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TITLE

FINITE ELEMENT BASED SPECTRAL METHODS FOR FATIGUE DAMAGE ASSESSMENT OF SHIP STRUCTURES

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Finite Element Based Spectral Methods for Fatigue Damage Assessment of Ship Structures

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ABSTRACT

The prediction of fatigue crack initiation or crack growth in a structure can be accomplished most accurately if the time history of loads is known and can be used to predict the time history of the structural response. In the case of ship structures the load history is usually only known in terms of the time spent at combinations of different ship speeds and headings in various sea environments. The sea environment may only be known in terms of a wave frequency spectrum which in combination with a given ship speed and heading can be used to derive a frequency spectrum of the local structural response (stress, strain or stress intensity factor) at a location or region of the ship. The resulting response spectra can then be applied in a fatigue damage assessment based on either spectral crack initiation or crack propagation analysis methods. This paper outlines a method for calculation of the frequency spectrum of the structural response based on the use of regular wave hull pressure loads and rigid body accelerations provided by PRECAL (a linear frequency domain hydrodynamics code based on 3D potential flow) and a top down quasi-static structural analysis procedure implemented in the VAST suite of finite element codes. The finite element quasi-static analyses are conducted for unit pressure loads on each hydrodynamic mesh facet and for unit rigid body acceleration load cases. This reduces considerably the number of quasi-static load cases required compared to using a direct method which applies a set of wave pressure loads and rigid body accelerations for each combination of regular wave frequency, ship speed and heading needed to represent a short term or long term ship operational profile.

INTRODUCTION

Traditional ship structural analysis, based on a static equivalent beam analysis of the ship hull girder balanced on a design wave, has worked reasonably well for design of conventional ship structures, but greatly oversimplifies a complex loading-response process and does not provide the rational means to assess through-life safety or develop more efficient designs. Recent advances in computing technology have resulted in improved methods for modelling the loads acting on a ship operating in a defined seaway and the corresponding response of the complex ship structure. Both naval and commercial sectors are developing analysis tools based on these physical modelling approaches. The major development project, Improved Ship Structural Maintenance Management (ISSMM) [1], and other Canadian Forces (CF) hull system life cycle management initiatives, require the capability to predict realistic structural response to sea loads for CF vessels.

The Hydronautics Section at the Defence Research Establishment Atlantic has been developing methods for prediction of sea loads and their application to finite element models of the hull structure to predict fatigue and ultimate strength performance. Through cooperative research with the NSMB (Netherlands Ship Model Basin), Cooperative Research Ships organization, a linear three-dimensional proprietary seakeeping code, PRECAL [2], has been developed to predict pressure loads for the ship hull operating in a seaway. The PRECAL code has been used in conjunction with the Department of National Defence structural finite element code VAST [3] to predict stress spectra at critical details in the ship hull for a single ship speed and heading in a seaway defined by one wave energy spectrum [4] and the hull pressure predictions from PRECAL have been validated through full scale measurement [5, 6]. The structural response in terms of stress spectra, strain spectra or stress intensity factor spectra can be used to predict fatigue and crack growth behaviour for a given operating profile of a vessel.

A realistic ship operational profile involves a large number of combinations of sea environments, ship speeds and ship headings (defined as operational cells), each of which produces a different hull pressure spectral load case. Application of classical random response methods for a full ship finite element model and the large number of wave load cases is impractical due to the large computational effort required to determine the structural response to all load cases. For crack propagation the finite element mesh must be changed after a certain increment of crack growth, and the structural response re-determined, further multiplying the computational effort required.

This paper proposes a new practical method for calculation of the structural response frequency spectrum based on the use of hull pressure response and rigid body acceleration response in regular waves (provided by PRECAL) and a top-down finite element structural analysis procedure.

CLASSICAL RANDOM LOAD METHOD

The classical equation for considering the forced response of a linear deterministic system subjected to stationary random loading is given by

$$[S_{\sigma\sigma}(\omega)] = [H_{\sigma F}(\omega)]\,[S_{FF}(\omega)]\,[H^*_{\sigma F}(\omega)]^T \qquad (1)$$

where $[S_{\sigma\sigma}]$ is the cross spectral density of the structural response, for example the components of stress at a particular location on the structure, and $[S_{FF}]$ is the cross spectral density of applied random loads as a function of the forcing frequency ω. The symbol * indicates the complex conjugate and $[X]^T$ indicates the transpose of the matrix $[X]$. $[H_{\sigma F}]$ is the deterministic transfer function between loads $\{f\}$ (harmonic with time t and represented by $\{F\}\exp(i\omega t)$) and the harmonic response $\{\sigma\}\exp(i\omega t)$ defined by

$$\{\sigma(\omega)\} = [H_{\sigma F}(\omega)]\,\{F(\omega)\} \qquad (2)$$

where $\{\ \}$ represents a column vector and i represents $\sqrt{-1}$. The elements of $\{\sigma\}$, $\{F\}$ and $[H_{\sigma F}]$ are complex numbers which account for phase differences between the harmonic quantities. Each harmonic quantity, for example the force component f_j, may be specified in terms of an amplitude $\hat{F}_j$ and phase angle α_j. In this case the complex force amplitude F_j used in Equation 2 is given by

$$F_j = \hat{F}_j\cos(\alpha_j) + i\hat{F}_j\sin(\alpha_j). \qquad (3)$$

A procedure has been developed in the VAST finite element code [7, 8] using the modal frequency response method or direct frequency response method to determine the response cross spectral density matrix $[S_{\sigma\sigma}]$ for a given cross spectral matrix of applied nodal forces $[S_{FF}]$. A method has also been developed to calculate this cross spectral density matrix of nodal

