Image Cover Sheet

CLASSIFICATION	SYSTEM NUMBER 502733
UNCLASSIFIED	

TITLE

APPROXIMATION TO BEAM PROPAGATION IN OCEAN WATER

System Number:

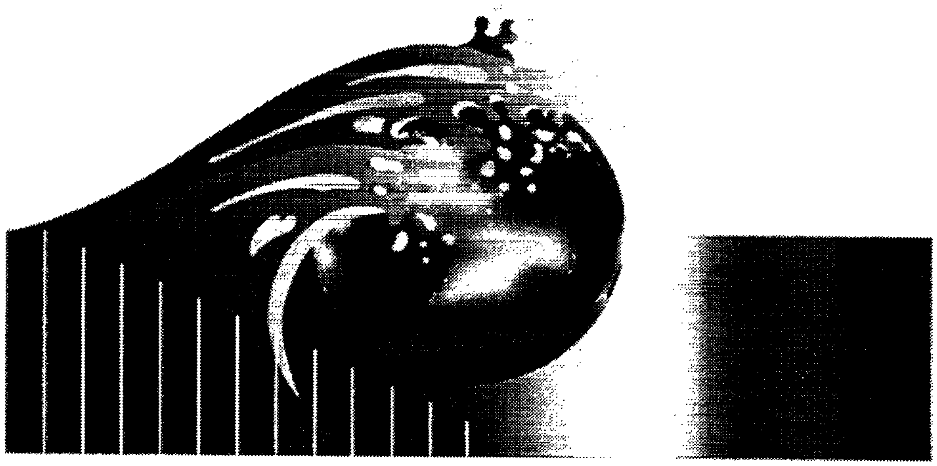
Patron Number:

Requester:

Notes:

DSIS Use only:

Deliver to: DK



Reprinted from

OCEAN OPTICS XIII

22–25 October 1996
Halifax, Nova Scotia, Canada

SPIE Volume 2963

Approximation to beam propagation in ocean water

G.R. Fournier

Defence Research Establishment Valcartier
Courcelette, Quebec, G0A 1R0, Canada

1. ABSTRACT

Using a modified form of the anomalous diffraction approximation we first derived in closed form an analytic expression for the phase function of Mie scatterers integrated over an inverse power law (Junge) size distribution . A simple analysis explained the apparent singularity seen experimentally at the forward scattering angle. Relationships were derived that related the inverse power law as a function of scattering angle in the near forward direction to the power law of the size distribution. The parameters of the formula are the relative index of refraction and the inverse power of the size distribution. Using the formula and path integrals, we have derived approximate analytic expressions that model laser beam propagation in ocean waters. The effect of strong absorption on beam shape and temporal spreading is accounted for. The results are currently being integrated into a comprehensive model of underwater active imaging systems.

Keywords: propagation, multiple scattering, phase function, underwater, path integral

2. INTRODUCTION

In previous work[1] a simple set of approximations involving the anomalous diffraction theory was used to derive from the basic physics an analytic phase function whose parameters can be related to a limited set of physically meaningful quantities. A comprehensive data base of oceanic phase functions[2] was then fitted to this form. The phase function is given by the following expression:

$$\beta(\theta) = C\pi\left(\frac{2\pi(n-1)}{\lambda}\right)^{\mu-3}\frac{1+\cos^2\theta}{8\sin(-\pi v)}\left[\frac{1}{(1-\delta^2)\delta^v}\right]$$

$$\left([v(1-\delta)-(1-\delta^v)]+\frac{4}{u^2}[(1-\delta^{v+1})-(v+1)(1-\delta)]\right). \quad (1)$$

Where

$$v = \frac{3-\mu}{2} \quad (2)$$

$$\delta = \frac{u^2}{3(n-1)^2} \quad (3)$$

$$u = 2\sin(\theta/2). \quad (4)$$

In these expressions, μ is the exponent of the inverse power law of the particle size distribution, λ is the wavelength and n is the index of refraction of water. From this form of the phase function it is easy to derive the behaviour at small angles. The constant of proportionality C depends on all the parameters but the inverse power ν of the phase function near $\theta = 0$ only depends on the inverse power μ of the particle size distribution function.

$$\nu = 5 - \mu \tag{5}$$

The slope of the phase function near $\theta = 0$ contains information only about the power law of the particle size distribution. The variation of the total scattering coefficient b is also an inverse power γ of the wavelength.

$$\gamma = \mu - 3 \tag{6}$$

Furthermore, equations 5 and 6 can be used to relate the wavelength dependence of the total scattering coefficient to the logarithm of the slope of the phase function as a function of angle in the near forward direction.

$$\gamma = 2 - \nu \tag{7}$$

The feature of interest in this formulation is the singularity in the forward scattering near $\theta = 0$. This implies that small angle forward scattering should be completely dominant in ocean waters. To verify this hypothesis, equation 1 was fitted to a data base of oceanic phase functions[2]. The distribution of values of the mean square scattering angle$< \theta^2 >$ for a large sample of measured phase functions confirmed the overwhelming dominance of forward scatter.

