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OPERATIONAL RESEARCH AND ANALYSIS

DIRECTORATE OF LOGISTICS ANALYSIS

ORA Project Report PR 703

LOGAN (LCC) V1.0: MATHEMATICAL THEORY
(Release 1.0)

by

M. Provencher
M. Vigneault

NOVEMBER, 1994

OTTAWA, CANADA

National Defence Défense nationale

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ORA PROJECT REPORT NO. PR 703

LOGAN (LCC) V1.0: MATHEMATICAL THEORY
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M. Provencher
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Work done under ORA Activity 42516-2

Recommended by: ____________ **D Log A**

Approved By ____________ **DORA**

OTTAWA, ONTARIO **NOVEMBER, 1994**

ABSTRACT

This paper describes the mathematics behind the Life Cycle Costing module of LOGAN (LOGistics ANalyzer). The Stochastic and Fuzzy Arithmetic approaches are explained.

RÉSUMÉ

Ce document décrit les mathématiques sur lesquelles le module de coût du cycle de vie de LOGAN (LOGistics ANalyzer) est basé. Les approches stochastique et de "logique floue" sont expliquées.

EXECUTIVE SUMMARY

1. The Directorate of Logistics Analysis (D Log A) conducts a wide range of analytical studies. Particularly, under activity 42516, D Log A designs, develops and evaluates a variety of decision support methods required for CF equipment life cycle costing.

2. This paper describes the mathematics behind the Life Cycle Costing (LCC) module of LOGAN (LOGistics ANalyzer) which is also called LOGAN(LCC). It describes how the life cycle cost of equipment is calculated and explains two approaches that can be used to account for the uncertainty in the LCC data. These two analyses, i.e. the stochastic and fuzzy arithmetic approaches, are explained in detail in this paper.

3. Although it is recognized that the approaches described are not necessarily the only or the best choices, the purpose of the paper is to objectively document the existing model. This will facilitate future changes.

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The authors would like to express their sincere appreciation to Mr. L.F. Kerzner who made parts of the Annexes of Reference [12] available for the production of this document.

THE MATHEMATICS OF LOGAN(LCC)

INTRODUCTION

1. The Directorate of Logistics Analysis (D Log A) conducts a wide range of analytical studies and provides decision assistance through the design, development and evaluation of specific decision support systems. Particularly, under activity 42516, D Log A designs, develops and evaluates a variety of decision support methods required for CF equipment life cycle costing, level of repair analysis, and spare parts analysis. This report concentrates on a Life Cycle Costing model called LOGAN(LCC).

2. This report does not provide a justification for the calculations done within LOGAN(LCC). Some mathematical methods used within this model are admittedly less than ideal. We will feel free to emphasize shortcomings in future reports discussing the necessary improvements. This paper documents the current model; there is no attempt to justify it.

3. Life Cycle Costing (LCC) is defined as the study of all the costs of an equipment or system arising over its entire life. LCC estimates the "cradle to grave" cost of a system. LCC is the process of evaluating various alternatives to choose the best way to employ resources.

4. In March 1978, Bell-Northern Research Ltd. delivered the original version of the LOGAN(LCC) model [1, 2, 3, 5]. The work had been done under contract for the Department of National Defence's Directorate of Engineering and Maintenance, Planning and Standardization (DEMPS). The object of the contract was to develop a model that would "form a common starting point for LCC evaluations, allow flexibility for inclusion of peculiar electronic and mechanical projects, provide programmable facility to compute cost dependent on effectiveness, and permit expenditure adjustments to a current dollar value" [1, foreword]. Later on, D Log A was asked to improve the model. The various improvements made to the program, then called DND LCC, have been documented in several reports [6, 7, 8, 9, 10, 11, 12, 13]. More information on the historical development of the model can be found in Reference [11].

5. This paper is the result of a review of the DND LCC program version 2.1 dated June 1993 and a review of previous documentation. The DND LCC program has been integrated into LOGAN (LOGistics ANalyzer), and the aim of this paper is to document the basic calculations done within the LCC module of LOGAN: LOGAN(LCC). The paper is largely based on Annexes A, B and C of [12]. It explains the mathematical LCC model.

AN ELEMENTARY COST ANALYSIS

6. A large number of LCC calculations involve a combination of summations and multiplications, i.e. summations of costs and multiplications by factors such as the inflation rate. The basic equations used within the model are explained below and are occasionally provided in different forms (some more readily used within the program).

