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Peter Tikuisis

Predicting survival time for cold exposure

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Abstract The prediction of survival time (ST) for cold exposure is speculative as reliable controlled data of deep hypothermia are unavailable. At best, guidance can be obtained from case histories of accidental exposure. This study describes the development of a mathematical model for the prediction of ST under sedentary conditions in the cold. The model is based on steady-state heat conduction in a single cylinder comprised of a core and two concentric annular shells representing the fat plus skin and the clothing plus still boundary layer, respectively. The ambient condition can be either air or water; the distinction is made by assigning different values of insulation to the still boundary layer. Metabolic heat production (M) is comprised of resting and shivering components with the latter predicted by temperature signals from the core and skin. Where the cold exposure is too severe for M to balance heat loss, ST is largely determined by the rate of heat loss from the body. Where a balance occurs, ST is governed by the endurance time for shivering. End of survival is marked by the deep core temperature reaching a value of 30° C. The model was calibrated against survival data of cold water (0 to 20° C) immersion and then applied to cold air exposure. A sampling of ST predictions for the nude exposure of an average healthy male in relatively calm air (1 km/h wind speed) are the following: 1.8, 2.5, 4.1, 9.0, and >24 h for −30, −20, −10, 0, and 10° C, respectively. With two layers of loose clothing (average thickness of 1 mm each) in a 5 km/h wind, STs are 4.0, 5.6, 8.6, 15.4, and >24 h for −50, −40, −30, −20, and −10° C. The predicted STs must be weighted against the extrapolative nature of the model. At present, it would be prudent to use the predictions in a relative sense, that is, to compare or rank-order predicted STs for various combinations of ambient conditions and clothing protection.

P. Tikuisis
Defence and Civil Institute of Environmental Medicine,
1133 Sheppard Avenue West, P.O. Box 2000, North York, Ontario, Canada M3M 3B9

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Introduction

Exposure to cold can be life-threatening to the ill-equipped or unprotected individual. Although appropriate levels of protection can usually be provided or foreseen for most situations, circumstances can lead to extreme and/or lengthy exposures. To analyse possible scenarios and to be better prepared for contingencies, the prediction of survival time (ST) in the cold is essential. Predictions are often based on experience; however, observations of controlled exposures to cold are usually limited to mild levels of hypothermia, of the order of a 2–3° C drop in central core temperature, which is far from any risk of death. Published data on cold survival are mostly limited to accidental cases and therefore as guidelines are applicable only to unique situations.

An alternative approach for the prediction of ST is by the use of mathematical models that simulate the physical and physiological processes underlying human response to cold. However, the lack of information on such processes, especially for deep hypothermia, makes this a difficult extrapolative task (Timbal et al. 1976; Maidment 1993). Some models are simply based on a linear extrapolation of body heat loss (Hall 1972) or rectal temperature cooling (Hayward and Eckerson 1984). Other more complex models (Tikuisis 1989; Wissler 1985) focusing on whole-body thermoregulation involve a large number of parameters, which would require considerable adjustment for extrapolation to conditions of deep hypothermia. An intermediate approach that concentrates on the balance between heat loss and heat generation may be more suitable for predicting ST.

The major determinant of ST is the relative severity of the cold stress. Under extreme conditions where body heat loss exceeds the individual's ability to generate heat, the predicted time to a low deep body temperature is largely reduced to a heat conduction problem and the

linear approximation made by Hayward and Eckerson (1984) may be applicable. If, on the other hand, heat generation balances heat loss, then ST essentially depends on how long such heat generations can be sustained. Information on the latter is almost unknown and, therefore, certainty in the prediction of ST will diminish with increased cold stress. At best, rational models can be constructed to reflect the present state of knowledge and calibrated with well-documented cases of survival in the cold. Greater certainty can be attached to predictions involving interpolation amongst known cases of survival than those involving extrapolation.

The objective of this study is to construct and calibrate a suitable model for predicting ST to cold exposure. Emphasis will be placed on the prediction of central body temperature and not of the extremities. Consequently, the model developed in this work will not be applicable for predicting cold injury, but instead will be configured to represent the trunk of the body and its rate of cooling. Despite the inherent uncertainties to the development of such a model, there is sufficient information to proceed. The need for predictions through a rationally derived model outweights the present reliance on the interpolation of scattered data. Furthermore, a rational model will help to identify the physical and physiological processes that are at present poorly understood and require further research, ultimately leading to more reliable model predictions.

Methods

The model has been developed as follows.

Configuration

The model is based on steady-state head conduction in a cylindrical core-multishell configuration, as depicted in Fig. 1. The shell is comprised of two distinct regions, the inner one representing the skin plus subcutaneous fat, and the outer one representing the external insulation. It is inherently assumed that properties are uniform within a region. The steady-state condition is a simplification that excludes the initial transient response to cold. This response is usually characterized by vasoconstriction which may delay the drop in deep body temperature for up to 1 h (Vallerand et al. 1993). Neglect of this transient period is not undesirable since the subsequent ST predictions should be conservative. Finally, a deep core temperature of 30° C is taken as the endpoint for ST in this study. Although death usually occurs when the deep body temperature is colder than 30° C, unconciousness develops near this value (Golden 1973) and without outside intervention, death is imminent with further cooling.

