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**CLASSIFICATION**

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**SYSTEM NUMBER**

507237



**TITLE**

VIBRATION ISOLATION RESEARCH FOR NAVAL SHIPS AT THE AUSTRALIAN DEFENCE FORCE  
ACADEMY

**System Number:**

**Patron Number:**

**Requester:**

**Notes:** Paper #34 contained in Parent Sysnum #507203

**DSIS Use only:**

**Deliver to:** DK



# Vibration Isolation Research for Naval Ships at the Australian Defence Force Academy

by Dr. Hugh Williamson

Acoustics and Vibration Unit  
School of Aerospace and Mechanical Engineering  
University College, The University of New South Wales  
Australian Defence Force Academy  
Canberra, ACT, Australia

To avoid detection in times of conflict, it is vitally important for naval ships to be designed so that acoustic signatures are minimised. The primary source of a ship's acoustic signature is on-board machinery. In order to reduce the acoustic signature, the transmission of structure-borne noise from machinery should be minimised. The minimisation of this transmission depends upon careful design of machinery isolators and supporting structures.

This paper will discuss research work in this area at the Australian Defence Force Academy which has been sponsored by Australia's Defence Science and Technology Organization. Part of the work concentrates on the measurement and characterisation of the performance of polymeric vibration isolators. Another aspect of the research concerns the characterisation and optimisation of machinery supporting structures for minimum transfer of vibrational energy to the ship's structure.

Contact between an isolator and supporting structure occurs over a finite contact area. Hence the true structural response is not well represented by the response due to a concentrated force, ie, point mobility. Research is proceeding on the concept of surface mobility, that is the response of supporting structures to forces distributed over a finite area. The ultimate aim is to find methods of predicting, measuring and optimising surface mobility of machinery support structures so that vibration isolation can be maximised.



# **Vibration Isolation Research for Naval Ships at the Australian Defence Force Academy**

**Dr Hugh Williamson**

**Acoustics and Vibration Unit  
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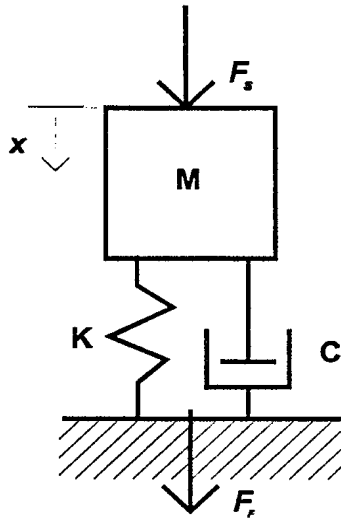
## **Introduction**

The Royal Australian Navy, like most navies, is concerned about reducing the acoustic signature of its ships in order to make detection by an unfriendly force as difficult as possible. In addition to the issue of detection by others, noise and vibration in and around a ship can interfere with sonar and other important equipment. As well, vibration and noise, particularly that from machinery, can be a source of annoyance to the crew of a vessel which in severe cases may become a health and safety problem. Hence in ship design, or in the design of modifications to a ship, effective vibration isolation of machinery is a very important consideration.

This paper gives an overview of a research program on vibration isolation at the Australian Defence Force Academy. The program is supported by Australia's defence research organisation, Defence Science and Technology Organisation, DSTO, through its Aeronautical and Maritime Research Laboratory in Melbourne.

## **Classical Vibration Isolation Theory**

The classical theory of vibration isolation of machinery [1] considers the machine to be a lumped mass mounted on a linear spring and linear damper as shown in Figure 1. As well as assuming linear spring and damper behaviour, the machine and its foundation are taken to be perfectly rigid.



