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RESEARCH NOTE 5/94

PARAMETER UNCERTAINTY IN RANGE SAFETY CALCULATIONS

by

E.J. Emond

August 1994

OTTAWA, CANADA

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ABSTRACT

In May 1994, DMS received a request from Maritime Command Operational Research to develop a mathematical approach to the problem of determining range safety offset distances when exact knowledge of accuracy parameters was not available. The results of this analysis are to be part of the development of a MARCOM range safety policy. The DMS paper uses an approach based on minimal assumptions and proceeds from simple cases to the most complex and general case. The analysis and results may be applied to many range safety situations, whether maritime or otherwise.

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PARAMETER UNCERTAINTY IN RANGE SAFETY CALCULATIONS

INTRODUCTION

1. This paper presents a mathematical approach to the problem of incorporating parameter uncertainty in assessing safe standoff distances for weapons trials. We assume that a single weapon is to be fired from a known fixed position with an intended target point some distance away. The range safety problem is to establish an area around the intended target point beyond which an observer or other bystander is declared to be safe from being physically harmed by the impact of the weapon. The quantitative interpretation of the word safe will be discussed in terms of the risk or probability of weapon impact beyond the stated range safety area. For example, a reasonable safety level for military trials might be 0.9999. Specifically this means that the probability of weapon impact outside of the range safety area around the target point is less than one in ten thousand or 0.0001.

2. There are two separate range safety situations or paradigms, representing logically distinct problems for the safety planner. The first paradigm, which is the subject of this paper, concerns the occurrence of random errors in the physical and environmental conditions of the trial which may cause the weapon to impact at a point other than the intended target point. Such factors as wind variability and manufacturing deviations typically contribute to random error. Because of the combining of many small random factors, this type of error is adequately modelled by assuming that the actual impact point is a two dimensional Normally distributed random variable with its probability distribution centred at the intended target point. The mathematical exposition of this case will be the subject of the majority of this paper.

3. The other safety paradigm which is distinct and separate from random error is the possible occurrence of catastrophic error. Examples are gross targeting error and severe mechanical malfunction. In such cases there is no safe offset range other than that provided by extremes of range or other physical limits. While this type of error must be considered in any range safety situation, it is not the subject of this paper and will therefore not be discussed further.

NORMAL ERROR MODEL

4. The general model to be discussed in this paper assumes that the along track and across track errors may be modelled as independent Normal random variables with zero means. The assumption of independence in this case is equivalent to the assumption that the along track and across track errors are uncorrelated. If sufficient data values are available, this can be verified by a statistical test of hypothesis for zero correlation. The Normality of the along track and across track errors is also verifiable with sufficient data. Finally, the zero means assumption is reasonable since any known aiming bias will obviously be removed before firing.

5. Rather than starting with the most general and therefore most complex case, this paper will reverse the pattern and begin with the easiest case. Progressively more complex cases will be considered in turn, in order to assist the reader in comprehending the increasingly complex mathematical models involved.

THE TWO DIMENSIONAL CIRCULAR NORMAL CASE WITH KNOWN VARIANCE

6. The simplest case is when the along track and across track errors are assumed to be independent Normal random variables with zero means and known equal variances. Then the two dimensional random variable representing the impact point of the missile has a circular Normal probability distribution centred at the target point. The range safety problem in this case reduces to finding the radius of a circle around the target point within which the missile will impact with a specified high probability.

7. For a circular Normal distribution of impact point, one can transform to polar coordinates and derive the distribution of the random variable representing the radial miss distance of the weapon from its target point. This is the Rayleigh distribution which is given in equation 1 below. The parameter σ represents the standard deviation of the two one-dimensional Normal random variables (across track and along track) which comprise the two dimensional circular Normal.

$$F(r) = 1 - e^{-\frac{r^2}{2\sigma^2}} \qquad r \geq 0 \;;\; \sigma > 0 \tag{1}$$

8. Equation 1 can easily be solved for the safety radius, r, given any desired safety level, F(r). Table I gives some examples. As in all the examples in this paper the safety radius is given in terms of a multiple of the standard deviation which acts as a scale parameter.

