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PROBABILITY OF DETECTION OF A MARITIME TARGET (U)

by

DR B.U. NGUYEN

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OTTAWA, CANADA

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Recommended by: ______________________ Approved by: ______________________
MORT TEAM LEADER DOR(CAM)

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ABSTRACT

This research note documents a formalism to compute the probability of detection of a target when the sensor range of the searcher is taken into account. A perfect sensor is assumed; that is, if the target comes within the sensor range of the searcher then it will be detected. However, more detailed detection models could be incorporated. Several types of targets have been investigated. It is shown that the searcher can expect higher probability of detection than Koopman's law of random search. This work is meant for the ORA community and will be useful for the evaluation of sensor systems such as SPOTSAR or IPS.

RÉSUMÉ

Ce document décrit un formalisme pour calculer la probabilité de détection d'une cible quand la portée du senseur est prise en considération. Nous considérons un senseur parfait c'est-à-dire que si la cible se trouve à l'intérieur de la portée du senseur alors elle est détectée. Néanmoins, des modèles plus détaillés de détection peuvent être incorporés. Plusieurs types de cibles ont été étudiés. Il est démontré, en général, que notre probabilité de détection est plus haute que celle de Koopman. Ce travail vise principalement la communauté d'ORA et pourrait être utile lors d'évaluation de nouveaux senseurs tels que SPOTSAR ou IPS.

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I. INTRODUCTION

1. Search theory has been developed since World War II. Koopman, Reference [1], recognised that the problem of search theory needed organised mathematical treatment. He provided models of detection in support of antisubmarine warfare operations in World War II. Koopman's law of random search is widely known and has often been applied and studied in defence studies, Reference [2]. His law of random search assumes only visual identification and therefore does not include the effect of a sensor coverage - nonzero area. To take into account the latter, this paper presents a formalism, developed by the author, to compute the probability of detection based on an integral-differential equation.

2. The detection probability was computed by Koopman for visual detection - a lateral detection range with zero area coverage. In such case, the probability of detection is equal to $1 - e^{-1}$(63 percent) if the platform sweeps through the area once. In this paper, we will show that the actual detection probability is greater than the one of Koopman when the non-zero sensor coverage is taken into account.

3. A realistic detection probability based on sensor coverage is helpful in many ways. For example, it is useful when planning search and rescue missions or for surveillance, Reference [3]. It is also useful when evaluating new radar systems such as SPOTSAR or IPS, Reference [4]. This piece of work is an outgrowth of the SPOTSAR project, number 013/96, and is intended mainly for the ORA community.

4. As our theory has its origin in Koopman's model, we will briefly review Koopman's law of random search.

Law of Random Search

5. The target position is assumed to be in the area of interest XY in which the searcher is moving. The probability, p, of visual detection can be derived on the basis of the following three assumptions:

a. The target position is uniformly distributed in XY;

b. the observer path is random in XY, that is, its different portions are independent of one another in XY and

c. on any portion of the path that is small relative to the total length of path but larger than the range of detection, the searcher always detects the target within the lateral range W/2 on either side of the path.

6. Dividing the searcher path L into n equal portions, the probability of detection for each portion is equal to WL/nXY. Under Koopman's assumption, the probability of detection can be written as:

$$\begin{aligned} P &= \lim_{n \to \infty} \left(1 - \left(1 - \frac{WL}{nXY}\right)^n\right) \\ &= 1 - \exp(-WL/XY) \end{aligned} \qquad (1)$$

7. The above formula implies that if the searcher sweeps through the equivalent of the total area XY once, the probability of detection is equal to 63 percent. We can see that this result can be applied incorrectly. For example, consider a scenario where the lateral visual range is equal to the vertical size of the search area, Figure 1. The shaded area is the already-swept area and the white area is the not-yet-swept area. If the searcher sweeps through the area horizontally once, then, under the assumption that the target remains inside the area, the probability of detection must be 100 percent. This is because there is no possible way that the target can move from the not-yet-swept area to already-swept area without being detected. This probability of detection is different from Koopman's since the searcher's path is not random. Koopman's random search would be better labelled as random encounter because his model is not applicable to any situation involving a systematic search. It is important to note that Koopman's result is only valid as long as the assumptions hold. In particular, if the area swept out is not small with respect to the total area, then Koopman's result may be inaccurate.

