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**EXECUTIVE SUMMARY  
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FOR  
NEAREST NEIGHBOR SEARCH

P. Zakarauskas — J.M. Ozard

Defence  
Research  
Establishment  
Atlantic



Centre de  
Recherches pour la  
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Atlantique

Canada



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# COMPLEXITY ANALYSIS FOR NEAREST NEIGHBOR SEARCH

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Head / Spinnaker Section

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## Executive Summary

Document: DREA Technical Memorandum 97/240

Title: Complexity Analysis for Nearest Neighbor Search

Authors: P. Zakarauskas and J. M. Ozard

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### Background

The Acoustic Analysis Group at Esquimalt Defence Research Detachment (EDRD) of Defence Research Establishment Atlantic is investigating the use of Matched-Field Processing (MFP) for the detection and localization of acoustic sources on and in the ocean. MFP consists of matching the acoustic data received on an underwater acoustic array, to predictions of the acoustic field for all possible source ranges, depths and bearings, instead of predictions for bearings alone as is the case for conventional processing. Consequently MFP requires an environmental model. For the processing and environmental costs MFP enhances array gain and localization accuracy. Three requirements must be met to outperform conventional processing. First, the sound propagation must be accurately represented by the propagation models incorporated in the software being developed, second, the sound field sampling by a practical array must be adequate for enhancing performance, and third, the system must operate in real time. Real time execution can be achieved if the sound propagation modelling and the matching are sufficiently fast. The first two requirements have been shown to be met for several scenarios investigated.

The analysis described here is in support of the development of a prototype MFP system in a collaborative venture with the private sector. MacDonald Dettweiler is the prime contractor and the prime private contributor in the private sector.

This report describes an investigation into the speedup that can be achieved by partitioning predictions for the received data so that predictions most like the data can be selected for matching. Separate simulation studies at low signal-to-noise ratios have shown a speedup of a factor of ten. Some theoretical guidance was desired to assist this work.

### Principal Results

Three methods for partitioning were investigated with similar speedups predicted under conditions of no noise. The robustness of the partitioning to nonuniformly distributed predictions for the received data was also investigated and found not to present a serious loss of speedup. Partitioning is appropriate for vertical arrays, however other methods for selecting predicted data before matching are indicated for horizontal arrays.

### Defence Significance and Future Research

This memorandum provides theoretical support for the results obtained in separate studies where methods for speeding up matching for vertical arrays have been substantiated for several scenarios. In these separate studies it was found that for vertical arrays restricting matching to nearest neighbours of the data resulted in an order of magnitude increase in the search region or number of frequencies that could be covered in real time. However since MFP is likely to see increased application to horizontal arrays, alternate nearest neighbors methods for selecting the propagation predictions prior to matching require investigation.

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In this technical memorandum we present cost estimates for finding the  $k$ -nearest neighbors to a test pattern according to a Minkowski  $p$ -metric, as a function of the size of the buckets in partitioning searching algorithms. Prior to finding the nearest neighbor, the search space must be ordered using either product or hierarchical partitioning. It is known that fast nearest neighbors algorithms find the  $k$  stored patterns closest to a test pattern in a time which is asymptotically constant with the total number of patterns. The asymptotic total cost is derived explicitly for four types of searches based on three different partitioning schemes: the  $k$ - $d$  tree partitioning, the ordered partitioning, and the product partitioning. The asymptotic expected number of operations performed to find the nearest neighbor is presented as a function of the average number of patterns per bucket  $n$  and the dimensionality of the search space  $d$ , and is shown to contain a global minimum. Total cost is explicitly calculated for the Euclidian metric,  $k = 1$ , and realistic computation costs. The optimum value of  $n$  is weakly dependent on  $d$ , but differs between algorithms. A computer simulation with a uniform distribution of patterns supports the theoretical results.

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