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Low-frequency transient (time domain) electromagnetic fields propagating in a marine environment

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SUMMARY

The propagation of electromagnetic waves in seawater is quite different from the propagation in free air. Both the velocity of propagation and the attenuation of the wave reveal a strong dependency on frequency. When using electromagnetic methods for underwater purposes, the low-frequency bands are preferable since high-frequency components are attenuated very quickly. Consider a transmitted electromagnetic pulse travelling outward from a submerged electric dipole. Such a pulse contains many frequency components. Since the different components travel with different velocity, the shape of the pulse will become more and more distorted as it travels from the source. In seawater, the distance of propagation is very limited to encourage practical applications. However, when the environment contains several electrically different media, the electromagnetic wave can propagate along the boundaries to a much longer distance and this property makes them valuable tools in geophysical prospecting and radio communication. The purpose of this study is twofold: first the paper outlines a technique that generalizes the solution of Maxwell's equations to an arbitrary number of layers; the second goal is to investigate the propagation of low-frequency transient electromagnetic fields in a marine environment modelled as a layered media. The frequency-domain formula for the travelling field of a horizontal electric dipole excited by a current pulse is Fourier transformed to obtain the solution for the field in time domain. Possible application to remote sensing in the ocean is considered. © Crown Copyright 2004. Reproduced with the permission & Her Majesty's Stationary Office. Published by John Wiley & Sons, Ltd.

KEY WORDS: electronicmagnetic; Maxwell's equations; seawater

1. INTRODUCTION

When an electromagnetic (EM) wave packet is transmitted into the air, it propagates with no change in shape since all frequencies in the spectrum of the modulated pulse travel with the velocity of light. In seawater, the propagation of a similar wave packet is a much more complicated phenomenon because the medium is dissipative and dispersive so that the wave number is complex and non-linear with frequency. Consequently, the shape of the wave packet is greatly modified as the pulse travels in the attenuating and dispersive seawater.

This article presents a method for the analysis of the transient EM field in a medium that may be modelled by a layered structure with parameters that change in the vertical direction. The EM field is generated by an electric dipole near the boundary between electrically different

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regions, such as the air over the ocean or seawater over the ocean floor. The presence of the boundary makes the field very different from that of the same dipole in an infinite homogeneous region. There are a number of applications such as EM sounding for geophysical exploration, EM propagation in shallow waters, borehole EM surveying, where this environment model is relatively accurate. Our interest in this study is related to the possible application of EM pulses to underwater communication and remote sensing in a marine environment.

In modelling low-frequency, transient EM fields in a conductive layered structure, the frequency-domain methods are prevalent. Time-domain methods like finite differences are not appealing mainly because of the computational effort involved, and they have predominantly been used in applications involving high-frequency propagation in dielectrics. This is partly due to a different physical background. Low-frequency EM propagation in conducting media such as seawater is a diffusive process that satisfies the heat conduction equation, while in the air the process obeys the classical wave equation. In the conducting media, the EM waves undergo both attenuation and dispersion. This means that, close to the source, the length and time scales are much shorter than in the far field. The scales are widened by the presence of different media with different conductivities. The length and time scales are of less concern in the frequency domain, but are of paramount importance in the time domain because they affect the stability conditions of the numerical updating process.

In this work, the EM field propagation problem in a layered media was solved in the frequency domain. Then, the time-domain solution was computed by Fourier synthesis. The objectives of the theoretical investigations are: (1) to form a sound basis for the numerical work, and (2) to reveal characteristic features of the electromagnetic field in a layered, lossy medium. The aim of the numerical implementations is an efficient determination of the electromagnetic wave field in this kind of medium.Both aspects are important in view of further development of, e.g. efficient and accurate imaging and inversion techniques.