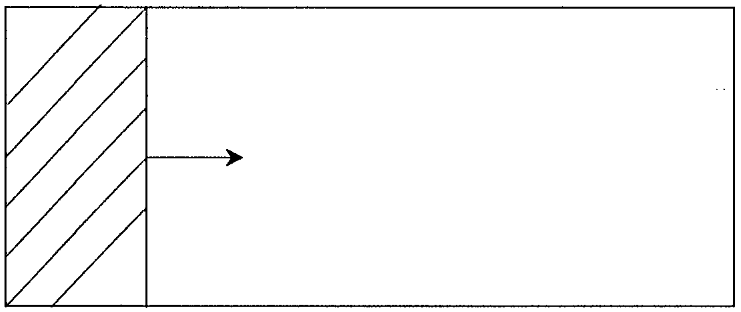

Figure 1 - Search when visual range is equal to the vertical size of the search area

II. METHODOLOGY

8. In a maritime context, the target consists of a vessel sailing at constant speed within a large area. We are interested in calculating the probability that the vessel comes within a distance R of a specific point whose position within the area may be modelled as uniform in two dimensions. *This probability is called the detection probability.* The target's path is not specified, but is assumed to wander inside the area without any specific pattern.

9. This note describes a formalism of how to obtain the detection probability when a platform searches for a target in a given area. We will consider the creeping line of advance tactic as our search pattern.

10. Figure 2 displays the route taken by the searcher to look for its target. In absence of prior information, we assume that the initial position of the target is randomly distributed within the search area. Denote P(t) the probability of detection at time t. As the searcher continues to look for the target, this probability of detection, P(t), approaches 100 percent as the time t goes to infinity (if the target remains inside the search area at all times).

$$\lim_{t\to\infty} P(t) = 1 \tag{2}$$

11. We assume that the target is detected if it is within the sensor range of the searcher. However, a target can be undetected if it goes to an area previously swept by the searcher.

General Model

12. The probability of detection is cumulative and is dependent on the target movement. Let A be the area covered by the sensor range at time t. A time evolution integral equation can be written for the detection probability at time t+dt:

$$P(t+dt) = P(t) + P_u(t) \int_{strip} \left(\int_{XY\setminus A} G(x,y;x',y';t)\; dx\,dy \right) dx'dy' \tag{3}$$

13. The first integral evaluates the probability of the target being inside the strip - the new area covered by the searcher sensor range as it moves. The second integral evaluates the probability of the target moving from all possible positions (except A) to the new area, Figure 3 where W (Koopman's notation) is equal to 2R (our notation). P_u is the probability that the target has not been detected and still remains in the search area. For example, if the target never exits the search area, P_u is equal to $(1-P(t))$.

14. The function G describes the movement of the target from (x,y) to (x',y') at time t. It also includes the type of density of the target position.