TABLE I - SAFETY RADII FOR VARIOUS PROBABILITY VALUES

Model : Circular Normal : Along track and across track errors independent and Normally distributed with zero means and known equal variances.

Probability	Safety Radius
0.999	3.717 σ
0.9999	4.292 σ
0.99999	4.799 σ

9. For the circular Normal case with known standard deviation, the range safety problem is solved by choosing a safety level and calculating the resulting safety radius as in Table I. For example, to ensure a safety level of 0.9999, the safety radius is given by 4.292 σ. This is interpreted in words as stating that if a standoff radius of 4.292 σ is used, there will be only one chance in ten thousand that any single weapon will impact outside of the safety radius due to random error.

THE ONE DIMENSIONAL CASE WITH KNOWN VARIANCE

10. In some cases the range safety problem reduces to one dimension because of the geometry of the situation. Most often this occurs when along track miss distance is not an issue because the firing range is cleared all along (and beyond) the intended track. Thus only across track miss distance is involved in the safety calculation.

11. The across track error is modelled as a one dimensional random variable having a Normal distribution centred on the target point and with known standard deviation. Equation 2 gives the well known one dimensional Normal distribution function, where F(x) is the probability that the across track error is less than x. It is customary to treat across track errors to the left of the target point as negative and to the right of the target point as positive, from the point of view of the firing position.

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{z^2}{2\sigma^2}} dz \qquad ; \sigma > 0 \qquad (2)$$

12. As in the two dimensional case, equation 2 can be solved numerically or mathematical tables can be used to obtain the safe standoff distance for any required probability level. Some examples are given in Table II.

TABLE II - STANDOFF DISTANCE FOR VARIOUS SAFETY LEVELS

Model : One Dimensional Normal : Across track error Normally distributed with zero mean and known variance.

Probability	Standoff Distance
0.999	3.291 σ
0.9999	3.891 σ
0.99999	4.417 σ

13. For the one dimensional Normal case with known standard deviation, the range safety problem is solved by choosing a safety level and calculating the resulting standoff distance as in Table II. For example, to ensure a safety level of 0.9999, the safety distance is given by 3.891 σ. This is interpreted in words as stating that if a standoff distance of 3.891 σ is used, there will be only one chance in ten thousand that any single weapon will impact beyond the safety distance due to random error.

THE TWO DIMENSIONAL NONCIRCULAR NORMAL CASE WITH KNOWN VARIANCE

14. We next consider the case in which the along track and across track errors may be modelled as independent Normal random variables with zero means and known but unequal variances. There are two ways of approaching this case mathematically. In the first approach, we transform the two dimensional noncircular Normal probability distribution into the unit variance circular Normal probability distribution by dividing the along track and across track errors by their respective standard deviations. We then calculate the required safety radius as in Table I. On transforming back to the original coordinates, we obtain an elliptical safety area with the required impact probability. The only drawback to this approach is that often an elliptical area is less useful practically than a circular or rectangular area. For this reason, the second approach, which results in a rectangular safety area, will be used in this paper.

15. The second approach to the two dimensional noncircular Normal case is to apply the one dimensional method of Table II to both the along track and across track dimensions independently, thereby producing a rectangular safety area with the desired probability. The only new wrinkle is that the overall probability of weapon impact inside the safety rectangle is the product of the across track and along track inclusion probabilities. For an overall probability of P, we must find along track and across track probabilities whose product is P. It is customary to use the square root of P for both the along track and across track probabilities although any two values whose product is P may be used. Some examples in which the along track and across track probabilities have been equalized are given in Table III. Note that the values displayed are offsets from the target point and are therefore equal to one half of the sides of the safety rectangle. The across track and along track standard deviations are given by σ_1 and σ_2 respectively.

TABLE III - STANDOFF DISTANCES FOR THE NONCIRCULAR NORMAL CASE

Model : Two dimensional noncircular Normal : Along track and across track errors independent and Normally distributed with zero means and known but unequal variances.