2. FOURIER ANALYSIS

The full vectorial analysis of the transient electromagnetic wave field in a layered media may be performed in a particular efficient way by first solving the problem in the frequency domain, and then obtaining the time-domain solution by Fourier inverse transform

$$\Phi(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(\mathbf{r}, \omega) \exp(\mathrm{j}\omega t)\, \mathrm{d}\omega \tag{1}$$

where ω is the frequency, and $\Phi(\mathbf{r}, \omega)$ the frequency-domain solution obtained via the method described below. The problem contains at least one lossy media, so that a closed-form time-domain solution is not possible. Thus, to obtain the transient solution, one has to integrate equation (1) numerically. Owing to the presence of the lossy medium, we can apply real-axis integration since there are no real poles. Moreover, because $\Phi(t)$ is real valued, it implies that $\Phi(-\omega) = \Phi^*(\omega)$ if ω is real. Hence, the infinite integral can be converted into a semi-infinite integral that is solved numerically using, for example, the inverse discrete Fourier transform (DFT):

$$\Phi(n) = \frac{1}{N} \sum_{k=0}^{N-1} \Phi(k) \exp\left(\mathrm{j}\, \frac{2\pi k n}{N}\right) \tag{2}$$

where N is the number of signal samples.

The fast Fourier transform (FFT) is an extremely efficient algorithm to perform the discrete Fourier transform. When using the DFT, the time of calculation is proportional to N^2, while using the FFT the time is proportional to $N \log_2(N)$ or even better. The algorithm itself is quite complicated and will not be treated here. Routines to perform FFT are available in the literature [1].

In order to transform the EM field to the time domain, the inverse FFT is used. At a certain distance from the electric dipole, the field is calculated for a number of frequencies using the mathematical model presented below. The frequency spectra must be long enough to hold all field values with reasonable amplitude. If the cut-off frequency is too low, vital information may be lost leading to an incorrect transformation. Numerically, imposing that 99% of the signal energy is located between the minimum and maximum frequency fulfils this requirement.

3. ELECTROMAGNETIC PROPAGATION IN A LAYERED STRUCTURE

For the problem considered in this work we used a synthetic marine environment consisting of five horizontal layers. The electromagnetic properties of each layer are considered to be homogeneous, linear and isotropic. It is understood that the magnetic permeability μ_i $(i = 0, 1, 2, 3, \ldots)$ for each layer can be replaced by the constant $\mu = \mu_0$, the permeability of the free space. Since we are considering time-harmonic cases, it is noted that the time factor $\exp(j\omega t)$ is used for the field quantities in the frequency domain and is omitted throughout. The EM propagation modelling is done in the frequency domain for low frequencies where the quasi-static approximation (displacement current neglected) is valid. We shall mainly be concerned with EM sources and observation points (sensors) within the water volume. To obtain the time-varying electromagnetic field at the sensors in seawater, one must solve Maxwell's equations with the boundary conditions imposed by a stratified media.

The x and y directions denote the horizontal plane. This is the plane in which the structure is uniform in its electromagnetic parameters. The z direction is the vertical direction pointing downwards in which the structure varies in its properties. The origin of this Cartesian reference frame is located on the interface of air and seawater, yielding positive z values in the layers of interest. It is convenient to introduce cylindrical polar co-ordinates (ρ, θ, z) with $x = \rho \cos\theta$, $y = \rho \sin\theta$. The very first and last layers of the propagation model are air $(\sigma_0 = 0)$ and the lithosphere $(\sigma_4 = 0.001 \text{ S/m})$, respectively. These layers are semi-infinite. Next layer from the top is seawater with the conductivity σ_1 for $0 < z < h_1$, where h_1 is the sea depth. Two layers of sediments with the spatial conductivity distributions $(\sigma_2, h_2, \sigma_3, h_3)$ are interposed between seawater and lithosphere.

Since the source and the sensors are placed within the seawater layer, the multi-layers problem can be solved by using the EM field solution for a two-layer conducting half-space, air–seawater–seabed and correcting the seabed conductivity to account for the stratified seabed structure. The two-layer conducting half-space EM solution was derived by Weaver [2]. We shall suppose that the current dipole is situated at the point $x = y = 0$, $z = a$ $(0 < a < h_1)$. The magnetic H and electric E fields are written as functions of the vector potential A:

$$\mathbf{H} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = i\omega\mu\mathbf{A} + \frac{1}{\sigma}\nabla\nabla \cdot \mathbf{A} \tag{3}$$

where μ is the permeability, and σ is the conductivity of the medium. The vector potential A_k, in medium k ($k = 0, 1, 2$), must satisfy the boundary conditions and the equation:

$$\nabla^2 \mathbf{A}_k - i\omega\mu\sigma_k \mathbf{A}_k = -I_k \delta(\mathbf{r}_0) \tag{4}$$

where $\mathbf{I}$ is the strength of the dipole, $I_0 = I_2 = 0$, $\sigma_0 = 0$ in air, δ is the Dirac delta function, and $\mathbf{r}_0$ is the distance from the source to the field point. To satisfy the boundary conditions it is necessary for the vector potential to have the form

$$\mathbf{A}_k = \widehat{\mathbf{x}} A_{k,X} + \widehat{\mathbf{y}} A_{k,Y} \tag{5}$$

Explicit expressions for the EM field solution in medium 1 (seawater) can be found in Reference [2]. As shown by Weaver, the solution can be written as an infinite series that refers to a simplified model of a conducting slab of thickness h_1 situated in vacuum, plus an additional term representing the effect of replacing the free space beneath the slab by a medium of conductivity σ_2. Since we shall be concerned only with the EM field in the region $0 < z < h_1$, the effect of seabed stratification may be accounted for by introducing an effective conductivity, σ_e, instead of σ_2 in the calculation of the term representing the contribution of the seawater–seabed interface.

To illustrate the procedure, the $A_{1,X}$ solution is taken as an example using a slightly different notation than Weaver:

$$\begin{aligned} A_{1,X} = {} & \frac{I}{4\pi r_0} \exp(-\alpha_1 r_0 \sqrt{i}) \\ & + \frac{I}{4\pi} \int_0^\infty d\lambda \frac{\lambda}{\sqrt{\lambda^2 + i\alpha_1^2}} J_0(\rho\lambda) \left\{ F \exp\left[-(z+a)\sqrt{\lambda^2 + i\alpha_1^2} \right] \right. \\ & \left. + P \exp\left[-(2h_i - z - a)\sqrt{\lambda^2 + i\alpha_1^2} \right] \right\} \end{aligned} \tag{6}$$

where we have put $\alpha_1 = (\mu\omega\sigma_1)$ and J_0 denotes the zeroth-order Bessel function of the first kind. The expression for P was determined from the boundary condition:

$$\begin{aligned} P = {} & \frac{G}{1 - FG \exp\left(-2h_1 \sqrt{\lambda^2 + i\alpha_1^2} \right)} \left[1 + F \exp\left(-z\sqrt{\lambda^2 + i\alpha_1^2} \right) \right] \\ & \left[1 + F\left(-2a\sqrt{\lambda^2 + i\alpha_1^2} \right) \right] \end{aligned} \tag{7}$$

Here we have defined the reflection coefficients for the seawater–air and seawater–seabed interfaces, respectively:

$$F = \frac{\sqrt{\lambda^2 + i\alpha_1^2} - \lambda}{\sqrt{\lambda^2 + i\alpha_1^2} + \lambda}, \quad G = \frac{\sqrt{\lambda^2 + i\alpha_1^2} - \sqrt{\lambda^2 + i\alpha_2^2}}{\sqrt{\lambda^2 + i\alpha_1^2} + \sqrt{\lambda^2 + i\alpha_2^2}} \tag{8}$$

The only parameter depending on σ_2 through $\alpha_2 = (\mu\omega\sigma_2)$ is G which will be corrected for the specific case considered here. To relate the reflected and transmitted fields to the incident field, it is necessary to express the complete field as the linear superposition of two components: the one with the electric field in the plane of incidence (E-polarized), the second with the magnetic field in the plane of incidence (H-polarized). For the case when the electric vector is perpendicular to the plane of incidence, the surface admittance at the water–seabed boundary may be defined by

$$Y_e = \frac{(\lambda^2 + i\omega\mu\sigma_e)^{1/2}}{i\omega\mu} = \frac{(\lambda^2 + i\omega\mu\sigma_2)^{1/2}}{i\omega\mu} Q_2 = -\left[\frac{H_{1x}}{E_{1y}}\right]_{z=h_2} \tag{9}$$

where Q_2 is the correction of the characteristic admittance of seabed to account for the presence of the lower layers. Similar relations exist for the seabed impedance, Z_e, in the case of the E-polarized field and it is interesting to note that $Y_e Z_e \neq 1$ [3].