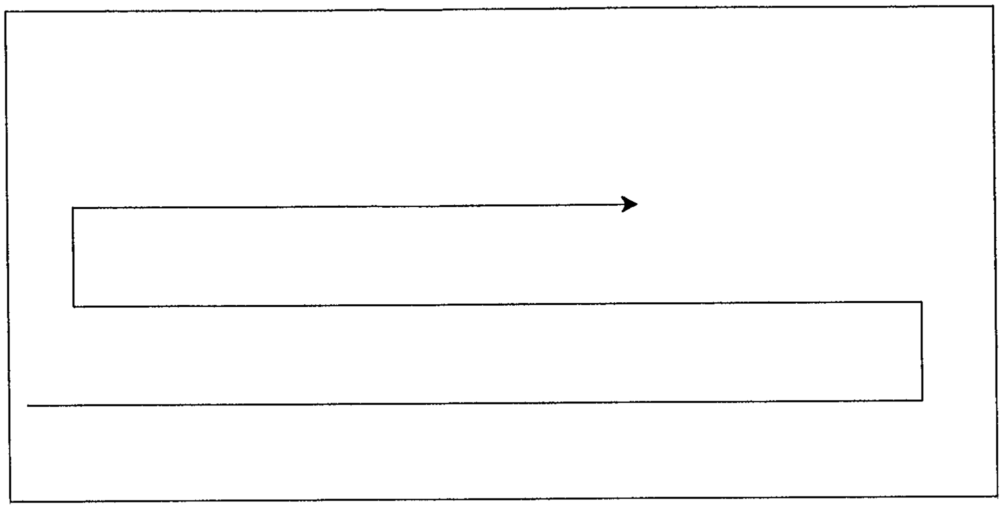

Figure 2 - Creeping line of advance

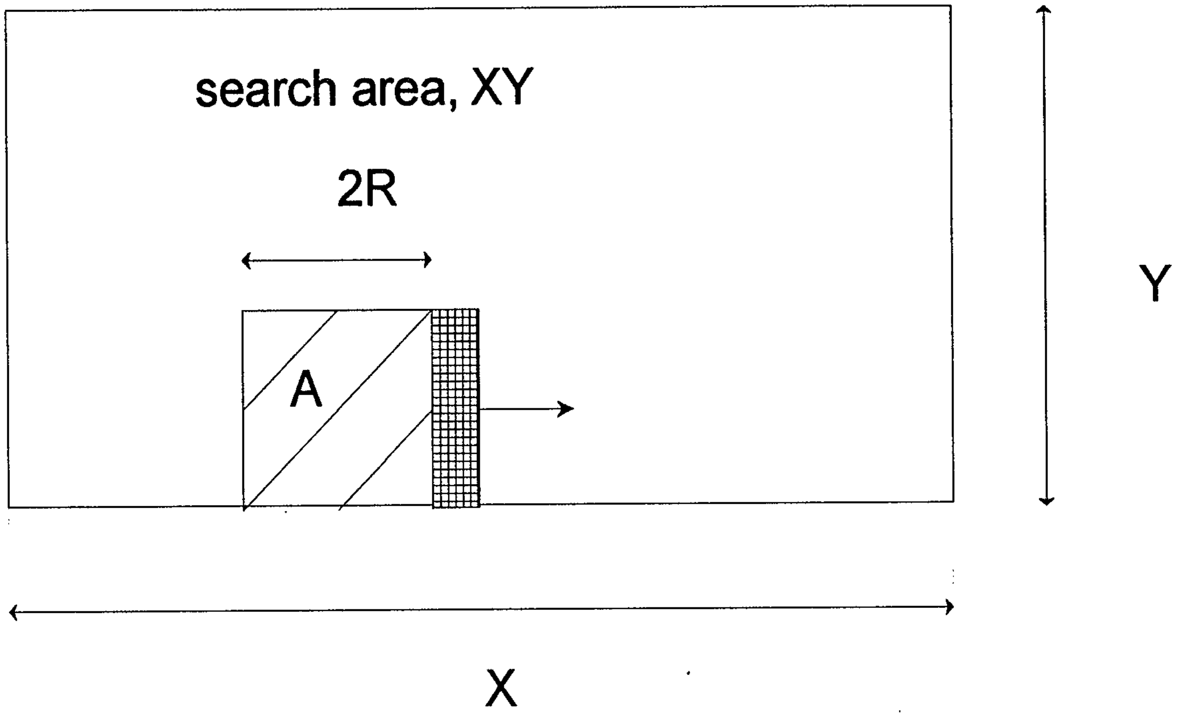

Figure 3 - Sensor coverage as a function of time

III. RESULTS

15. We apply the general model, in Annex A, to four different types of target: stationary, transiting, Koopman-type and slow-moving target. The following deterministic results were obtained.

