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Out-of-sequence measurements filtering using forward prediction

F. Rhéaume
A. Benaskeur
DRDC Valcartier

Defence R&D Canada – Valcartier
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Principal Author

F. Rhéaume & A. Benaskeur

Approved by

Éloi Bossé
Head/A Section

Approved for release by

C. Carrier
Chief Scientist

Abstract

In target tracking applications, there may be situations where measurements from a given target arrive out of sequence at the processing center. This problem is commonly referred to as Out-of-Sequence Measurements (OOSMs). So far, most of the existing solutions to this problem are based on retrodiction, where backward prediction of the current estimated state is used to incorporate the OOSMs at appropriate time instants. This document suggests a new method for tackling the OOSMs problem without backward prediction. Based on a forward prediction and decorrelation approach, the method has proved to compare favorably to the best retrodiction-based methods, while requiring less data storage in most cases.

Résumé

Dans le domaine du pistage de cibles, des situations peuvent se produire où les mesures sur une cible particulière arrivent hors séquence au centre de traitement. Ce problème est désigné sous le nom de Mesures Hors Séquence (MHS), tiré de l'anglais "Out-Of-Sequence Measurements" (OOSMs). Jusqu'ici, la plupart des solutions à ce problème sont basées sur le concept de rétrodiction. Cette méthode utilise la projection dans le passé (ou rétroprojection) de l'état courant estimé afin d'incorporer les MHS. Une nouvelle méthode pour résoudre le problème des MHS est suggérée, laquelle n'utilise pas de la de rétroprojection. Elle se base, plutôt, sur la prédiction directe (vers l'avant) et la décorrélation de l'information, et se compare favorablement aux meilleures méthodes utilisant la rétrodiction, tout en exigeant la même capacité de mémoire, et dans la plupart des cas, une quantité moindre.

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Executive summary

Out-of-sequence measurements filtering using forward prediction

F. Rhéaume , A. Benaskeur ; DRDC Valcartier TR 2005-485; Defence R&D Canada – Valcartier; August 2007.

The tracking of dynamic targets involves a measurement process that collects and processes data provided by one or more sources. Because of the temporal aspect of the tracking operation, the chronological appearance of the measurements is of prime importance. It is looked at and usually considered by using a time stamp related to each measurement. This time stamp indicates the time at which the measurement was collected. In many applications, where most are concerned with multiple-sensor tracking, there is a time delay between the instant at which the measurement is collected from the environment and the instant at which it is received and processed by the tracking system. It may happen that the delay is large enough so that a measurement at a given time arrives at the tracking system after the target track has already been updated with one, or more, more recently collected measurements. Such a delayed measurement is referred to as an Out-Of-Sequence Measurement (OOSM).

So far, most of the existing solutions to this problem are based on retrodiction. The latter uses backward prediction of the current estimated state in order to incorporate the OOSMs at appropriate time instants. A new method to tackle the OOSMs problem is suggested, without the need of backward prediction. It is based on forward prediction and decorrelation, and it has proved to compare favorably to the best retrodiction-based algorithms, while requiring, in most cases, less data storage. A pseudo-track (or a tracklet) is created using the OOSM and track value at a time prior to the OOSM date. Before being fused with the actual track, the created tracklet is predicted forward and de-correlated from the actual track using a track decorrelation method similar to the information filter approach. This new proposed method does not require the state transition matrix to be invertible and provides optimal performance whether the measurements are regularly or asynchronously spaced in time, and no matter if delays are anticipated or not.

Finally, in order to decide whether an OOSM should be processed or discarded, the impact of missing measurements on the track accuracy should be evaluated. In short, the impact depends on the process noise, the measurement noise, the state error covariance prior to the OOSM, the sampling interval and the age of the missing measurement (the number of measurement processed since the OOSM time). It is shown that there is a complex relation between age of the missing measurement, process noise, measurement noise and impact of a missing measurement on track accuracy.

Sommaire

Out-of-sequence measurements filtering using forward prediction

F. Rhéaume , A. Benaskeur ; DRDC Valcartier TR 2005-485; Recherche et Développement pour la Défence Canada – Valcartier; août 2007.

Le pistage de cibles dynamiques comporte une étape de traitement des mesures qui rassemble et traite les données fournies par une ou plusieurs sources. En raison de l'aspect temporel de l'opération de pistage, l'ordre chronologique des mesures prend toute son importance. Dans plusieurs applications, où la plupart sont à sources multiples, il peut y avoir des délais entre l'instant où la mesure est effectuée dans l'environnement et l'instant où elle est reçue et traitée par le système de pistage. Il se peut que le retard soit assez grand pour qu'une mesure effectuée à un temps donné arrive au système de pistage après que la piste d'une cible ait déjà été mise à jour avec une ou plusieurs mesures plus récentes. Une mesure ainsi retardée est désignée sous le nom de Mesure Hors Séquence (MSH), tiré de l'anglais "Out-Of-Sequence Measurements" (OOSM).

Jusqu'ici, la plupart des solutions à ce problème sont basées sur le concept de rétrodiction. Cette méthode utilise la projection vers l'arrière de l'état courant estimé afin d'incorporer les MHS. Une nouvelle méthode pour résoudre le problème des MHS est suggérée, laquelle n'utilise pas de projection vers l'arrière dans le temps. Elle se base plutôt sur la prévision vers l'avant et la dé-corrélation de l'information, et se compare favorablement aux meilleures méthodes utilisant la rétrodiction, tout en exigeant la même capacité de mémoire, et dans la plupart des cas, une capacité moindre. Une pseudo-piste (ou "tracklet") est créée à partir de la MHS et de l'estimation dans la piste à un temps précédant celui de la MHS. Avant d'être fusionné avec la plus récente estimation dans la piste, la pseudo-piste créée est prédite dans le temps et decorrélée de la plus récente estimation dans la piste par une méthode de décorrélation similaire à celle par le filtre d'information ("information filter"). Aussi, la méthode proposée n'exige pas que la matrice de transition d'état soit inversible et elle fournit des résultats optimaux que les mesures soit synchrones ou asynchrones, et peut fonctionner que les mesures hors séquences soient prévues ou non.

En conclusion, afin de déterminer si une MHS devrait être traitée ou simplement laissée de côté, l'impact des mesures sur la qualité d'une piste devrait être évalué. En bref, l'impact d'une mesure dépend du bruit de processus, du bruit dans les mesures, de la covariance de l'erreur sur l'état estimé avant le temps du MHS, de l'intervalle de prélèvement de mesures et de l'âge de la mesure hors séquence, qui est représenté par le nombre de mesures traitées depuis le temps de la MHS.

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List of Symbols

B	Track storage interval
DR	Determinant ratio
$\boldsymbol{F}$	State transition matrix
$\boldsymbol{\Gamma}$	Process noise gain matrix
h	Sampling period
$\boldsymbol{H}$	Measurement matrix
$\boldsymbol{I}$	Identity matrix
l	Lag
n	Dimension of the state vector
$\hat{\boldsymbol{P}}$	Estimated state error covariance matrix
q	Process noise spectral density
$\boldsymbol{Q}$	Process noise covariance matrix
r	Range
$\boldsymbol{R}$	Measurement noise covariance matrix
$\mathcal{R}$	The set of real numbers
σ	Standard deviation
$\boldsymbol{x}$	State vector
$\hat{\boldsymbol{x}}$	Estimate of the state vector
t	Time label (continuous)
θ	Azimuth
$\boldsymbol{v}$	Process noise
$\boldsymbol{w}$	Measurement noise
$\boldsymbol{W}$	Filter gain
$\boldsymbol{z}$	Measurement
Z^k	Measurement sequence up to time instant k

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1 Introduction

The tracking of dynamic targets involves a measurement process that collects and processes data provided by one or more sources. This tracking process aims at producing a mathematical model of the time history for the target kinematics [22]. It uses the past measurements to estimate the temporal equations of the observed state (track update) and also to predict, in time, the state expected to be observed (track prediction). Because of the temporal aspect of tracking operation, the chronological appearance of the measurements is of prime importance to the estimation process. The measurements are usually considered by using a time stamp related to each measurement. This time stamp indicates the time at which the measurement was collected.