Once the effective conductivity of seabed is calculated for the two separate cases, the problem reduces to the previously known results for the two-layer conducting half-space, substituting σ_2 with σ_e, the new conductivity of the bottom. Explicitly, it follows from that (9) that

$$i\omega\mu\sigma_e = i\omega\mu\sigma_2 Q_2^2 + \lambda^2(Q_2^2 - 1) \tag{10}$$

for the H-polarized field. In this equation, 'λ' can take any value and it can be identified with $k_1 \sin\varphi$, where φ is the (complex) angle of incidence and k_1 is the wave number. Q_2 can be calculated using the downward recursion rule applied here for a three-layered sea bottom [3]:

$$\begin{aligned}
Q_2 &= \frac{Y_3 + N_2 \tanh u_2 h_2}{Y_3 \tanh u_2 h_2 + N_2}, \quad N_2 = \frac{u_2}{i\omega\mu}, \quad u_2 = (\lambda^2 + i\omega\mu\sigma_2)^{1/2} \\
Y_3 &= \frac{(\lambda^2 + i\omega\mu\sigma_3)^{1/2}}{i\omega\mu} Q_3 \\
Q_3 &= \frac{Y_4 + N_3 \tanh u_3 h_3}{Y_4 \tanh u_3 h_3 + N_3}, \quad N_3 = \frac{u_3}{i\omega\mu}, \quad u_3 = (\lambda^2 + i\omega\mu\sigma_3)^{1/2} \\
Y_4 &= \frac{(\lambda^2 + i\omega\mu\sigma_4)^{1/2}}{i\omega\mu}
\end{aligned} \tag{11}$$

The value of the effective conductivity of the seabed in both cases of E and H field polarization becomes complex and depends on the continuous (integration) variable 'λ'. When the symbol 'λ' is identified with $k_1 \sin\varphi$, the effective admittance (10) is given by a single set E and H of plane waves of angle of incidence φ. From this point of view, the multi-layer bottom conductivity will affect the reflection coefficient at the seawater–seabed interface at every angle of incidence.

4. ELECTROMAGNETIC PULSE IN SEAWATER

To verify the algorithm, we modelled the propagation in seawater of a low-frequency EM field in the form of a sine wave train or a 'radar' pulse generated by an electric dipole. The field is calculated at various distances in the equatorial plane of the dipole source. King [4] investigated this problem analytically. The carrier frequency is sufficiently small to permit a significant distance of travel. Since the medium is dissipative and dispersive, the shape of the wave packet is greatly modified as the pulse travels in seawater.

The strength of the dipole is 1A-m, the frequency of the signal is 25.5 Hz, its duration is 1 s and, with the water conductivity $\sigma = 4$ S/m, $\sigma/\omega\varepsilon \sim 10^3$. The initial and final transients are generated in the rise form and decrease to zero of the 25.5-cycle wave packet (Figure 1). King [4] identified four distinctive regions as functions on the propagation distance 100, 223, 446 and 892 m, respectively, where the x-component of the electric field of the EM pulse has specific shapes. These shapes were evaluated numerically and presented in Reference [4, Figure 4(a)–(d)].

We reproduced the same calculation for the electric field in x-direction (Ex) using the Fourier synthesis of the frequency-domain solutions and the results are shown in Figure 2(a)–2(d). The pulse shapes are identical, except for the one calculated at 100 m distance, which is slightly different at the first cycle. A possible explanation for this discrepancy is that the King's calculation for the transient near the ends is not exact owing to an approximation (Equation (29) in Reference [4]) made in their evaluation. But what seems to be more important, the radar pulse plotted in Reference [4, Figure 4(a)] has 180° phase shift compared with our calculation. Also, in Figure 2(a)–2(c) the arrows indicate the position of the central peak that is retarded in its time of arrival at the respective distances, in perfect agreement with the values given by Equation (61) in Reference [4]. Moreover, the electric field is plotted in units of μV/m, while in Reference [4] it has an arbitrary amplitude scale.