$$F_s = \text{source force} = F_s \cos 2\pi f t$$

$M$  = machine mass

$K$  = isolator stiffness

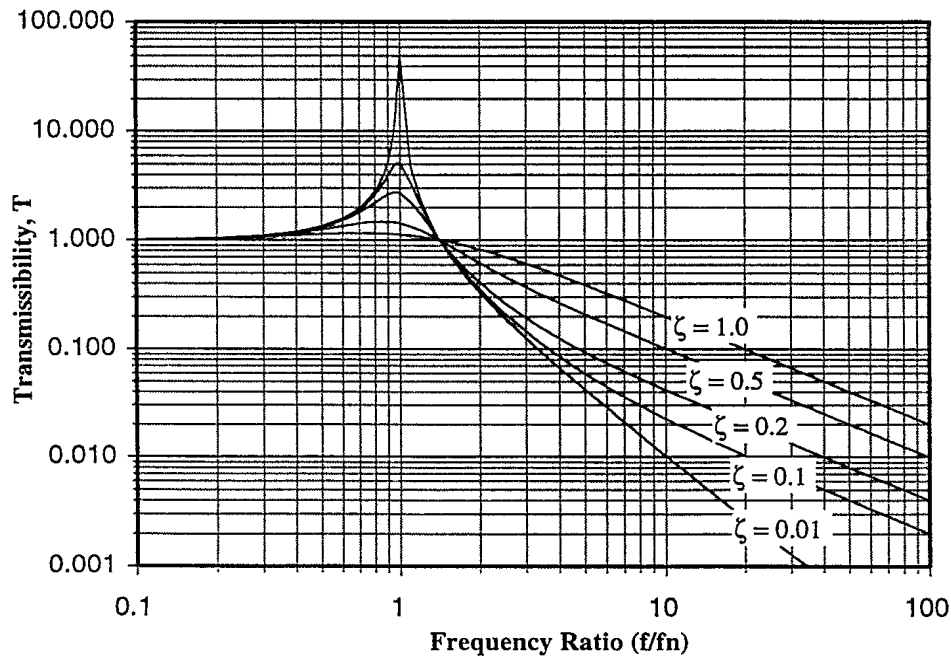
$C$  = isolator damping

$$F_f = \text{transmitted force} \\ = Kx + Cx'$$

**Figure 1 Classical one dimensional model of machinery isolation**

In this simple model  $F_S$  represents the source of vibrations, that is all the forces within the machine which cause vibrations, for example out of balance forces generated in rotating parts, and  $F_F$  represents the forces transmitted to the foundation by the isolator. Applying the equations of motion to this model leads to the calculation of transmissibility,  $T$ , the ratio of the force transmitted to the foundation to the source force.

$$T = \frac{F_F}{F_S} \quad (1)$$



**Figure 2 Transmissibility as predicted by the classical theory**

Transmissibility, as shown in Figure 2, is a function of frequency ratio,  $f/f_n$ , and damping ratio,  $\zeta$ .

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K}{M}} \quad \zeta = \frac{C}{2\sqrt{KM}} \quad (2)$$

As suggested by the shape of the transmissibility curves, isolation should occur when the frequencies of the exciting forces in the machinery are significantly above the natural frequency,  $f_n$ , of the machine-isolator system. In many cases this simple approach will lead to adequate vibration isolation, however in many other instances poor isolation is achieved because factors not considered in this simple model have been ignored. Such factors include the following.

- Isolator materials are not adequately modeled by a simple linear spring and linear damper. For polymeric isolators in particular, behaviour is often dependent on applied load, frequency and temperature. Additionally the mass of an isolator needs to be taken into account since this leads to wave effects at higher frequencies.
- The simple lumped mass model for the machinery does not take into account the fact that vibrations can occur in various directions or as rotational oscillations about various axes. Machinery is normally mounted on a number of isolators in various locations in order to accommodate all directions of vibration and rotational oscillation.
- Neither the mounting, points on the machine, nor the foundation are perfectly rigid. This lack of rigidity at the mounting points on the machine and foundation can have a very significant effect on the effectiveness of isolation.

### Isolator Model Including Foundation Mobility

The model of vibration isolation discussed below has been developed [2,3] to give a more adequate description of vibration isolation, particularly in terms of accounting for the non-rigidity of the foundation and the non-rigidity of the machinery mounting points. The isolator is represented using the four-pole parameter model which allows a more complete description of isolator behaviour including isolator mass and wave effects.