7. Let C_0 be the present cost of an item and r, the inflation rate. Suppose the cost is deferred m years and let C_m denote the item cost in year m. Then

$$C_m = C_0(1+r)^m \qquad (1)$$

where $m = 0,1,2,...$

8. Next, suppose C_0 recurs for the next n years , instead of being deferred. Then after n years, the total of the costs, T_n, is given by

$$T_n = C_0[1+(1+r)+(1+r)^2+...+ (1+r)^n] \qquad (2)$$

where $n = 0,1,2,...$

9. This equation may also be expressed as:

$$T_n = C_0[1+(1+r)[1+(1+r)[....]]] \qquad (3)$$

where the term $1+(1+r)$ in the last formula is repeated n times.

10. These formulae are algebraically equivalent to:

$$T_n = C_0\frac{(1+r)^{n+1}-1}{r} \qquad \text{if } r > 0 \text{ and}$$

(4)

$$T_n = (n+1)C_0 \qquad \text{if } r = 0.$$

11. Next, let $\alpha_0,...,\alpha_{k-1}$ be a sequence of k non-negative real numbers whose sum is 1. Each α_i will represent the proportion of a cost incurred during year i. This sequence is used, for example, in the procurement of a fleet of vehicles over k years. In this case, α_i refers to the fraction purchased in year i.

12. Costs are incurred per unit purchased. However, the actual cost spent in any one year must be inflated to that year's unit cost. Let C_0 represent the present cost of the entire activity. The amount spent on the entire activity over k years, T_{k-1}, is

$$T_{k-1}=C_0[\alpha_0+\alpha_1(1+r)+...+\alpha_{k-1}(1+r)^{k-1}]. \qquad (5)$$

13. The most general situation combines factors from the above equations. Suppose the maintenance cost of a vehicle fleet in year 0 dollars, is known. The fleet will be purchased over k+1 years in proportions given by α_i , i = 0, 1,..., k and the first purchase will occur in year m. The vehicles are expected to have a service life of n years. Then the maintenance cost, T_L, associated with the present cost of maintenance, C_0, will be:

$$T_L = C_0(1+r)^m[\alpha_0+\alpha_1(1+r)+...+ \alpha_k(1+r)^k][1+(1+r)+...+ (1+r)^{n-1}]. \qquad (6)$$

14. LOGAN(LCC) first determines the cost of each activity in a way specific to each activity. Then, the costs with the same inflation rate are summed together. Inflation rates are then applied to the appropriate sums to find their contribution to the life cycle cost. The total of these contributions determines the project's life cycle cost.

15. Two types of uncertainty analyses can be done to see how uncertainties impact on the project's life cycle cost. This is done because of the uncertainty associated with a number of input values used in the calculation of the project life cycle cost. One analysis is stochastic and considers the input values as random variables from a triangular distribution, while the other uses fuzzy arithmetic and considers the input values as fuzzy numbers.

THE STOCHASTIC APPROACH

16. To model the uncertainty associated with cost estimation, a stochastic approach can be used. The user provides the most likely value of a given parameter along with lower and upper values. The stochastic approach considers the parameter (i.e. a cost or a parameter contributing to the calculation of a cost) as a random variable whose probability density function is triangular. The moments of the triangular distribution can be easily calculated. The project life cycle cost is a summation and multiplication of these costs. Therefore, its cumulative distribution function can be approximated, assuming independence between costs.

17. The assumption of independence between costs is not realistic. Costs are not the result of a controlled experiment. However the assumption is often necessary to simplify or even permit the estimation.

THE MOMENTS OF A DISTRIBUTION

18. In this section definitions related to raw and central moments of a distribution are given. Equivalent expressions are also presented and are used in LOGAN(LCC) to determine the central moments of the triangular distribution based on its raw moments. A more complete discussion of the moments of a distribution can be found in [4].

19. If X is a continuous random variable with a probability density function f(x), the expected value of the mathematical function h(X) is defined by:

$$E(h(X)) = \int_{-\infty}^{+\infty} h(x)f(x)dx \ . \tag{7}$$

Note that for $h(X) = X^k$ we obtain $\mu'_k = E(X^k)$, the k^{th} raw moment of the distribution.

20. When $h(X) = X$, the resulting term, $\mu'_1 = E(X)$, is called the mean of the distribution. The mean gives a descriptive measure of the location of the distribution. The k^{th} moment about the mean or central moment is defined as

$$\mu_k = E[(X-\mu'_1)^k] \ . \tag{8}$$

Note that $\mu_1 = 0$.