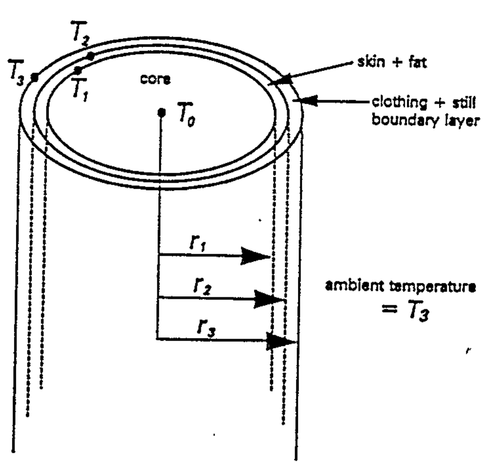

Fig. 1 Schematic representation of the core-multishell model (see text for explanation of variables

Steady-state heat loss (Q) is predicted by

$$Q = \frac{T_0 - T_3}{R_{eff}} \tag{1}$$

where T_0 and T_3 refer to cylindrical centre (deep core) and ambient temperatures, respectively (see Fig. 1), and R_{eff} (in units of °C · m² per W) is the effective thermal resistance for bodies having uniform internal heat production in the core region only and given by (see the Appendix for derivation):

$$R_{eff} = \frac{r_3}{2 \cdot k_{co}} + \frac{r_3}{k_{sf}} \cdot \ln\frac{r_2}{r_1} + \frac{r_3}{k_{il}} \cdot \ln\frac{r_3}{r_2} \tag{2}$$

where r_i is the compartment radius, k is the thermal conductivity, and *co*, *sf*, and *il* refer to the core, skin plus subcutaneous fat, and external insulation layer (clothing plus an unstirred ambient boundary layer), respectively. For brevity, the three consecutive terms of Eq. 2 will be designated as a_1, a_2, and a_3.

If the volume and surface area of the model cylinder are matched to the corresponding values of a human body, then the model radius would be considerably smaller than the radius of the human trunk. Also, the surface area of the body that actually exchanges heat with the environment ranges from 50 to 80% of the total body surface area (Fourt and Hollies 1970). To reflect this and to obtain a closer correspondence between the model and human trunk dimensions, the model radius is first determined by matching characteristics to the whole body (i.e., twice the volume of the body divided by its surface area) and then increasing this value by a factor of (1+α), i.e.,

$$r_2 = (1+\alpha) \cdot \left(\frac{2 \cdot wt}{\rho \cdot A_D} \right) \tag{3}$$

where wt, ρ, and A_D are the weight, density, and surface area of the body, respectively. The value of α will be treated as a model parameter and determined via the calibration of the model. The skin plus subcutaneous fat thickness ($r_2 - r_1$) is based on a regression equation for young males (Durnin and Womersley 1974) and a body density determined from the percentage of body fat (Siri 1961). The 'standard' anthropometric values used in this study are a body fatness (*bf*) of 17.7%, and height and weight of 1.77 m and 73.9 kg, respectively, based on males aged from 20 to 29 years (CFLRI 1988). The latter values yield a body surface area (A_D) of 1.90 m² (DuBois and DuBois 1916).

Thermal conductivities of tissue are 0.60 and 0.35 W/m per °C for core and for skin plus subcutaneous fat, respectively (Sekins and Emery 1982); the latter value pertains to cold tissue. The thermal conductivity of clothing is assumed to be close to the value for still air, i.e., 0.042 W/m per °C (Danielsson 1993). Therefore, 1 Clo (0.155° C · m² per W) of insulation is represented by a thickness of 0.0065 m. The thickness of the still boundary layer is considered further below.

Temperatures

The deep core temperature is assumed to begin at 37.0° C. At steady state, the skin temperature is given by

$$T_2 = T_3 + a_3 \cdot Q \tag{4}$$

and the core-subcutaneous fat boundary temperature is given by

$$T_1 = T_2 + \frac{r_2}{r_3} \cdot a_2 \cdot Q \tag{5}$$

Note that a_3^1 represents the combined radiative and convective heat transfer coefficient. Mean core temperature is given by (Sekins and Emery 1982)

$$\overline{T}_{co} = \frac{T_0 + T_1}{2} \tag{6}$$

and mean skin plus subcutaneous fat temperature is approximated by

$$\overline{T}_{sf} = \frac{T_1 + T_2}{2} \tag{7}$$

Finally, the mean body temperature is given by

$$\overline{T}_b = f_{co} \cdot \overline{T}_{co} + (1 - f_{co}) \cdot \overline{T}_{sf} \tag{8}$$

where f_{co} is the fraction by volume of the core compartment. The change in the body mean temperature is determined by

$$\Delta \overline{T}_b = \frac{(M - Q) \cdot \Delta t}{cb} \tag{9}$$

where M is the metabolic rate, cb is the body heat capacity, and Δt is the time step in the numerical procedure. Hence, using the relationships of Eqs. 1 and 4–8, the resultant deep core temperature is given by

$$T_0 = \frac{a_2 \cdot T_3 + \left(\frac{r_3}{r_2}\right) \cdot \left[2 \cdot \overline{T}_b \cdot R_{eff} - (2 - f_{co}) \cdot T_3 \cdot (R_{eff} - a_3)\right]}{a_2 + \left(\frac{r_3}{r_2}\right) \cdot \left[f_{co} \cdot R_{eff} + (2 - f_{co}) \cdot a_3\right]} \tag{10}$$

The numerical procedure used assumes a quasi-steady state. That is, while model temperature and heat variables are calculated assuming steady state at each time step, any heat imbalance resulting in a non-zero value of $\Delta \overline{T}_b$ (Eq. 9) will lead to changes in the values of the model variables in the next time step.