16. The probability of detection of a stationary target rises linearly with time. Independently of the sensor range, the probability of detection of a stationary target is 100 percent after one complete sweep.

17. The probability of detection of a transiting target also rises with time. However, it is modified by a quadratic correction describing the fact that the target can move out of the search area.

18. The Koopman result was re-derived using this model. We have two new results deriving from different rates of detection: one based on a random target (RT), and the other based on a slow moving target (SMT). RT is used for a target whose position is random within the search area at any time. SMT is used for a target whose speed is much less than the searcher's. Figures 4 and 5 display the cumulative probability of detection as a function of time for two different sensor ranges, 25 kilometers (km) and 50 km respectively. These sensor ranges are equivalent to radar ranges at altitude roughly equal to 30 meters (m) and 200 m. The assumed search area is a square of 200 km on each side. The searcher (such as a maritime patrol aircraft, MPA) speed is set equal to 100 m/s or roughly 200 knots (effective speed parameter is equal to 90 m/s). We can see on both Figures 4 and 5 that SMT and RT provide higher probability of detection than Koopman's result.

19. Figure 6 presents the probability of detection after one complete sweep of the search area (200 km by 200 km) as a function of sensor range. SMT produces the highest probability of detection, then RT, then Koopman's result. We can see that SMT and RT probabilities of detection increase rapidly as a function of the sensor range while Koopman's probability of detection is a constant independent of the sensor range.