3. PATH INTEGRAL SOLUTION TO BEAM PROPAGATION

Multiple scattering can be viewed as a sophisticated random walk problem and as such is particularly amenable to the path integral method. Feynman and Hibbs[4] used the path integral approach to solve the neutron scattering problem in the small forward angle regime. Their solution, with an appropriate change in notation, is directly applicable to light scattering in waters where forward scattering dominates. For the time independent one dimensional case, Feynman and Hibbs show that:

$$P(X, \theta_x, l) = Ce^{-\frac{1}{2b<\theta_i^2>}\int_0^l (\frac{d^2x}{dz^2})^2 dz} \tag{8}$$

where P is the probability of finding a photon that started at $z = 0, x = 0, \theta = 0$ at $z = l, x = X, \theta = \theta_x$, b is the total scattering coefficient and $< \theta_i^2 >$ is the one dimensional mean square scattering angle

$$< \theta_i^2 >= \int_0^\infty f(\theta_i)\theta_i^2 d\theta_i. \tag{9}$$

In equation 8 $x(z)$ is the solution of the differential equation

$$\frac{d^4x}{dz^4} = 0 \tag{10}$$

with the boundary conditions

$$x(0)=0,\left(\frac{dx}{dz}\right)_{z=0}=0, x(l)=X,\left(\frac{dx}{dz}\right)_{z=l}=\theta_x \tag{11}$$

This solution is given by:

$$x(z)=(3X-\theta_x l)\left(\frac{z}{l}\right)^2+(\theta_x l-2X)\left(\frac{z}{l}\right)^3 \tag{12}$$

$$\frac{dx(z)}{dz}=\frac{2}{l}(3X-\theta_x l)\left(\frac{z}{l}\right)+\frac{3}{l}(\theta_x l-2X)\left(\frac{z}{l}\right)^2 \tag{13}$$

$$\frac{d^2x(z)}{dz^2}=\frac{2}{l^2}(3X-\theta_x l)+\frac{6}{l^2}(\theta_x l-2X)\left(\frac{z}{l}\right) \tag{14}$$

Using equations 8 and 12, and taking into consideration the fact that the scattering in the x and y planes is independent, the two dimensional photon probability distribution is given by:

$$P(X,Y,\theta_x,\theta_y,l)=\frac{3}{\pi^2u^2}\exp\left\{\frac{-1}{ul}\left[3(X-\theta_x l/2)^2+3(Y-\theta_y l/2)^2+(\theta_x l)^2+(\theta_y l)^2\right]\right\}, \tag{15}$$

where

$$u=b<\theta^2>l^2 \tag{16}$$

and

$$<\theta^2>=\frac{1}{2}\int_0^\infty \beta(\theta)\theta^3\,d\theta. \tag{17}$$

Note that the factor of 1/2 in front of the integral in equation 17 is there to account for the one dimensional nature of equation (9). Equation 15 is equivalent to the expression found by van de Hulst and Kattawar for the time contracted distribution[5]. The only difference is in the method of derivation. The various contracted distributions can be obtained from equation 15 by simple integration over the desired variables. All the resulting formulas are analytic and will not be quoted here as it would take too much space.