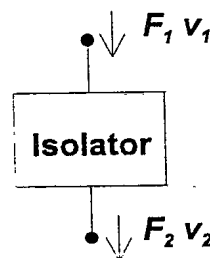


Figure 3 Schematic representation of an isolator

Figure 3 shows schematically, the four-pole parameter representation of an isolator with forces and velocities shown at the input and output of the isolator. Note that in this section, time varying forces, velocities and other quantities are in complex notation. The characteristics of the isolator are then described in terms of the four-pole parameters,  $\alpha_{11}$ ,  $\alpha_{12}$ ,  $\alpha_{21}$  and  $\alpha_{22}$ , as shown in equation (3). Using principles of reciprocity and symmetry it can be shown that  $\alpha_{11} \alpha_{22} - \alpha_{12} \alpha_{21} = 1$  and that  $\alpha_{11} = \alpha_{22}$  respectively, thus leaving only two independent parameters [2]. The four-pole parameters are in general complex quantities which depend on frequency. They provide a considerably more complete description of isolator behaviour than  $K$  and  $C$  of the classical description. In particular they do not assume that force is the same at each end of the isolator, a necessary requirement for considering wave effects in an isolator. Measurements of the four-pole parameter properties of large machinery isolators have been carried out recently by Norwood and Dickens [4] of DSTO.

$$\begin{aligned} F_1 &= \alpha_{11} F_2 + \alpha_{12} V_2 \\ v_1 &= \alpha_{21} F_2 + \alpha_{22} V_2 \end{aligned} \quad (3)$$

Figure 4 shows schematically, two situations: (a) a machine or source which is mounted on an isolator, and (b) a machine mounted directly to a foundation. The non-rigidities of both the foundation and the machinery mounting points are represented by point mobilities,  $M_F$ , and  $M_s$ .

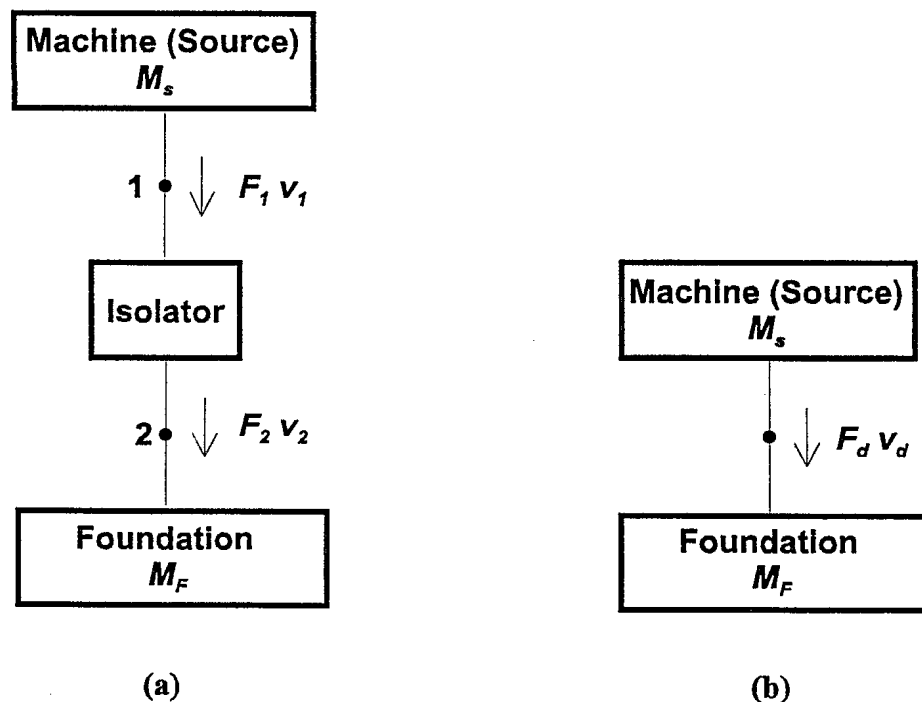


Figure 4 (a) Schematic of a machine mounted on an isolator, (b) Schematic of a machine mounted directly to the foundation



The non-rigidity of the foundation is represented by its mobility,  $M_F$ , where mobility is defined in equation (4).

