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TITLE

GENERALIZED ELF PROPAGATION

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Generalized ELF Propagation

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Abstract— **Applications in marine electromagnetics require knowledge of the electromagnetic field propagation from an electric dipole embedded in a multi-layered infinite conducting half space. Various solutions to Maxwell's equations exist for between one and four layers with parallel planar interfaces. Shallow water locations require solutions for multiple sea-bed layer situations and for a non-parallel sea bottom. Applications in northern latitudes require solutions in the presence of a low conductivity ice layer. The present paper outlines a technique which generalizes the solution of Maxwell's equations to an arbitrary number of layers, and which can be extended to a sloping seabed.**

Keywords— **ELF Propagation, Maxwell's Equations, Sloping Layer, Fredholm Integral Equation, Iterative Algorithms**

I. INTRODUCTION

Research in marine electromagnetics requires computation of the propagation characteristics of the extremely low frequency (ELF) electromagnetic (EM) radiation arising from an *ac* electric-current source embedded in a conducting medium. Two cases considered are ELF propagation in both offshore and inshore locations.

For offshore locations, the solution to Maxwell's equations for a three parallel-layered medium (air, seawater and seabed) developed by Weaver[1] has proved sufficient. New solutions are required for areas with multiple layers such as stratified ocean, ice cover or sub-bottom sediments, as well as in inshore regions with sloping seabed. Although several authors[1], [2] have developed successful solutions for parallel-layered media, EDRD is investigating a spectral technique that can be adapted to multiple layers and arbitrary interfaces. The present report outlines this technique.

II. THEORETICAL BACKGROUND

Computation of ELF propagation requires solution of Maxwell's equations:

$$\begin{aligned} \text{curl } \mathbf{E} &= -\dot{\mathbf{B}} && (1)\\ \text{curl } \mathbf{H} &= \dot{\mathbf{D}} + \mathbf{J} && (2)\\ \text{div } \mathbf{D} &= \rho && (3)\\ \text{div } \mathbf{B} &= 0 && (4) \end{aligned}$$

where **E** and **H** are the electric and magnetic field intensities, **D** and **B** are the electric and magnetic flux densities, and ρ and **J** are the charge and current densities. For an isotropic medium $\mathbf{D} = \varepsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$ where ε is the permittivity and μ the permeability.

We now introduce the vector and scalar potentials $\mathbf{A}'$ and φ defined by

$$\text{curl } \mathbf{A}' = \mathbf{B} \qquad (5)$$

$$\mathbf{E} + \dot{\mathbf{A}}' = -\text{grad } \varphi \qquad (6)$$

and write the current density **J** in the conducting media in the form $\mathbf{J} = \sigma\mathbf{E} + \mathbf{J}^{ex}$ where $\mathbf{J}^{ex}$ is the density of the extrinsic (impressed) source currents and σ is the electrical conductivity. We can then show that the vector potential $\mathbf{A} = \mathbf{A}'/\mu$ satisfies the Helmholtz equation

$$\triangle\mathbf{A} + k^2\mathbf{A} = -\mathbf{J}^{ex} \qquad (7)$$

where $k^2 = \omega^2\varepsilon\mu - \imath\omega\sigma\mu$, and

$$\begin{aligned} \mathbf{E} &= -\imath\omega\mu(\text{grad } k^{-2}\text{div } \mathbf{A} + \mathbf{A}) && (8)\\ \mathbf{H} &= \text{curl } \mathbf{A} && (9) \end{aligned}$$

where **A** is assumed to have the time dependence $e^{\imath\omega t}$.

We note these expressions maintain the *displacement* terms (involving ε) in anticipation of solving in the presence of low conductivity dielectric layers. If the current sources oscillate with sufficiently low frequency, then displacement currents may be neglected. Then the forms for **E** and **H** are identical to Weaver's.

III. GENERAL SCHEME

We consider the geometry of Fig. 1 which consists of parallel layers perpendicular to the z-axis, together with one layer that slopes along the x-axis. We assume that a single source, consisting of any arbitrary system of electric currents, is exterior to the sloping layer, and that we are also interested in solutions for **B** and **E** outside of the sloping layer.

