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TITLE

A PE-BASED BACKPROPAGATION ALGORITHM FOR MATCHED-FIELD PROCESSING

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A PE-Based Backpropagation Algorithm for Matched-Field Processing

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Abstract: We examine a matched-field processor for source localization that combines measured data with parabolic equation (PE) starting fields to backpropagate an ambiguity surface outwards from a vertical line array. For an N-element system, this (unnormalized) processor generates the ambiguity surface N times faster than the standard Bartlett correlator. The effectiveness of the backpropagation algorithm is assessed by comparing it to the Bartlett processor for multi-tonal data obtained during the Hudson Canyon experiment.

INTRODUCTION

In applications of matched-field processing (MFP) to passive sonar, signals measured on a hydrophone array due to a source at some unknown position $\mathbf{x}_s = (r_s, z_s)$ are correlated with signals predicted by a propagation model (replicas) due to a source at a given position $\mathbf{x} = (r, z)$. This matching is carried out for many assumed $\mathbf{x}$'s within a search region to form a (normalized) ambiguity surface whose peak value provides an estimate of $\mathbf{x}_s$ (1). For an N-element array, reciprocity is invoked to reduce the computational effort to computing N replica fields at each point on the search grid. In this paper, we examine a matched-field processor proposed several years ago by Tappert *et al.* (2) that combines measured data with PE starting fields to effectively backpropagate an ambiguity surface outwards from the receiving array. Although unnormalized, this processor generates an ambiguity surface N times faster than the normalized one.

For tonal signals, the basic data that is input to the matched-field processor consists of the vector $\mathbf{d}$ of complex-valued pressures obtained from an array of N hydrophones. The conventional (Bartlett) scheme correlates these data to replica fields $\mathbf{r}(r, z)$ by forming the scalar product

$$B_r(r,z) = |\mathbf{d}^*\mathbf{r}(r,z)|^2 \equiv \mathbf{r}^*\mathbf{C}\mathbf{r}, \tag{1}$$

for each search grid location (r, z). Here, $\mathbf{C} = \mathbf{d}\mathbf{d}^*$ is the cross-spectral matrix, * denotes the conjugate transpose, and the vector $\mathbf{r} = \mathbf{p}/||\mathbf{p}||$ is a normalized prediction of the acoustic field on the array due to a source at (r, z).

THEORY

The outgoing 2D PE field $\psi(r,z) = p(r,z)\exp(ik_0 r)\sqrt{r}$ can be marched from r to $r + \Delta r$ in the form

$$\psi(r+\Delta r, z) = \exp\left\{-i\delta + i\delta\sqrt{1+X}\right\}\psi(r,z) \approx \psi(r,z) + \sum_{j=1}^{J}\psi_j(r+\Delta r, z), \tag{2}$$

where p is the acoustic pressure, $\delta = k_0\Delta r$, k_0 is a reference wavenumber, and $X = N^2 - 1 + k_0^{-2}\rho\partial_z\left(\rho^{-1}\partial_z\right)$ is a depth-dependent operator. Here, $N = n(1+i\alpha)$, $n = c_0/c$ is the refractive index, and c, ρ and α denote the sound speed, density and absorption, respectively. The field ψ satisfies a radiation condition as $z \to \infty$. In the split-step Padé algorithm (3) for solving Eq. (1), each ψ_j satisfies a tri-diagonal system of the form

$$(1 + b_j X)\,\psi_j(r+\Delta r, z) = a_j X \psi(r,z), \tag{3}$$

where a_j and b_j are the jth coefficients of the Padé series approximation to the propagator.

Efficient replica calculations for MFP applications make use of reciprocity so that only N replica surfaces $\mathbf{p}(r, z)$ corresponding to a point source located at each of the N hydrophone positions on the receiving array need to be generated. For a vertical line array (VLA), the backpropagated PE algorithm combines all N replicas at the same time by forming the scalar product $\mathbf{d}^*\mathbf{p}(0, z)$ at the array for each point z on the PE computational grid and backpropagating a correlation surface outward from the array toward the potential target locations. Instead of the normalized correlation function given in Eq. (1), the backpropgation method yields the unnormalized correlation measure

$$B_p(r,z) = |\mathbf{d}^*\mathbf{p}(r,z)|^2. \tag{4}$$

Calculations of B_p based on Eq. (4) proceed N times faster that those for B_r based on Eq. (1) where N individual replicas must be computed and saved before the vector norm $||\mathbf{p}|| = \sqrt{\mathbf{p}^*\mathbf{p}}$ can be evaluated.