4. CENTRAL PATH APPROXIMATION FOR ABSORPTION

The particular virtue of the path integral approach is the ease with which it allows one to include the effects of strong absorption. Multiplying equation (14) by the following factor accounts for the absorption of the photon along the various paths.

$$P(X,Y,\theta_x,\theta_y,l)=e^{-aS(X,Y,\theta_x,\theta_y,l)}P(X,Y,\theta_x,\theta_y,l) \tag{17}$$

In this expression a is the absorption coefficient and S is the mean path length covered by the photon. This mean path length can be expressed as follows:

$$S(X,Y,\theta_x,\theta_y,l)=\int_0^l\sqrt{1+\left(\frac{dx}{dz}\right)^2+\left(\frac{dy}{dz}\right)^2}\,dz. \tag{18}$$

One can then use equation 13 to obtain an explicit expression for dx/dz and dy/dz. The exact solution of equation 18 leads to a complex set of elliptic integrals. For small values of the path derivatives, which is always the case for small forward angle scattering, the binomial expansion can be used to obtain the expression:

$$S(X, Y, \theta_x, \theta_y, l) = l + s(X, Y, \theta_x, \theta_y, l), \tag{19}$$

where

$$s(X, Y, \theta_x, \theta_y, l) = \left(\frac{1}{2}\right) \int_0^l \left(\frac{dx}{dz}\right)^2 dz + \left(\frac{1}{2}\right) \int_0^l \left(\frac{dy}{dz}\right)^2 dz. \tag{20}$$

5. RESULTS

By substituting equations 19-20 into equation 17 and integrating over the appropriate variables one obtains the following expressions for the contracted distributions:

$$P(\theta^2, l) = \frac{e^{-al}}{u(au/5+2)} \exp\left[\frac{(a^2u^2/8 + 13au/3 + 10)}{2u(au+10)} l\theta^2\right] d(l\theta^2), \tag{21}$$

with

$$\theta^2 = \theta_x^2 + \theta_y^2$$

and

$$P(r^2, l) = \frac{3e^{-al}}{u(au/15+2)} \exp\left[3\frac{(a^2u^2/8 + 13au/3 + 10)}{2u(au+30)} (\frac{r^2}{l})\right] d(\frac{r^2}{l}), \tag{22}$$

with

$$r^2 = X^2 + Y^2,$$

and

$$P(l) = \frac{10e^{-al}}{(a^2u^2/8 + 13au/3 + 10)}. \tag{23}$$

Equations 21 and 22 have some significant consequences. If we write the half width of the distributions σ^2 in terms of their half width in the limiting case of no absorption σ_s^2 we obtain

$$\sigma^2 = \sigma_s^2 \left(\frac{au + 10}{a^2u^2/8 + 13au/3 + 10}\right) \tag{24}$$

for the angular distribution and

$$\sigma^2 = \sigma_s^2 \left(\frac{au + 30}{3(a^2u^2/8 + 13au/3 + 10)}\right) \tag{25}$$

for the radial distribution. Note that the correction factor for absorption can be entirely expressed in terms of the dimensionless variable

$$au = ab < \theta^2 > l^2 \tag{26}$$

The importance of this parameter has been noted before and the present derivation clarifies its origin. If we take the limit of equation 24 as $l \to \infty$ we obtain:

$$\sigma^2 \to \frac{16}{al}. \tag{27}$$

Equation 27 shows that when absorption is significant in the small angle approximation the angular distribution of radiation will narrow indefinitely. This implies that the existence of an asymptotic radiance distribution in the ocean is not a direct consequence of a balance between the beam expansion due to scattering and the narrowing due to absorption. This asymptotic state must instead be reached by a complex feedback between the backward and forward scattered radiances.

In order to verify the accuracy of equations 21 and 22, we used a previously developed Monte-Carlo code. The accuracy of this code was verified against the data of Tyler for Lake Pend Oreille[6]. Since this code gives the radiance distribution, we compared its results with equation 21. Figure 1 shows a comparison of the ratio of the half width with absorption to that without absorption for a uniform scattering function between 0° and 5°. As can be seen the match is excellent and confirms the accuracy of our results.

6. CONCLUSIONS

Using path integrals, we have derived simple formulas for the beam spread function in the case of small forward angle scattering in the presence of the strong absorption typical of ocean waters. We have compared our results with a Monte-Carlo code which itself had been verified against high quality experimental data. We are currently in the process of simplifying the formulas for the time dependent solutions. At this point, the results of van de Hulst and Kattawar[5] can be used in the case of strong absorption by simply multiplying them by the following factor:

$$\exp\left(\frac{-at}{nc}\right)$$

Where t is the time, n is the index of refraction of water and c is the speed of light. This approach is exact since both the effect of absorption and the delay in the time of arrival are directly proportional to the path length. We are presently implementing these results in a comprehensive model of underwater active imaging systems.

7. REFERENCES

1. G.R. Fournier and J.L. Forand, "Analytic phase function for ocean water", *Proceedings of SPIE*, Vol. 2258, pp. 194-201, Bergen, Norway, 1994.