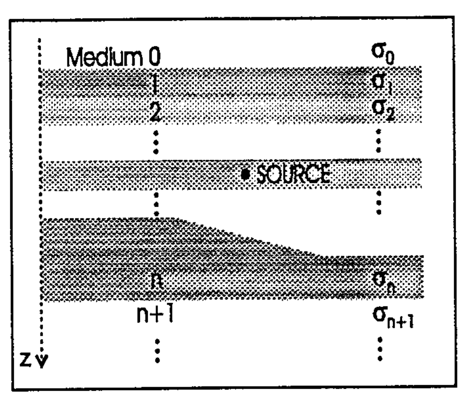

Fig. 1. A slice of the layered and sloping media.

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Since constructing the appropriate analytic Green's function for this model is extremely difficult, we use an alternative scheme which consists of:

- solving the problem for the case of parallel layers
- using the parallel layer solutions to determine the fields within the sloping layer "n", and
- determining the solutions in any region outside of the sloping layer.

Each of these processes will now be described briefly.

A. PARALLEL LAYERS

For solving the more commonly studied parallel layer situation (7) is generally written in component form

$$\nabla^2 A_\alpha + k^2 A_\alpha = -J_\alpha^{ex}(x,y,z) \tag{10}$$

and solved subject to the boundary and Sommerfeld radiation conditions

$$A_{\alpha\imath} - A_{\alpha(\imath-1)} = 0, \qquad (\alpha = x,y,z) \tag{11}$$

$$\frac{\partial A_{\alpha\imath}}{\partial z} - \frac{\partial A_{\alpha(\imath-1)}}{\partial z} = 0, \qquad (\alpha = x,y) \tag{12}$$

$$k_\imath^{-2}\text{div } A_\imath - k_{\imath-1}^{-2}\text{div } A_{(\imath-1)} = 0 \tag{13}$$

$$A_{\alpha\imath} \to 0 \qquad r \to \infty. \tag{14}$$

Here $\imath$ refers to the $\imath$-th parallel layer of the medium.

In the present approach the x and y-derivatives are removed from (10) by performing a two-dimensional Fourier transform to yield a simpler ordinary second order differential equation

$$\frac{d^2\tilde{A}_\alpha}{dz^2} - q^2\tilde{A}_\alpha = -\tilde{J}_\alpha^{ex}, \tag{15}$$

where $q^2 = \nu_1^2 + \nu_2^2 - k^2$, and where $\tilde{A}_\alpha(\nu_1,\nu_2,z)$ and $\tilde{J}_\alpha^{ex}(\nu_1,\nu_2,z)$ are Fourier transforms of A_α and J_α^{ex} respectively. Eqn. (15) with its boundary conditions form a boundary value problem that can be solved more easily than the corresponding boundary problem for (10).

The vector potential **A**, **E** and **H** in the spatial domain are then obtained through inverse transforms, such as

$$A_\alpha(\mathbf{r}) = (2\pi)^{-2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{A}_\alpha e^{\imath(\nu_1 x+\nu_2 y)}d\nu_1 d\nu_2. \tag{16}$$

B. SLOPING LAYERS

For a y-independent geometry (*ie. cylindrical* in mathematical terminology) the electric fields E_α within the sloping "n"-layer are connected to the fields E_α^n in the parallel layered medium through the expression

$$E_\alpha^n(\mathbf{r}) + \imath\omega\mu\int\int_S \Delta\sigma E_\alpha(\mathbf{r}')G_\alpha(\mathbf{r},\mathbf{r}')d\mathbf{r}' = E_\alpha(\mathbf{r}) \tag{17}$$

where G_α is the appropriate 2-D Green's function for the parallel-layered medium, S is the cross-section of the sloping layer, and $\Delta\sigma$ is the perturbation in the conductivity caused by the sloping layer. The magnetic field can either be computed from the first of Maxwell's equations (1) or by substituting $\nabla' \times \mathbf{G}$ for G in (17).

Consequently, within the sloping layer S (i.e., $\mathbf{r}' \in S, \mathbf{r} \in S$), E_α is obtained by solving a Fredholm integral equation of the second kind. This assumes that the appropriate analytic Green's function can be found and a suitable algorithm developed for the numerical solution. We note that in practice it is adequate to solve only for some finite sequence for the two dimensional integral equations, thus obtaining an ensemble of 2-D sections for each of the 3-D functions E_α within the sloping layer.