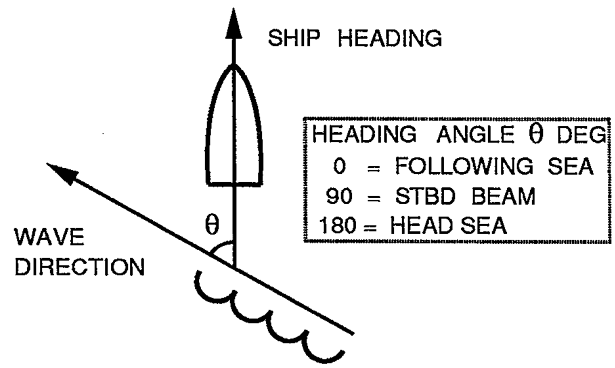


Figure 1: Convention for the the relative heading angle between wave direction and ship heading used in PRECAL

forces $[S_{FF}]$ for a given directional wave spectrum $S_{\eta\eta}$ and ship speed V based on the equation

$$[S_{FF}(\omega,\theta)] = \{H_{F\eta}(\omega,\theta)\}\,S_{\eta\eta}(\omega,\theta)\,\{H^*_{F\eta}(\omega,\theta)\}^T \qquad (4)$$

where the transfer function vector $\{H_{F\eta}\}$ between the nodal forces $\{F\}$ and a regular wave of amplitude η is calculated using PRECAL and algorithms to transfer between PRECAL hydrodynamic mesh hull pressures and the finite element mesh nodal forces. Since a directional wave spectrum is considered, the matrices shown in Equation 4 are considered as function of the wave frequency ω and the relative heading angle θ between the wave direction and ship heading. The relative heading angle can be defined as shown in Figure 1 based on the convention used in PRECAL. The resulting response spectral density $[S_{\sigma\sigma}]$ from Equation 1 also becomes a function of wave frequency and heading angle. For a given ship speed, heading angle and wave frequency, a dynamic analysis can be conducted at the encounter frequency ω_e, the forcing frequency seen by the structure, to determine the structural response.

When applied to a full ship finite element model, the above method is computationally intensive. Also the entire finite element analysis would have to be repeated for each ship speed, heading angle and regular wave frequency combination (potentially 500 to 1000 load cases for linear wave loading and possibly combined with several wave heights if nonlinear sea loads are considered resulting in several thousand load cases). In a top-down modelling scenario it would be desirable to compute and store the global model displacement response spectra $S_{UU}(\omega)$ for each operational cell. Using the classical random response method the cross spectral density of displacement would have to be stored for each of the degrees of freedom (DOF) in the global model for each operational cell. If the global model contained 10000 DOF this would require storage of 10^8 spectral values $S_{UU}(\omega)$ for each operational cell. Clearly the method is likely to be impractical based on the computation time requirements and disk storage requirements.

Since most of the wave spectral energy is usually at frequencies well below the lowest hull vibration mode, the low frequency response can be considered in a quasi-static analysis outlined in the following sections to calculate the transfer function $[H_{\sigma F}]$ between the hull loads and structural response. Since the cross spectral density matrix of loads is derived from a single wave spectrum it is also not necessary to use this transfer function explicitly. Combining Equation 1 and Equation 4 gives

$$[S_{\sigma\sigma}] = [H_{\sigma F}]\{H_{F\eta}\} S_{\eta\eta} \{H^*_{F\eta}\}^T [H^*_{\sigma F}]^T \quad (5)$$

$$= ([H_{\sigma F}]\{H_{F\eta}\}) S_{\eta\eta} ([H_{\sigma F}]\{H_{F\eta}\})^{*T}$$

$$= \{H_{\sigma\eta}\} S_{\eta\eta} \{H^*_{\sigma\eta}\}^T . \quad (6)$$

Thus, if the transfer function $\{H_{\sigma\eta}\}$ from regular wave to the structural response is calculated from

$$\{H_{\sigma\eta}\} = [H_{\sigma F}]\{H_{F\eta}\} \quad (7)$$

then the cross spectral density of the response $[S_{\sigma\sigma}]$ can be obtained using Equation 6 above without calculating the cross spectral density of loads matrix $[S_{FF}]$ explicitly. It is implicitly included by the middle three terms in Equation 5.

GLOBAL MODEL ANALYSIS

In the proposed method a top-down modelling procedure is employed involving a global coarse finite element model of the entire ship and a local detailed finite element model of the area of interest. The global model is used to calculate global displacements which are subsequently used in the local analysis. This section will consider analysis of the global model only. In the previous section the transfer function from wave to structural response $\{H_{\sigma\eta}\}$ was calculated based on Equation 7 which requires the transfer function $[H_{\sigma F}]$ between hull nodal forces (or element pressures) and the structural response. Each column of $[H_{\sigma F}]$ represents the global response calculated from a finite element analysis with a unit force applied to one of the wetted hull nodes (or alternatively with a unit pressure applied to one of the finite elements on the wetted surface of the hull). Since the number of hull elements is smaller than the number of nodes, using pressure loads on each element would result in fewer finite element runs required to determine $[H_{\sigma F}]$ than if nodal forces were used. Since the number of facets in the PRECAL hydrodynamic mesh is likely to be significantly lower than the number of wetted hull structural finite elements, further reduction can be obtained by formulating the transfer function between the regular wave and the structural response as

$$\{H_{U\eta}\} = [H_{UP}]\{H_{P\eta}\} \quad (8)$$

where $[H_{P\eta}]$ is the transfer function between the regular wave and the hydrodynamic mesh facet pressures, which can be computed by PRECAL, and $[H_{UP}]$ is the transfer function between the hydrodynamic mesh facet pressures and the structural response which in this case is taken to be the displacements $\{U\}$ at all nodal DOF in the global model. Each column of $[H_{UP}]$ represents the complex displacement amplitudes calculated from the global finite element model with a unit amplitude harmonic pressure applied over the finite elements of the hull corresponding to one of the hydrodynamic mesh facets. Since the forcing frequencies from wave loading are well below the first natural frequency of vibration of the structure, a quasi-static approach can be employed.