Probability	Across Track Distance	Along Track Distance
0.999	3.481 σ_1	3.481 σ_2
0.9999	4.056 σ_1	4.056 σ_2
0.99999	4.565 σ_1	4.565 σ_2

16. For the two dimensional noncircular Normal case with known but unequal variances, a rectangular range safety area may be obtained as in Table III by considering the along track and across track errors separately. For example, a safety rectangle centred at the target point with across track standoff distance of 4.056 times the across track standard deviation and along track standoff distance of 4.056 times the along track standard deviation will give a safety level of 0.9999. The probability of a weapon impact outside this safety rectangle is only one in ten thousand or 0.0001.

CIRCULAR NORMAL CASE WITH UNKNOWN VARIANCE

17. We next consider the case in which the along track and across track errors are independent and Normally distributed with zero means and equal but unknown variances. In this case the two dimensional circular Normal distribution applies but the parameter σ is unknown. This situation may be handled by considering σ to be a random variable so that uncertainty about its value can be incorporated into safety calculations. Suppose therefore that σ is a random variable with probability density function given by h(σ) and constrained to be positive. For a given level of safety, say P, we can use the law of total probability to establish an equation involving the safety radius, C, by integrating the Rayleigh distribution over all possible values of σ as follows.

$$P = \int_0^{\infty} h(\sigma)\,(1 - e^{-\frac{C^2}{2\sigma^2}})\,d\sigma \qquad ; C > 0 \qquad (3)$$

18. In order to make use of the above equation we need information about the distribution of σ. The most natural source of information consists of a random sample of prior observations of radial miss distances : $r_1, r_2, \ldots, r_n$. (For an impact point with along track error x and across track error y, the radial miss distance, r, is the square root of the sum of x squared plus y squared.) Given n independent previous observations of radial miss distances, we can use Bayes' Theorem with a non-informative prior[1] to obtain an expression for h(σ) as follows.

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[1] In this paper the non-informative prior for σ is taken for the purpose of being conservative to be the uniform density on (0, ∞). Since σ is a scale parameter, the theoretically correct non-informative prior is h(σ) = σ^{-1} on (0, ∞), known as the Jeffreys prior or the invariant scale prior. The effect of using the Jeffreys prior rather than the uniform prior would be equivalent to increasing n, the number of previous observations, by ½ in the circular case and by 1 in the one dimensional case.

$$h(\sigma) \propto \frac{1}{\sigma^{2n}} e^{-\frac{\sum_{i=1}^{n} r_i^2}{2\sigma^2}} \quad ; \sigma > 0 \tag{4}$$

$$h(\sigma) = \frac{2\left(\frac{\sum_{i=1}^{n} r_i^2}{2}\right)^{n-\frac{1}{2}}}{\Gamma\left(n-\frac{1}{2}\right) \sigma^{2n}} e^{-\frac{\sum_{i=1}^{n} r_i^2}{2\sigma^2}} \quad ; \sigma > 0 \tag{5}$$

19. Equation 5 is derived from equation 4 by adding the constraint that the area under h(σ) must be unity. This expression for h(σ) may next be substituted into equation 3. After integrating and then simplifying, we obtain equation 6.

$$P = 1 - \left(\frac{\sum_{i=1}^{n} r_i^2}{\sum_{i=1}^{n} r_i^2 + C^2}\right)^{n-\frac{1}{2}} \quad ; C > 0 \tag{6}$$

20. In order to see the effect of increasing the number of observations, n, the safety radius, C, in equation 6 can be expressed in terms of the unbiased maximum likelihood estimate of σ which is given in equation 7.

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} r_i^2}{2n}} \qquad (7)$$

$$P = 1 - \left(\frac{1}{1 + \frac{C^2}{2n\hat{\sigma}^2}} \right)^{n-\frac{1}{2}} \qquad (8)$$

21. Using equation 8 we can obtain the required safety radius for any specified value of P. Table IV gives the safety radius for several values of P in terms of the maximum likelihood estimate of σ for various values of n. Note the convergence to the case where σ is known as n increases.

TABLE IV - SAFETY RADIUS FOR VARIOUS VALUES OF P AND N

Model : Circular Normal, Unknown Variance : Along track and across track errors independent and Normally distributed with zero means and equal but unknown variances.