Once the components of the electric field have been obtained within the sloping layer, we can consider (17) outside the sloping layer (i.e., when $\mathbf{r} \in R^2\backslash S$) as an ordinary integral with respect to the right-hand side $E_\alpha(\mathbf{r}), \mathbf{r} \in R^2\backslash S$. Thus $\mathbf{r}$ can be any location outside the layer, and in principle any quadrature formula can be used to perform the double integral.

It is not necessary to store all sections of the 3-D functions E_α within the sloping layer because the double integral can be calculated on the plane $y = y_0$ and the the value of y_0 stepped leading to computational efficiency.

For general sloping layers (17) is three-dimensional.

IV. PARALLEL LAYER SOLUTIONS - HED

The solution of the boundary-value problem (10-14) can be obtained in closed form for some cases and here we consider a point horizontal electric dipole (HED) aligned with the x-axis as shown in Fig. 2. There are two approaches to solving the problem, the spectral technique detailed above, and a spatial representation method outlined in §IV-B.2. Although the two approaches are theoretically equivalent, the computational effectiveness of their implementation can differ substantially.

A. SPECTRAL SOLUTIONS

The spectral approach can be demonstrated by considering two and four-layered media.

A.1 Two-Layer Media

Although the layers of Fig. 2a are labelled "a" and "w" for eventual correspondence with air and sea, the equations are solved for $\sigma_a \neq 0$ implying a general two-conducting-layer model. The system of equations (15) becomes

$$\frac{d^2\tilde{A}_x}{dz^2} - q^2\tilde{A}_x = -\tilde{J}_x^{ex} \tag{18}$$

$$\frac{d^2\tilde{A}_z}{dz^2} - q^2\tilde{A}_z = 0. \tag{19}$$

The inhomogeneous equation can be solved using the method of variation of parameters[3] to give

$$\tilde{A}_x = (2q)^{-1}[e^{qz}(B_1 - \int_0^z \tilde{J}^{ex}e^{-q\zeta}d\zeta) + e^{-qz}(B_2 + \int_0^z \tilde{J}^{ex}e^{q\zeta}d\zeta)] \tag{20}$$

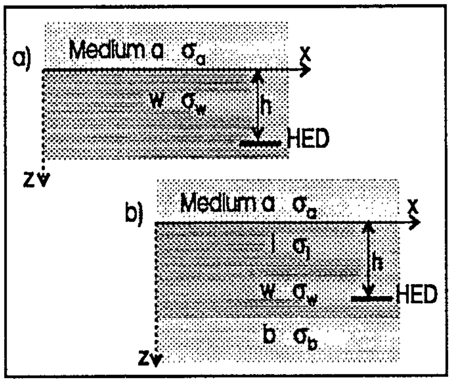

Fig. 2. Geometry for two and four-layered media.

and the source integral evaluated to yield

$$\int_0^z \tilde{J}_x e^{\pm q\zeta} d\zeta = \begin{cases} e^{\pm q_w h} & \text{if } z > h \\ 0 & \text{if } z < h. \end{cases} \qquad (21)$$

Setting the dipole moment to 1 for convenience the general solution of the inhomogeneous and homogeneous equations (18) and (19) for $\tilde{A}_x$ and $\tilde{A}_z$ respectively can be written

$$\tilde{A}_x = (2q_w)^{-1}[e^{q_w z}(B_1^w - e^{-q_w h}) + e^{-q_w z}(B_2^w + e^{q_w h})], \quad z > h \qquad (22)$$

$$\tilde{A}_x = (2q_w)^{-1}(B_1^w e^{q_w z} + B_2^w e^{-q_w z}), \quad 0 < z < h \qquad (23)$$

$$\tilde{A}_x = (2q_a)^{-1}(B_1^a e^{q_a z} + B_2^a e^{-q_a z}), \quad z < 0 \qquad (24)$$

$$\tilde{A}_z = C_1^w e^{q_w z} + C_2^w e^{-q_w z}, \quad z > 0 \qquad (25)$$

$$\tilde{A}_z = C_1^a e^{q_a z} + C_2^a e^{-q_a z}, \quad z < 0 \qquad (26)$$

where the indices "w" or "a" denote the layer.