Heat generation and endurance

This study considers the subject in a sedentary condition only. Consequently, the sources of heat generation are limited to the resting and shivering metabolic rates. Heat generation through shivering for healthy uninjured individuals (in W/m²) is predicted according to Tikuisis et al. (1988) as:

$$M_{shiv} = \frac{6.4 \cdot (32 - T_{2sig})^2 + 75.8 \cdot (32 - T_{2sig}) \cdot (37 - T_0)}{A_D \cdot bf} \tag{11}$$

where $T_{2\,sig}$ is the temperature signal of the skin equal in value to T_2. To avoid an unrealistically high efferent shivering command when T_2 is very low (which represents an extrapolation of the original data from which Eq. 11 was regressed), the predicted value of M_{shiv} is not allowed to exceed 200 W/m² ($M_{shiv\,max}$). Coupled with the thermoneutral resting metabolic rate (M_{rest}) fixed at 50 W/m², the resultant maximum allowable M represents five times the resting value in concurrence with that reported by Iampietro et al. (1960).

In addition, as the core temperature decreases below 32° C, it is assumed that the overall shivering command weakens. This decrease is modelled through the following empirical reduction factor defined by a hyperbolic secant function that smootly and sigmoidally reduces the shivering heat production by a factor of 100 as the core temperature decreases from 32 to 30° C as follows:

$$M_{shiv} \times sech[2 \cdot (T_0 - 32)^{1.4}] \tag{12}$$

This is a departure from the linear decrease assumed by Timbal et al. (1976) as deep core temperature decreases from 35 to 30° C.

The endurance time of shivering in relation to shivering intensity has been assumed to be analogous to the relationship between endurance time and exercise intensity (Wissler 1985). With increased exercise intensity, endurance time decreases exponentially. When applied to shivering, Wissler (1985) modelled endurance time (in h) as:

$$t_{end} = \frac{18}{L_r} \cdot e^{-4.0 \cdot L_r} \tag{13}$$

where L_r represents the relative effort of shivering as measured by the ratio $M_{shiv}/M_{shiv\,max}$. Although it is known that fat is also metabolized during shivering (Vallerand and Jacobs 1989), glycogen storage in the above calculation serves as a reference point and the value of 18 h in Eq. 13 is a calibration factor that corresponds to observed fatigue during shivering (Wissler 1985). Accordingly continuous shivering at 25, 50, 75 and 100% of maximum shivering intensity have estimated endurance times of about 26.4, 4.9 1.2, and 0.33 h, respectively.

To account for a variable shivering intensity, the end of endurance is predicted when $\sum \Delta t / t_{end}$ equals unity where Δt is the time step and t_{end} is the endurance time corresponding to the shivering intensity during that time step. Once the summation exceeds unity shivering is assumed to continue but at an intensity that is also smoothly reduced by the following factor:

$$M_{shiv} \times sech\left(\frac{\sum \Delta t / t_{end} - 1}{\beta}\right) \tag{14}$$

where β is the second model calibration parameter. Decreasing the value of β enhances the decrease in shivering intensity. The resultant shivering metabolic rate is then added to M_{rest} for the total heat generation (M). For completeness, M_{rest} is also subject to a reduction, in this case due to the Q_{10} effect (Werner and Buse 1988) which states that metabolism decreases with decreasing tissue temperature. In the present model, the factor is

$$M_{rest} \times 2^{(\overline{T}_{co} - 28)/10} \tag{15}$$

if mean core temperature $\overline{T}_{co}$<28° C, otherwise no reduction is assumed. Finally, M is not allowed to exceed heat loss, Q, otherwise mean body temperature would rise contrary to the usual observation.

Calibration

The transfer of heat to the environment has been modelled by conduction only and, therefore, no restriction is placed on the medium to which the model is applied. That is, the thermal insulation of the still boundary layer, whether it be air or water, is the determinant of heat loss to the environment. Consequently, exposure to water was chosen for the calibration of the model since survival times have been more clearly delineated from accidental immersions than from cold air exposure. Figure 2 shows the survival curves based on the reports of Molnar (1946) and Veghte (1972), and the model values for nude and wet clothed conditions.

To obtain the model results for the data of Fig. 2, external insulation values had to be determined and the parameters α (Eq. 3) and β (Eq. 14) were fitted. Assuming a heat transfer coefficient of about 160 W/m² per °C (Boutelier 1979) and a thermal conductivity of 0.6 W/m per °C (Sekins and Emery 1982), the resultant still water boundary layer of about 4 mm provides an insulation of 0.0- Clo. To simulate clothing that may have been worn but is assumed to be thoroughly soaked, the additional insulation was assumed to be 0.07 Clo per intrinsic dry Clo value based on the ratio of thermal conductivities of air to water (i.e., 0.042:0.60). To obtain S- curves that fell reasonably within those of Molnar (1946) and Veghte (1972), the two calibration factors, α and β, were chosen as 0.5 and 0.38, respectively. The former value leads to a model radius that is 50% larger than the average for the whole body, bu