$$M_F = \frac{v_2}{F_2} = \frac{v_d}{F_d} \quad (4)$$

The machine (source) is characterised by the free velocity,  $v_0$ , at the mounting point, that is the velocity at the mounting point which occurs when the machine is freely suspended and the force at the mounting point is zero. When the machine is connected to the isolator, a force of  $-F_1$  is applied to the machinery mounting point. The free velocity,  $v_0$ , is then modified by  $M_s (-F_1)$  as shown in (5). The same relationship between forces and velocities also applies when the machine is mounted directly to the foundation, also shown in (5).

$$v_1 = v_0 - M_s F_1 \quad \text{or} \quad v_d = v_0 - M_s F_d \quad (5)$$

Considering the situation where the machine is mounted on an isolator (3), (4) and (5) can be solved to give expressions for the force and velocity at the foundation. One measure of isolator performance is the ratio of the magnitude of the velocity at the foundation with the isolator present to the magnitude of the free velocity (6).

$$\frac{|v_2|}{|v_0|} = \frac{|\text{foundation velocity}|}{|\text{free velocity}|} = \left| \frac{M_F}{\alpha_{11} M_s + \alpha_{12} M_s M_F + \alpha_{21} + \alpha_{22} M_F} \right| \quad (6)$$

An alternative measure of isolator performance is called effectiveness,  $E$ , which is the magnitude of the ratio of the force transmitted to the foundation with the isolator present, Figure 4(a), to the force that would be transmitted if the machine were connected directly to the foundation, Figure 4(b).

$$E = \left| \frac{F_2}{F_d} \right| = \left| \frac{M_s + M_F}{\alpha_{11} M_s + \alpha_{12} M_s M_F + \alpha_{21} + \alpha_{22} M_F} \right| \quad (7)$$

### Foundation Mobility

Equations (6) and (7) demonstrate that foundation mobility can have a significant effect on the performance of a vibration isolation situation. In a ship or other structure it is normal to design a foundation to be as rigid and substantial as possible, however, there are practical limitations to this and it is clear from the above analysis that foundation mobility can have a major effect on isolation effectiveness.

Research at the Australian Defence Force Academy has been directed towards obtaining a better understanding of foundation mobility, with the long term aim of developing techniques for optimising foundation design from a vibration isolation point of view. Mobility as used in equation (4) is defined in terms of the response of a structure at the point of application of a concentrated force, hence such a mobility is referred to as a *point mobility*. However in a practical case, isolators tend to be of a significant size with contact between the isolator and the foundation occurring over a significant area. Theoretical values of point mobility have been calculated for simple structures such as plates and beams [5], and point mobility can be measured by standard techniques such as an instrumented impact hammer or shaker with an impedance head, however these predictions or measures do not represent the actual situation where contact occurs over a significant area. It will be shown below that a contact region is of significant size when it has a dimension which is larger than approximately 10 percent of the governing wavelength of the foundation. Thus there is a need for considering mobility over a finite region of contact, that is a need for the concept of *surface mobility*.

Hammer and Peterson [6,7] considered a similar problem in building acoustics and vibration where contact between a wall and a floor occurs over a narrow strip. They suggested two approaches to defining surface mobility, an average power definition, designated below as  $M^{sp}$ , and a definition based on effective point mobility, designated below as  $M^{se}$ . Current research at the Acoustics and Vibration Unit is developing theoretical expressions, and carrying out measurements, of the surface mobility of simple anechoic plates where the contact region is either circular or rectangular. Techniques and concepts developed in these studies will be later applied to the measurement and prediction of surface mobilities for typical foundation structures.

### Definitions of Surface Mobility

The average power definition of surface mobility is based on the vibrational power flow,  $Q$ , which occurs at the point or region of contact between an isolator and a structure. A schematic diagram of the contact region on an arbitrary body is shown in Figure 5.