Application of the Fourier transformed boundary and radiation conditions leads to evaluation of the coefficients and determination of the spectral vector potential in both air and seawater. The seawater solution is found to be

$$\tilde{A}_x = \frac{1}{2q_w(q_w+q_a)}[(q_w+q_a)e^{q_w h} + (q_w-q_a)e^{-q_w h}]e^{-q_w z}, \quad z > h \qquad (27)$$

$$\tilde{A}_x = \frac{1}{2q_w(q_w+q_a)}[(q_w+q_a)e^{q_w z} + (q_w-q_a)e^{-q_w z}]e^{-q_w h}, \quad 0 < z < h \qquad (28)$$

$$\tilde{A}_z = \imath\nu_1 \frac{q_w - q_a}{q_w k_a^2 + q_a k_w^2} e^{-q_w h} e^{-q_w z}, \quad z > 0 \qquad (29)$$

which is identical to that of Baños[4] when the Hertz potential Π and $\mathbf{A}$ are linked through $\Pi = \imath\omega k^{-2}\mathbf{A}$.

By setting $\sigma_a = \sigma_w$ we obtain the one-layer solutions

$$\tilde{A}_x = \frac{e^{-q_w|z-h|}}{2q_w}; \quad \tilde{A}_z = 0 \qquad (30)$$

which are consistent with earlier work.

The spectral solutions for the three-layer media can be derived by analogy.

A.2 Four-Layer Medium

The four-layer medium is shown in Fig. 2b where "a", "$\imath$", "w" and "b" correspond to air, ice, sea and bottom, although once again σ_a is general. To preserve space we concentrate solely on the "w"-layer.

The solutions to the inhomogeneous and homogeneous differential equations are:

$$\tilde{A}_x = (2q_w)^{-1}[e^{q_w z}(B_1^w - e^{-q_w h}) + e^{-q_w z}(B_2^w + e^{q_w h})], \quad z_2 > z > h \qquad (31)$$

$$\tilde{A}_x = (2q_w)^{-1}(B_1^w e^{q_w z} + B_2^w e^{-q_w z}), \quad z_1 < z < h \qquad (32)$$

$$\tilde{A}_z = C_1^w e^{q_w z} + C_2^w e^{-q_w z}, \quad z_2 > z > z_1. \qquad (33)$$

Applying the boundary and radiation conditions to the inhomogeneous solution and solving the corresponding system of linear equations, we obtain

$$B_1^w = -\frac{D_4}{D_1 D_4 - D_2 D_3}E \quad B_2^w = \frac{D_3}{D_1 D_4 - D_2 D_3}E, \qquad (34)$$

with

$$E = [a_{2w}^2 e^{q_w h}(\frac{q_b}{q_w} - 1) - e^{-q_w h}(\frac{q_b}{q_w} + 1)] \qquad (35)$$

$$D_1 = \frac{q_b}{q_w} + 1 \qquad D_2 = a_{2w}^2(\frac{q_b}{q_w} - 1) \qquad (36)$$

$$D_3 = a_{3w}(\frac{q_i}{q_w} - \frac{a_{1i} + a_{2i}p}{a_{1i} - a_{2i}p}) \qquad (37)$$

$$D_4 = a_{4w}(\frac{q_i}{q_w} + \frac{a_{1i} + a_{2i}p}{a_{1i} - a_{2i}p}) \qquad (38)$$

$$p = \frac{q_i - q_a}{q_i + q_a} \qquad (39)$$

where $a_{1i} = e^{q_i z_1}$, $a_{2i} = e^{-q_i z_1}$, $a_{2w} = e^{-q_w z_2}$, $a_{3w} = e^{q_w z_1}$ and $a_{4w} = e^{-q_w z_1}$.

By using the continuity of both $\tilde{A}_z$ and $k^2(i\nu_1\tilde{A}_x + \frac{\partial \tilde{A}_z}{\partial z})$ on every interface, the analogous formulae for the homogeneous equation yield

$$C_1^w = \frac{Q_1 R_4 - Q_2 R_2}{R_1 R_4 - R_2 R_3} \qquad C_2^w = \frac{Q_2 R_1 - Q_1 R_3}{R_1 R_4 - R_2 R_3} \qquad (40)$$

where

$$Q_1 = \frac{\imath\nu_1}{2q_w}[(B_2^w + e^{q_w h})e^{-2q_w z_2} + (B_1^w - e^{-q_w h})](k_b^2 - k_w^2) \qquad (41)$$

$$Q_2 = a_{2i}k_w^2 q_i s(\frac{a_{1i} - a_{2i}t}{a_{1i} + a_{2i}t} + 1) - \imath\nu_1 \tilde{A}_x^{wi}(k_i^2 - k_w^2) \qquad (42)$$