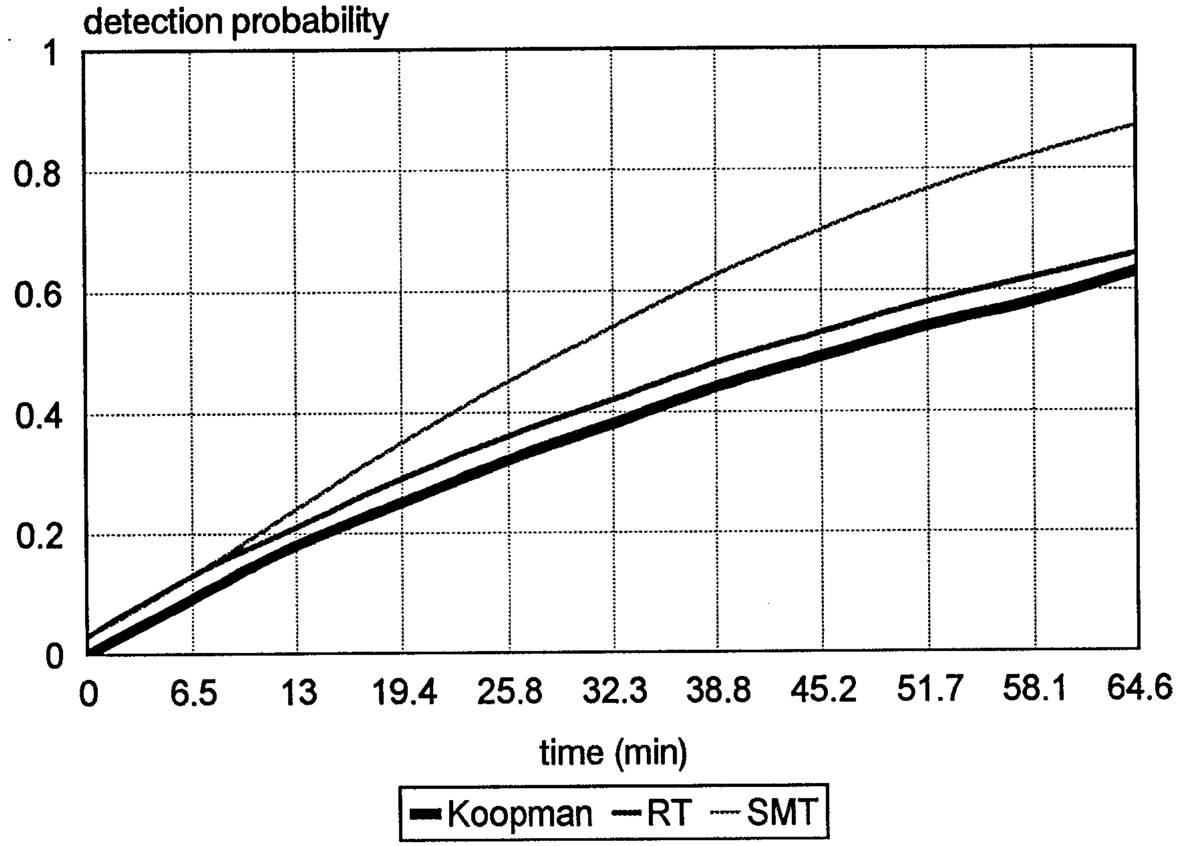

Figure 4 - Probability of detection as a function of time
(Sensor range, 25km. Area size, 40.000 km sq)

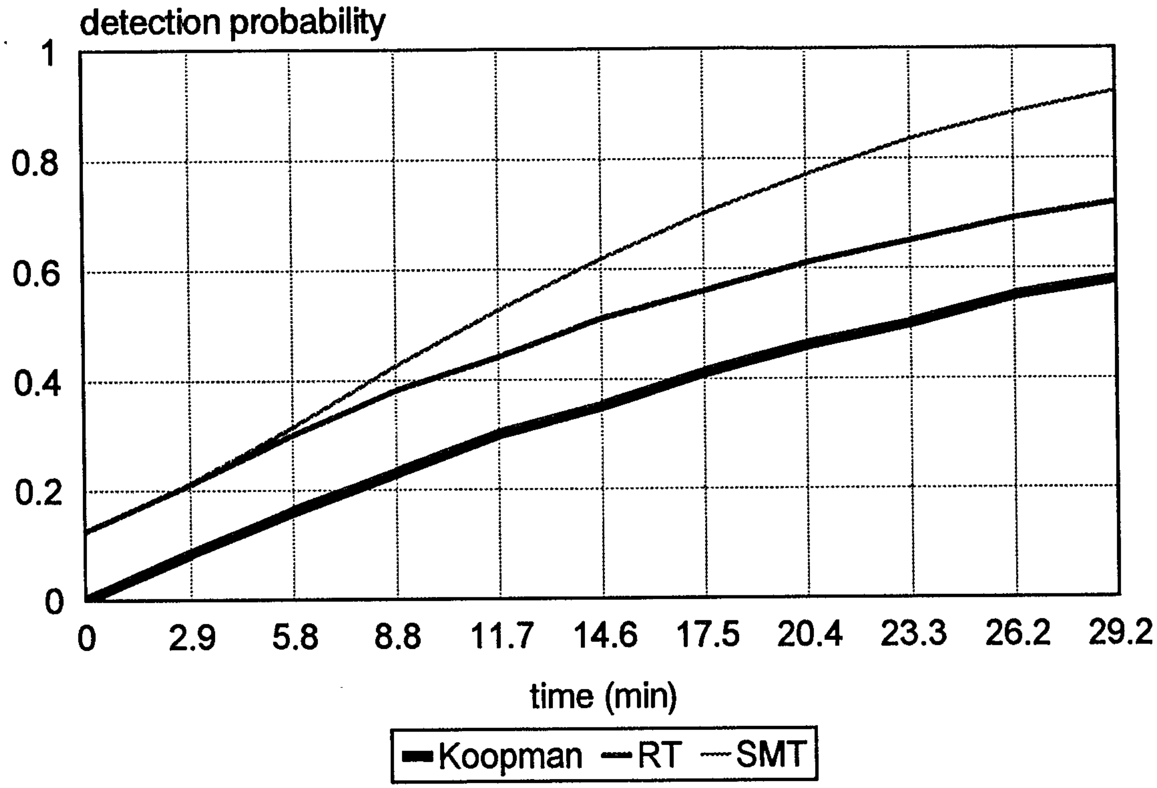

Figure 5 - Probability of detection as a function of time
(Sensor range, 50 km. Area size 40.000 km sq)