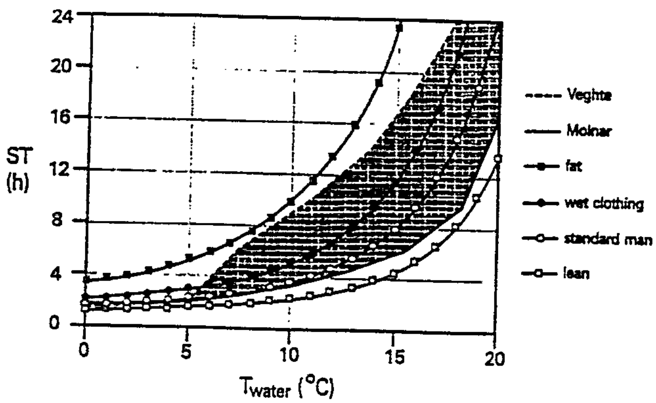

Fig. 2 Survival times (ST in h) plotted against water temperature (T_{water}) based on the reports of Molnar (1946) and Veghte (1972), which are separated by the *shaded region*, and from the model following calibration. The standard man, lean, and fat profiles are based on a nude condition; 'wet clothing' represents clothing that has an intrinsic dry insulation value of 1 Clo reduced to 0.07 Clo in the completely wet condition (for the standard man)

close to the trunk dimension, and the latter value reduces shivering intensity by one-half when the end of shivering endurance is exceeded by 50%.

Effect of wind

Under all cold air exposure conditions, the model assumes the existence of a still air boundary layer on the outermost surface, whether nude or clothed. The effect of wind is to reduce the thickness of this boundary layer. If clothing is worn, there are the additional factors of compression and/or penetration leading to further reductions in insulation due to the disturbance of the internal air layer. The following calculation of the resultant surface insulation follows the method outlined by Danielsson (1993).

Firstly, the effect of wind on the nude condition is addressed. The thermal resistance (in °C · m² per W) of the external still air boundary layer, taking into account radiative and convective heat transfer, is predicted by

$$R_{a,e} = (4.5 + h_{c,e})^{-1} \quad (16)$$

where $h_{c,e}$ is the external heat transfer coefficient (in W/m² per °C) given by

$$h_{c,e} = 12.9 \cdot v^{0.55} \quad (17)$$

and where v is the windspeed in m/s. The conversion to Clo units is made simply by dividing R by 0.155. For example, the insulation value of the still air boundary is approximately 0.5 Clo for a windspeed of 0.5 m/s (1.8 km/h) and 0.1 Clo when wind is increased by a factor of 50 to 25 m/s (90 km/h). These values are consistent with the prediction of Burton and Edholm (1969).

The effect of wind on the insulation of clothing has been examined primarily by empirical methods and theoretical considerations. Using such an integrated approach, Danielsson (1993) has developed a model to calculate the total thermal resistance (including the external still air boundary) of multi-layer clothing ensembles in loose and tight-fitting configurations; fabrics under consideration included blends of cotton and polyester. Key equations based on whole-body application are presented below, but the reader is referred to the original reference for details.
The total thermal resistance is given by

$$R_t = R_{cl} + \frac{R_{a,e}}{1 + 2 \cdot R_{cl}} \quad (18)$$

where $R_{a,e}$ is given by Eq. 16 (except that $h_{c,e}$=11.7·$v^{0.57}$ for loose clothing) and R_{cl} is the intrinsic thermal resistance of the clothing given by

$$R_{cl} = n_l^{0.51} \cdot \left(\frac{1}{4.2 + h_{cl}} + 0.024 \cdot d_f \right) \quad (19)$$

where n_l is the number of air layers (assumed equal to the number of clothing layers), h_{cl} is the convective heat transfer coefficient (4.0 and 5.7 W/m² per °C for loose and tight-fitting clothing, respectively), and d_f is the average material thickness (in mm). It is important to note that Eq. 19 is not applicable for multi-layer clothing where d_f has a high value relative to the thickness of the internal air layer. In such cases, R_{cl} is simply assigned the lower physical limit value of $24 \cdot d_f \cdot n_l$.

As an example, consider a loose-fitting three-layer clothing ensemble having an average layer thickness of 1 mm. According to Eq. 19, the ensemble's intrinsic thermal resistance is 0.256 °C · m² per W or 1.65 Clo. Under windspeed conditions of 0.5 and 25 m/s, the external heat transfer coefficient ($h_{c,e}$) becomes 0.081 and 0.013° units should be W/m² per °C, respectively. From Eq. 18, the respective total insulation (clothing plus still air boundary layer) values are thus calculated as 2.0 and 1.7 Clo.

Results

Test cases

The model was tested against very diverse conditions of air exposure: thermoneutrality, nude exposure at 5° C, and accidental exposures at –20 and –22° C. The first exposure represents no thermal stress and tests the model's steady-state prediction. The second exposure represents a well-documented experimental condition that tests the model's initial prediction during cold stress. The accidental exposures test the model's long-term predictions during severe cold exposure. All conditions assume sedentary activity superimposed by any shivering that may occur.

Two thermoneutral conditions were tested assuming calm air [0.25 m/s (<1 km/h)], nude at 28° C and 1 Clo of insulation (excluding the still are boundary layer) at 21° C. The predicted steady-state metabolic rates and skin temperatures for the standard man (central core temperature remains fixed at 37° C) are 43 and 44 W/m², and 32.2 and 31.8° C, respectively. These values agree reasonably well with the observations of Parsons (1993). The experimental condition involved a 3 h seated nude exposure at 5° C and a windspeed of about 0.5 m/s (Vallerand et al. 1993). Mean body characteristics were 1.74 m in height, 74.3 kg in weight, and 14.0% body fat. The model predicts values of 163 W/m², 35.4° C, and 17.6° C after 3 h of exposure for M, T_0, and T_2, respectively.