21. Most distributions are completely specified once all their raw moments are known. However many distributions can be adequately described by their first four raw moments. We will thus concentrate on them.

22. The second moment about the mean is a measure of dispersion. It is known as the variance and is denoted as

$$V(X) = \mu_2.$$

An equivalent definition in raw moments is

$$V(X) = \mu'_2 - (\mu'_1)^2. \tag{9}$$

23. The third moment about the mean is related to the asymmetry or skewness of a distribution and is denoted as μ_3. A useful formula for μ_3, in terms of its raw moments, is

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2(\mu'_1)^3. \tag{10}$$

The quantity

$$\beta_1 = \frac{\mu_3}{(\mu_2)^{3/2}} \tag{11}$$

measures the skewness of the distribution relative to its spread. This standardization permits the comparison of the symmetry of two distributions whose scales of measurement differ.

24. The fourth moment about the mean is related to the peakedness or the kurtosis of the distribution and is denoted as μ_4. As above, a useful formula for μ_4 is

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6(\mu_1')^2\mu_2' - 3(\mu_1')^4 . \qquad (12)$$

The quantity

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} \qquad (13)$$

is a relative measure of kurtosis, independent of scale.

THE THREE POINT COST ESTIMATION

25. Under the stochastic approach for LOGAN(LCC), each parameter supporting estimates of lower, most likely and upper values is assumed to follow a triangular distribution. This assumption was made to simplify the burden of data gathering. The triangular distribution generally provides an imperfect representation of the actual distribution, but it is simple and convenient to use. Its use is based on the premise that very little information is available about the actual distribution, and that only subjective estimates of "high", "low" and "most likely" values are available.

26. A triangular distribution is specified by three distinct parameters: the lower bound L, the most likely value M and the upper bound H. We define h by

$$h = \frac{2}{H-L} , \qquad (14)$$

where h is the height of the triangle with base length H-L and of unit area. The triangular probability density function compatible with this information is given by

$$f(x) = \frac{h}{M-L}(x-L) \qquad L \leq x \leq M$$

$$f(x) = \frac{h}{H-M}(H-x) \qquad M \leq x \leq H \tag{15}$$

and is shown in Figure 1.

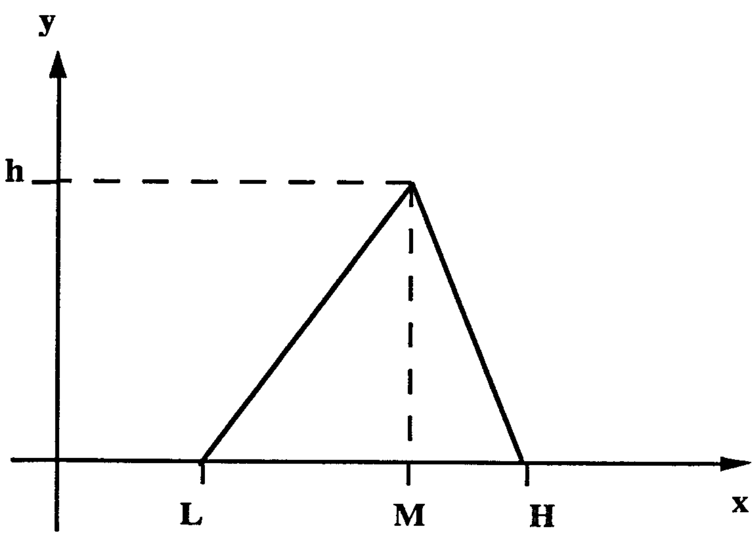

Figure 1: Triangular Distribution

27. The m^{th} raw moment or moment about zero for the triangular distribution is evaluated directly.

$$\int_L^H x^m f(x)dx = \int_L^M x^m f(x)dx + \int_M^H x^m f(x)dx$$

$$= \frac{h}{M-L}(x-L)\frac{x^{m+1}}{m+1}\Big|_L^M - \frac{h}{M-L}\int_L^M \frac{x^{m+1}}{m+1}dx$$

$$+ \frac{h}{H-M}(H-x)\frac{x^{m+1}}{m+1}\Big|_M^H + \frac{h}{H-M}\int_M^H \frac{x^{m+1}}{m+1}dx. \tag{16}$$