In the quasi static approach the real and imaginary components of the hull pressure loads and acceleration forces due to rigid body motion are applied to the global finite element model and the real and imaginary components of the resulting deformation displacements are calculated from static finite element analyses. PRECAL can be used to provide the amplitudes and phase angles for both the hydrodynamic mesh facet pressures $\{P\}$ and the six components of rigid body acceleration $\{A\}$ (surge, sway, heave, roll, pitch, or yaw) of the ship center of gravity (CG) for a unit amplitude regular wave at a given wave frequency, heading angle and ship speed. Conversion of the amplitudes and phase angles to complex amplitudes yields the transfer function from wave height to hydrodynamic facet pressures and rigid body accelerations $\left\{H_{\frac{A}{P}\eta}\right\}$. For a given wave amplitude η, the complex facet pressures and acceleration of the ship CG are then given by

$$\left\{ \begin{array}{c} \{A\} \\ \{P\} \end{array} \right\} = \left\{H_{\frac{A}{P}\eta}\right\}\eta. \quad (9)$$

The wave to global model nodal displacement transfer function can then be defined as

$$\{H_{U\eta}\} = \left[H_{U\frac{A}{P}}\right]\left\{H_{\frac{A}{P}\eta}\right\} \quad (10)$$

where $\left[H_{U\frac{A}{P}}\right]$ is the transfer function relating the hydrodynamic facet pressures and CG rigid body accelerations to the displacements of the nodes of the

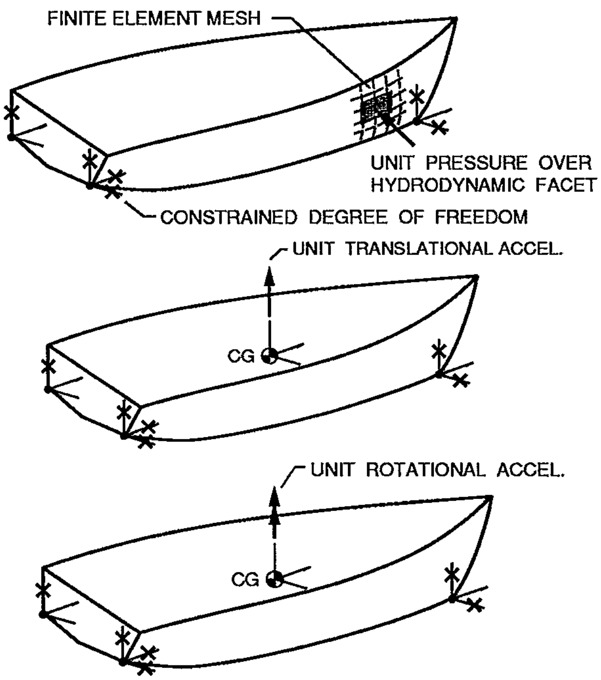


Figure 2: Example unit facet pressure and unit acceleration load cases for quasi-static global analyses

global finite element model. Each column of $\left[H_{U\frac{A}{P}}\right]$ can be determined by running a static finite element analysis with either a unit pressure over one hydrodynamic facet or a unit rigid body acceleration load (a translational acceleration component or a rotational acceleration component about the CG). The number of facets in the PRECAL hydrodynamic mesh is likely to be on the order of 200. This method would require, in the case of 200 hydrodynamic facets, running quasi-static finite element analyses for 206 finite element load cases and storing the nodal displacements for each case. Typical load cases are given in Figure 2 including one hydrodynamic facet unit pressure case, a rigid body translational acceleration in the vertical direction and a rigid body rotational acceleration about the vertical axis. Six finite element nodal displacement DOF have been constrained to eliminate rigid body modes.

The application of translational acceleration loads is relatively straight forward. A rigid body acceleration in the x direction a_x, for example, would result in inertial static loads of $-m_i a_x$ applied at node i, for $i = 1$ to N_S (the number of nodes in the global model) where m_i is the nodal mass. VAST incorporates an option to specify the acceleration component and will automatically generate the translational inertial loads required.