The predicted values of heat production and mean skin temperature agree closely with the measured values at the end of the experiment. The predicted central core temperature is, however, about 1° C lower than the measured rectal temperature. This can be attributed to the assumption of uniform heat production in the model; that is, if the intensity of heat production is actually higher towards the core as should be expected, then the core

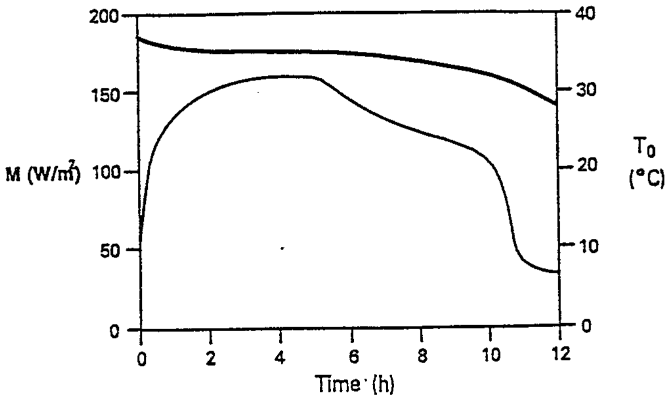

Fig. 3 Model-predicted metabolic rate (*M*; *thin line*) and deep core temperature (T_0; *thick line*) plotted against time for a nude exposure to 5° C air at 0.5 m/s windspeed under sedentary conditions

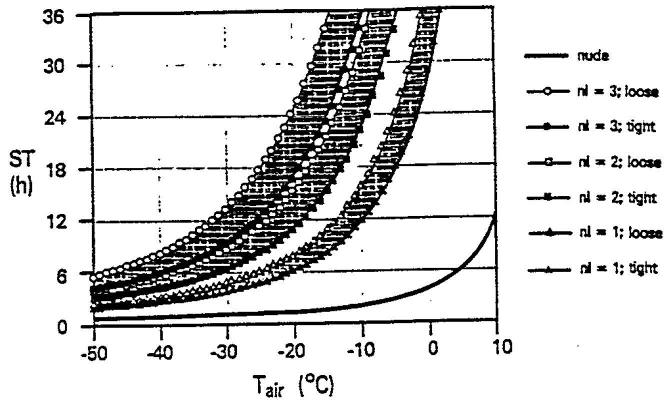

Fig. 4 Model-predicted survival times plotted against air temperature (T_{air}) for different numbers of clothing layers (*nl*; each 1 mm thick) on the standard man exposed to a 5 km/h wind. The *shaded regions* represent the difference in survival times (STs) between loose and tight clothing configurations

temperature should be correspondingly higher. Figure 3 shows the evolution of the model-predicted metabolic rate and deep core temperature. In this case, ST is predicted to be 11.2 h (when T_0=30° C). Note the pronounced decreases in *M*, first when the endurance time for shivering is exceeded after about 5 h of exposure and then when the core temperature drops below 32° C after 10 h exposure.

The first accidental case involved the crash of a CF Hercules 130 near Alert, Northwest Territory, Canada (Canadian Forces 1991). During the approximately 36 h before rescue arrived, the average air temperature was –20° C and the wind changed from calm to blizzard conditions; however, in the shelter of the damaged fuselage where most of the casualties rested with mats on the floor and with the entrance draped by a tarpaulin, a low wind speed can be assumed. Most of the casualties huddled or were snow-covered and their clothing, which included a parka in many cases, was partially wet. At the time of rescue, the causualties were considered near the end of their endurance. For the purposes of simulation, it will be assumed that their deep body core temperature was 35° C at rescue time (the predicted fall in body temperature from this point forward is progressively rapid; see Fig. 3). A numerical search was conducted to determine the corresponding level of insulation consistent with this scenario; its value, including the external still air boundary layer, was predicted to be 2.55 Clo. This estimate is quite reasonable under the given circumstances, especially considering that one member who succumbed to hypothermic death was less insulated and suffered greater exposure than those who survived.

The second accidental case involved a 31-month-old girl found outside her home after 5.7 h of exposure to –22° C in a wind of 33 km/h (J. Dobson; see the Acknowledgements). Her deep core temperature measured 1.5 h later was 14.2° C. Wearing only one boot, a diaper, nightgown, and an open coat, her estimated insulation (excluding the external air boundary layer) is 0.7 Clo (McCullough et al. 1984). Given her height and weight of 0.80 m and 14.5 kg, respectively, and assuming a body fatness of between 15 and 20%, the predicted time to a deep core temperature of 14° C lies between 5.5 and 5.8 h. Noting that wind speed decreases with increasing proximity to the ground (Steadman 1971), these estimates must be consisered low. Halving the wind speed increases the predicted ST values to 5.9–6.3 h. While further body cooling was halted at the time of the girl's discovery, it is unlikely that her deep core temperature would have warmed greatly during the following 1.5 h at room temperature considering the effects of thermal inertia and afterdrop (Romet 1988). Under these circumstances, the model estimates are quite reasonable.