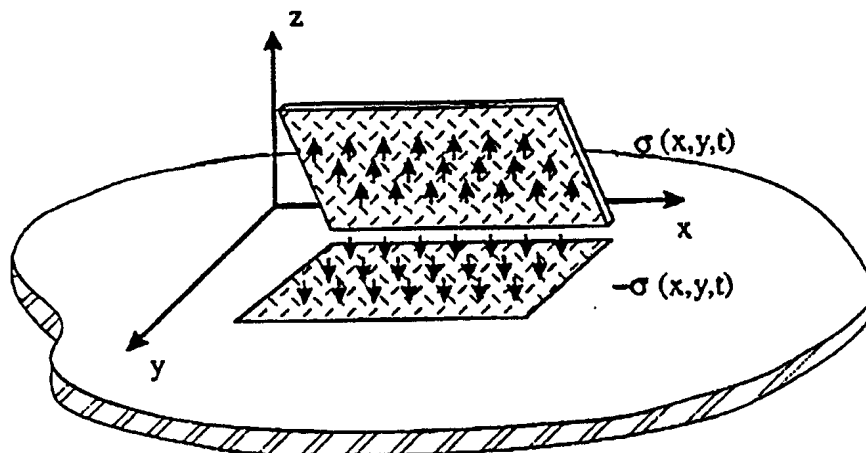


Figure 5 Schematic diagram of a contact region on the surface of a body

For the general case the contact force, represented by stress,  $\sigma(x,y)$ , and the velocity,  $v(x,y)$ , vary continuously over the contact region. Note that  $\sigma$  and  $v$  are complex amplitudes of time varying quantities which also vary spatially.

For point contact between two bodies, the time averaged power flow can be expressed in terms of mobility as follows.

$$Q = \frac{1}{2} \text{Re}\{F^* \cdot v\} = \frac{1}{2} |F|^2 \text{Re}\{M\} \quad (8)$$

For contact over a region, the time averaged power flow, and the total force,  $F$ , are given by the following.

$$Q = \frac{1}{2} \text{Re} \left\{ \int_s \sigma^*(x,y) \cdot v(x,y) ds \right\} \quad (9)$$

$$F = \frac{1}{2} \int_s \sigma^*(x,y) ds \quad (10)$$

Thus by analogy the surface mobility based on average power is given by (11).

$$M^{sp} = \frac{2Q}{|F|^2} = \frac{1}{|F|^2} \text{Re} \left\{ \int_s \sigma^*(x,y) \cdot v(x,y) ds \right\} \quad (11)$$

Note that while surface mobility,  $M^{sp}$ , and power flow,  $Q$ , could be defined as complex quantities, only the real part which is the actual time averaged power flow has physical meaning. (Both the real and imaginary parts of the point mobility have meaning because the phase angle of the mobility is equal to the phase difference between the force and velocity at the point of contact. However with distributed forces and velocities over a contact region, the phase angle of the surface mobility is no longer meaningful.)

The definition of surface mobility,  $M^{se}$ , based on effective point mobility is founded on the basis of multi-point contact between two bodies [6]. For power transmission in multi-point coupled systems where there are  $N$  contact points, effective point mobility at point  $j$  is given by

$$M_j^e = \frac{v_j}{F_j} = \frac{\sum_{i=1}^N M_{ij}}{F_j} \quad (12)$$

where  $M_{ij}$  = the ordinary transfer mobility between points  $i$  and  $j$ .

If the contact points are distributed over an area,  $s$ , the effective point mobility,  $M^e$ , at the point  $(x_i, y_i)$  is given by

$$M^e(x_i, y_i) = \frac{\int \mathbf{M}(x_i, y_i | x_j, y_j) \sigma(x_j, y_j) ds}{\sigma(x_j, y_j)} \quad (13)$$

and the total time averaged power flow through the contact area is given by the following.

$$Q = \frac{1}{2} \int_s |\sigma(x_i, y_i)|^2 \cdot \text{Re}(M^e(x_i, y_i)) ds \quad (14)$$

Now surface mobility is defined again by reference to (8).

$$M^{se} = \frac{\frac{1}{2} \int_s |\sigma(x_i, y_i)|^2 \cdot \text{Re}(M^e(x_i, y_i)) ds}{\left| \int_s \sigma(x_i, y_i) ds \right|^2} \quad (15)$$

Implementation of either of the above definitions of surface mobility requires the determination of the force distribution over the contact area.

### Surface Mobility of an Infinite Plate

Current research has concentrated on determining the surface mobility for relatively simple situations such as a uniform con-phase force distribution applied over a circular or rectangular region of an infinite (i.e. anechoic) plate [8,9]. These predictions can be duplicated experimentally and following confirmation of the techniques, the theory will form the basis for development of methods for determining surface mobility of practical support structures.