$$R_1 = q_b k_w^2 + q_w k_b^2 \qquad R_2 = a_{2w}^2(q_b k_w^2 - q_w k_b^2) \qquad (43)$$

$$R_3 = a_{3w}(q_i k_w^2 \frac{a_{1i} - a_{2i}t}{a_{1i} + a_{2i}t} - q_w k_i^2) \qquad (44)$$

$$R_4 = a_{4w}(q_i k_w^2 \frac{a_{1i} - a_{2i}t}{a_{1i} + a_{2i}t} + q_w k_i^2) \qquad (45)$$

$$s = \frac{k_i^2 - k_a^2}{q_i k_a^2 + q_a k_i^2}\imath\nu_1 \tilde{A}_x^{ai} \qquad t = \frac{q_i k_a^2 - q_a k_i^2}{q_i k_a^2 + q_a k_i^2} \qquad (46)$$

and the terms on the interface are

$$\tilde{A}_x^{wi} = (2q_i)^{-1}(B_1^{(i)}e^{q_i z_1} + B_2^{(i)}e^{-q_i z_1}) \quad (47)$$

$$\tilde{A}_x^{ai} = (2q_i)^{-1}(B_1^{(i)} + B_2^{(i)}) \quad \text{where} \quad (48)$$

$$B_1^i = \frac{a_{3w}}{2a_{1i}}(\frac{q_i}{q_w}+1)B_1^w + \frac{a_{4w}}{2a_{1i}}(\frac{q_i}{q_w}-1)B_2^w \quad (49)$$

$$B_2^i = \frac{a_{3w}}{2a_{2i}}(\frac{q_i}{q_w}-1)B_1^w + \frac{a_{4w}}{2a_{2i}}(\frac{q_i}{q_w}+1)B_2^w. \quad (50)$$

Suitable substitution leads to both $\tilde{A}_x^w$ and $\tilde{A}_z^w$, the spectral solutions in the layer containing the HED.

B. SPATIAL DOMAIN SOLUTIONS

B.1 FFT Approach

It can be seen that use of the spectral domain leads to closed form solutions for the vector potential of two, three and four-layered media. Extension to larger numbers of layers is relatively straightforward.

To determine the spatial domain solutions, the inverse Fourier transforms of $\mathbf{\tilde{A}}$, or of (8) and (9) could be calculated either by analytic evaluation, by numerical computation, or by a combination of both. To demonstrate the usefulness of the spectral approach, we have computed the three and four-layer spatial domain solutions using the fast Fourier transform (FFT).

In performing the FFT, two factors must be considered, namely the choice of maximum or Nyquist frequency used in the transform, and the various derivatives required in (8) and (9). We have found no *a priori* elegant way of determining the appropriate discretization, and we have determined this in practice through experimentation.

The various first and second derivatives of the vector potential can be computed either after or before performing the FFT. If computed afterwards, $\tilde{A}_x$ and $\tilde{A}_z$ are transformed and the various derivatives needed for **E** and **H** evaluated through spline interpolated finite difference methods. If the derivatives are first performed in the frequency domain then many of the terms can be written down by inspection since $\frac{\partial A_x}{\partial x} = \mathcal{F}^{-1}[\imath\nu_1\tilde{A}_x]$ *etc.* which simplifies (8) and (9) to

$$\tilde{E}_x = -(\imath\omega\mu + \nu_1^2(\sigma_w)^{-1})\tilde{A}_x + \imath\nu_1(\sigma_w)^{-1}\frac{\partial\tilde{A}_z}{\partial z} \quad (51)$$

$$\tilde{E}_y = -\nu_1\nu_2(\sigma_w)^{-1}\tilde{A}_x + \imath\nu_2(\sigma_w)^{-1}\frac{\partial\tilde{A}_z}{\partial z} \quad (52)$$

$$\tilde{E}_z = (q_w^2(\sigma_w)^{-1} - \imath\omega\mu)\tilde{A}_z + \imath\nu_1(\sigma_w)^{-1}\frac{\partial\tilde{A}_x}{\partial z} \quad (53)$$

$$\tilde{H}_x = \imath\nu_2\tilde{A}_z \qquad \tilde{H}_y = (\frac{\partial\tilde{A}_x}{\partial z} - \imath\nu_1\tilde{A}_z)$$
$$\tilde{H}_z = -\imath\nu_2\tilde{A}_x. \quad (54)$$

