A Piecewise Matched-Field Tracking Algorithm

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Abstract: Matched-field tracking algorithms, using as input a set of ambiguity surfaces generated by matched-field processing, have been successfully applied to track sources moving linearly or on a circular path at constant speed and heading. These algorithms assume that the track start and end surfaces are known *a priori*; this restriction is removed in the Piecewise Tracking Algorithm (PTA). In simulated data it is shown that using the weighted PTA the start and end surfaces of the the statistically significant tracks, can be found with a mean absolute error of less than two, provided the track signal-to-noise ratio is greater than 13. A discussion of the PTA applied to data from the PACIFIC SHELF 93 trials is presented.

INTRODUCTION

Matched-Field Processing (MFP) matches a measured acoustic field with a predicted field, based on a model of the environment and a propagation model, for all possible locations of an acoustic source in a search region (1). The result is an ambiguity surface whose value at a position represents the likelihood that the source is present at that position in the search region. If MFP is repeated for data collected over a length of time, the result is a series of ambiguity surfaces whose information may be combined to track a source moving linearly at constant speed and heading (2). The Piecewise Tracking Algorithm (PTA) described in the next section was designed so that the start and end surfaces of the linear track are estimated, rather than assumed known *a priori*.

PIECEWISE TRACKING ALGORITHM

The linear PTA consists of four sets of computations, the first three of which are the same as that for the previously described Matched-Field Tracking (MFT) algorithm (2). The computations are: (i) for each of the N_s surfaces the positions of the largest N_{pk} peaks are determined; (ii) all possible linear tracks joining two peaks on different surfaces are found. These are called combinatoric tracks; (iii) a constraint to realistic maximum speeds for the target is imposed to reduce the combinatoric tracks to the physically possible tracks; (iv) the most significant section of a track is then determined by finding I and J where $1 \leq I < J \leq N_s$ such that the weighted track SNR

$$\widehat{SNR}_{pw} = \max_{\substack{1 \leq I \leq i < \\ j \leq J \leq N_s}} \frac{1}{s\sqrt{\sum_{k=I}^{J} c_k^2}} \sum_{k=I}^{J} c_k B'(\mathbf{r}_k). \tag{1}$$

is maximized. Here i and j are the surfaces of the the two peaks used to form a track, where, without loss of generality, it is assumed $i < j$. I and J are the start and stop surfaces of the significant track. The conditions $I \leq i$ and $J \geq j$ ensure the track passes through the positions associated with the peaks on surfaces i and j used to form the track. This condition could be removed. $B(\mathbf{r}_k)$ is the Bartlett output for the normalized replica vector $\mathbf{r}_k$ associated with the position of the possible source of the k^{th} surface along the track. Also $B'(\mathbf{r}_k)$ is defined as $B(\mathbf{r}_k) - \overline{x}$, and $\overline{x}$ and s are the average value and standard deviation respectively of the Bartlett statistic in noise alone. Finally, c_k is the Bartlett output (signal only) at the k^{th} point along the track normalized over the N_s points in the track:

$$c_k = \frac{\lambda_k}{\sum_{k=1}^{N_s} \lambda_k} \tag{2}$$

where λ_k is the theoretical unnormalized Bartlett output (signal only) associated with the position of the possible source on the k^{th} surface along the track. In other words λ_k is proportional to the magnitude of the unnormalized replica vector. In the unweighted or uniform weighted algorithm, $c_k = \frac{1}{N_s}$.

Table 1: Average mean absolute error in locating track start and end surfaces for various SNR's and σ_w's for the weighted and unweighted algorithms. Four data samples were used to form Bartlett statistics on the 80 ambiguity surfaces.

Track	Average estimated mean absolute errors for various σ_w and *SNR* ranges.							
	$\sigma_w = 0.0$		$\sigma_w = 0.8$		$\sigma_w = 1.6$		$\sigma_w = 2.4$	
SNR	weighted	uniform	weighted	uniform	weighted	uniform	weighted	uniform
1–3	13.99	13.99	14.03	13.93	14.01	13.91	13.77	13.78
3–5	7.701	7.642	7.891	7.705	8.170	7.913	8.257	7.959
5–7	3.727	3.723	4.185	3.924	4.821	4.410	5.079	4.908
7–9	1.736	1.739	2.120	2.017	2.818	2.693	3.363	3.513
9–11	0.955	0.945	1.229	1.205	1.863	1.925	2.485	2.836
11–13	0.616	0.620	0.777	0.821	1.248	1.526	1.826	2.483
13–15	0.467	0.471	0.551	0.667	0.842	1.382	1.269	2.336
15–17	0.388	0.383	0.430	0.565	0.642	1.309	0.965	2.334
17–19	0.319	0.323	0.358	0.493	0.522	1.239	0.796	2.356

PTA PERFORMANCE ON SIMULATED DATA

An indication of the PTA's performance was obtained using simulated data. The noise was assumed to be spatially white with Gaussian amplitude and the signal, if present, a constant with unknown uniformly distributed phase. These idealized assumptions are used to model many environments (1). Propagation effects were simulated by making the signal propagation a normal random variable with standard deviation σ_w. Bartlett statistics were formed from these data. They represent the values on a straight line through a series of ambiguity surfaces. Performance is defined as the mean absolute error to which the start and end surfaces of the track are found as a function of the track SNR, number of samples used to form the Bartlett statistics, number of ambiguity surfaces and signal propagation parameter σ_w. Table 1 gives a typical result.

It is seen that for fixed SNR the mean absolute error increases as the propagation standard deviation increases. In this case, and all cases studied, to minimize errors in determinining the start and end track surfaces, the unweighted algorithm should be used for very low track SNR and the weighted algorithm for high track SNR. For the cases studied the weighted algorithm determined the start and end surfaces of the source track with a mean absolute error less than two when the track SNR was 13 or more and σ_w was 2.4 or less.

SUMMARY OF PTA PERFORMANCE OF PACIFIC SHELF 93 DATA

The PACIFIC SHELF 93 trial included a series of ocean experiments conducted at a site on the continental slope and shelf regions off Vancouver Island, in the North-East Pacific. The experiments and MFT results are described in Ozard *et al* (2). The weighted three dimensional PTA, when run on the same set of data, yielded almost identical results to that obtained using MFT. The weighted track SNR was 15-19 dB. The PTA and MFT depth and range estimates were in good agreement with the true source values. The bearing estimates were not so accurate as they were determined solely from environmental symmetry splitting and an extremely small tilt of the vertical line array. These results suggest that for sources with these track SNR's, the start and end positions of linear track segments need not be known *a priori*, but can be estimated during the tracking process.

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