The application of rotational accelerations is not as straight forward. Assume a harmonic rigid body rotation about one of the global axes, for example ϕ_x about the x axis, given by $\phi_x(t) = \Phi_x \cos(\omega_e t)$. Also consider the plane perpendicular to the axis of rotation through node i (at rectangular coordinates x_i,y_i,z_i) and the projection of the line between the CG and node i in this plane. The length of the projected line will be called R_p (see Figure 3). Application of the rotational acceleration about the CG results in a rotational acceleration at node i of the same value and a translational acceleration in the plane perpendicular to the axis of rotation which can be considered in terms of a radial acceleration component a_r along the projection line and a tangential component a_t perpendicular to the projection line. If the amplitude Φ_x of the rotation is small compared to one radian then

$$\begin{aligned} a_t(t) &= -R_p \Phi_x \omega_e^2 \cos(\omega_e t) = R_p \ddot{\phi}_x \\ &= R_p A_x \cos(\omega_e t) \qquad (11) \\ a_r(t) &= R_p \Phi_x^2 \omega_e^2 \cos(2\omega_e t) \\ &= -R_p \Phi_x A_x \cos(2\omega_e t) \qquad (12) \end{aligned}$$

where $\ddot{\phi}_x$ is the angular acceleration about the x axis through the CG with amplitude A_x. The radial centrifugal component is a non-linear component proportional to the square of the rotational amplitude and occurs at twice the frequency of the CG angular rotation. Comparing equations 11 and 12 shows that the ratio of radial acceleration amplitude to the tangential acceleration amplitude A_r/A_t is equal to the amplitude of the angle of rotation Φ_x in radians. Given a rotational amplitude of 5 degrees the radial acceleration component amplitude is 9 percent of the tangential component. The tangential component is easily incorporated into the quasi-static approach since VAST includes an option to supply a constant rotational acceleration about a specified point and calculate the resulting structural deformation and stresses. The non-linear radial component resulting from the harmonic rotational accelerations about the ship CG supplied by PRECAL will not be included in the method, although it is acknowledged that it could be significant compared to the tangential component in severe seas, especially in conditions giving large roll angles.

As previously indicated, to run a quasi-static finite element analysis, the model must be constrained from rigid body motion which requires constraint of six displacement DOF. If the hull pressure and inertial acceleration loads balance, as ideally they should, then the reaction forces at the constrained DOF should be zero. Differences between the discretization of mass

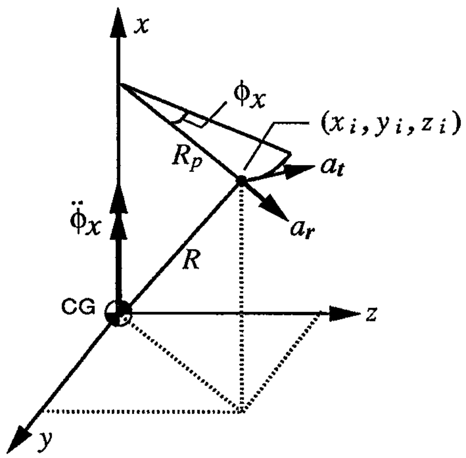


Figure 3: Translational accelerations a_t and a_r at point (x_i, y_i, z_i) due to a rigid body rotational acceleration about the x axis through the ship CG

and how loads are applied in the structural finite element model compared to the hydrodynamic model, may lead to some unbalance of quasi-static forces. The balance of forces can be checked or adjusted with the following method. During running of the VAST static analyses with unit load cases, the six reaction forces at constrained DOF can be used to form the transfer functions $\left[H_{R_P^A}\right]$ relating rigid body accelerations and facet pressures to the six reaction forces at the constrained DOF. These can be combined with any PRECAL wave pressure and acceleration load case to give the wave to reaction transfer function:

$$\{H_{R\eta}\} = \left[H_{R_P^A}\right]\left\{H_{{}_P^A\eta}\right\}. \qquad (13)$$

Thus for any PRECAL regular wave load case, the reaction forces $\{H_{R\eta}\}$ can be checked to see if they are close to zero.

It is possible to calculate the accelerations required to achieve zero reactions for a given set of PRECAL facet pressures. This can be done by partitioning $\left[H_{R_P^A}\right]$ between reactions due to unit acceleration loads and reactions due to unit facet pressure loads as

$$\left[H_{R_P^A}\right] = \left[\begin{array}{cc} H_{RA} & H_{RP} \end{array}\right] \qquad (14)$$

and partitioning the facet pressures and CG accelerations due to a unit amplitude wave as

$$\left\{H_{{}_P^A\eta}\right\} = \left\{\begin{array}{c} H_{A\eta} \\ H_{P\eta} \end{array}\right\}. \qquad (15)$$

Setting the reactions to zero (the LHS of Equation 13) then gives the following equation for the required rigid body accelerations,

$$\{H_{A\eta}\} = -\left[H_{RA}\right]^{-1}\left[H_{RP}\right]\{H_{P\eta}\}. \qquad (16)$$

This can be checked against the original PRECAL rigid body accelerations for each load case. In theory the location of the six constrained DOF is not critical as long as only rigid body modes are constrained and there is no constraint of deformation of the structure. Some unbalance could likely be tolerated as long as the resulting reaction forces are not too large and are not close to areas of the model where the local analysis is to be undertaken. Some calculations will be carried out to determine the degree of unbalance likely to occur based on typical hydrodynamic and structural models and to quantify the effect of the unbalance on structural response predictions.

In summary, prior to conducting any local response analysis, separate global static finite element analyses would be conducted with either a unit pressure load on each of the hydrodynamic mesh facets or a unit rigid body acceleration load. The resulting displacements at all global model DOF would form, for each load case, a column of the transfer matrix $\left[H_{U_P^A}\right]$ (real elements) relating applied facet pressures $\{P\}$ and rigid body acceleration components $\{A\}$ to the global displacements $\{U\}$. The transfer matrix $\left[H_{U_P^A}\right]$ would be calculated once and stored. At the same time transfer functions $\left[H_{R_P^A}\right]$ relating the applied facet pressures and rigid body acceleration components to the six reaction forces providing rigid body motion constraints would be calculated and stored.