Applications

The last test case demonstrated the model's predictive applicability to diverse body types. On should expect a decrease in ST for thin and less fat individuals since both a decreased body radius and decreased fatness impose less thermal resistance to heat loss, and vice versa. To demonstrate further this relationship, predictions were made for body types representing lean and fat individuals (<10th and >90th percentile of a normal male population; CFLRI 1988) using the following respective anthropometric values of height 1.77 m, weight 66.3 kg, and 11.2% body fat, and 1.77 m, 88.2 kg, and 28.6% body fat. Predicted STs for nude water immersion are shown in Fig. 2). These values are close to the boundary limits indicated by the data of Molnar (1946) and Veghte (1972), and are in agreement with the observations of Hayward and Keatinge (1981).

An interesting application of the model is to examine the benefit of increased clothing insulation and loose versus tight configurations. Figure 4 shows the predicted STs for three different numbers of clothing layers (*nl*=1, 2, and 3), each having a thickness of 1 mm, for the standard man exposed to a 5 km/h wind. Clearly, clothing offers substantial protection over the nude condition and

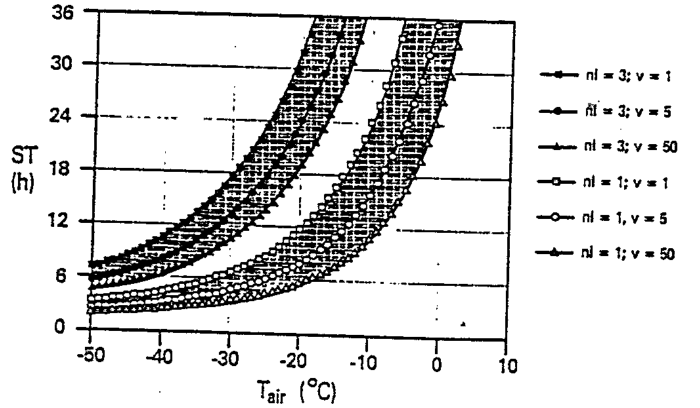

Fig. 5 Model-predicted survival times plotted against air temperature for windspeeds (v) from 1 to 50 km/h (range indicated by the *shaded regions*) and for two different numbers of clothing layers (nl; each 1 mm thick) in a loose configuration worn by the standard man

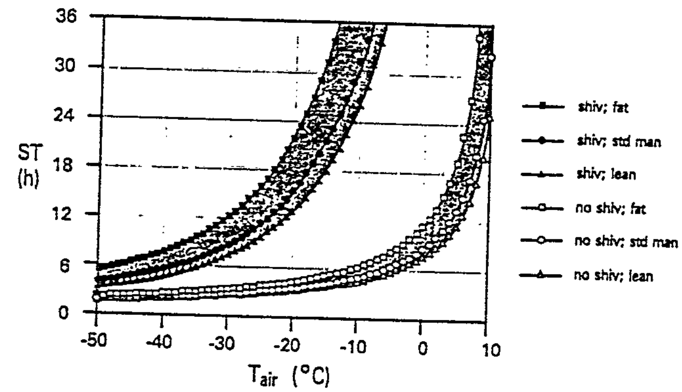

Fig. 6 Model-predicted survival times plotted against air temperature for shivering and non-shivering conditions. The standard (std) man is exposed to a windspeed of 5 km/h with two layers of clothing (each 1 mm in thickness) in a loose configuration. The *shaded regions* represent the difference in STs under the same conditions for body types ranging from lean to fat

further protection increases as the number of layers increase. Also seen is an important increase in ST with loose versus tight configurations. The total calculated insulation values in ascending order from the nude condition are 0.32, 1.07, 1.21, 1.39, 1.59, 1.64, and 1.88 Clo (see Fig. 4).

A further interesting application is the examination of the effect of wind on clothing insulation. Figure 5 shows the predicted STs for two levels of clothing insulation (nl=1 and 3, each 1 mm thick) in a loose configuration for the standard man exposed to a 5 km/h windspeed. Not surprisingly, ST decreases with increased wind speed and the effect is more pronounced with the lower level of insulation. The finding that wind has an increased impact with decreased insulation can be attributed to a relatively greater compression and/or penetration of the clothing material. Total calculated insulation values in ascending order from the nude condition are 1.03, 1.21, 1.43, 1.72, 1.88, and 2.07 Clo (see Fig. 5).

An important comparison in predicted STs can be made by considering shivering versus non-shivering cases. The latter is an extreme condition that may accompany injury or extreme fatigue simulating a worst case scenario. Figure 6 illustrates the predicted STs under these conditions for body types representing lean to fat individuals exposed to a 5 km/h wind and insulated with two layers (each 1 mm thick) of clothing in a loose configuration. Not surprisingly, the benefit of shivering is demonstrated by a marked increase in the predicted ST.

Discussion

In general, ST is predicted to increase hyperbolically with increasing air temperature and with either decreasing wind speed or increasing insulation. This is in agreement with the general trend observed in sea survival cases where ST increases hyperbolically with increasing water temperature (Molnar 1946; Veghte 1972). Indeed, the model was calibrated using such cases and is well-suited for predicting ST for water immersion. ST predictions of more than 36 h for air exposure are not given since the certainty in their values is greatly diminished due to uncertainty in the long-term endurance to generate heat. Considering the speculative nature of the model predictions, care should be exercised in their use: the values may be more meaningful in relative comparisons than in absolute terms. For example, in the case of an exposure to –20° C in a 5 km/h wind, the predicted ST (see Fig. 4) is approximately doubled by doubling the number of clothing layers. It should also be noted that the wind speed used must correspond to the inividual's location and depending on their position above the ground, this value may vary considerably from the meteorological report (Steadman 1971).