	SAFETY RADIUS		
Number of Observations (n)	P = 0.999	P = 0.9999	P = 0.99999
2	19.90 $\hat{\sigma}$	43.04 $\hat{\sigma}$	92.77 $\hat{\sigma}$
3	9.44 $\hat{\sigma}$	15.26 $\hat{\sigma}$	24.37 $\hat{\sigma}$
4	7.04 $\hat{\sigma}$	10.16 $\hat{\sigma}$	14.37 $\hat{\sigma}$
5	6.03 $\hat{\sigma}$	8.21 $\hat{\sigma}$	10.91 $\hat{\sigma}$
6	5.49 $\hat{\sigma}$	7.21 $\hat{\sigma}$	9.24 $\hat{\sigma}$
7	5.15 $\hat{\sigma}$	6.61 $\hat{\sigma}$	8.26 $\hat{\sigma}$
8	4.92 $\hat{\sigma}$	6.22 $\hat{\sigma}$	7.63 $\hat{\sigma}$
9	4.75 $\hat{\sigma}$	5.93 $\hat{\sigma}$	7.19 $\hat{\sigma}$
10	4.62 $\hat{\sigma}$	5.72 $\hat{\sigma}$	6.87 $\hat{\sigma}$
15	4.28 $\hat{\sigma}$	5.16 $\hat{\sigma}$	6.03 $\hat{\sigma}$
20	4.12 $\hat{\sigma}$	4.91 $\hat{\sigma}$	5.67 $\hat{\sigma}$
25	4.04 $\hat{\sigma}$	4.78 $\hat{\sigma}$	5.48 $\hat{\sigma}$
50	3.87 $\hat{\sigma}$	4.52 $\hat{\sigma}$	5.12 $\hat{\sigma}$
100	3.79 $\hat{\sigma}$	4.40 $\hat{\sigma}$	4.95 $\hat{\sigma}$
Infinity	3.72 $\hat{\sigma}$	4.29 $\hat{\sigma}$	4.80 $\hat{\sigma}$

22. Inspection of Table IV shows that the safety radius decreases rapidly as the number of observations increases from 2 through 10 and that the decrease is less rapid as the number of observations grows larger. This is intuitively reasonable since as n increases the uncertainty in the value of σ decreases.

23. In conclusion, for the circular Normal case in which n values of radial miss distance are available as prior information, we can use equation 6 to solve for the safety radius for any specified safety level. The safety radius is always larger than if the true value of σ were known, but the difference decreases as the number of prior observations increases. (Note that the case in which σ is known is equivalent to the case where infinitely many observations are available.) For example, to achieve a safety level of 0.9999 when only 5 previous observations of radial miss distances are available, a safety radius of 8.21 times the estimated standard deviation is required. This compares to 4.29 times the true but unknown standard deviation, reflecting the risk added because of the uncertainty over the value of a vital parameter.

THE ONE DIMENSIONAL CASE WITH UNKNOWN VARIANCE

24. We now return to the case in which only the across track error is under consideration because the range has been cleared in the along track dimension. Recall that the across track error is modelled as a one dimensional Normal random variable with zero mean. In this case however we do not assume that the standard deviation is known. We treat the unknown standard deviation, σ, as a random variable which is restricted to be positive and whose probability distribution is given by h(σ). For a given level of safety, say P, we can find an equation involving the safety distance, C, by integrating the Normal probability density function over all values of σ as follows.

$$P = \int_0^{\infty} h(\sigma) \int_{-C}^{C} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{z^2}{2\sigma^2}} \, dz \, d\sigma \qquad ; C > 0 \qquad (9)$$

25. As before, let $x_1, x_2, \dots, x_n$ be a set of independent previous observations of across track errors. We can apply Bayes' Theorem with a non-informative prior as follows in equation 10. Equation 11 is derived from equation 10 by adding the constraint that the area under the probability density function must be one.

