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A CASE AGAINST LINEAR PROGRAMMING FOR STOCKPILE PLANNING

BY

Mr. Ivan Taylor, JSORT 2

JUNE 1998

OTTAWA, CANADA

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OTTAWA, ONTARIO

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ABSTRACT

The ACROSS model is a linear programming optimization of the stockpile planning system for NATO. As one of the Canadian scientific advisors to J4 Logistics, the author was asked by J4 Logistics Doctrine to provide scientific support on the ACROSS model. After quick introduction to the model, the author conducted sensitivity analysis runs using the example data set. In this process, a serious bug was found in the algorithm. For some reasonable parameter settings, the program stated that the problem was infeasible. Furthermore, the linear programming approach appears to be highly unstable in that small changes to the input can lead to large changes in the output. The program also appeared to make illogical recommendations concerning the stockpile mix. This is caused by an inherent flaw in the linear programming approach; a marginal analysis optimization approach is proposed as an alternative.

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A CASE AGAINST LINEAR PROGRAMMING FOR STOCKPILE PLANNING

INTRODUCTION

1. ACROSS is a linear programming model developed for SHAPE to determine optimal ammunition stockpiles for all NATO Nations (Ref 1). The author was quite sceptical of a linear programming approach to stockpile planning optimization. It has been noted by a number of researchers that highly optimized systems are inherently unstable. A sensitivity analysis was conducted on the ACROSS model to determine if this hypothesis was true.

2. The example data set for the Canadian Forces was run through the model for varying amounts of total investment and the list of ammunition stock that the model suggested was examined. It was quickly found that the results did appear unstable. Furthermore, there appears to be a bug in the program so that in some cases increased investment actually reduces the objective that was being maximized, namely, the Percent Target Value Destroyed. For some reasonable parameter settings, the program returned infeasible results, namely, negative ammunition stockpiles. Finally, the author presented the results to an experienced logistician and the logistician recognized that the mix of ammunition for the various investment levels appeared to be illogical. With more investment, the model suggested selling more valuable and scarce Howitzer 155 ammunition and buying cheaper mortars in large numbers.

3. The author has a great deal of experience with the heuristic optimization approach called Marginal Analysis (Refs. 2 and 3). Although, the results using this approach are not as highly optimized, they are very stable and the algorithm is completely intuitive. A Marginal Analysis approach will be presented at the end of the paper that can accommodate the NATO Planning Situations that will be used in future stockpile planning guidance.

THE BUG

4. The Canadian portion of the example data set when run with "unbounded" investment produces a mix of ammunition that costs $80,125,545 and provides a Percent Target Value Destroyed of 71.39% using among other things 1,926 Howitzer 155mm Heat rounds and 22,619 Mortar 81mm Heat rounds. When various other investment targets between $79.5M and $80M were investigated the only changes to the ammunition mix were generated in these two types of ammunition with more Mortars being recommended and less 155mm rounds being recommended as the amount of investment increased (see Table I).

TABLE I
SENSITIVITY ANALYSIS WITH CANADIAN DATA SET

Budget	%Target Value Destroyed	AM-HOW-155-HE Rounds Recommended	AM-MOR-81-HE Rounds Recommended
79,500,000	70.75	2414	18389
79,600,000	71.02	2339	19052
79,700,000	71.38	2299	19571
79,800,000	71.63	2191	20370
79,900,000	70.86	2149	20895
80,000,000	71.10	2043	21691
80,125,545 (unbounded)	71.39	1926	22619

5. Table I shows all of the details of the inherent problems with the ACROSS model.

6. Notice the maximum Percent Target Value Destroyed is 71.63% at $79.8M. When the investment is increased to $79.9M, the Percent Target Value Destroyed decreased to 70.86%. In fact, when the investment target was left unbounded the total investment was suggested to be over $80M but the Percent Target Value Destroyed was found to be 71.39% which is less than when the investment was set at $79.8M.