Also prior to conducting the local response analysis PRECAL would be run for each regular wave load case (a combination of wave frequency ω_i, heading angle θ_j and ship speed V_k), likely to occur in the cells of wave spectra, ship heading and speed forming the ship operational profile, to determine the the transfer functions $\left\{H_{{}_P^A\eta}\right\}_{ijk}$ relating the hydrodynamic pressures and ship CG rigid body accelerations to unit amplitude regular waves. $\left\{H_{{}_P^A\eta}\right\}_{ijk}$ would be stored for each ω_i, θ_j, V_k combination. Once the PRECAL and global finite element runs are completed and the information stored then any number of local analyses could be conducted at any location in the ship without having to repeat the global finite element analyses or PRECAL analyses.

LOCAL FINITE ELEMENT ANALYSIS

A local finite element analysis will be used to determine the stress or strain spectra at a location (or locations) for a given cell of wave spectrum, ship speed and heading which can be used in a fatigue crack initiation calculation or to determine the stress intensity factor spectra to be used in the calculation of an increment of crack growth. When the crack increment reaches a length which is likely to cause a significant change in the stress intensity factor, then the local model will have to be re-meshed and the local finite element analysis repeated. The following discussion will consider the local response in terms of the stress components $\{\sigma\}$ but will also apply to strain $\{\epsilon\}$ or stress intensity factor components $\{K\}$. The nodes on the boundary of the local model common to the global model will be considered as master nodes where global displacement boundary constraints will be applied to the local model. All other nodes of the local model on the common boundary but which are not in the global model will be treated as 'slave' nodes and constraint equations used to define the slave node displacements in terms of master node displacements.

The overall approach involves determining transfer functions $\{H_{\sigma\eta}\}_{ijk}$ between wave amplitude and the local stress components for all combinations of wave frequency ω_i, heading angle θ_j and ship speed V_k that are in the cells forming the ship operational profile. The directional wave spectral density $S_{\eta\eta}(\omega,\theta)$ will be defined discretely as $S^{ij}_{\eta\eta}$ over a number of wave frequencies ω_i and headings θ_j for a cell of the ship operation at speed V_k. The corresponding cross spectral density of local response $[S_{\sigma\sigma}]_{ij}$ is then given by

$$[S_{\sigma\sigma}]_{ij} = \{H_{\sigma\eta}\}_{ijk} \, S^{ij}_{\eta\eta} \, \{H^*_{\sigma\eta}\}^T_{ijk} \, . \qquad (17)$$

There are several approaches that can be taken to compute the transfer function component $\{H_{\sigma\eta}\}_{ijk}$. These include a direct approach involving a local finite element analysis for each regular wave load case and two methods involving either application of unit displacements to the local model boundary master nodes or application of unit pressures to hydrodynamic facets. The efficiency of the three methods will depend mainly on the number of local finite element static load cases that must be employed in each method in the process of obtaining $\{H_{\sigma\eta}\}_{ijk}$ for all the PRECAL load cases (combinations of discrete wave frequency ω_i, heading angle θ_j and ship speed V_k. The number of PRECAL load cases n_l is given by

$$n_l = n_\omega \times n_\theta \times n_V \, , \qquad (18)$$

the product of the number of discrete wave frequencies, heading angles and ship speeds needed to represent the operational profile. The direct method requires $2n_l$ load cases since separate analyses are required for the real and imaginary components. The local unit load method requires $n_u + n_p + 6$ load cases where n_u is the number of local model boundary master node DOF and n_p is the number of hydrodynamic facets intersecting the local model (likely 0 to 4). The unit facet pressure method requires $n_P + 6$ load cases where n_P is the number of the facets used in the PRECAL hydrodynamic mesh.

If the ship operational profile is very simple, defined for example, in terms of a single operational cell for long crested waves (one heading angle, one ship speed and perhaps twenty discrete wave frequencies), then the direct method would require 40 load cases. A realistic short term or long term operation could involve perhaps 8 headings, 20 frequencies and 5 ship speeds resulting in 1600 load cases. Even if operational cells are defined in terms of 5 headings (head, bow, beam, quartering, following seas) dividing these headings between port and starboard would result in 8 heading angles.

Based on a typical hydrodynamic mesh containing perhaps 200 facets, the unit facet pressure method would require 206 local load cases (200 unit pressure load cases plus 6 unit rigid body acceleration load cases). If the global finite element model is relatively coarse then the local model could contain a relatively small number of boundary master nodes, perhaps 40, giving 240 DOF which would result in about 250 load cases if the local unit load method is employed. This would indicate that the unit facet pressure method would likely require the fewest local finite element analyses. Since the unit facet pressure method is an extension of the global analysis method and will use similar algorithms for applying facet pressures it is likely to require less effort to implement. In the detailed computer implementation of either method further advantages or disadvantages may be discovered and in the end it will likely be desirable to implement both the local unit load and unit facet pressure methods. Only the unit facet pressure method will be presented in detail in this paper.