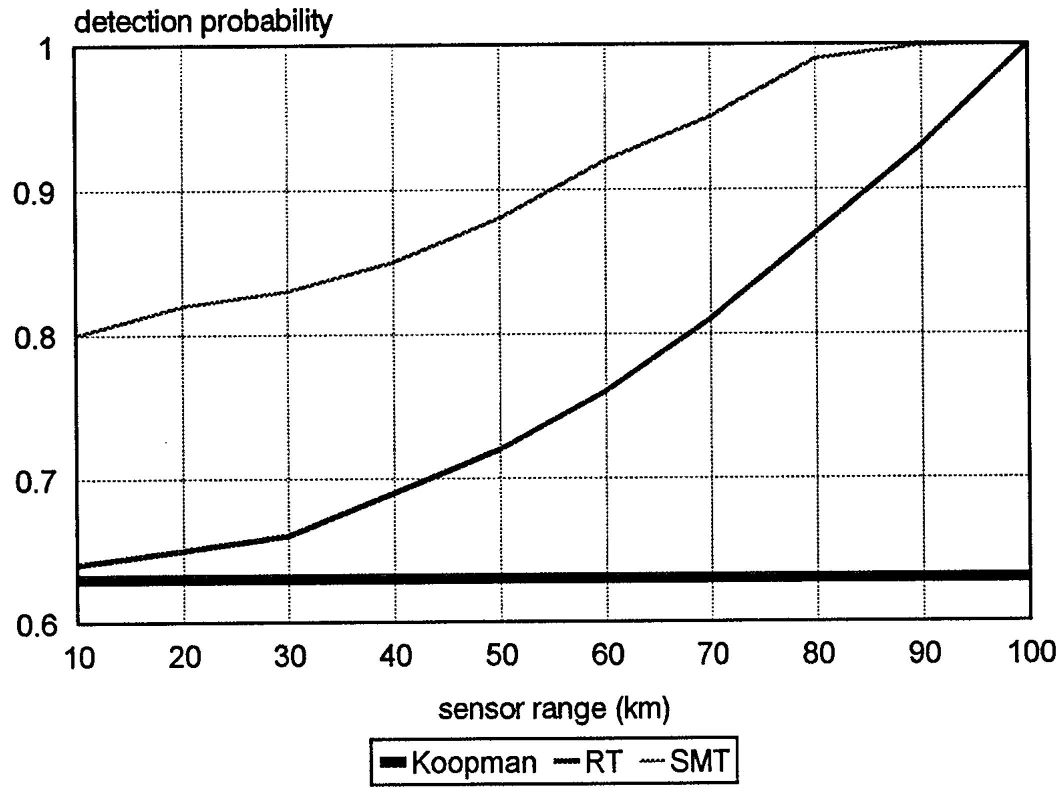

Figure 6 - Probability of detection as a function of sensor range

IV. DISCUSSION

20. The formalism presented is general, and has been developed by the author. This approach can be used for all types of targets such as transiting targets or randomly moving targets. Although, it has not been done in this study, a Brownian motion target such as a fishing boat can be studied. Each different type of target would produce a different rate of detection as a function of the target speed, the searcher speed, the sensor range and the size of the search area.

21. This formalism also includes a probability that the target leaves the search area. Given the type of movement of the target and the initial distribution of the target, such probability could be computed. In practice, the type of movement of a target is not only a function of the target itself but is also a function of the sea state. For example, a lost object's motion at sea is completely dependent on the sea.

22. When information about the currents and wave action, and the target type are given, the formalism could produce the probability of detection as a function of time either numerically or analytically.

23. A more detailed probability model as a function of distance could be considered. We have only considered a model where, if the target is within the sensor range then it will be detected. In reality, a target is more easily detected if it is closer to the searcher, Reference [5]. This could be incorporated into the general model if required.