In many applications, where most are concerned with multiple-sensor tracking, there is a time delay between the instant at which the measurement is collected from the environment and the instant at which it is received and processed by the tracking system. Such a delay, that may arise from many factors, such as the sensor diversity, communication delays and unsteady pre-processing times of the observed data, depending on the system load, can vary from one measurement to another. It may happen that the delay is large enough so that a measurement taken at a time t_τ arrives at the tracking system at a time $t_d > t_k$, that is after the target track has already been updated with one, or more, more recently collected measurements. Such a delayed measurement is referred to as an Out-Of-Sequence Measurement (OOSM) [2]. The problem of the OOSM comes to update the track corresponding to the given target, at the current time t_d, using the older measurement from time t_τ, given that the track has already been updated by measurements collected after time t_τ. This problem is also referred to as the problem of tracking with random sampling and delays [15] or negative-time measurement update [2].

A direct solution to the OOSM problem is simply to ignore and discard the OOSM in the tracking process. This solution leads obviously to a loss of the information contained in the discarded OOSM. To avoid such a drawback, a simple alternative consists in reprocessing, in chronological order, all measurements that are collected from the OOSM time t_τ to the time t_d [7]. This solution yields optimal track quality. Nevertheless, it is inefficient because of its high computation and storage requirements. In most cases, it is even unfeasible since most tracking systems keep only the current state estimate and the corresponding error covariance matrix.

Several methods have been proposed in the literature to deal more efficiently with the OOSM problem. Most of these methods are based on backward prediction of the current state to incorporate the OOSMs, also referred to as retrodiction approach [3, 6, 2, 7, 21, 15, 11, 9, 16, 19, 25, 18, 17]. All the retrodiction-based methods vary in how the process noise is accounted for during the retrodiction. The suggested methods in [7, 21, 15] compensate partially for the process noise, or simply ignore it. The major difficulty here is that there is a strong dependency between the retrodicted process noise and the current state in backward prediction.

In this report, a new method that does not rely on retrodiction is proposed. The main idea consists in combining forward prediction with track decorrelation, as used in the information filter approach [12], to optimally incorporate the OOSMs. This is why the method is referred to as Forward-Prediction Fusion and Decorrelation (FPFD). The proposed method has proved to provide performance that compares favorably to the best retrodiction-based algorithms, while requiring, in most cases, less data storage.

This report is organized as follows. The problem of OOSMs is stated in Chapter 2. In Chapter 3 are discussed the retrodiction-based methods. Before addressing the OOSM solution, a study of the impact of missing measurements on tracks is presented in Chapter 4. Then the proposed FPFD approach is described in Chapter 5. In Chapter 6, comparison results and performance discussion of the FPFD method are provided. Finally, concluding remarks about the proposed method are given in Chapter 7.

2 Problem statement

The dynamical model of the target of interest is assumed to be

$$\boldsymbol{x}(k) = \boldsymbol{F}(k, k-1)\boldsymbol{x}(k-1) + \upsilon(k, k-1) \tag{1}$$

with $\boldsymbol{F}(k, k-1)$ the state transition matrix from time t_{k-1} to time t_k and $\upsilon(k, k-1)$ represents the effect of the process noise from time t_{k-1} to time t_k. The measurement model is

$$\boldsymbol{z}(k) = \boldsymbol{H}(k)\boldsymbol{x}(k) + \omega(k) \tag{2}$$

where $\boldsymbol{H}(k)$ is the measurement matrix and $\omega(k)$ is the measurement noise. The process noise $\upsilon(k, k-1)$ and the measurement noise $\omega(k)$ are assumed to be white with zero mean and covariance matrices $\boldsymbol{Q}(k, k-1)$ and $\boldsymbol{R}(k)$, respectively. Details on $\boldsymbol{F}(k, k-1)$ and $Q(k, k-1)$ are given in Section A. Also, note that throughout this work the continuous time representation of a discrete time i is noted t_i.

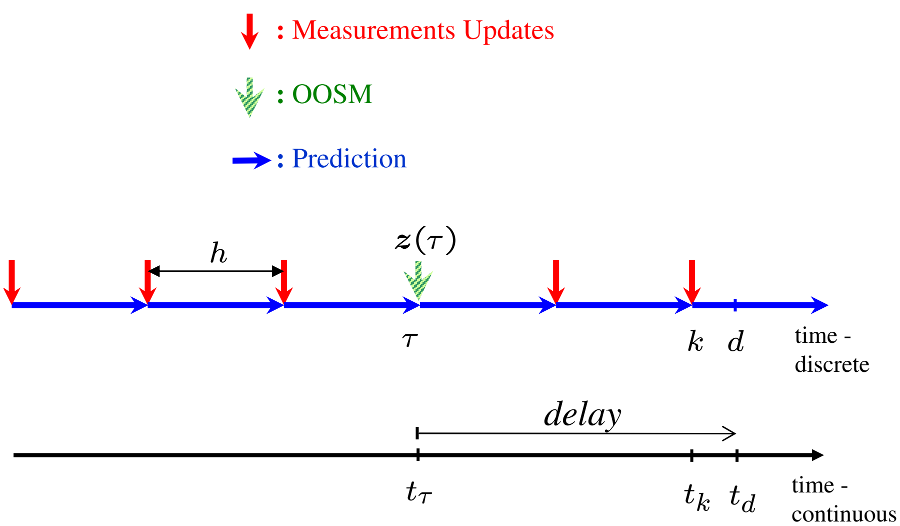

Figure 1: Out-of-sequence measurement

The role of the tracking system is to update the target track composed of a state estimate $\hat{\boldsymbol{x}}$ and an estimation error covariance matrix $\boldsymbol{P}$. It is assumed that a measurement z is collected and used to update the track at the time interval h. At time t_k, the tracking system computes the state estimate $\hat{\boldsymbol{x}}(k|k)$ and its corresponding estimated error covariance matrix $\boldsymbol{P}(k|k)$

$$\hat{\boldsymbol{x}}(k|k) = E\left[\boldsymbol{x}(k)\middle|Z^k\right] \tag{3}$$

$$\boldsymbol{P}(k|k) = \text{cov}\left[\boldsymbol{x}(k)\middle|Z^k\right] \tag{4}$$

where Z^k corresponds to the measurement sequence up to time instant t_k, excluding a measurement $z(\tau)$ with a time stamp $t_\tau < t_k$, as shown in Figure 1. Once $z(\tau)$ arrives at the processing center, with a certain delay, the problem comes to calculate $\hat{\boldsymbol{x}}(k|k,\tau)$ and $\boldsymbol{P}(k|k,\tau)$ that take into consideration the OOSM $z(\tau)$, as shown below.

$$\hat{\boldsymbol{x}}(k|k,\tau) = E\left[\boldsymbol{x}(k)\middle|Z^k, z(\tau)\right] \tag{5}$$

$$\boldsymbol{P}(k|k,\tau) = \operatorname{cov}\left[\boldsymbol{x}(k)\middle|Z^k, z(\tau)\right] \tag{6}$$

Furthermore, depending on the delay time τ, an OOSM can have different step lag values. Then, a l-step lag OOSM is defined as an OOSM for which

$$l = k - \tau \tag{7}$$

3 Backward prediction methods

A widely used approach to tackle the problem of OOSMs in tracking is based on the "prediction" of the state estimate back to the time of the delayed measurement. Depending on the authors, this approach may be referred to as retrodiction, backward prediction or reverse-time prediction. It sums up to predict the track from the last update time k backward to the time τ of the OOSM, and then update the track using the OOSM and then predict forward from time instant τ to time instant k.