In the global model analysis the transfer functions $\left[H_{U^A_P}\right]$ are to be stored. Each column of this matrix represents the global model nodal displacements for either a unit acceleration component applied at the ship CG or a unit pressure applied over a hydrodynamic mesh facet. Extraction of the rows of $\left[H_{U^A_P}\right]$ corresponding to the local model boundary master node displacements $\{u\}$ gives the matrix $\left[H_{u^A_P}\right]$. The transfer function $\left[H_{\sigma^A_P}\right]$ relating the facet pressures and CG accelerations to the local response can be obtained by applying in turn each column of $\left[H_{u^A_P}\right]$ as boundary displacements on the local model in conjunction with the unit CG acceleration if the column is associated with a global unit acceleration load or a unit facet pressure if the column is associated with a facet pressure and that facet is completely or partially contained in the local model. The local model load cases are illustrated in Figure 4 and a flow chart of the method is shown in Figure 5. The resultant structural response for each load case forms a column of the transfer function $\left[H_{\sigma \frac{A}{P}}\right]$ which can be used inconjunction with transfer function $\left\{H_{\frac{A}{P}\eta}\right\}_{ijk}$ obtained with PRECAL to calculate the transfer function $\{H_{\sigma\eta}\}_{ijk}$ for a given PRECAL load case using the equation

$$\{H_{\sigma\eta}\}_{ijk} = \left[H_{\sigma \frac{A}{P}}\right] \left\{H_{\frac{A}{P}\eta}\right\}_{ijk}. \tag{19}$$

ENCOUNTER FREQUENCY SPECTRA

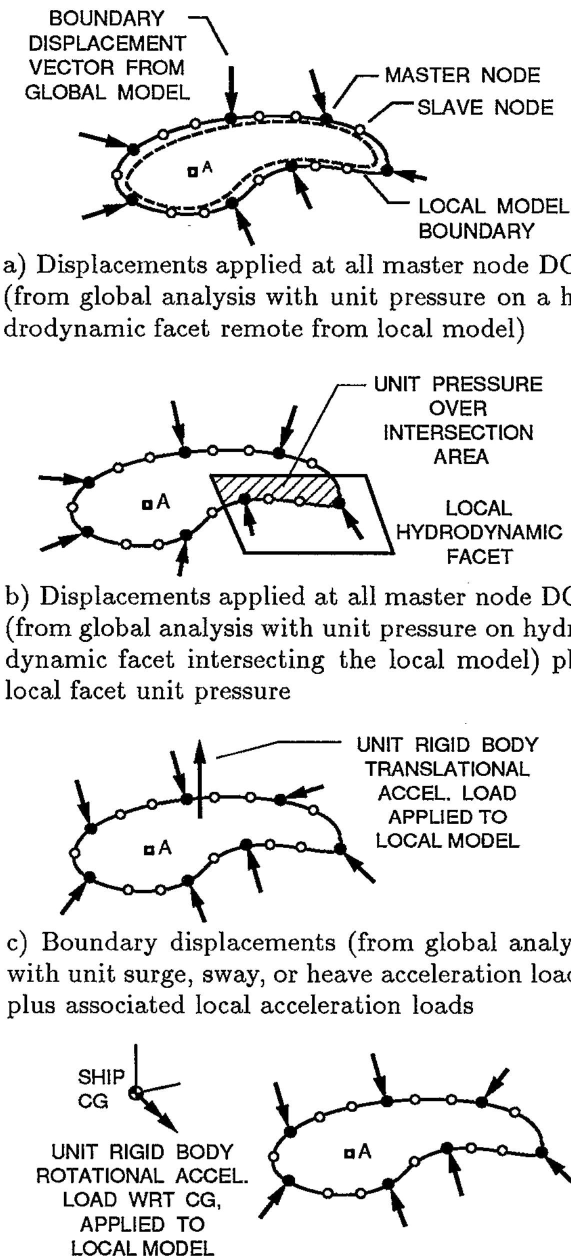
For use in fatigue and crack growth analysis it is desirable to consider the local response in terms of encounter frequency ω_e (the actual frequency seen by the structure) based on the response spectra $[S_{\sigma\sigma}(\omega_e)]$ for each operational cell. The encounter frequency is given by

$$\omega_e = |\omega - \omega^2 V \cos(\theta)/g| \tag{20}$$

where g is the acceleration of gravity and V is the ship speed.

For an operational cell with ship speed V_k, and wave directional spectral density $S_{\eta\eta}(\omega, \theta)$, defined over discrete frequencies ω_i, $i = 1, n_\omega$ and heading angles θ_j, $j = 1, n_\theta$ as $S_{\eta\eta}^{ij}$, Equation 17 can be used to calculate the local response $[S_{\sigma\sigma}(\omega, \theta)]$ again defined discretely as $[S_{\sigma\sigma}]_{ij}$. $[S_{\sigma\sigma}(\omega_e)]$ can be obtained by integrating the response spectral energy density $S_{\sigma\sigma}(\omega, \theta)$ between lines of constant encounter frequency with spacing δ_{ω_e}. Figure 6 shows plots of a hypothetical spectrum in terms of contours of constant $S_{\sigma\sigma}$ overlaid with lines of constant encounter frequency 0.01 Hz apart centered at 0.05, 0.10, and 0.20 Hz for a ship speed of 15 knots. Integration of the spectral energy density in the shaded area between each set of lines would give the spectral energy in a bandwidth of 0.01 Hz about the specified center frequencies. The figure shows that wave energy over a broad range of wave frequencies can contribute to the response energy in a narrow band of encounter frequency.