Noting that $\tilde{A}_x$ and $\tilde{A}_z$ are given by (22), (23) and (25), the first derivatives are simply

$$\frac{\partial\tilde{A}_x}{\partial z} = \begin{cases} [(B_1^w - e^{-q_w h})e^{q_w z} & \\ \quad -(B_2^w + e^{q_w h})e^{-q_w z}]/2 & z_2 > z > h \\ (B_1^w e^{q_w z} - B_2^w e^{-q_w z})/2 & z_1 < z < h \end{cases} \quad (55)$$

$$\frac{\partial\tilde{A}_z}{\partial z} = q_w(C_1^w e^{q_w z} - C_2^w e^{-q_w z}). \quad (56)$$

These expressions for $\mathbf{\tilde{E}}$ and $\mathbf{\tilde{H}}$ now contain analytically expressed derivatives and can be inverse transformed to yield **E** and **H**.

Examples of E_x and H_z calculated for the four-layer medium are shown in Fig. 3, and as can be seen are consistent with other works.

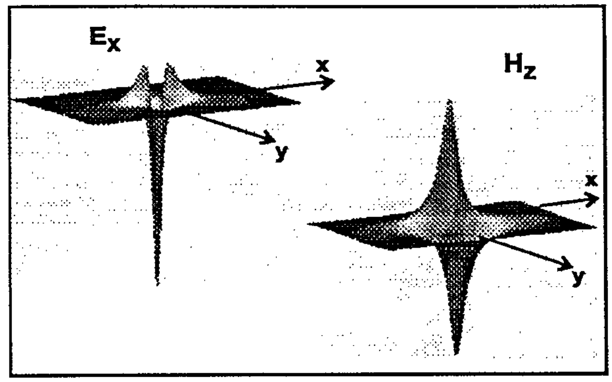

Fig. 3. Calculated E_x and H_z for four layer model.

B.2 General Spatial Domain Approach

An alternative scheme for finding spatial solutions is to modify (10) in the case of the HED to give

$$\Delta A_x + k^2(z)A_x = -\delta(\mathbf{r}) \quad (57)$$

$$\text{div }(\sigma(z)^{-1}\text{grad}A_z) + \imath\omega\mu A_z = -\frac{\partial A_x}{\partial x}\frac{\partial}{\partial z}\sigma(z)^{-1}. \quad (58)$$

The solutions to these equations can then be written explicitly as:

$$A_x = (4\pi)^{-1}\int_0^\infty J_0(r\xi)p(z,\xi)\xi d\xi \quad (59)$$

$$A_z = (4\pi)^{-1}\int_0^\infty \frac{x}{r}J_1(r\xi)[\frac{\partial p}{\partial z} - \xi g(z,\xi)]d\xi \quad (60)$$

where the one-parameter functions $p(z,\xi), g(z,\xi)$ are defined by solving the Cauchy problems for the Riccati equations

$$\frac{1}{\sqrt{\omega}}u'(z) - u^2(z) = \imath\mu\sigma(z) - \eta^2, \qquad 0 < z < z_N \quad (61)$$

$$u(z) = \sqrt{\eta - \imath\mu\sigma(z)}, z = z_N \qquad \text{and} \quad (62)$$

$$\frac{1}{\sqrt{\omega}}v'(z) - \sigma(z)v^2(z) = \imath\mu - \frac{\eta^2}{\sigma(z)}, \qquad 0 < z < z_N \quad (63)$$

$$v(z) = \frac{\sqrt{\eta^2 - \imath\mu\sigma(z)}}{\sigma(z)}, \qquad z = z_N \quad (64)$$

with $\eta = \frac{\xi}{\sqrt{\omega}}$, $u(z) = -\frac{p'(z)}{\sqrt{\omega p(z)}}$, $v(z) = -\frac{g'(z)}{\sqrt{\omega\sigma(z)v(z)}}$ and where z_N is the lowermost interface. The Cauchy problems can be solved numerically for an arbitrary piecewise continuous function $\sigma(z)$. We note the expressions of Burke and Jones[2] for generalizing Weaver's method can be obtained from this approach when $\sigma(z)$ is piecewise constant.