The value of M used in Eq. 9 relates to the body surface area (A_D) which is larger than the model surface area (see Eq. 3). Thus, the metabolic rate applied to the model underestimates the body total heat production. For example, if M is 100 W/m^2, then assuming the standard anthropometric values, total heat production (MA_D) is 190 W. However, 100 W/m^2 applied to the model cylinder yields a total heat production [$M \cdot 2\pi \cdot r_2 \cdot l$ where $l = wt/(\pi \cdot \rho \cdot r_2^2)$] of 127 W for the standard man. The difference or amount of heat production not accounted for by the model (63 W in this example) may represent other avenues of body heat loss such as through respiration and convective heat transfer by the blood from the core to the extremities. The latter's contribution is supported by the finding that while the trunk produces most of the body heat during shivering (Bell et al. 1992), the limbs are responsible for most of the body heat loss (Tikuisis 1989).

A primary unknown quantity in the development of the model is the individual's endurance to generate heat through shivering. The present approach adopted the non-linear approximation of Wissler (1985) based on the relationship between exercise intensity and endurance time. Timbal et al. (1976) choose to decrease the shivering metabolic rate linearly when either the deep core temperature dropped below 35° C or the metabolic heat production from shivering exceeded 337.5 Wh per m^2. The model of Timbal et al. predicts, however, a very narrow range of ST for cold air exposure which is inconsistent with the wide range of respective values predicted for cold water immersion. For example, STs of 7.0 and 9.1 h are predicted for a nude exposure to 10° C air for lean (height 1.80 m, weight 65 kg, 4 mm skinfold thickness) and fat (1.70 m, 85 kg, 20 mm skinfold thickness) individuals, respectively. While the predicted value of 7.8 h for the lean individual from the present model agrees closely to the Timbal et al. (1976) prediction, the present model prediction of 26.1 h for the fat individual is considerably larger but consistent with the wide range found for cold water immersion.

Prediction comparisons with other 'survival' models are conjectural because of the lack of data to test the predictions. In cases such as is described above, rational arguments are used to defend one prediction against another. In other cases, instructive comparisons can be made, for example, with the prediction models of Hall (1972) and Hayward and Eckerson (1984). Hall (1972) predicted tolerance times in cold water based on a linear extrapolation of the body heat loss. Assuming a fixed M of 87.2 W/m^2, the reported predicted times to a deep core temperature of 32.8° C are 27 and 97 min for average individuals immersed in 5° C water and protected with a net effective insulation value (after taking compression and wetness into account) of 0.8 and 1.5 Clo, respectively. These times contrast sharply with the present model prediction of more than 24 h, which is attributed to the allowance of additional heat production through shivering that Hall (1972) acknowledged but did not implement.

Closer agreement in predictions are found with the model of Hayward and Eckerson (1984) by assuming a linear rate of fall in deep core temperature. Their prediction for individuals wearing light clothing (0.8 Clo when dry) immersed in ice water is a 50% mortality within 1.5 h. The present model predicts STs of 1.4, 1.9, and 3.9 h for lean, average, and fat individuals, respectively. However, it is doubtful whether this level of agreement would continue with less stressful conditions, especially where the individual could balance heat loss with heat production, in which case the deep core temperature should stabilize.

While experiments may never be conducted to exhaust completely an individual's shivering capacity, long-term exposure to cold can yield important information regarding metabolite utilization and endurance. Also of prime concern is the mechanism whereby the pathophysiology of injury affects the body's thermoregulatory defence mechanisms against cold in terms of vasomotor response and thermogenesis. Several other factors also require future consideration. These include the impact of nutritional and hydration status, fatigue, activity, posture, contact with surfaces, and wetness of clothing. As cautioned earlier, the prediction of ST in cold air is based on theoretical considerations and observations of individuals in states far from hypothermia. Clearly, model predictions are subject to adjustments as more information becomes available for calibration. However, the present cocurrence with various diverse examples of air exposure is supportive given the extrapolative nature of the model.

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Appendix: Derivation of R_{eff}

The bioheat equation (Pennes 1948) expresses the temperature distribution of tissue with internal heat generation, and its steady-state form in cylindrical coordinates is given by

$$\frac{\partial}{\partial r}\left(r\cdot k\frac{\partial T}{\partial r}\right)=-q\cdot r \qquad \text{(A1)}$$

where T is temperature and q is the intensity (i.e., per unit volume) of heat generated. Integrating Eq. A1 leads to

$$r\cdot k\frac{\partial T}{\partial r}\bigg|_{r_i}^{r}=\int_{r_i}^{r} q\cdot\rho\cdot\partial\rho \qquad \text{(A2)}$$

where the limits of integration represent the region of internal heat generation. Continuing,

$$r\cdot k\frac{\partial T}{\partial r}-r_i\cdot k_i\frac{\partial T}{\partial r}\bigg|_{r_i}=-\int_{r_i}^{r} q\cdot\rho\cdot\partial\rho \qquad \text{(A3)}$$

$$\partial T=\frac{\partial r}{r\cdot k}\left(r_i\cdot k_i\frac{\partial T}{\partial r}\bigg|_{r_i}\right)-\frac{\partial r}{r\cdot k}\left(\int_{r_i}^{r} q\cdot\rho\cdot\partial\rho\right) \qquad \text{(A4)}$$