28. The lower limit of the first term and the upper limit of the third term of the last equation reduce to zero. Furthermore, the remaining components are equal but of opposite

sign. Therefore what remains to be evaluated are the two integral terms. Substituting for h we get

$$\int_L^H x^m f(x)dx = \frac{2}{(m+1)(m+2)(H-L)}\left\{\frac{H^{m+2}-M^{m+2}}{H-M} - \frac{M^{m+2}-L^{m+2}}{M-L}\right\}. \quad (17)$$

29. That expression may be written as

$$\int_L^H x^m f(x)dx = \frac{2}{(m+1)(m+2)(H-L)} \sum_{j=0}^{m} [H^{j+1}-L^{j+1}]M^{m-j}$$

$$= \frac{2}{(m+1)(m+2)} \sum_{j=0}^{m} \sum_{k=0}^{j} H^k L^{j-k} M^{m-j}$$

$$= \frac{2}{(m+1)(m+2)} \sum_{j+k+p=m} H^j L^k M^p \quad . \quad (18)$$

It can be shown that this last expression is true even if L, M and H are not distinct.

30. It is important to note here that the lower and upper values provided by the user are respectively the 5^{th} and 95^{th} percentiles as opposed to the lower bound L and the upper bound H (Figure 2). The values of L and H are derived iteratively from the percentiles as suggested in [14].

31. This choice of the 5^{th} and 95^{th} percentiles is based on the numerical comparison between the approximation of the mean and variance by different triangular models, for a set of Beta distributions [15]. The Beta distribution occurs rather often in statistics. The idea is that assuming that the data for the approximations are assessed with perfect accuracy, we can compare the estimates of the mean and the variance with the actual values i.e. the mean and variance of the Beta distribution. This simulates the situation where only those points required by each approximation are elicited from the expert (with perfect accuracy) while the type of distribution remains unknown. In [15] it was shown that using the 5^{th} and 95^{th} percentile and the most likely value or mode gives a better approximation of the mean and variance, than if the lower bound L, upper bound H and the mode had been used.

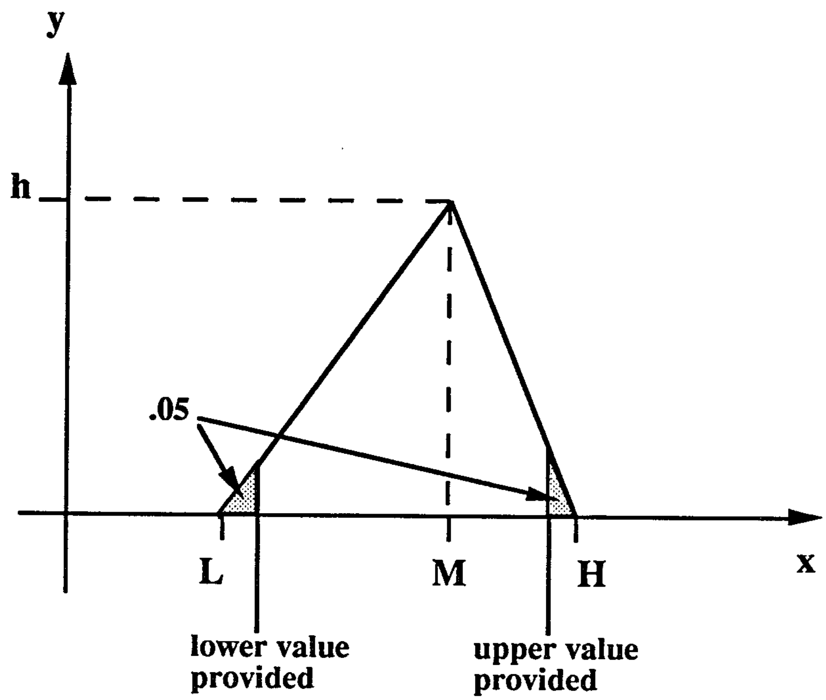

Figure 2: Triangular Distribution with lower and upper values provided

SUMMATION AND MULTIPLICATION OF VARIABLES AND THEIR STATISTICAL DISTRIBUTION

32. If we let $Z = \sum_{i=1}^{p} X_i$ where $X_i, i=1, \ldots, p$ are independent random variables, then we can show that

$$\begin{aligned} E(Z) &= \sum_{i=1}^{p} E(X_i) \\ V(Z) &= \sum_{i=1}^{p} V(X_i) \\ \mu_3(Z) &= \sum_{i=1}^{p} \mu_3(X_i) \\ \mu_4(Z) &= \sum_{i=1}^{p} \mu_4(X_i) + 6\sum_{i=1}^{p} \sum_{j=i+1}^{p} V(X_i)V(X_j). \end{aligned} \tag{19}$$

The results for the first four central moments of the sum of two independent random variables in terms of each variable's central moments are obvious.