Various retrodiction methods have been presented in the literature recently [3, 6, 2, 7, 21, 15, 11, 9, 16, 19, 25, 18, 17]. In [2], a retrodiction method is suggested, which is presented as an exact solution compared to those in [7, 21, 15], insofar as the process noise is accounted for entirely during the retrodiction. This solution has proved to give optimal results for the case where the OOSM lies between the last two measurements, which is also called the 1-step lag case. The problem with this algorithm, compared to those presented in [21, 15], where the process noise in the retrodiction process is only partially compensated for, is that it requires storage of the innovation. The optimal approach described in [2] and the suboptimal approaches in [21, 15] were extended in [3] to the case of an OOSM with an arbitrary lag, that is an OOSM whose time stamp can be earlier than the last sampling interval. The approach in [3] for an arbitrary lag, which was originally presented in [6], uses an equivalent measurement concept. This concept was first presented in [13, 14]. The equivalent measurement combines together all the measurements that are more recent than the OOSM. This concept was used to extend the algorithms in [2], the optimal and the suboptimal, to the case of an n-step lag OOSM. The authors concluded in [3] that the suboptimal algorithm, which ignores the retrodicted noise, is a good compromise between accuracy and cost.

All of the above-discussed retrodiction-based methods have some advantages and some drawbacks. The optimal solutions presented in [19, 25] require a large storage capacity compared to other suboptimal OOSM methods. The equivalent measurement concept with backward prediction presented in [3] also necessitates a considerable amount of data storage. Even in the case where the retrodicted noise is ignored, covariance of the equivalent innovation needs to be computed. This requires storage of the error covariance matrices corresponding to the state estimates based on all measurement time stamps that are subsequent to the OOSM. The same applies to the Augmented State Kalman Filter method presented in [9], which also uses equivalent measurements.

4 Impact of missing measurements on track quality

In the presence of OOSMs, an interesting issue concerns the evaluation of the accuracy that is gained or lost whether a track has a measurement update or not. In other words, it should be determined how useful is the inclusion of an OOSM to the current estimate, in order to decide whether the OOSM should be included or discarded. To answer that question, the impact of a particular measurement on track accuracy must be determined. In [20], it is suggested that the decision to include or not an OOSM should be based on a utility measure of the measurement. Here are studied the factors that influence the impact of missing measurements on track accuracy. The study separates in two cases: the 0-step miss case and the multi-step miss case. Figure 2 shows both cases, where the 1-step and 2-step miss cases corresponds to the multi-step case. It will be shown that the impact of missing measurements on track quality is influenced differently depending wheth-er the miss is 0-step or multi-step.

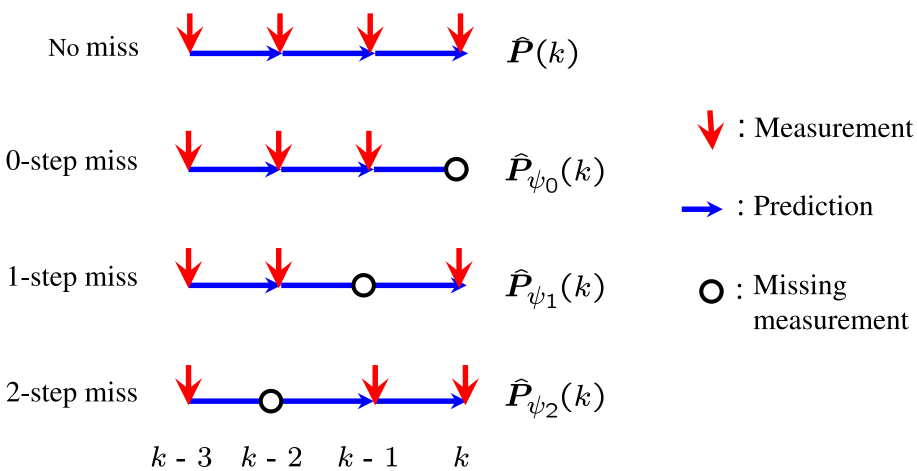

Figure 2: Track comparison between no miss, miss at current time (0-step lag), 1-step lag, 2-step lag and 3-step lag. All tracks originate from the same one prior to time $k-3$.

4.1 Factors

There are some different factors that determine the degree of the impact of missing measurements on the track accuracy. With dynamic processes, the most obvious one is the age of the missing measurement, in that older missing measurements should have less impact on the track accuracy than the newer ones. The influence of age can be explained by the Kalman filter recursiveness. Therefore, the age of a measurement is determined by the number of subsequent measurements. The degree of impact should also be related in some way to the process noise, the measurement noise, the state estimated error covariance prior to the time of the missing measurement, and the sampling interval.

Suppose that at time $k-1$ a track has a related estimated error covariance matrix $\boldsymbol{P}(k-1)$.

Here $\boldsymbol{P}$ will be referred to as the estimated covariance matrix of the track that would have had all the measurements, while $\boldsymbol{P}_\psi$ will be defined as the estimated covariance matrix of the track that misses the measurement. Furthermore, in order to simplify calculations, let define the measurement error covariance matrix in terms of a unique measurement error covariance σ_R^2 for all of the space dimensions

$$\boldsymbol{R} = \sigma_R^2 \boldsymbol{I} \tag{8}$$

Also,

$$\Upsilon = \Gamma\Gamma^T \tag{9}$$

where Γ is the process noise gain matrix such that

$$\boldsymbol{Q} = \sigma_v^2 \Upsilon \tag{10}$$

and where σ_v^2 is the process noise variance. Finally, let us have

$$\boldsymbol{\zeta} = [\boldsymbol{H}^T \boldsymbol{H}]^{-1} \tag{11}$$

where $\boldsymbol{H}$ is the measurement matrix.

0-step miss

The 0-step miss case is the one where a measurement is missing at the current update time k. In that case, the last measurement inclusion happened before the missing measurement time.

Suppose that at time k, a measurement update is scheduled, given that a measurement is available. In the case where the measurement is available (**No miss** case in Figure 2), the track accuracy expressed in the information space is

$$\boldsymbol{P}(k)^{-1} = \left[\boldsymbol{F}\boldsymbol{P}(k-1)\boldsymbol{F}^T + \sigma_v^2\Upsilon\right]^{-1} + \left[\sigma_R^2\boldsymbol{\zeta}\right]^{-1} \tag{12}$$

In the case where the measurement at time k is missing (**0-step lag** case in Figure 2), there is no measurement update at time k and the track accuracy is only defined by the filter prediction

$$\boldsymbol{P}(k)_\psi^{-1} = \left[\boldsymbol{F}\boldsymbol{P}(k-1)\boldsymbol{F}^T + \sigma_v^2\Upsilon\right]^{-1} \tag{13}$$

According to (12) and (13), the ratio of the track estimated error covariance without a measurement update at time k ($\boldsymbol{P}(k)_\psi$) to the track estimated error covariance with a measurement update at time k ($\boldsymbol{P}(k)$) is:

$$\boldsymbol{P}_\psi \boldsymbol{P}^{-1}\Big|_k = \left[\boldsymbol{F}\boldsymbol{P}(k-1)\boldsymbol{F}^T + \sigma_v^2\Upsilon\right]\left(\left[\boldsymbol{F}\boldsymbol{P}(k-1)\boldsymbol{F}^T + \sigma_v^2\Upsilon\right]^{-1} + \left[\sigma_R^2\boldsymbol{\zeta}\right]^{-1}\right) \tag{14}$$

$$= \boldsymbol{I} + \boldsymbol{F}\boldsymbol{P}(k-1)\boldsymbol{F}^T\sigma_R^{-2}\boldsymbol{\zeta}^{-1} + \sigma_v^2\sigma_R^{-2}\Upsilon\boldsymbol{\zeta}^{-1} \tag{15}$$

According to (15), the impact of a 0-step missing measurement on the track accuracy is influenced by

1. $\sigma_\upsilon^2 \sigma_R^{-2}$, which represents the ratio of the process noise variance to the measurement noise variance.