For possible applications to underwater communication or detection of targets in the ocean, the propagation of the pulse through the seawater is not efficient because the amplitude of the wave packet attenuates rapidly with the distance. However, in shallow waters when the source and the receiver are located a few skin depths from the air–sea surface or sea–seabed interface, long-range propagation is still possible by lateral (surface) waves. To investigate this possibility, the EM propagation of the pulse was modelled in an environment consisting of four horizontal layers: air–seawater–sediment–rock. The seawater has the conductivity 3 S/m and 17 m depth. The layer of sediments having conductivity 0.03 S/m and 3 m in thickness is interposed between

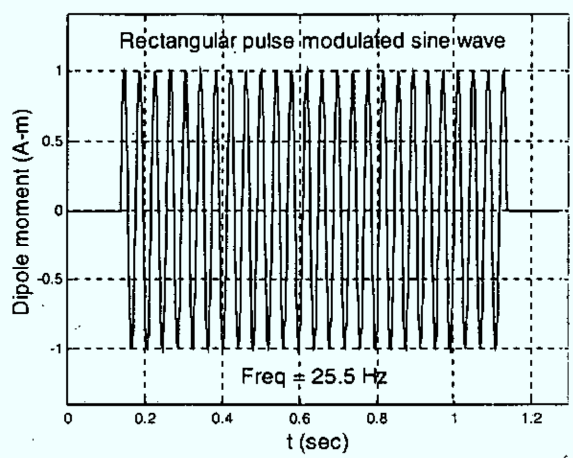

Figure 1. Current in the electric dipole in seawater.

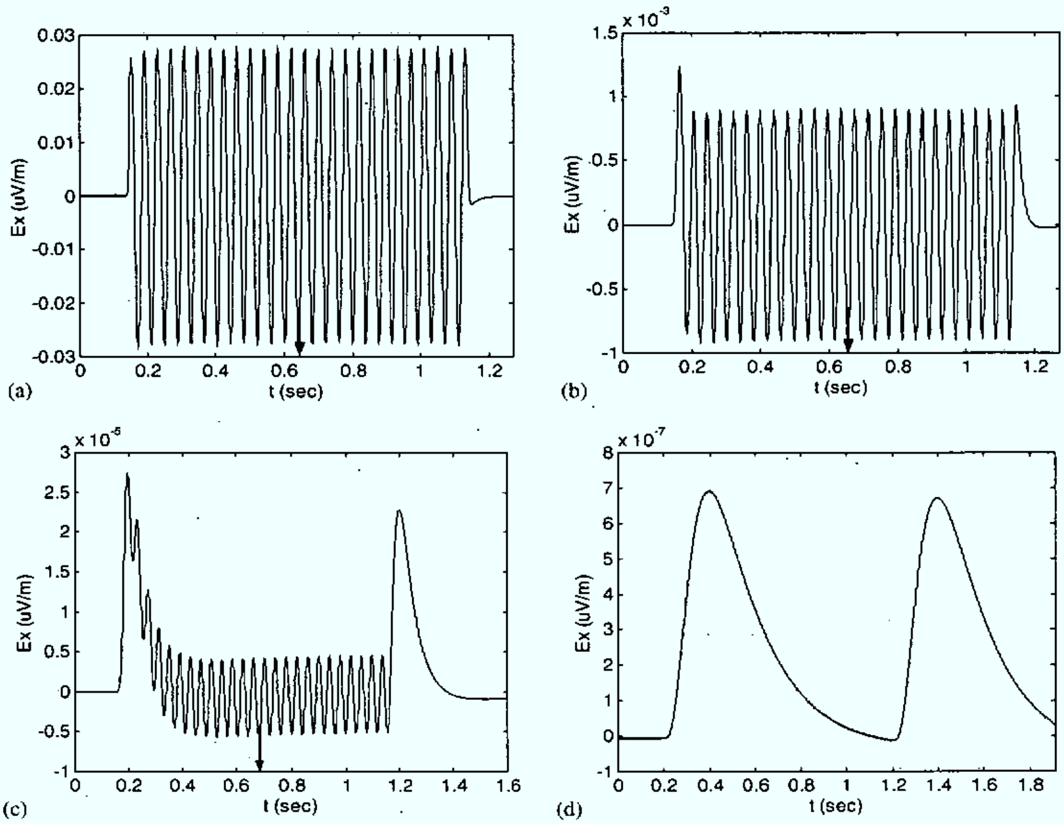

Figure 2. The Ex component of EM pulse in the seawater at distances (a) 100, (b) 223, (c) 446 and (d) 892 m.