V. SLOPING GEOMETRY

As noted earlier, solution when the sloping layer is present requires specification of the Green's function for the parallel layer case followed by solution of the Fredholm integral equation of the second kind.

A. GREEN'S FUNCTION DETERMINATION

It is well-known that the dyadic Green's function for the isotropic homogeneous medium has the form

$$\mathbf{G}(\mathbf{r},\mathbf{r}') = [\mathbf{I} + \frac{\nabla'\nabla'}{k^2}]G_h(\mathbf{r},\mathbf{r}') \qquad (65)$$

where

$$G_h(\mathbf{r},\mathbf{r}') = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}.$$

One spectral form of the Green's function of the parallel layered medium is given in [5] (see Eq. (7.4.14), p.414), but this form is inconvenient because of difficulties in using the FFT. To avoid these problems we use (65) to compute the Green's function although it is necessary to change the scalar function G_h to a function G_l that describes the parallel layered medium. Following Dmitriev[6], this function can be constructed as follows. Let us suppose that the source and observer are located at (x', y') and (x, y) in the m-th and n-th layer respectively. The two-dimensional Green's scalar function G_l can then be written

$$G_l(\mathbf{r},\mathbf{r}') = \begin{cases} (2\pi)^{-1}\int_0^\infty R_b(z)P_b(z')\cos\lambda(x-x')d\lambda & \text{if } z \le z' \\ (2\pi)^{-1}\int_0^\infty R_t(z)P_t(z')\cos\lambda(x-x')d\lambda & \text{if } z \ge z' \end{cases} \qquad (66)$$

where

$$R_b(z) = \frac{R^b_{n+1}}{1+b_n e^{-q_n d_n}}(e^{q_n(z-z_{n+1})} + b_n e^{-q_n(z-z_n)}) \qquad (67)$$

$$R_t(z) = \frac{R^t_n}{1+a_n e^{-q_n d_n}}(e^{q_n(z-z_n)} + a_n e^{-q_n(z-z_{n+1})}) \qquad (68)$$

$$P_b(z') = q_m^{-1}\frac{1+b_m e^{-q_m d_m}}{1-a_m b_m}(e^{-q_m(z'-z_{m+1})} + a_m e^{q_m(z'-z_m)}) \qquad (69)$$

$$P_t(z') = q_m^{-1}\frac{1+a_m e^{-q_m d_m}}{1-a_m b_m}(e^{q_m(z'-z_m)} + a_m e^{-q_m(z'-z_{m+1})}) \qquad (70)$$

$$a_n = e^{-q_n d_n}\frac{\xi_n + a_{n+1}e^{-q_{n+1}d_{n+1}}}{1+\xi_n a_{n+1}e^{-q_{n+1}d_{n+1}}} \qquad (71)$$

$$b_n = e^{-q_n d_n}\frac{\zeta_n + b_{n-1}e^{-q_{n-1}d_{n-1}}}{1+\zeta_n b_{n-1}e^{-q_{n-1}d_{n-1}}} \qquad (72)$$

$$\xi = \frac{q_n - q_{n+1}}{q_n + q_{n+1}} \qquad \zeta = \frac{q_n - q_{n-1}}{q_n + q_{n-1}} \qquad (73)$$

$$R^b_n = \frac{R^b_{n+1}}{1+b_n e^{-q_n d_n}}(e^{-q_n d_n} + b_n) \qquad (74)$$

$$R^t_n = \frac{R^t_{n-1}}{1+a_{n-1}e^{-q_n d_n}}(e^{-q_{n-1}d_{n-1}} + a_{n-1}) \qquad (75)$$

and where $d_n = z_{n+1} - z_n$. For an N-layered medium, all coefficients can be computed recursively, starting from $b_0 = 0$, $a_N = 0$, $R^b_{m+1} = 1$ and $R^t_m = 1$. It is easy to see that the integrands in (66) are continuous enabling the layer Green's functions to be evaluated.

B. THE INTEGRAL EQUATION METHOD

Although there are several computational algorithms for solving Fredholm equations of the second kind, many are based on the method of successive approximations which converges if $|\lambda = i\omega\mu| < ||\Delta\sigma G_l||^{-1/2}_{L_2}$. Estimating this norm for the finite region S, we obtain the condition

$$\omega\mu \cdot meas(S)\max|\Delta\sigma| \ll 1. \qquad (76)$$

which is satisfied only if the source is located sufficiently far from the sloping layer. Since $\Delta\sigma E^n$ can be interpreted as the excess current density that perturbs the initial field, this inequality implies that inductive interactions are ignored and that no skin-effect exists within the sloping layer. This assumption is not always valid.