2. $\boldsymbol{F}\boldsymbol{P}(k-1)\boldsymbol{F}^T\sigma_R^{-2}$, which involves the transition matrix and the estimated error covariance at time $k-1$ and the measurement noise variance.

To have a more explicit analysis of the factors that influence the impact of a 0-step missing measurements on track accuracy, assume the following 1-dimension constant velocity model with white noise acceleration

$$\boldsymbol{x}(k+1) = \boldsymbol{F}\boldsymbol{x}(k) + \boldsymbol{\Gamma}\boldsymbol{\upsilon}(k) \tag{16}$$

where

$$\boldsymbol{F} = \begin{bmatrix} 1 & h \\ 0 & 0 \end{bmatrix} \tag{17}$$

and

$$\boldsymbol{\Gamma} = \begin{bmatrix} h^2/2 \\ h \end{bmatrix} \tag{18}$$

and where h is the sampling interval time. The process noise covariance matrix is

$$\boldsymbol{Q} = \sigma_\upsilon^2 \begin{bmatrix} h^4/4 & h^3/2 \\ h^3/2 & h^2 \end{bmatrix} \tag{19}$$

Also, let $\boldsymbol{P}(k-1)$ be defined as

$$\boldsymbol{P}(k-1) = \begin{bmatrix} \sigma_x^2 & \sigma_{x\dot{x}}^2 \\ \sigma_{x\dot{x}}^2 & \sigma_{\dot{x}\dot{x}}^2 \end{bmatrix} \tag{20}$$

According to that, in the case where a measurement update occurs at time k, the position error variance $\sigma_x^2(k)$ at time k is

$$\sigma_x^2(k) = \left[(\sigma_x^2 + 2\sigma_{x\dot{x}}^2 h + \sigma_{\dot{x}\dot{x}}^2 + \frac{1}{4}\sigma_\upsilon^2 h^4)^{-1} + \sigma_R^{-2} \right]^{-1} \tag{21}$$

where σ_R^2 is the measurement variance. In the case where a measurement update does not occur at time k, the position error variance $\sigma_{x_\psi}^2(k)$ at time k is

$$\sigma_{x_\psi}^2(k) = \sigma_x^2 + 2\sigma_{x\dot{x}}^2 h + \sigma_{\dot{x}\dot{x}}^2 + \frac{1}{4}\sigma_\upsilon^2 h^4 \tag{22}$$

Using (21) and (22), the ratio of the position estimated error variance with a measurement update at time k to the position estimated error variance without a measurement update at time k is

$$\left. \frac{\sigma_{x_\psi}^2}{\sigma_x^2} \right|_k = 1 + \frac{\sigma_x^2 + 2\sigma_{x\dot{x}}^2 h + \sigma_{\dot{x}\dot{x}}^2 h^2 + \frac{1}{4}\sigma_\upsilon^2 h^4}{\sigma_R^2} \tag{23}$$

Equation (23) shows the influence of σ_υ^2, σ_R^2 and h as well as the state error covariances σ_x^2,$\sigma_{x\dot{x}}^2$ and $\sigma_{\dot{x}\dot{x}}^2$. Note that

$$\frac{\sigma_{x_\psi}^2(k)}{\sigma_x^2(k)} > 1 \tag{24}$$

for the 0-step lag case shown in Figure 2.

Therefore, according to the above analysis, it follows that

1. The higher σ_v/σ_R is, and
2. the higher the initial estimated error covariance is,

the more a 0-step missing measurement will have an impact on the track accuracy.

Multi-step miss

The multi-step miss case is the one where a measurement has been missing and where new measurements have been included since then. In that case, the last measurement inclusion happened after the missing measurement time. Unlike the 0-step miss case and because the last measurement inclusion happened after the missing measurement time, it is believed that the σ_v/σ_R ratio will not have the same impact on track accuracy. Because of the recursive nature of the Kalman filter, this multi-step miss case will not be developed analytically (see Section B for some brief analysis). Rather, it will be studied quantitatively in Section 4.2. It will also be shown that the later the missing measurement occurs in the track update sequence, the more its impact over track accuracy.

4.2 Quantitative analysis

Tests were made to quantify how σ_v, σ_R and age of the missing measurement influence the track estimated error covariances. The 0-step miss case and the multi-step miss case are presented.

The impact of a missing measurement on track accuracy is measured in terms of the determinant ratio (DR), defined as follows

$$\mathrm{DR}(l) = \frac{\det(\boldsymbol{P}_{\psi_l}(k))}{\det(\boldsymbol{P}(k))} \tag{25}$$

The determinant ratio is a way to compare the estimated error covariance matrix of the track that misses a measurement ($\boldsymbol{P}_{\psi_l}(k)$) with the estimated error covariance matrix of the track that does not miss the measurement ($\boldsymbol{P}(k)$) [23]. Note that $\boldsymbol{P}_{\psi_l}(k)$ refers to the state estimated covariance matrix at time k for which a measurement is missing at time $k-l$, as shown in Figure 2. l expresses the age of the missing measurement, or its lag, defined as the number of update intervals passed since the missing measurement time and up to the current time. DR is always greater than 1, and as DR approaches 1, the missing measurements have less impact on the track accuracy.

0-step miss

With the 0-step miss case, the impact of a missing measurement over track accuracy is expected to increase as the ratio σ_v/σ_R increases. Tests made with different σ_v/σ_R values verify this statement, as shows Table 1. In fact, for a noise ratio σ_v/σ_R of 0.02, the DR is about 2. This represents a relatively small impact compared with the case of $\sigma_v/\sigma_R = 50$ and where the DR is about 513950. Thus track uncertainty of tracks that misses the last measurement is much more high for high σ_v/σ_R ratios than for low ones.

σ_v/σ_R	DR
.02	2.0
.1	2.5
1	16.0
10	2206.9
50	513950.6

Table 1: DR for the 0-step miss case and for different noise ratio values (σ_v/σ_R)

Multi-step miss

With the multi-step miss case, the impact of a missing measurement over track accuracy varies depending on σ_v/σ_R, although it is hard to find the relation between σ_v/σ_R and the impact. A more extensive study would be needed.

Anyhow, Table 2 shows that there are σ_v/σ_R values for which the impact is not negligible even for higher step misses. With low σ_v/σ_R ratios we have DR > 1 even for 4-step, 5-step and 6-step missing measurement. For example, with $\sigma_v/\sigma_R = 0.005$ and with a 5-step missing measurement the determinant ratio is 1.19. Low σ_v/σ_R ratios have less impact than high σ_v/σ_R ratios for the small step miss cases (such as 3-step, 2-step and 1-step missing measurements). As the number of steps increases, it seems that the more σ_v/σ_R gets close to 1, the less is the impact of the missing measurement over track accuracy. Figure 3 illustrates the variation of DR for different σ_v/σ_R values and for different step misses.

l		6	5	4	3	2	1	0
σ_v/σ_R	.005	1.225	1.191	1.193	1.231	1.312	1.448	1.665
	.02	1.164	1.154	1.173	1.221	1.307	1.447	1.669
	.1	1.046	1.074	1.117	1.179	1.274	1.433	1.743
	1	1.000	1.001	1.003	1.009	1.058	1.320	2.142
	10	1.000	1.000	1.002	1.011	1.077	1.600	10.023
	50	1.981	2.413	3.178	4.699	8.359	21.084	146.121

Table 2: DR for different miss lags and different noise ratio values (σ_v/σ_R)

From the above discussion, and from the results of Tables 1 and 2, the following conclusions can be drawn with respect to the impact of the missing measurements on track accuracy.