For an infinite plate, the theoretical predictions of surface mobility are based on the solution for the vibration field caused by a point force [5]. The solution of the thin plate vibration equations leads to the following relationship between the exciting force field,  $\sigma(x_0, y_0)$ , and the velocity,  $\mathbf{v}(x, y)$ , at some arbitrary location [8].

$$\mathbf{v}(x, y) = \int_s \sigma(x_0, y_0) \cdot \mathbf{M}_0 \cdot \Pi(kr) ds \quad (16)$$

where

$$\Pi(kr) = \mathbf{H}_0^{(2)}(kr) - \mathbf{H}_0^{(2)}(-jkr)$$

$\mathbf{H}_0^{(2)}$  is a zero-order Hankel function,

$r$  = distance between  $(x_0, y_0)$  and  $(x, y)$ ,  
 $k$  = wave number,

$M_0 = \frac{k}{8c_B \rho h}$  is the point mobility of an infinite thin plate,

$c_B$  = bending wave speed in the plate,

$\rho$  = mass density of the plate, and

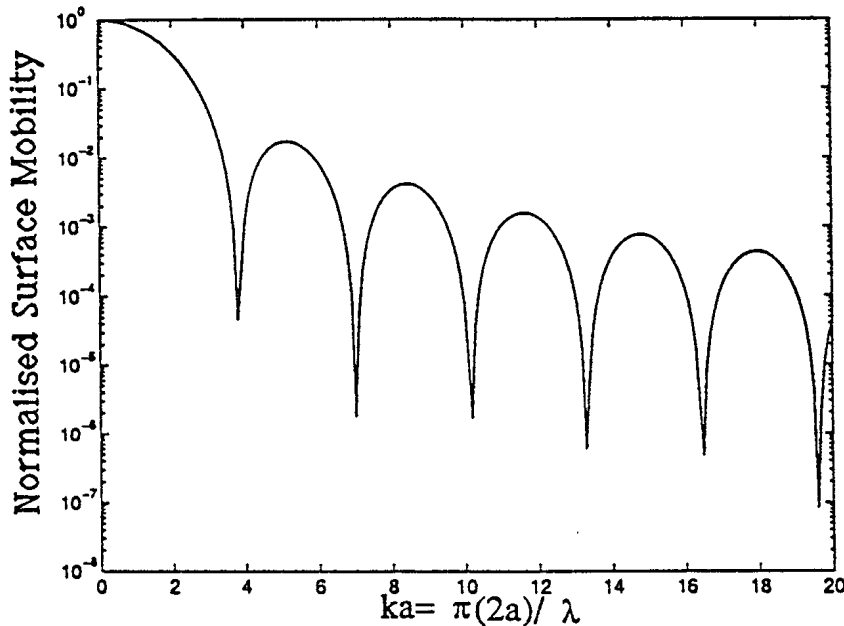
$h$  = thickness of the plate.

The following plots are examples of the predictions obtained of surface mobility based on the averaged power definition given above. All plots are for cases where it has been assumed that a uniform con-phase force distribution is acting over the contact area.

Figure 6 gives the theoretical prediction of normalised surface mobility,  $M^{sp}/M_0$ , for a contact region of radius  $a$ . Normalised surface mobility is plotted against Helmholtz Number which physically represents  $\pi$  times the ratio of the diameter of the contact region to the governing wavelength of the bending waves in the plate.

$$\text{Helmholtz Number} = ka = \pi(2a)/\lambda \quad (17)$$

where  $\lambda = 2\pi/k = \text{wavelength}$



**Figure 6** Normalised surface mobility,  $M^{sp}/M_0$ , for a circular contact region on an infinite plate subject to a uniform con-phase force distribution [8,9].

It can be seen in Figure 6 that when the contact region is small relative to the wavelength, i.e. small values of  $ka$ , the surface mobility approaches the point mobility. However for shorter wavelengths relative to the diameter of the contact region, mobility significantly decreases. Additionally, the mobility exhibits distinct minima or dips whenever a multiple of the wavelength is approximately equal to the contact diameter.