seawater and rock (0.001 S/m). Both the electric dipole and the sensor are oriented in the x-axis direction and placed close to the seawater–seabed interface at 892 m separation distance. The radar pulse for this geometry is shown in Figure 3. As one can see, the initial and final transients affect the shape of the wave packet in a relatively insignificant degree. Owing to the presence of low conductivity seabed, the signal is well transmitted at this distance. It looks similar to the one plotted in Figure 2(b) for a propagation distance of 223 m in seawater, but the phase is changed 180°.

The application of transient EM method to remote sensing in seawater is limited by the fact that, due to the low phase velocity, the reflected pulses from a target close to the source arrive before the generating current pulse drops to zero. At distances great enough to make the time of travel greater than the pulse length, the amplitude of the wave packet becomes undetectable. A different situation was presented in this paper, where the EM lateral waves are employed. Consider a scenario where the transmitter, the receiver in the vicinity of the transmitter and the target (a metal cylinder) are on the ocean floor. Since the lateral waves travel through a less conductive media, their velocity of propagation is large enough to separate at the receiver the reflected pulse from the cylinder from the direct pulse. For other scenarios where the target is in

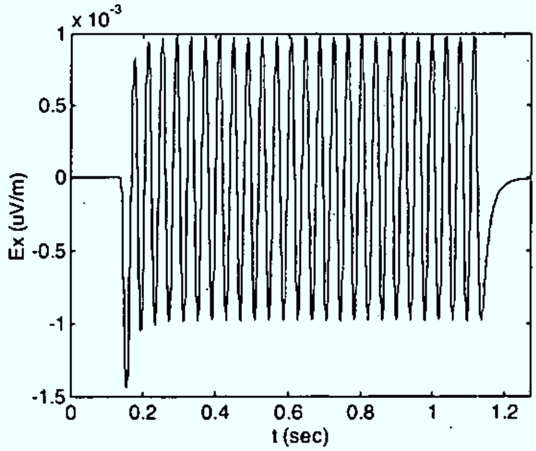

Figure 3. The Ex component of EM pulse propagated at 892 m in the air–seawater–seabed environment.

between the air and seafloor boundaries, a complete investigation of the range of relevant depths and pulse widths is necessary and it can be made using the presented method.

5. CONCLUSION

The main purpose of this study was to present a method for analysing low-frequency transient EM field propagation in a marine environment. This environment is quite accurate modelled as a horizontally layered structure. Since the procedure uses the frequency-domain solution, the fields from an elemental electric dipole were calculated in a layered structure by solving the Maxwell's equations. To transform the field to the time domain by the use of FFT, the whole frequency spectrum must be included. Thus, thousands of field values must be calculated for one specific distance. Even so, the computation on a Pentium III computer is done in a few seconds.

The method presented here is closely related to the previous studies on the propagation of EM pulses in dispersive media [4,5]. The novelty of the present study is that it considers the EM surface waves, which can propagate along the boundaries between electrically different media. Moreover, the investigation in this article is concerned with the complete field and especially the near field generated by the rectangular pulse modulated current in an electric dipole in seawater. The main disadvantage of this method is its incapability to simulate EM wave propagation in complex geometries. In this case, one must rely on numerical methods.

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AUTHOR'S BIOGRAPHY

Marius Birsan received the MSc degree from the Technical University of Iasi, Romania, and the PhD degree (1995) from McGill University, Montreal, both in Electrical Engineering. During the PhD programme he has done graduate work at the Atomic Energy of Canada nuclear research facility in Chalk River, Ontario. From 1995 to 1997, he worked as a post-doctoral researcher at California Institute of Technology and Jet Propulsion Laboratory (NASA) in Pasadena, and then worked as R&D engineer for UCAR International in Cleveland, Ohio. Since 1999 he is a scientist with Defence R&D Canada Atlantic in the Signatures Section. His research has been in the area of magnetic materials, magnetism and electromagnetic propagation in marine environment.