The energy spectra $[S_{\sigma\sigma}(\omega_e)]$ can be obtained numerically by simply calculating terms $[S_{\sigma\sigma}]_{ij}\, \delta_\omega \delta_\theta$ where δ_ω and δ_θ are the wave frequency and heading angle point spacing. This response energy is added to the appropriate frequency bin based on the encounter frequency $\omega_e^{ij}(\omega_i, \theta_j)$. For experimental wave



a) Displacements applied at all master node DOF (from global analysis with unit pressure on a hydrodynamic facet remote from local model)

b) Displacements applied at all master node DOF (from global analysis with unit pressure on hydrodynamic facet intersecting the local model) plus local facet unit pressure

c) Boundary displacements (from global analysis with unit surge, sway, or heave acceleration loads) plus associated local acceleration loads

d) Boundary displacements (from global analysis with unit roll, pitch, and yaw acceleration loads) plus local acceleration loads due to associated accelerations about ship CG

Figure 4: Example load cases for quasi-static local analyses with the Unit Facet Pressure Method to determine structural response at internal node A

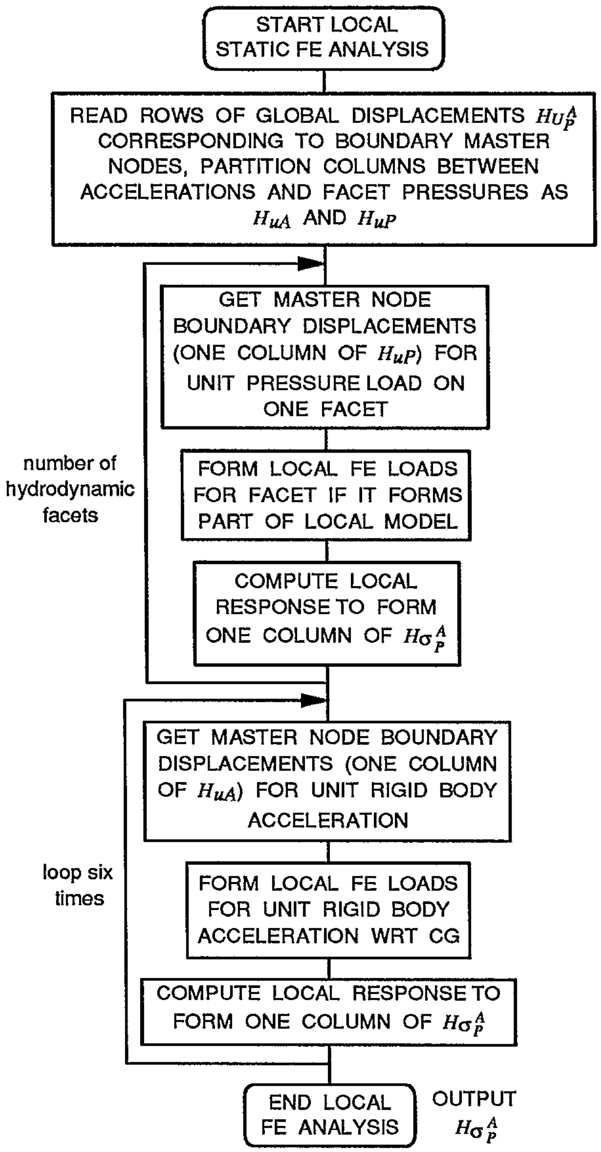


Figure 5: Flow chart for the Unit Facet Pressure Method of calculating the transfer function relating the hydrodynamic facet pressures and rigid body accelerations to the local response

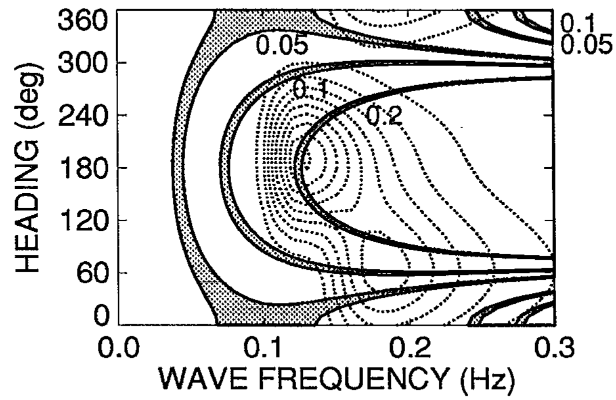


Figure 6: Plot of directional spectra $S_{\sigma\sigma}(\omega, \theta)$ overlaid with 0.01 Hz wide bands of encounter frequency at 0.05 Hz, 0.1 Hz, and 0.2 Hz; $\cdots$ contours of response spectral density, — bounds of shaded frequency bands

directional spectra this approach tended to produce 'jagged' spectra [5]. Smooth spectra were obtained if bilinear interpolation of the $[S_{\sigma\sigma}]_{ij}$ points was used to make δ_ω and δ_θ smaller. Reducing the angular and frequency spacing to $\delta_\omega/4$ and $\delta_\theta/4$ and maintaining bins of encounter frequency of width δ_{ω_e} equal to the original wave frequency spacing δ_ω produced smooth spectra. It is possible to employ higher order integration schemes but it was found that this did not work well for measured directional spectra which tended to have sharp spikes with high energy at some points and low energy at surrounding points.

If the cell wave spectra are unidirectional then obtaining $[S_{\sigma\sigma}(\omega_e)]$ is simpler. This may be the case in ISSMM where a cell is planned to be defined in terms of the Bretschneider two-parameter model for long crested waves with spectral density $S_{\eta\eta}(\omega)$ defined by the modal period and significant wave height. In this case, for a given ship speed V_k, heading angle θ_j, and wave spectral density $S_{\eta\eta}^i$ defined at discrete wave frequencies ω_i, $i = 1, n_\omega$, the response spectral density is given by

$$[S_{\sigma\sigma}]_i = \{H_{\sigma\eta}\}_{ijk} \, S_{\eta\eta}^i \, \{H_{\sigma\eta}^*\}_{ijk}^T, i = 1, n_\omega \qquad (21)$$