Fortunately, by considering the Fredholm integral equation of the second kind in the form of a more general operator equation of the first kind, one obtains a family of iterative processes that utilize the general spectral approach for constructing regularizing operators used to solve linear ill-posed problems [7]. A converging iterative process can be constructed using the following scheme.

Eqn. (17) is first rewritten in the more convenient form

$$f(x) + \lambda\int_\Omega K(x,s)y(s)ds = y(x) \qquad (77)$$

where x, s are two-dimensional vectors, and where f, $y(x)$ and $K(x,s)$ denote E^n_α, E_α and $\Delta\sigma G_l$ respectively. Following discretization using any quadrature formula, we note the operator $I - \lambda\int_\Omega K(x,s)ds$ will generate some hypermatrix A, and the two-dimensional vectors f and y can be transformed to a corresponding regular matrix and one-dimensional vectors, respectively. For convenience, we denote these transformations by the same symbols and obtain the system of linear equations

$$Ay = f \qquad (78)$$

where A is determined by $\Delta\sigma$ and G_l. We note that A can be large and that (78) can be ill-conditioned when (76) is not satisfied so that some regularization technique must be used for solving the simultaneous equations. Since A is a bounded operator in the Euclidean space it is appropriate to use the spectral approach [7] for constructing the regularizing operators for (78) so that we obtain the iterative procedure

$$y_{k+1} = (I - \gamma A^* A) y_k + \gamma A^* f \tag{79}$$

where A^* implies the conjugate matrix and which converges for $0 < \gamma < \frac{2}{\|A^* A\|}$ with the initial approximation that $y_0 = 0$. This particular process requires the norm of the superposition of matrices A^* and A to be computed.

If γ does not satisfy the above inequality, an alternative iterative process can be used, namely

$$y_{k+1} = (I + \gamma A^* A)^{-1} y_k + (I + \gamma A^* A)^{-1} A^* f \tag{80}$$

which converges for any $\gamma > 0$.

Since the field E^n_α is computed approximately, we need to choose an appropriate number of iterations that depend on the error level. Regularizing properties of the iterative processes are guaranteed under certain realizable conditions.

VI. Concluding Remarks

In summary we have proposed a general scheme for computing the electromagnetic field in the presence of a sloping layer. The scheme included solving the parallel layer situation followed by solving a Fredholm integral equation of the second kind to produce the solution in the sloping layer. The solution for any arbitrary region outside of the sloping layer was then determined by evaluation of an integral. Although the scheme works with any type of inhomogeneity within the parallel layered media, computational effectiveness is greatest for y-independent inhomogeneities (*ie. cylindrical* in mathematical terminology) since the model is then two-dimensional.

Two approaches were analysed for constructing Green's functions for the Helmholtz equation that describes the electromagnetic field in parallel layer media. The first was related to the spectral representation of the dyadic Green's function, whereas the second was based on constructing the scalar Green's function. Since the scalar Green's function has no singularity points, its determination is reduced to integration of a rapid oscillatory function. This second approach therefore seems preferable for determining the Green's function for parallel layered media.

Some computational methods for solving the Fredholm integral equation of the second kind were also analysed. For cylindrical inhomogeneities the problem of computing the three-dimensional electromagnetic field in the presence of a sloping layer was reduced to solving a collection of two-dimensional integral equations. Attention was focussed on iterative procedures that provide reasonable computational effectiveness. The analysis suggests use of regularizing procedures that guarantee convergence.

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At 13:59 -0400 13/06/97, you (peter.holtham@EDRD.DND.CA) wrote:
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Please make the following minor changes:

Abstract:
First sentence: delete the word "often".
Third sentence: delete the phrase "Increasing interest in" and "now demands" so that the sentence begins "Shallow water locations require solutions for multiple .."

Introduction:
First sentence: delete the word "often".
Second sentence: delete "to the Esquimalt Defence Research Detachment (EDRD)" and change "areas of interest" to "cases considered" so that this sentence reads:
"Two cases considered are ELF propagation in both offshore and and inshore locations."
Fifth sentence: Delete "More recently however interest has increased in areas " and replace with:
"New solutions are required for areas.."

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