First, in terms of track accuracy it is always better to include a measurement than discarding it if the measurement accuracy does not change through time. Still, the impact on track accuracy can variate depending on several parameters, that include the σ_v/σ_R ratio, the initial estimated error covariance, the sampling interval h, and the missing measurement lag l. From the results shown in Tables 1 and 2, there is an evidence that there may be situations where older measurements have a non-negligible impact on track accuracy. The impact is not the same depending on the age of the missing measurement.

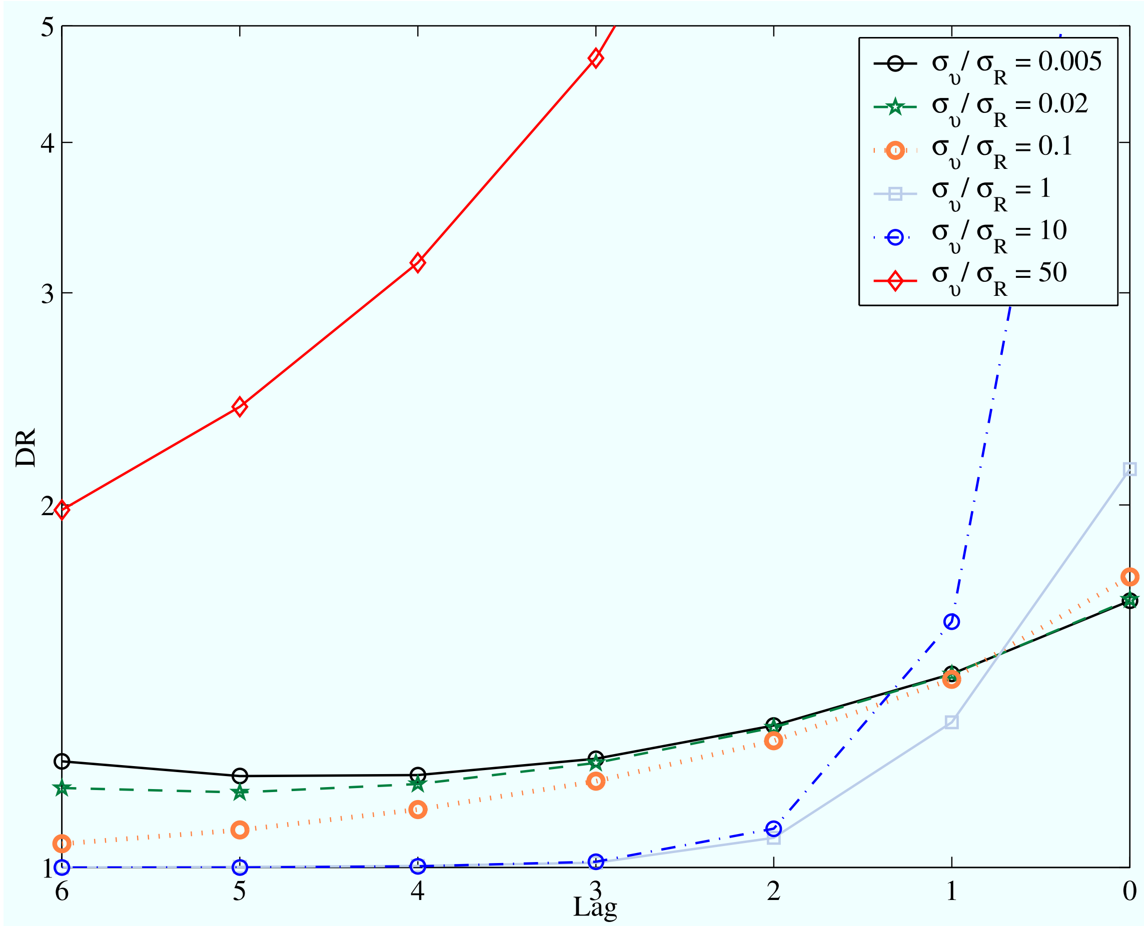

Figure 3: DR for different miss lags and different noise ratio values (σ_υ/σ_R)

Based on these remarks and following a more extensive study, the σ_υ/σ_R ratio could be used to decide whether an OOSM should be discarded or not, depending on its predicted impact on track accuracy. Moreover, the decision to reject or not an OOSM may ultimately be based on computation and memory requirements. Chapter 5 presents an new algorithm that reduces the memory requirements for the inclusion of OOSMs.

5 Forward-Prediction Fusion and Decorrelation

The Forward-Prediction Fusion and Decorrelation (FPFD) method, proposed in this document, uses a decorrelation technique originally presented in [12] and exploited in the information filter used for track-to-track fusion. A forward predicted version of a tracklet that uses the OOSM is fused with the current track using the information form of the Kalman Filter. A tracklet is used, instead of a complete track, to remove the correlation between the two pieces of information and created by their common history. Such a decorrelation aims at making the OOSM-based tracklet independent of the current track.

5.1 Algorithm

At time k, before the OOSM is processed, the state estimate and its error covariance matrix are represented by $\hat{\boldsymbol{x}}(k|k)$ and $\boldsymbol{P}(k|k)$ respectively. As shown in Figure 4, the OOSM is predicted forwards up to time k in order to incorporate the OOSM $\boldsymbol{z}(\tau)$ into the current state estimate $\hat{\boldsymbol{x}}(k|k)$. This requires knowledge of the state estimate at a time $t_b < t_\tau$.

The forward predicted OOSM $\hat{\boldsymbol{x}}(k|\tau)^*$ and the corresponding covariance matrix $\boldsymbol{P}(k|\tau)^*$ are used to update the track $\hat{\boldsymbol{x}}(k|k), \boldsymbol{P}(k|k)$ at time k. The goal is to have a forward predicted OOSM $\hat{\boldsymbol{x}}(k|\tau)^*$ and its covariance matrix $\boldsymbol{P}(k|\tau)^*$ such that the state estimate obtained by using the forward predicted OOSM will be the same as when all the measurements (including the OOSM) are processed sequentially. This requires independence of $\hat{\boldsymbol{x}}(k|\tau)^*$ and $\hat{\boldsymbol{x}}(k|k)$. Consequently, the covariance update for the forward predicted OOSM is given by

$$\boldsymbol{P}^{-1}(k|k)^* = \boldsymbol{P}^{-1}(k|k) + \boldsymbol{P}^{-1}(k|\tau)^* \tag{26}$$

$$= \boldsymbol{P}^{-1}(k|k,\tau) \tag{27}$$

where $\boldsymbol{P}^{-1}(k|k,\tau)$ represents the inverse of the covariance matrix after the OOSM has been included and $\boldsymbol{P}^{-1}(k|k)^*$ represents the inverse of the covariance matrix of the state estimate $\hat{\boldsymbol{x}}(k|k)^*$. $\hat{\boldsymbol{x}}(k|k)^*$ results from the in-sequence reprocessing of all the measurements (including the OOSM).

Equation (26) represents the optimal solution where $\hat{\boldsymbol{x}}(k|\tau)^*$ and $\hat{\boldsymbol{x}}(k|k)$ are independent. The followings will show our approach to get $\hat{\boldsymbol{x}}(k|\tau)^*$.