7. This appears to the author as a serious 'bug' in the algorithm. If the objective is to maximize the Percent Target Value Destroyed subject to an investment constraint, it should not decrease with more investment when everything else is held constant. If the objective function is not accurately represented by the Percent Target Value Destroyed,

the correct measure of the objective function should be displayed on the run screen, so that users can determine when to stop their sensitivity analysis. This implementation of a linear programming algorithm is obviously flawed.

THE INSTABILITY OF THE ALGORITHM

8. The author was not looking for a bug when he did the sensitivity analysis. He was looking for instability in the solution. In particular, the results in Table I show that as more investment is added, the stockpile of AM-HOW-155-HE should decrease according to the ACROSS model. That is, if we had $79.5M to start with we would buy 2414 rounds of AM-HOW-155-HE. If we later found another $0.5M to invest, the model suggests we buy only 2043 rounds of AM-HOW-155-HE ammunition. Therefore, if the acquisition of 2414 rounds were already made, we would need to sell 371 rounds of AM-HOW-155-HE ammunition to obtain the optimal solution. Linear programming will always result in instability in the solution space because it does not consider the past or the future in the acquisition program. One could introduce additional constraints into the formulation to account for previous buys but to consider a sequence of buys would be quite labour intensive. The author's Marginal Analysis technique, on the other hand, builds on previous results which can be easily traced from past to present to future.

THE ILLOGIC IN THE ALGORITHM

9. When these results were shown to an experienced logistician, he noted that the AM-HOW-155-HE ammunition was being reduced with increased investment while the AM-MOR-81-HE ammunition was being increased. He stated that this was illogical because the AM-HOW-155-HE ammunition was more sophisticated and effective than the AM-MOR-81-HE and therefore with more money we should be able to buy more of it and improve our effectiveness.

INFEASIBLE SOLUTIONS

10. For some of the initial runs of the ACROSS model, the program returned a statement so that the problem was infeasible. However, the investment target was fairly large, therefore there should have been an optimal buy of ammunition with this money to maximize the Percent Target Value Destroyed. The marginal analysis approach does not have this problem.

THE MARGINAL ANALYSIS TECHNIQUE

11. Marginal analysis is a well known optimization heuristic that if used in the ACROSS model would overcome the problems noted above. The algorithm starts with a user-specified minimum amount of ammunition. This could be zero or it could be the current inventory. Then the algorithm considers all the ammunition types to determine which type provides the best return on investment. Then it buys one round of that type and re-computes the return on investment for the next buy. It continues to buy ammunition round-by-round until the investment limit is reached or the objective function target is reached. In this way, the program can be traced as shown in Figure 1 from low investment/low confidence to high investment/high confidence in a completely logical manner (see Ref. 2).