V. CONCLUSIONS

24. We have shown that the searcher can in general expect a higher probability of detection than the typical Koopman's law of random search. Our model assumption is more realistic and can provide a more accurate probability of detection whether for planning search and rescue missions or for the evaluation of sensor systems.

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APPLICATIONS OF THE GENERAL MODEL

Stationary Target

1. Assume that the search area is a rectangle of width exactly equal to twice the sensor range. The searcher enters the search area from the left and travels to the right end (Figure 1 of main text). Since the target is stationary, G is independent of time and can be expressed by Dirac delta functions:

$$P_u(t)\,G(x,y;x',y';t)=\frac{1}{XY}\,\delta\,(x-x')\,\delta\,(y-y') \qquad \textbf{(4)}$$

2. Evaluating the detection probability equation (3) from the general model (paragraph 11 of main text), we get the following simple differential equation:

$$P(t+\Delta t)=P(t)+\frac{2Rv_s\Delta t}{XY} \qquad \textbf{(5)}$$

where R is the sensor range, v_s the searcher speed, X and Y the width and length of the search area.

3. The boundary condition at time equal to zero is:

$$P(0)=\frac{2R^2}{XY} \qquad \textbf{(6)}$$

This is equivalent to a searcher starting his sweep at the edge of the search area. At time zero, a half of his sensor coverage is within the search area and the other half is outside of the search area. This type of boundary is used in Reference [4].

4. Therefore, in general, the probability of detection can be expressed in the following way:

$$P(t) = \frac{2R^2}{XY} + \frac{2Rvt}{XY} \tag{7}$$

5. Equation (7) indicates that the probability of detection reaches 100 percent when the searcher completes his sweep, at the following time:

$$t = \frac{XY-2R^2}{2vR} \tag{8}$$

6. The probability of detection in equation (7) is valid for time, t from zero to (X-R)/v. Note that this result and the next one for transiting target can be obtained by simple geometry, they are done this way only to illustrate the methodology.

Transiting Target

7. We assume that the target is initially in the search area and moves with speed v_t along the positive y-axis. This scenario applies to barrier patrol. We again assume that the search area is a rectangle of width exactly equal to twice the sensor range, and the search is conducted by a single sweep of this area. The function G can again be expressed by Dirac delta functions as follows:

$$P_u(t)\, G = \frac{1}{XY}\, \delta\,(x-x')\, \delta\,(y-y'-v_t t) \tag{9}$$

8. The differential equation becomes

$$\frac{dP}{dt} = \frac{v_s}{XY}\,(2R-v_t t) \tag{10}$$

with solution

$$P(t) = \frac{2R^2}{XY} + \frac{v_s t}{XY}\,(2R-\frac{v_t t}{2}) \tag{11}$$

9. In this case, the time t is restricted from zero to the minimum of $(X-R)/v_s$ and $2R/v_t$. We can see that the probability of detection of the target is now decreased by a quadratic term in t as compared to the probability of detection of a stationary target. This accounts for the probability that the target exits the search area before it is detected.

Koopman-like Target

10. Koopman assumes the target position is random inside the entire area so that his probability of detection satisfies the following differential equation as an application of equation (3):

$$\frac{dP}{dt} = (1 - P(t))\,\frac{2\,v_s R}{XY} \tag{12}$$

11. The second term on the right hand side is called the rate of detection. The solution of the Koopman line-of-sight differential equation is equal to the following:

$$P(t) = 1 - \exp\left(-\frac{2 v_s R t}{XY}\right) \tag{13}$$

with P(0)=0 (as Koopman assumes only a lateral visual range and not a sensor coverage).