33. Similarly if $Z = \prod_{i=1}^{p} X_i$ where X_i, i=1, ..., p are independent random variables, it can be shown that

$$
\begin{aligned}
E(Z) &= \prod_{i=1}^{p} E(X_i) \\
V(Z) &= \prod_{i=1}^{p} E(X_i^2) - \prod_{i=1}^{p} E^2(X_i) \\
\mu_3(Z) &= \prod_{i=1}^{p} E(X_i^3) - 3\prod_{i=1}^{p} E(X_i^2)E(X_i) + 2\prod_{i=1}^{p} E^3(X_i) \\
\mu_4(Z) &= \prod_{i=1}^{p} E(X_i^4) - 4\prod_{i=1}^{p} E(X_i^3)E(X_i) + 6\prod_{i=1}^{p} E^2(X_i)E(X_i^2) \\
&\quad - 3\prod_{i=1}^{p} E^4(X_i).
\end{aligned}
\tag{20}
$$

34. The results for the first four central moments of the product of two independent random variables in terms of each variable's central moments are, upon reduction:

$$
\begin{aligned}
E(X_1X_2) &= E(X_1)E(X_2) \\
\mu_2(X_1X_2) &= \mu_2(X_1)E(X_2)^2+\mu_2(X_2)E(X_1)^2 \\
&\quad +\mu_2(X_1)\mu_2(X_2) \\
\mu_3(X_1X_2) &= \mu_3(X_1)E(X_2)^3+\mu_3(X_2)E(X_1)^3 \\
&\quad +3\mu_3(X_2)\mu_2(X_1)E(X_1)+3\mu_3(X_1)\mu_2(X_2)E(X_2) \\
&\quad +6E(X_1)\mu_2(X_1)E(X_2)\mu_2(X_2)+\mu_3(X_1)\mu_3(X_2) \\
\mu_4(X_1X_2) &= \mu_4(X_1)E(X_2)^4+\mu_4(X_2)E(X_1)^4 \\
&\quad +6\mu_2(X_2)\mu_2(X_1)(E(X_1)E(X_2))^2 \\
&\quad +12E(X_1)E(X_2)[E(X_1)\mu_2(X_1)\mu_3(X_2)+E(X_2)\mu_2(X_2)\mu_3(X_1)] \\
&\quad +12E(X_1)E(X_2)\mu_3(X_1)\mu_3(X_2)+6\mu_4(X_2)\mu_2(X_1)E(X_1)^2 \\
&\quad +6\mu_4(X_1)\mu_2(X_2)E(X_2)^2 \\
&\quad +4E(X_1)\mu_3(X_1)\mu_4(X_2)+4E(X_2)\mu_3(X_2)\mu_4(X_1) \\
&\quad +\mu_4(X_1)\mu_4(X_2).
\end{aligned}
\tag{21}
$$

35. As mentioned previously, LCC calculations are a combination of summations and multiplications. Each cost is seen as a random variable following a triangular distribution. Assuming independence between costs, we find the first four central moments of the random variable "project life cycle cost".

36. We are able to determine the form of the cumulative distribution function of the project life cycle cost X using the Gram-Charlier Type A series [4]. The distribution function f(x), centralized about its mean, may be expanded formally as:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\left[1 + \frac{(\mu_2-1)}{2} H_2(x) + \frac{\mu_3}{6} H_3(x) + \frac{(\mu_4-6\mu_2+3)}{24} H_4(x) + \ldots\right] . \quad (22)$$

37. Here, $H_r(x)$ is the Chebyshev-Hermite polynomial of order r, defined by the relationship:

$$\left(-\frac{d}{dx}\right)^r e^{-x^2/2} = H_r(x)\, e^{-x^2/2} . \quad (23)$$

It follows that

$$\begin{aligned} H_0(x) &= 1 \text{ by convention} \\ H_1(x) &= x \\ H_2(x) &= x^2-1 \\ H_3(x) &= x^3-3x \\ H_4(x) &= x^4-6x^2+3 \\ H_5(x) &= x^5-10x^3+15x \\ &\ldots \end{aligned} \quad (24)$$