Integrating both sides of Eq. A4 from the inner boundary to the surface leads to

$$T_s-T_i=\int_{r_i}^{r_s}\frac{\partial\eta}{\eta\cdot k}\left(r_i\cdot k_i\frac{\partial T}{\partial r}\bigg|_{r_i}\right)-\int_{r_i}^{r_s}\frac{\partial\eta}{\eta\cdot k}\left(\int_{r_i}^{\eta} q\cdot\rho\partial\rho\right) \qquad \text{(A5)}$$

If the inner surface is insulated, as assumed in this development, then $\frac{\partial T}{\partial r}\big|_{r_i}=0$ and the first term in Eq. A5 goes to zero as r_i goes to zero (note that at steady state, T is constant in the region from r=0 to r_i if no heat is generated in that region). Therefore,

$$T_i-T_s=\int_{r_i}^{r_s}\frac{\partial\eta}{\eta\cdot k}\left(\int_{r_i}^{\eta} q\cdot\rho\cdot\partial\rho\right) \qquad \text{(A6)}$$

Through an integration by parts (i.e., $\int u\cdot\partial v=u\cdot v-\int v\cdot\partial u$) after setting

$$u=\int_{r_i}^{\eta} q\cdot\rho\cdot\partial\rho \quad \& \quad \partial v=\frac{\partial\eta}{\eta\cdot k} \qquad \text{(A7)}$$

so that

$$\partial u=q\cdot\rho\cdot\partial\rho \quad \& \quad v=\int_{r_i}^{\rho}\frac{\partial\eta}{\eta\cdot k} \qquad \text{(A8)}$$

Eq. A6 becomes

$$T_s - T_i = \int_{r_i}^{\eta} q\cdot\rho\cdot\partial\rho \int_{r_i}^{\rho} \frac{\partial\eta}{\eta\cdot k}\Big|_{\eta,\rho=r_i}^{r_s} - \int_{r_i}^{r_s} q\cdot\rho\left(\int_{r_i}^{\rho}\frac{\partial\eta}{\eta\cdot k}\right)\partial\rho \tag{A9}$$

Evaluating the first term at its limits and expanding the second term through $\int_{r_i}^{\rho} = \int_{r_i}^{r_s} - \int_{\rho}^{r_s}$ leads to

$$T_s - T_i = \int_{r_i}^{r_s} q\cdot\rho\cdot\partial\rho \int_{r_i}^{r_s} \frac{\partial\eta}{\eta\cdot k} - \int_{r_i}^{r_s} q\cdot\rho\left(\int_{r_i}^{r_s}\frac{\partial\eta}{\eta\cdot k}\right)\partial\rho + \int_{r_i}^{r_s} q\cdot\rho\left(\int_{r_i}^{r_s}\frac{\partial\eta}{\eta\cdot k}\right)\partial\rho \tag{A10}$$

Noting that the last two terms cancel and that thermal resistance, R, is defined by

$$R = r_s \int_{r}^{r_s} \frac{\partial r}{r\cdot k} \tag{A11}$$

Eq. A10 simplifies to

$$T_i - T_s = \frac{1}{r_s}\int_{r_i}^{r_s} q\cdot r\cdot R\cdot\partial r \tag{A12}$$

Noting also that steady-state heat flux is given by

$$Q = \frac{1}{r_s}\int_{r_i}^{r_s} q\cdot r\cdot\partial r \tag{A13}$$

then

$$\frac{T_i - T_s}{Q} = \frac{\int_{r_i}^{r_s} q\cdot r\cdot R\cdot\partial r}{\int_{r_i}^{r_s} q\cdot r\cdot\partial r} \tag{A14}$$

Since $T_0 = T_i$ at steady state, extending the limits of integration to $r=0$ will lead to

$$\frac{T_0 - T_s}{Q} = \frac{\int_{0}^{r_s} q\cdot r\cdot R\cdot\partial r}{\int_{0}^{r_s} q\cdot r\cdot\partial r} \tag{A15}$$

which is defined herein as the effective thermal resistance, R_{eff}.

For a core-multishell cylindrical model where no heat is assumed to be generated in the shell region, the effective thermal resistance is obtained by integration over the core region only, expressed as

$$R_{eff} = \frac{\int_{0}^{r_1} q\cdot r\cdot R_{co}\cdot\partial r}{\int_{0}^{r_1} q\cdot r\cdot\partial r} \tag{A16}$$

where r_1 is the radius of the core region and R_{co} is its thermal resistance. Further assuming that thermal conductivity values are uniform within each region, then R_{co} can be obtained from the discretized form of Eq. A11 following the substitution of the subscript 3 (representing the external insulation layer radius) for s (external surface of cylindrical model), i.e.,

$$R_{co} = r_3 \sum_{n=1}^{3} \frac{1}{k_n}\cdot\ln\frac{r_n}{r_{n-1}} = \frac{r_3}{k_{co}}\cdot\ln\frac{r_1}{r} + \frac{r_3}{k_{sf}}\cdot\ln\frac{r_2}{r_1} + \frac{r_3}{k_{il}}\cdot\ln\frac{r_3}{r_2} \tag{A17}$$

where the subscripts are defined in the text (see Eq. 2). Finally, assuming uniform heat generation in the core region (i.e., q=constant), the substitution of the above equation into Eq. A16 and subsequent integration leads to

$$R_{eff} = \frac{r_3}{2\cdot k_{co}} + \frac{r_3}{k_{sf}}\cdot\ln\frac{r_2}{r_1} + \frac{r_3}{k_{il}}\cdot\ln\frac{r_3}{r_2} \tag{A18}$$

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