12. A more careful application of equation (3) for sensor analysis gives us the same result except that XY is changed to $XY\text{-}4R^2$. This difference is because, although the target position is random in the area, it could not be within the sensor range and not be detected. In other words, the target random position at time t+dt is only within the entire area minus the sensor range at time t. With our boundary condition, our Koopman-like probability of detection is equal to:

$$P(t) = 1 - \left(1 - \frac{4R^2}{XY}\right)\exp\left(-\frac{2 v_s R t}{XY - 4R^2}\right) \tag{14}$$

13. In reality, as the searcher starts sweeping at the edge of the search area, our Koopman-like probability of detection must be described by two solutions: one before the complete sensor coverage is entered inside the search area and one after.

$$P(t) = \begin{cases} 1 - \frac{1}{XY}(XY - 2R^2 - 2Rv_s t) & t < R/v_s \\ 1 - \left(1 - \frac{4R^2}{XY}\right)\exp\left(-\frac{2R(v_s t - R)}{XY - 4R^2}\right) & t > R/v_s \end{cases} \quad (15)$$

14. We call the above probability of detection the random target (RT) result.

Slow-moving Target

15. In practice, if a target speed is slow compared to the searcher speed or if the searcher sensor coverage is comparable to the search area, then the chance that a target moves away from the region not-yet-swept by the searcher to the region already-swept and remains undetected is small.

16. Large Radar Range. When the radar range is equal to the vertical size of the search area, that probability is zero. Thus, in that case, the probability of detection satisfies the following differential equation:

$$\frac{dP}{dt} = (1 - P(t))\,\frac{2Rv_s}{XY - 2R^2 - 2Rv_s t} \quad (16)$$

with solution

$$P(t) = 1 - \frac{1}{XY}(XY - 2R^2 - 2Rv_s t) \quad (17)$$

17. This solution means that the detection probability is 100 percent after one sweep when the radar range 2R is equal to the vertical size of the search area independently of the target speed. In this case, the probability of detection of a target when the sensor range is large is the same as the probability of detection of a stationary target.

18. Smaller Radar Range. We will now model the rate of detection of a target. In Koopman's model it is equal to $2Rv_s/XY$. This was argued to be incorrect for our search route. We know that at time zero, this rate must be equal to $2Rv_s/(XY-2R^2)$ (equation (16)). It is true that as time increases, the rate of detection increases because the more we sweep the better

chance we find the target in a systematic search in the sense that we will eventually corner the target due to speed superiority. As time approaches infinity, the rate of detection also approaches infinity because the target's speed is assumed to be much less than the searcher's speed. The following model is proposed for a rate of detection satisfying the previous boundary conditions:

$$\begin{cases} \dfrac{2Rv_s}{XY-2R^2-2Rv_s t} & t \le R/v_s \\ \dfrac{2Rv_s}{(XY-4R^2)\exp\left(-\dfrac{2Rv_{eff}t}{XY-4R^2}\right)} & t > R/v_s \end{cases} \tag{18}$$

19. v_{eff} is a modelling parameter. It indicates how effective, in terms of relative speeds, the searcher can detect the target. From dimensional analysis, we know that it is a function of the searcher speed, v_s, and of the target speed, v_t. When substituting the above rate of detection into the differential equation, we obtain the following probability of detection as a function of time, t:

$$P(t) = \begin{cases} 1 - \dfrac{1}{XY}(XY-2R^2-2Rv_s t) & t \le R/v_s \\ 1 - \left(1 - \dfrac{4R^2}{XY}\right)\exp\left(-\dfrac{v_s}{v_{eff}}\left(\exp\left(\dfrac{2Rv_s t}{XY-4R^2}\right) - \exp\left(\dfrac{2R^2}{XY-4R^2}\right)\right)\right) & t > R/v_s \end{cases}$$

20. This probability of detection is named the slow-moving target (SMT) probability of detection.

21. As time goes to infinity, the SMT probability of detection approaches 100 percent. As time goes to zero, the probability of detection is equal to $2R^2/(XY)$. When the target speed is large, v_{eff} is small and when the target speed is small, v_{eff} is large. And when v_{eff} is large, the rate of detection is large.

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DETECTION

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PROBABILITY

SENSOR

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