Let us consider a forward predicted OOSM $\hat{\boldsymbol{x}}(k|\tau)^\Psi$ and its covariance matrix $\boldsymbol{P}(k|\tau)^\Psi$. These are given based on the state estimate $\hat{\boldsymbol{x}}(b)$, with $t_b < t_\tau$, and the corresponding error covariance matrix $\boldsymbol{P}(b)$

$$\boldsymbol{P}(\tau|b) = \boldsymbol{F}(\tau,b)\boldsymbol{P}(b)\boldsymbol{F}(\tau,b)^T + \boldsymbol{Q}(\tau,b) \tag{28}$$

$$\boldsymbol{P}^{-1}(\tau|\tau)^\Psi = \boldsymbol{P}^{-1}(\tau|b) + \boldsymbol{H}(\tau)^T\boldsymbol{R}^{-1}(\tau)\boldsymbol{H}(\tau) \tag{29}$$

$$\boldsymbol{P}(k|\tau)^\Psi = \boldsymbol{F}(k,\tau)\boldsymbol{P}(\tau|\tau)^\Psi\boldsymbol{F}(k,\tau)^T + \boldsymbol{Q}(k,\tau) \tag{30}$$

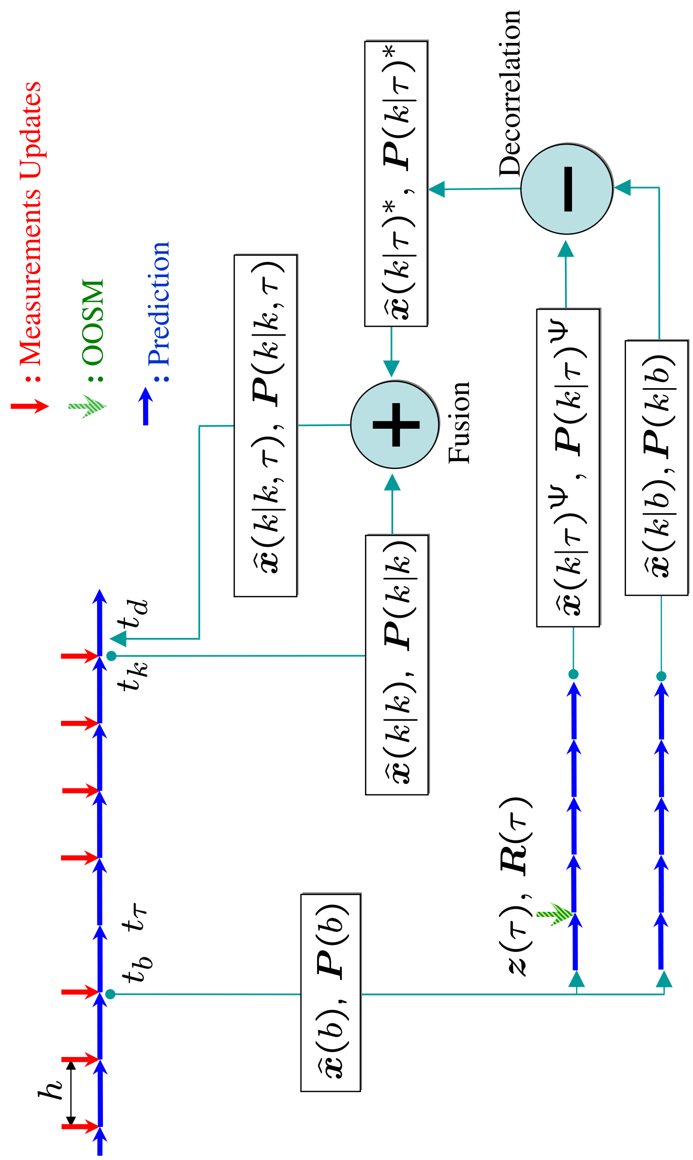

Figure 4: Forward-Prediction Fusion and Decorrelation method for OOSM track update

and

$$\hat{\boldsymbol{x}}(\tau|b) = \boldsymbol{F}(\tau, b)\hat{\boldsymbol{x}}(b) \tag{31}$$

$$\boldsymbol{P}^{-1}(\tau|\tau)^{\Psi}\hat{\boldsymbol{x}}(\tau|\tau)^{\Psi} = \boldsymbol{P}^{-1}(\tau|b)\hat{\boldsymbol{x}}(\tau|b) + \boldsymbol{H}(\tau)^T\boldsymbol{R}^{-1}(\tau)\boldsymbol{z}(\tau) \tag{32}$$

$$\hat{\boldsymbol{x}}(k|\tau)^{\Psi} = \boldsymbol{F}(k, \tau)\hat{\boldsymbol{x}}(\tau|\tau)^{\Psi} \tag{33}$$

where $\boldsymbol{F}(j, i)$ represents the state transition matrix from time t_i to time t_j and $\boldsymbol{Q}(j, i)$ is the process noise error covariance matrix for the time interval $[t_i, t_j]$.

$\hat{\boldsymbol{x}}(k|\tau)^{\Psi}$ and $\hat{\boldsymbol{x}}(k|k)$ are correlated since they share the same history (*i.e.*, $\hat{\boldsymbol{x}}(b)$) and both rely on the same prediction model between t_b to t_k. To have fusion of independent pieces of information, the redundant information between the forward predicted OOSM $\hat{\boldsymbol{x}}(k|\tau)^{\Psi}$ and the state estimate $\hat{\boldsymbol{x}}(k|k)$ must be removed. Let this redundant information be represented by the forward prediction of the state estimate $\hat{\boldsymbol{x}}(b)$ from time t_b to t_k, denoted $\hat{\boldsymbol{x}}(k|b)$. The prediction of the state estimate from time t_b to t_k and the corresponding error covariance matrix are given by

$$\hat{\boldsymbol{x}}(k|b) = \boldsymbol{F}(k, b)\hat{\boldsymbol{x}}(b) \tag{34}$$

$$\boldsymbol{P}(k|b) = \boldsymbol{F}(k, b)\boldsymbol{P}(b)\boldsymbol{F}(k, b)^T + \boldsymbol{Q}(k, b) \tag{35}$$

Using the information form [12], the de-correlation of the forward predicted OOSM $\hat{\boldsymbol{x}}(\tau|b)$ from the state estimate $\hat{\boldsymbol{x}}(k|b)$ is given by

$$\boldsymbol{P}^{-1}(k|\tau)^* = \boldsymbol{P}^{-1}(k|\tau)^{\Psi} - \boldsymbol{P}^{-1}(k|b) \tag{36}$$

$$\boldsymbol{P}^{-1}(k|\tau)^*\hat{\boldsymbol{x}}(k|\tau)^* = \boldsymbol{P}^{-1}(k|\tau)^{\Psi}\hat{\boldsymbol{x}}(k|\tau)^{\Psi} - \boldsymbol{P}^{-1}(k|b)\hat{\boldsymbol{x}}(k|b) \tag{37}$$

where $\hat{\boldsymbol{x}}(k|\tau)^*$ is the resulting de-correlated measurement and $\boldsymbol{P}(k|\tau)^*$ is its corresponding covariance matrix.

Finally, the current state estimate $\hat{\boldsymbol{x}}(k|k)$ is updated with the forward predicted OOSM $\hat{\boldsymbol{x}}(k|\tau)^*$. Using the information fusion form as in (26), the covariance update for the forward predicted OOSM is

$$\boldsymbol{P}^{-1}(k|k, \tau) = \boldsymbol{P}^{-1}(k|k) + \boldsymbol{P}^{-1}(k|\tau)^* \tag{38}$$

and the state update is

$$\boldsymbol{P}^{-1}(k|k, \tau)\hat{\boldsymbol{x}}(k|k, \tau) = \boldsymbol{P}^{-1}(k|k)\hat{\boldsymbol{x}}(k|k) + \boldsymbol{P}^{-1}(k|\tau)^*\hat{\boldsymbol{x}}(k|\tau)^*. \tag{39}$$

5.2 Impact of the correlation between the process noise and the current state

An OOSM algorithm is optimal if it performs exactly as the in-sequence measurements reprocessing method. In order to have the FPFD method optimal, the fusion of the forward predicted OOSM $\hat{\boldsymbol{x}}(k|\tau)^*$ at time t_k must be equivalent to the fusion of the OOSM $\boldsymbol{z}(\tau)$ at time t_τ.