2. G.R. Fournier and M. Jonasz, "Data base of oceanic optical properties", *Proceedings of SPIE*, Vol. 2258, pp. 576-587, Bergen, Norway, 1994.

3. T.J. Petzold, "Volume Scattering Functions for Selected Ocean Waters," Scripps Institute of Oceanography, SIO Ref. 72-28, 1972.

4. R.P. Feynman and A.R. Hibbs, *Quantum Mechanics and Path Integrals* , pp. 337-339, McGraw-Hill, New York, 1965.

5. H.C. van de Hulst and G.W. Kattawar, "Exact spread function for a pulsed collimated beam in a medium with small-angle scattering", *Applied Optics* **33**(24), pp. 5820-5829 (1994)

6. G.R. Fournier, J.L. Forand, G. Pelletier and P. Pace, "Ground-truthing the oceanic light field using an in-situ tunable transmissometer-nephelometer", 16th Canadian Symposium on Remote Sensing, Sherbrooke, Canada, 1993.

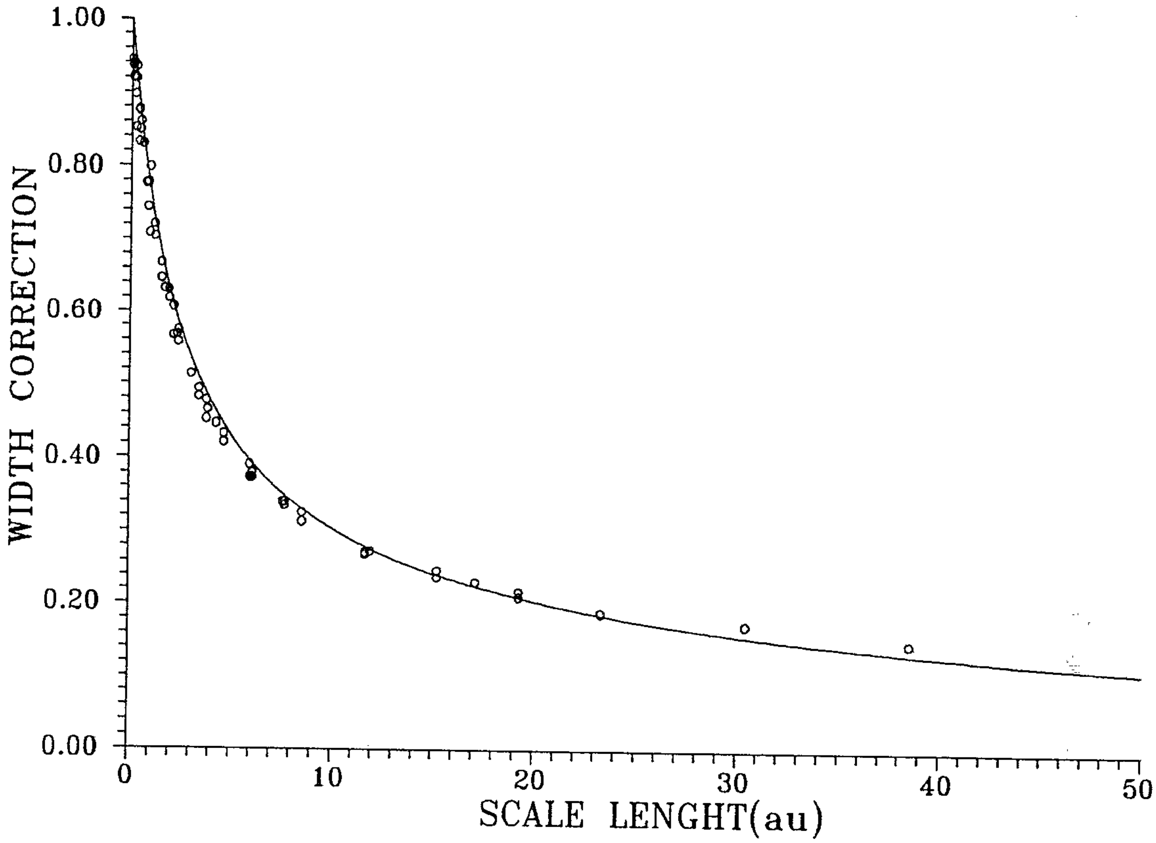


Figure 1. Comparison the analytic formula for the half width ratio with the results of the Monte Carlo code for a uniform phase function between 0° and 5°. The horizontal axis is in units of *au*.

#502733