38. The cumulative distribution function associated with a known set of moments can then be expressed as:

$$\int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}dx$$

$$-\frac{1}{\sqrt{2\pi}} e^{-x^2/2}\left[\frac{(\mu_2-1)}{2} H_1(x) + \frac{\mu_3}{6} H_2(x) + \frac{(\mu_4-6\mu_2+3)}{24} H_3(x) + \ldots\right] . \quad (25)$$

In LOGAN(LCC) it is Edgeworth's form of the Type A series [4] which is used. Specifically the cumulative distribution function is approximated by

$$\int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}dx - \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\left[\frac{\beta_1}{6} H_2(x) + \frac{(\beta_2-3)}{24} H_3(x) + \frac{\beta_1^2}{72} H_5(x) \right], \quad (26)$$

where β_1 and β_2 are given by (11) and (13) respectively.

39. Using this approximation, we will be able to evaluate the probability that the project does not exceed a specified level of life cycle cost. We will refer to this risk analysis as the stochastic approach as opposed to the fuzzy approach which will be discussed in the following section.

FUZZY ARITHMETIC

40. A Fuzzy Number can be considered as an extension of a confidence interval. Formally, a Fuzzy Number A is a nested family of closed intervals (decreasing with increasing index α)

$$A_\alpha = [a_1(\alpha),a_2(\alpha)], \quad \alpha \in [0,1] \quad (27)$$

so that the interval at $\alpha = 1$ is a single point.

41. The value of the index associated with an interval used in defining a Fuzzy Number specifies a plausibility rating for the interval. This rating is the minimum degree of certainty that the measured attribute is associated with values in the interval. The point associated with the plausibility rating $\alpha = 1$ means that the point is a certainly realizable instance of the measured attribute. On the other hand, the interval associated with the plausibility rating zero indicates that no possible realization of the attribute exists outside

this interval. Intermediate index values are associated with intervals at intermediate states of possibility of attribute realization.

42. The sum of two Fuzzy Numbers A and B is given through the following operation on each of their associated intervals A_α and B_α:

$$\begin{aligned} A_\alpha \,(+)\, B_\alpha &= [a_1(\alpha),a_2(\alpha)] + [b_1(\alpha),b_2(\alpha)] \\ &= [a_1(\alpha)+b_1(\alpha),a_2(\alpha)+b_2(\alpha)] \ . \end{aligned} \tag{28}$$

Similarly, the product of two Fuzzy Numbers is given through:

$$\begin{aligned} A_\alpha \,(*)\, B_\alpha = [&\min\{a_1(\alpha)b_1(\alpha),a_1(\alpha)b_2(\alpha),a_2(\alpha)b_1(\alpha),a_2(\alpha)b_2(\alpha)\}, \\ &\max\{a_1(\alpha)b_1(\alpha),a_1(\alpha)b_2(\alpha),a_2(\alpha)b_1(\alpha),a_2(\alpha)b_2(\alpha)\}] \ . \end{aligned} \tag{29}$$

The operations given here can be used to calculate life cycle costs from Fuzzy inputs. The result would be a specification of life cycle cost as a Fuzzy Number.

43. The easiest way of specifying a Fuzzy Number is to use a three point estimation procedure. A number corresponding to the point associated with Index One should be selected. Two other quantities, bracketing this number, should also be selected to correspond to the bounds of the interval for Index Zero. Let the lower bound, the central value and the upper bound of a quantity be given by L, C and U respectively. Then for a specific intermediate Index α, let its interval be given by:

$$[L+\alpha(C-L),U+\alpha(C-U)] \ . \tag{30}$$

The set of these intervals, indexed on α, defines a Fuzzy Number compatible with the given three point estimation information.

44. Using Fuzzy Arithmetic in the LOGAN(LCC) model requires the same data specified for stochastic risk analysis. The most likely value of a given cost is provided by the user along with a lower and upper value to represent respectively C, L and U. Internally, the model carries the locations of the intervals associated with an evenly spaced

set of index values. The summation and product operations, applied to each of these intervals, produce interval results. Eventually, the intervals associated with the project life cycle cost are derived.

45. Stochastic risk analysis produces narrower bounds for the range of potential life cycle costs when compared to Fuzzy risk analysis. However, the wider bounds of Fuzzy Arithmetic are better suited for those situations where solid data is lacking.

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This paper describes the mathematics behind the Life Cycle Costing module of LOGAN. The stochastic and Fuzzy Arithmetic methods are explained.

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