When many observations $\mathbf{d}_\ell$ of the field on the array are available, an improved estimate of the cross-spectral matrix $\mathbf{C}$ can be obtained by forming the average $\langle\mathbf{C}\rangle = (1/L)\sum_{\ell=1}^{L}\mathbf{d}_\ell\mathbf{d}_\ell^*$. In this case, it can be shown that the best estimate of the VLA signal vector $\tilde{\mathbf{d}}$ that can be determined from the averaged cross-spectral matrix $\langle\mathbf{C}\rangle$ is given by the eigenvector corresponding to its largest eigenvalue (4).

EXAMPLE

We examine the localization capability of the unnormalized processor in Eq. (4) by applying it to some measured data obtained from the Hudson Canyon experiment that was carried out at a shallow-water (73 m) site off the New Jersey coast. The environmental profiles that were used to generate replicas are described elsewhere (4), (5). For the outgoing run considered here, four tonals were transmitted at frequencies of 50, 175, 375 and 425 Hz. The nominal depth of the towed source was 36 m. The receiving VLA consisted of 24 hydrophones spaced at 2.5-m intervals between the depths of 14.95 and 72.45 m. Michaelopoulou and Porter (5) have analyzed this data set in the context of MFP localization and demonstrated that the tracking capability is significantly improved by averaging the correlation surfaces over frequency.

In Fig. 1, both the backpropagated PE and Bartlett localization estimates are compared to the true source positions for the outgoing Hudson Canyon run. All PE replica computations used $J = 2$, $\Delta z = 0.5$ m and $\Delta r = 5$ m. Each $\langle\mathbf{C}\rangle$ and $\tilde{\mathbf{d}}$ was based on $L = 10$. The backpropagated processor obtained correct localizations for eight of the ten positions along the track. Otherwise, the processor peaked at nearby sidelobes.

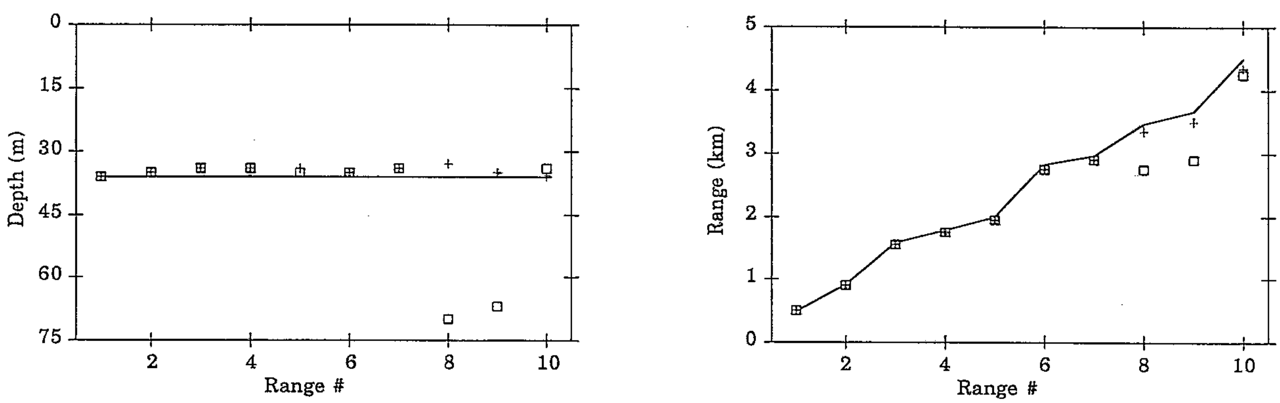

FIGURE 1. Bartlett (+) and backpropgated (□) localization estimates for the outgoing track (—).

SUMMARY

We have shown that a PE-based, backpropagation MFP processor can successfully track the multitone towed source used in the Hudson Canyon experiment. Although unnormalized, the backpropagated localizations agreed well with the Bartlett estimates and required only $1/N$ of the computing effort.

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