When no measurements have contributed to the state estimate between $]t_b, t_\tau[$ and $]t_\tau, t_k[$, the FPFD method is optimal. This corresponds to the 1-step lag OOSM case. The optimality comes from the fact there is no correlation between the process noise and the state estimate over the intervals $]t_b, t_\tau[$ and $]t_\tau, t_k[$:

$$E\left[\hat{\boldsymbol{x}}(\tau|b)\upsilon(\tau, b)^T\right] = 0, \text{ if } Z_{b+1}^{\tau-1} = \{\emptyset\}, \tag{40}$$

$$E\left[\hat{\boldsymbol{x}}(k|\tau)\upsilon(k, \tau)^T\right] = 0, \text{ if } Z_{\tau+1}^{k-1} = \{\emptyset\}, \tag{41}$$

where $Z_{b+1}^{\tau-1}$ and $Z_{\tau+1}^{k-1}$ are the cumulative sets of measurements from time t_{b+1} to time $t_{\tau-1}$ and from time $t_{\tau+1}$ to time t_{k-1} respectively. In that case the estimation of the state $\boldsymbol{x}$ over the intervals $]t_b, t_\tau[$ and $]t_\tau, t_k[$ uses forward prediction only, so that it does not depend on the process noise. Then, the forward prediction of the OOSM $\boldsymbol{z}(\tau)$ provides an optimal fusion since the process noise does not influence the state estimate $\hat{\boldsymbol{x}}$ between $]t_b, t_\tau[$ and $]t_\tau, t_k[$.

However, when there are one or more measurements between $]t_b, t_\tau[$ or between $]t_\tau, t_k[$, there is some correlation between the process noise and the state estimates for the corresponding intervals $]t_b, t_\tau[$ or $]t_\tau, t_k[$. This corresponds to the multiple-step lag OOSM case.

$$E\left[\hat{\boldsymbol{x}}(j|j)\upsilon(j, j-1)^T\right] \neq 0, \text{ if } \exists \boldsymbol{z}(j), \tag{42}$$

with

$$j = b+1, \ldots, \tau$$

or

$$j = \tau+1, \ldots, k$$

and where $\hat{\boldsymbol{x}}(j|j)$ has been updated with measurement $\boldsymbol{z}(j)$. Equations (1) and (2) demonstrate that $\boldsymbol{z}(j)$ depends on $\boldsymbol{x}(j)$, and that $\boldsymbol{x}(j)$ depends on $\boldsymbol{\upsilon}(j, j-1)$. Therefore $\hat{\boldsymbol{x}}(j|j)$ and $\upsilon(j, j-1)$ are correlated since $\hat{\boldsymbol{x}}(j|j)$ has been updated with $\boldsymbol{z}(j)$.

The estimation of the state $\boldsymbol{x}$ over the intervals $]t_b, t_\tau[$ and $]t_\tau, t_k[$ depends on the measurements $Z_{b+1}^{\tau-1}$ and $Z_{\tau+1}^{k-1}$ and on the forward prediction model. Therefore the state estimate $\hat{\boldsymbol{x}}$ over the intervals $]t_b, t_\tau[$ and $]t_\tau, t_k[$ does depend on the process noise cumulated over $]t_b, t_\tau[$ and $]t_\tau, t_k[$. This correlation between the process noise and the state estimate is not taken into account by $\hat{\boldsymbol{x}}(k|\tau)^*$. Hence, the fusion of $\hat{\boldsymbol{x}}(k|\tau)^*$ and $\hat{\boldsymbol{x}}(k|k)$ is not optimal when there are one or more measurements between $]t_b, t_\tau[$ or between $]t_\tau, t_k[$.

Let $\boldsymbol{e}$ be the discrepancy caused by ignoring the process noise in $\hat{\boldsymbol{x}}(k|\tau)^*$ and $\boldsymbol{E}$ be its corresponding covariance matrix. As a comparison with the optimal case shown in (26) and according to $\boldsymbol{e}$ and $\boldsymbol{E}$, we have

$$\boldsymbol{P}^{-1}(k|k)^* = \boldsymbol{P}^{-1}(k|k) + \boldsymbol{P}^{-1}(k|\tau)^* + \boldsymbol{E}^{-1} \tag{43}$$

$$= \boldsymbol{P}^{-1}(k|k, \tau) \tag{44}$$

and

$$\boldsymbol{P}^{-1}(k|k)^*\hat{\boldsymbol{x}}(k|k)^* = \boldsymbol{P}^{-1}(k|k)\hat{\boldsymbol{x}}(k|k) + \boldsymbol{P}^{-1}(k|\tau)^*\hat{\boldsymbol{x}}(k|\tau)^* + \boldsymbol{E}^{-1}\boldsymbol{e} \tag{45}$$

$$= \boldsymbol{P}^{-1}(k|k,\tau)\hat{\boldsymbol{x}}(k|k,\tau). \tag{46}$$

Note that the equivalent measurement retrodiction solution presented in [3] faces the same correlation problem for multiple-step lag OOSMs.

5.3 Data storage

Data storage may be a relevant issue depending on the storage capacity available in a tracking system. The storage requirements of the retrodiction-based algorithms Al1 and Bl1 from [3], as well as of the optimal algorithm presented in [25], will be compared with the storage requirements of algorithm FPFD.

As mentioned in [3], all OOSM algorithms necessitate, at least, the storage of the following data: i) a scalar for the time stamp of the next update; ii) n scalars for the state estimate; and iii) $n(n+1)/2$ scalars for the covariance of the state estimate. Furthermore, algorithm FPFD requires; i) a scalar for the time stamp t_b associated with track $\hat{\boldsymbol{x}}(b), \boldsymbol{P}(b)$; ii) n scalars for the state estimate $\hat{\boldsymbol{x}}(b)$ held in memory at time instant t_b; iii) $n(n+1)/2$ scalars for the covariance $\boldsymbol{P}(b)$ of the state estimate.

Based on the numbers given above, algorithm FPFD requires a storage of n^2+3n+2 scalars. Because the delay of an OOSM cannot be predicted in advance, a maximum number of lags l_{max} needs to be determined. This maximum number of lags l_{max} has a direct impact on the storage requirements of the retrodiction-based algorithms. As shown below, the storage requirements for the FPFD approach do not depend on l_{max}[1]. Some details about the storage requirements for algorithms Al1 and Bl1 are provided in [3]. The total number of scalars stored for each OOSM algorithm is given below

$$\textbf{FPFD} : n^2 + 3n + 2 \tag{47}$$

$$\text{A}l1 : \left[\frac{l_{max}+1}{2}\right](n^2 + 3n + 2) \tag{48}$$

$$\text{B}l1 : \left[\frac{l_{max}+1}{2}\right](n^2 + 3n + 2) - nl_{max} \tag{49}$$

The data storage requirements of the FPFD, Al1 and Bl1 algorithms are less than those of the optimal algorithm presented in [25], which requires at least[2]

$$\left[\frac{4l_{max}-1}{2}\right]n^2 + \left[\frac{8l_{max}-1}{2}\right]n + l_{max} \tag{50}$$

[1]Although the storage requirements of algorithm FPFD do not depend on a predetermined maximum number of lags l_{max}, its performance in terms of track accuracy does depend on l_{max}. This is discussed in Section 5.4.

[2]Data storage requirements for *Case I : Perfect knowledge about* τ *at time* $j+1$ in [25].

scalars.

Table 3 shows the data storage requirements in terms of l_{max} for a state vector of 4 dimensions.

l_{max}	1	2	3	4
FPFD	30	30	30	30
Al1	30	45	60	75
Bl1	26	37	48	59
Algorithm I in [25]	39	88	137	186

Table 3: Number of scalars needed to be stored in terms of l_{max} and for a state vector of 4 dimensions

5.4 Determination of the storage time of the state estimate

Algorithm FPFD requires the storage of the state estimate and its covariance matrix at a time $t_b < t_\tau$. Depending on the context of the tracking application, different approaches can be used to determine the storage time t_b of the state estimate. For example, in the case where the sampling rate is constant, the estimate can be stored in memory after each measurement update and up until an anticipated measurement update is missed due to a delay (of the OOSM). This would ensure that the estimate is always stored at a time $t_b < t_\tau$, with $t_b = t_\tau - h$.