$$h(\sigma) \propto \frac{1}{\sigma^n} e^{-\frac{\sum_{i=1}^{n} x_i^2}{2\sigma^2}} \quad ; \sigma > 0 \tag{10}$$

$$h(\sigma) = \frac{2\left(\frac{\sum_{i=1}^{n} x_i^2}{2}\right)^{\frac{n-1}{2}}}{\Gamma\left(\frac{n-1}{2}\right)\sigma^n} e^{-\frac{\sum_{i=1}^{n} x_i^2}{2\sigma^2}} \quad ; n > 1 \,,\, \sigma > 0 \tag{11}$$

26. The expression for $h(\sigma)$ as given in equation 11 can now be used in equation 9. In this case there is no closed form solution but equation 9 can be solved numerically in an iterative manner to find the value of the safety distance, C, for any desired safety level, P. We again make the problem dimensionless by expressing the safety distance, C, in terms of the maximum likelihood unbiased estimate of σ, which is given in equation 12.

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n-1}} \quad ; n > 1 \tag{12}$$

27. Table V gives the safety distance in terms of the estimated value of σ for various values of n and safety level, P. It can be seen in Table V that as the number of prior observations increases, the safety distance decreases towards the value in the case where σ is known. The convergence is slower than in the circular Normal case. This reflects the fact that in the two dimensional circular Normal case, each observation is actually equivalent to two observations, one along track and one across track.

TABLE V - SAFETY DISTANCE FOR VARIOUS VALUES OF P AND N

Model : One Dimensional Normal, Unknown Variance : Across track error Normally distributed with zero mean and unknown variance.

	STANDOFF DISTANCE		
Number of Observations (n)	P = 0.999	P = 0.9999	P = 0.99999
3	31.40 $\hat{\sigma}$	94.70 $\hat{\sigma}$	236.0 $\hat{\sigma}$
4	12.93 $\hat{\sigma}$	27.97 $\hat{\sigma}$	59.67 $\hat{\sigma}$
5	8.62 $\hat{\sigma}$	15.54 $\hat{\sigma}$	27.58 $\hat{\sigma}$
6	6.87 $\hat{\sigma}$	11.18 $\hat{\sigma}$	17.81 $\hat{\sigma}$
7	5.96 $\hat{\sigma}$	9.09 $\hat{\sigma}$	13.52 $\hat{\sigma}$
8	5.41 $\hat{\sigma}$	7.89 $\hat{\sigma}$	11.20 $\hat{\sigma}$
9	5.05 $\hat{\sigma}$	7.13 $\hat{\sigma}$	9.78 $\hat{\sigma}$
10	4.79 $\hat{\sigma}$	6.60 $\hat{\sigma}$	8.82 $\hat{\sigma}$
15	4.15 $\hat{\sigma}$	5.37 $\hat{\sigma}$	6.71 $\hat{\sigma}$
20	3.89 $\hat{\sigma}$	4.90 $\hat{\sigma}$	5.96 $\hat{\sigma}$
25	3.75 $\hat{\sigma}$	4.66 $\hat{\sigma}$	5.57 $\hat{\sigma}$
50	3.51 $\hat{\sigma}$	4.24 $\hat{\sigma}$	4.93 $\hat{\sigma}$
100	3.40 $\hat{\sigma}$	4.06 $\hat{\sigma}$	4.66 $\hat{\sigma}$
Infinity	3.29 $\hat{\sigma}$	3.89 $\hat{\sigma}$	4.42 $\hat{\sigma}$

28. For the one dimensional Normal error case in which the value of the standard deviation is not known, equations 9 and 11 can be used to calculate the required standoff distance for any specified safety level by making use of prior observations. If fewer than 10 prior observations are available, the safety distances using this method are considerably larger than in the case where σ is known. For example with only 5 previous observations of across track errors from previous trials, the standoff distance required for a safety level of 0.9999 is 15.65 times the estimated across track standard deviation. This compares to 3.89 times the value of the true but unknown standard deviation, reflecting the risk added by uncertainty in the value of a vital parameter.

THE TWO DIMENSIONAL NONCIRCULAR NORMAL CASE WITH UNKNOWN VARIANCE

29. We next consider the general case in which the along track and across track errors are modelled as independent Normal random variables with zero means and unknown and unequal variances. It is assumed that n pairs of observations of along track and across track errors from previous trials are available. By applying the one dimensional method discussed above to both the along track and across track errors independently, we can obtain a safety rectangle around the target point for any desired level of safety. As in the earlier case of this type (when the variances were known), to achieve a safety level P we set both the along track and across track safety levels to the square root of P.