While it is simpler to convert $[S_{\sigma\sigma}(\omega)]$ to $[S_{\sigma\sigma}(\omega_e)]$ than converting $[S_{\sigma\sigma}(\omega, \theta)]$ to $[S_{\sigma\sigma}(\omega_e)]$ it is not trivial since at some headings three values of wave frequency can contribute energy to the same encounter frequency. A numerical method similar to that proposed for treating $[S_{\sigma\sigma}(\omega, \theta)]$, but reduced to one dimension, could be employed. An overall flow chart summarizing the global and local analyses is shown in Figure 7 for the case where the operational cell is

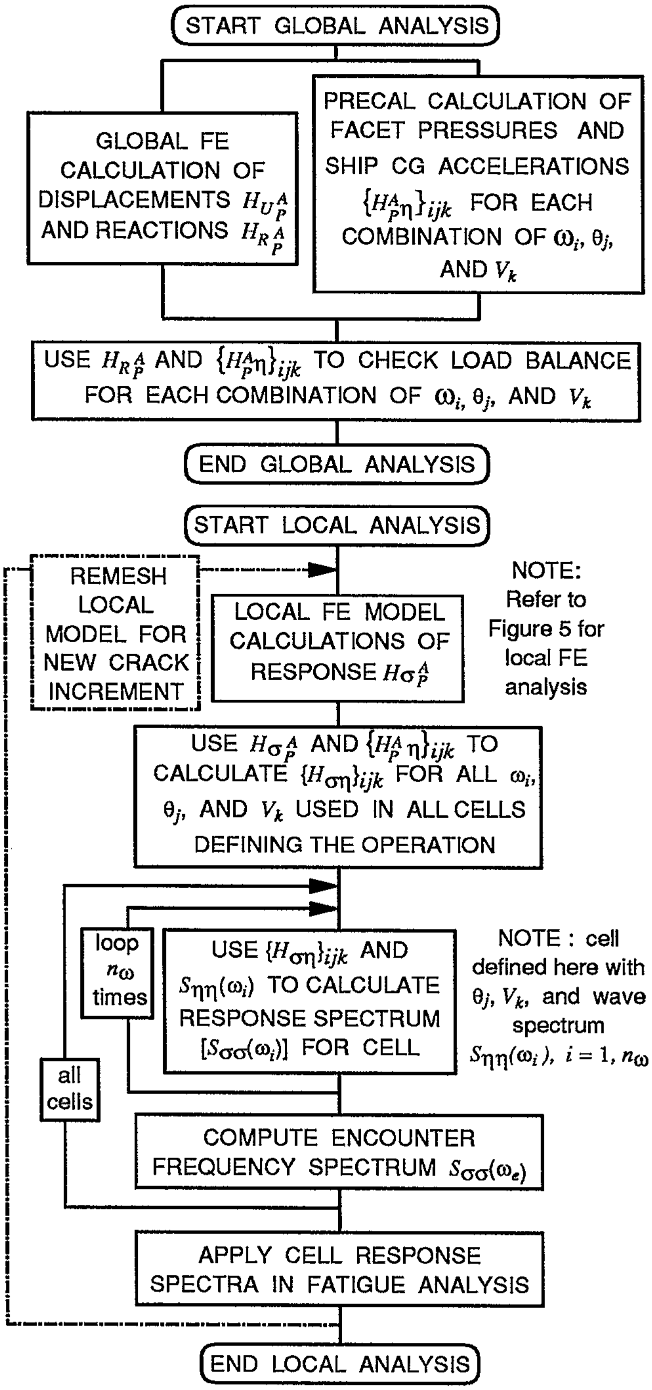


Figure 7: Overall flow chart for global and local analyses for the case where an operational cell is defined for a single heading angle θ, ship speed V, and a unidirectional wave spectrum $S_{\eta\eta}(\omega)$

defined for long-crested seas. A loop is shown for remeshing the local model, after an increment of crack growth for the case of a crack propagation analysis, although in this case as previously mentioned, the local response spectrum would be based on stress intensity factor K and not the stress σ shown in the figure.

SUMMARY AND CONCLUSIONS

This paper proposes a top-down finite element procedure for the prediction of structural response frequency spectra from wave spectra and regular wave pressure loads predicted with a 3D linear hydrodynamics code. These spectra can be used for assessment of fatigue strength and possibly ultimate strength based on realistic ship operational profiles.

The method employs a quasi-static approach based on finite element computations for unit pressures applied to hydrodynamic mesh facets. Transfer functions between regular waves and the structural response are employed, eliminating the need to calculate huge cross spectral density matrices of hull pressure loads as used in classical random response methods. Separate global static finite element analyses are conducted with a unit pressure on each facet of the hydrodynamic mesh and for six rigid body acceleration load cases. The resultant global model displacements can be stored and then used for any subsequent local model analyses with any number of operational cells, employing different wave spectra and ship speeds and headings, without having to repeat the global finite element analysis.

The quasi-static approach requires constraint of the global model against rigid body motion. The method also leads to a relatively efficient procedure for checking reaction forces and correcting inertial loads to achieve a static load balance for any of the PRECAL regular wave load cases employed in the analyses.

Of three methods considered for the local detailed model analyses, the unit facet pressure method is likely to be the most efficient and easiest to implement.

It is anticipated that the proposed approach should make spectral methods, based on hull pressure wave loads, sufficiently efficient to be practically applied within ISSMM. The approach is being implemented in the VAST suite of codes and tested for application to fatigue analysis as part of the first phase of ISSMM. If the method is found to give realistic predictions with an acceptable computational effort, it will be refined and integrated into the ISSMM software. An investigation of the use of the method to handle nonlinear sea loads will also be undertaken.

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