced a ceiling effect on the RT obtained for each problem type. However, the effects of problem type on the CR obtained for dimensional conditions 3D and 2D also supported the Mental Models principle hypothesis while countering that of the Formal Rules theory. Thus, problems type 2 which described indeterminate spatial locations among objects were systematically more difficult to reason about than problems type 3 which described determinate spatial locations. This order of difficulty prevailed despite the fact that problems types 2 required shorter formal derivations than problems type 3.

Overall, the results strongly indicate that spatial deductive reasoning is based on the model-theoretic approach in logic as proposed by the Mental Models theory. The theory is in fact well positioned to account for the problem of indeterminacy in spatial reasoning, a problem which is ubiquitous in Air-to-Air combat maneuvers. Further research should investigate dynamic 3D spatial reasoning where geometrical positions must be integrated with variables such as object velocity and travel distance. Since 3D spatial reasoning is an essential logical activity for combat pilots, it could also be properly measured and trained.

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