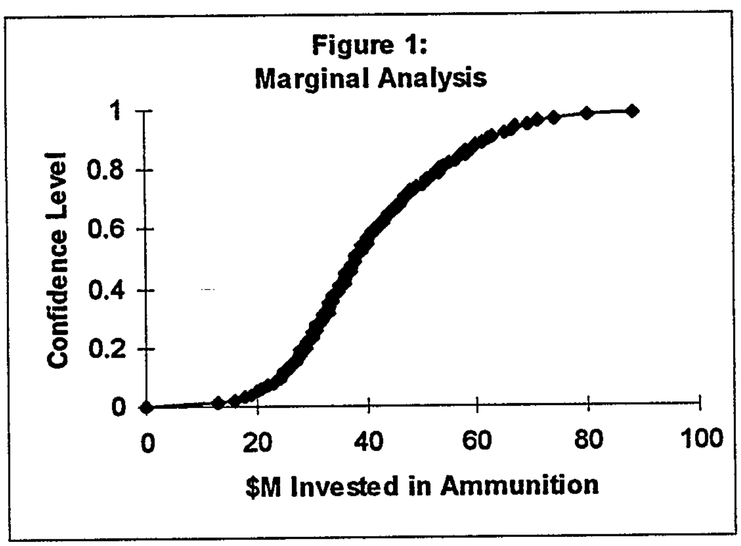

Figure 1: Marginal Analysis

MARGINAL ANALYSIS WITH SCENARIOS

12. The author has developed a technique in which marginal analysis can be combined with scenarios (Ref. 3). In Canada, we have 11 fundamental scenarios for force planning. These scenarios range from Search and Rescue and Disaster Relief to Aid to the Civil Power, Peace Support Operations and Collective Defence. With any particular force, the model can quantify the probability of mission success (confidence) in these scenarios. Or alternatively, for any level of investment in defence it will minimize the probability of failure (risk) in these scenarios. These scenarios can be grouped and prioritized and acceptable risk levels can be established by senior leadership. The tasks associated with these scenarios can be identified and a matrix developed showing the relationship. The weapon systems associated with each task can also be shown using a matrix. Finally, the demand (or expected usage) of the weapon system in each task can be identified in a matrix. The marginal analysis process would work with the first priority scenarios and optimize the ammunition buy for each weapons system assuming a statistically distributed usage rate and the desire to avoid running out of ammunition. The fact that there is more than one scenario in the first priority group leads to the assumption that these scenarios can be run concurrently. Once the target risk level is reached, the algorithm can consider the second priority scenarios. However, these would be assumed not ever to occur concurrently with the first priority scenarios. Therefore, ammunition bought for the first priority scenarios could be used in the second priority scenarios if necessary and the initial risk level may not be as high when the marginal analysis begins to consider the second priority scenarios. Figure 2 shows example results for two sets of scenarios, possibly, Peacetime and Wartime. The discontinuity occurs when the Peacetime scenarios are satisfied and the algorithm begins considered the Wartime group. Notice in this graph we wish to minimize risk rather than maximize confidence.

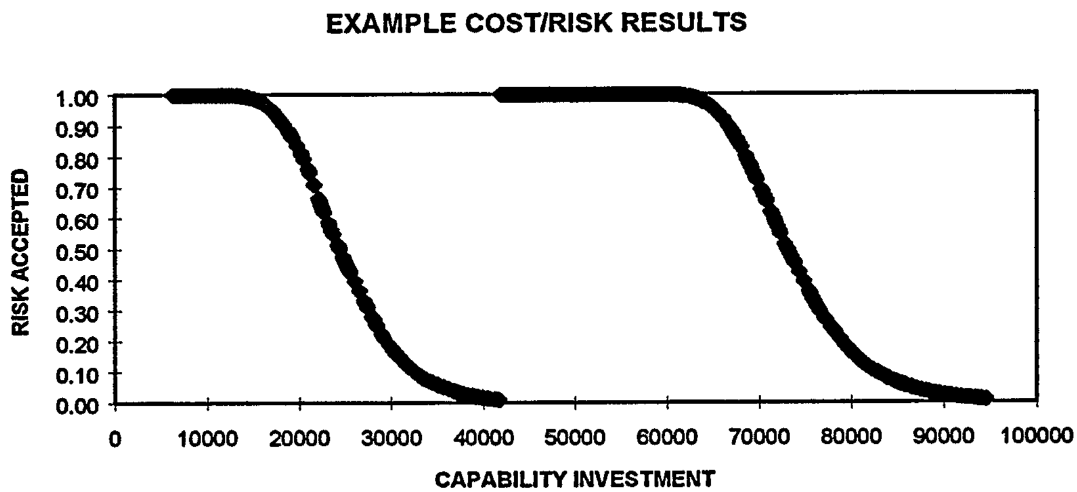

Figure 2: Marginal Analysis With Scenarios

SUMMARY

13. We have identified a number of problems with the current ACROSS Model and its application of linear programming to stockpile planning. We have also described an alternative approach based on marginal analysis that could be used to minimize risk subject of cost constraints in a manner that would avoid the errors, instabilities, infeasabilities and illogical results of the ACROSS model. The marginal analysis approach also has been adapted to the scenario approach and a scenario-based approach will be used in future stockpile planning for NATO.

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3. Taylor, I.W.; A Marginal Analysis Approach to Risk Management Under the Scenario Planning Framework; Directorate of Operational Research (Joint and Land) Research Note 9814; May 1998.

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