Also in Figure 6 it is clear that once the diameter is greater than approximately 10 percent of the governing wavelength, point mobility is no longer a good approximation for the surface mobility, see also discussion of point mobility of plates in [5].

In a similar way Figure 7 gives the theoretical prediction of normalised surface mobility,  $M^{sp}/M_o$ , for rectangular regions, length  $l$  and width  $w$ , of various aspect ratios. Again it can be seen that when the contact region is small relative to the wavelength, the surface mobility approaches the point mobility, and that for relatively small wavelengths, i.e. higher frequencies, mobility decreases. Mobility also decreases somewhat with higher aspect ratios and again dips in mobility are seen whenever a multiple of the wavelength is approximately equal to the width of the contact region.

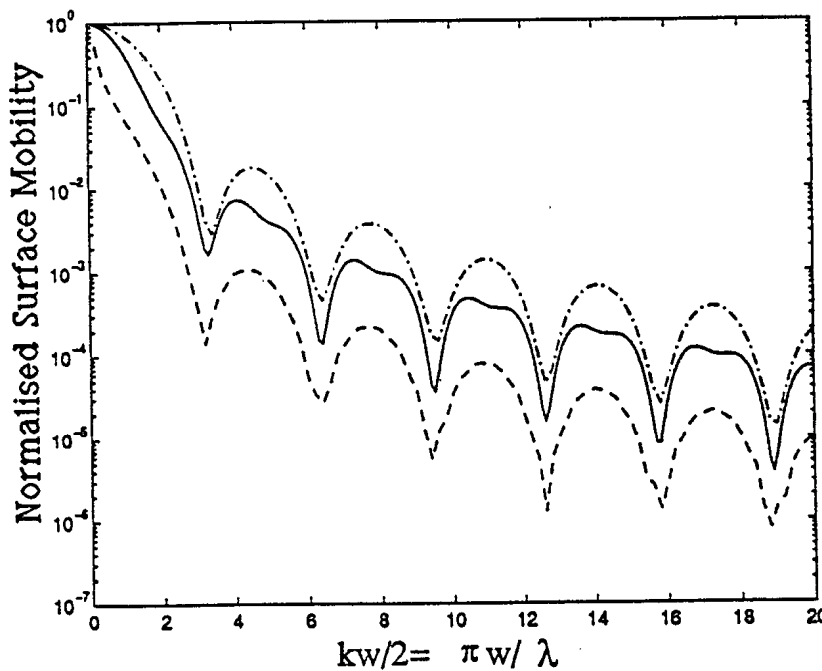


Figure 7 Normalised surface mobility,  $M^{sp}/M_o$ , for rectangular contact regions of various aspect ratios on an infinite plate, subject to a uniform con-phase force distribution [9]. Symbols: - - - -  $l/w = 1$ , ———  $l/w = 2$ , - · - ·  $l/w = 10$ .

As well as the above theoretical results for surface mobility, theoretical work [10, 11] is also proceeding on cases where a uniform velocity distribution is assumed. Additionally, experimental work [8, 12] is proceeding which is in the process of confirming various theoretical predictions. Use of the finite element method for predicting surface mobility is also being explored [10] as this could provide a ready tool for predicting the surface mobility of practical foundation structures.

## Summary and Discussion

Vibration isolation is a critical issue in naval ships where the generation of excessive underwater noise can lead to detection by an enemy. Additionally, poor vibration isolation in any structure, a vehicle or a building, can lead to unwanted noise and disturbance. A theoretical framework has been set out above for taking into account, not only the characteristics of the vibration isolator, but also the mobility or lack of rigidity in the foundation which supports machinery. Research being carried out at the Australian Defence Force Academy is studying methods of characterising and measuring foundation mobility. The long term aim of the work is to develop better methods of assessing machinery foundations as well as methods for optimising foundation design.

## Acknowledgments

The financial and in-kind contributions of the Defence Science and Technology Organisation, the Australian Research Council and The University of New South Wales are gratefully acknowledged. In particular the author's colleagues and collaborators, Chris Norwood, John Dickens, Yuejin Li, Dai Jue and Jiye Zhao are particularly thanked for their support, encouragement and hard work.

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