30. Table VI gives the along track and across track standoff distances in terms of the estimated along track and across track standard deviations for several values of both safety level P and number of prior observations n. As an example of how to use Table VI, suppose a safety level of 0.9999 is desired and data from 15 previous trials are available:

Observation Number	Along Track Error (km.)	Across Track Error (km.)
1	-0.04	-0.98
2	-1.23	-1.00
3	2.01	0.33
4	-0.20	-0.44
5	-0.11	0.57
6	0.01	0.97
7	-0.57	0.37
8	0.70	-0.89
9	0.07	-0.37
10	-0.32	-0.31
11	1.35	0.37
12	0.12	-0.07
13	-1.06	0.09
14	0.41	-0.42
15	1.89	-0.17

Using equation 12 we estimate the along track standard deviation as 0.97 km. and the across track standard deviation as 0.60 km. From Table VI we find that the multiplier for a safety level of 0.9999 when 15 data values are available is 5.76. Therefore the along track offset distance is 5.59 km. and the across track offset distance is 3.46 km. Using these offset distances will ensure that there is less than one chance in ten thousand that any individual weapon will impact in the area outside of the range safety rectangle.

TABLE VI - SAFETY DISTANCE FOR VARIOUS VALUES OF P AND N

Model : Noncircular Normal, Unknown Variances : Across track and along track errors Normally distributed with zero means and unknown variances. Values in table multiply both along track and across track standard deviation estimates.

	STANDOFF DISTANCE		
Number of Observations (n)	P = 0.999	P = 0.9999	P = 0.99999
3	44.15 $\hat{\sigma}$	128.5 $\hat{\sigma}$	291.4 $\hat{\sigma}$
4	16.33 $\hat{\sigma}$	35.21 $\hat{\sigma}$	74.45 $\hat{\sigma}$
5	10.31 $\hat{\sigma}$	18.50 $\hat{\sigma}$	32.64 $\hat{\sigma}$
6	7.98 $\hat{\sigma}$	12.88 $\hat{\sigma}$	20.42 $\hat{\sigma}$
7	6.79 $\hat{\sigma}$	10.26 $\hat{\sigma}$	15.19 $\hat{\sigma}$
8	6.09 $\hat{\sigma}$	8.79 $\hat{\sigma}$	12.40 $\hat{\sigma}$
9	5.62 $\hat{\sigma}$	7.86 $\hat{\sigma}$	10.72 $\hat{\sigma}$
10	5.30 $\hat{\sigma}$	7.22 $\hat{\sigma}$	9.59 $\hat{\sigma}$
15	4.51 $\hat{\sigma}$	5.76 $\hat{\sigma}$	7.15 $\hat{\sigma}$
20	4.19 $\hat{\sigma}$	5.22 $\hat{\sigma}$	6.29 $\hat{\sigma}$
25	4.03 $\hat{\sigma}$	4.93 $\hat{\sigma}$	5.85 $\hat{\sigma}$
50	3.74 $\hat{\sigma}$	4.45 $\hat{\sigma}$	5.13 $\hat{\sigma}$
100	3.61 $\hat{\sigma}$	4.25 $\hat{\sigma}$	4.84 $\hat{\sigma}$
Infinity	3.48 $\hat{\sigma}$	4.06 $\hat{\sigma}$	4.57 $\hat{\sigma}$

DISCUSSION

31. The methods described in this report allow for the calculation of range safety areas with any desired level of safety when both along track and across track standard deviations are estimated using data from previous trials. The mathematical derivation incorporates the conservative assumption that no other knowledge exists about the two standard deviations. Thus the only source of information about these values is the set of data from previous trials. As can be seen from Tables V and VI, when less than 10 data values are available, the penalty to be paid because of lack of information can be large in terms of the size of the range safety area. On the other hand, the absence of any assumptions about the along track and across track standard deviations provides a high level of assurance that the areas calculated are sufficiently safe in all cases.

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