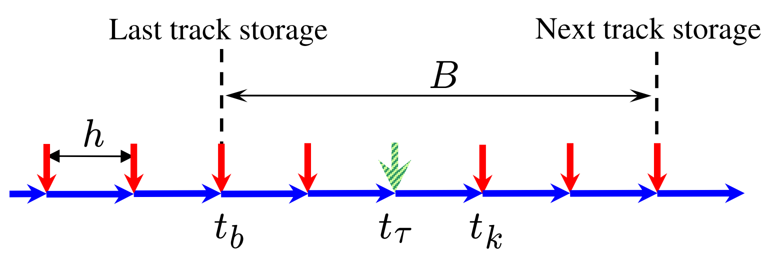

Figure 5: Case where the storage time t_b of the state estimate is determined according to a time interval of length B. The state estimate and its covariance matrix are stored in memory after each time interval of length B, where $B > h$.

In a more general case, the state estimate could be stored in memory after each time interval of length B, where $B > h$. In that case the storage time t_b of the state estimate changes after each time interval of length B, as illustrated in Figure 5. B corresponds to the maximum number of lags l_{max} that are taken into consideration. Note that the state estimate is not stored at the time of the OOSM t_τ.

The value of B has some influence on the performance of algorithm FPFD. On average, the performance will deteriorate as B increases. More exactly, the performance deteriorates as the difference $t_\tau - t_b$ increases, where we have

$$\max(t_\tau - t_b) = B \tag{51}$$

The deterioration of the performance for increasing $t_\tau - t_b$ differences is due to the correlation between the process noise and the current state, as discussed in Section 5.2.

5.5 Data association

Faced with the problem of data association and OOSM, the most straightforward but costly solution is to include the OOSM by reprocessing in chronological order all the measurements that happened after the OOSM time. Besides its computational cost, this method works with any data association algorithm. With more efficient OOSM algorithms that do not reprocess the sequence of the past measurements, and therefore do not store the past measurements, data association becomes a more complicated problem. Some methods use the probabilistic data association filter (PDAF) [10, 24], but they require some large amount of storage and processing. PDAF could be used jointly with the FPFD method, though this will not be investigated in this work. In short the approach would come down to save all tracks at t_W, predict up to the OOSM time, compute the association weight of the OOSM for each track, update the tracks with the OOSM, predict the tracks up to the current time, fuse and decorrelate the same way as described in equations (34) to (39). Note that this method, that combines PDAF and FPFD, would not yield optimal results since PDAF normally requires the innovation for all pairs of measurements and tracks. Besides PDAF, one of the most widely used multi-target tracking and data association approach is the multiple hypotheses tracking algorithm (MHT) [1, 7]. The combination of FPFD and MHT has not been studied, and it looks to be a very challenging problem. As more, no solution have been presented yet in the literature that combines MHT and an OOSM algorithm.

6 Results and discussion

To demonstrate the performance of the FPFD method, the latter is compared with the retrodiction-based algorithms presented in [2, 3]. The results reported therein are compared with those yielded by the FPFD method, using the same test scenarios. First, a scenario with a delayed measurement whose time stamp is within the last sampling interval (Example 2 of [2]) is presented. It will be referred to as the 1-step lag scenario. Afterward, we present a multiple-step lag scenario that is identical to the one used in [3]. This scenario considers the case of delayed measurements whose time stamps are within one or more sampling intervals. Finally, we simulate three OOSM scenarios with 2D nonlinear measurement model and sensor communication delays. The three scenarios are similar to some of the real-world examples presented in [3]. In order to preserve symmetry of the covariance matrices, the Joseph form is used for the covariance update [8]. Also, the storage of the state estimate for algorithm FPFD is made such that $t_b = t_\tau - h$, except for the results shown in Tables 14 and 15, where different values of the storage interval B are tested.

6.1 1-step lag scenario

The considered scenario (from [2]) uses a discrete-time dynamical system with three different values for the continuous time process noise variance ($q = 0.5$, 1, 4). The state equation is given by

$$\boldsymbol{x}(k) = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \boldsymbol{x}(k-1) + \boldsymbol{v}(k, k-1) \tag{52}$$

where h is the sampling interval, and where the process noise $\boldsymbol{v}(k, k-1)$ is assumed white with a zero mean and a covariance matrix

$$E\left[\boldsymbol{v}(k, k-1)\boldsymbol{v}(k, k-1)'\right] = q \begin{bmatrix} h^3/3 & h^2/2 \\ h^2/2 & h \end{bmatrix} = \boldsymbol{Q} \tag{53}$$

The measurement $\boldsymbol{z}(k)$ of the state $\boldsymbol{x}(k)$ is given by

$$\boldsymbol{z}(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \boldsymbol{x}(k) + \boldsymbol{w}(k) \tag{54}$$

where $\boldsymbol{w}(k)$ is white measurement noise, with a zero mean and a covariance matrix

$$E\left[\boldsymbol{w}(k)^2\right] = R = 1. \tag{55}$$

The estimation starts at time $k = 1$, with initial covariance matrix

$$\boldsymbol{P}(1|1) = \begin{bmatrix} R & R/h \\ R/h & 2R/h^2 \end{bmatrix}. \tag{56}$$

An OOSM with time stamp $\tau = 1.5$ has to be processed at time $k = 2$.

Based on the above-described scenario, the FPFD method is compared with the in-sequence measurements reprocessing method (In-seq), OOSM discarding, and with the algorithms[3]

[3] Algorithm A is also referred to as the "Y-algorithm" in [10].

A, B, and C from [2]. Algorithm A is referred to as the optimal retrodiction algorithm in that it accounts entirely for the process noise. Algorithms B and C are called suboptimal retrodiction algorithms since they ignore the retrodicted noise. Algorithm C is a simpler version of algorithm B. Furthermore, the in-sequence measurement reprocessing method is the simple approach that reprocesses all the past measurements chronologically starting from the OOSM time. It provides the optimal solution[4]). The discard approach simply ignores the OOSM.

1. **Performance** — As shown in Table 4, algorithm A and FPFD are both optimal since they yield the same $\hat{P}(k|k,\tau)$ as the in-sequence measurements reprocessing method. This applies for different values of the process noise. Table 5 shows that discarding the OOSM has a significant impact on the track quality, since the trace of $\hat{P}(k|k,\tau)$ is 6.1% to 18.4% higher than in the case of the optimal methods.

q	**4**	**1**	**0.5**
In-Seq	$\begin{bmatrix} .6825 & .7396 \\ .7396 & 2.5725 \end{bmatrix}$	$\begin{bmatrix} .6248 & .5018 \\ .5018 & 1.0539 \end{bmatrix}$	$\begin{bmatrix} .6129 & .4526 \\ .4526 & .7626 \end{bmatrix}$
Discard	$\begin{bmatrix} .8636 & .6818 \\ .6818 & 2.5909 \end{bmatrix}$	$\begin{bmatrix} .8421 & .5526 \\ .5526 & 1.0658 \end{bmatrix}$	$\begin{bmatrix} .8378 & .5270 \\ .5270 & .7872 \end{bmatrix}$
FPFD	$\begin{bmatrix} .6825 & .7396 \\ .7396 & 2.5725 \end{bmatrix}$	$\begin{bmatrix} .6248 & .5018 \\ .5018 & 1.0539 \end{bmatrix}$	$\begin{bmatrix} .6129 & .4526 \\ .4526 & .7626 \end{bmatrix}$
A	$\begin{bmatrix} .6825 & .7396 \\ .7396 & 2.5725 \end{bmatrix}$	$\begin{bmatrix} .6248 & .5018 \\ .5018 & 1.0539 \end{bmatrix}$	$\begin{bmatrix} .6129 & .4526 \\ .4526 & .7626 \end{bmatrix}$
B	$\begin{bmatrix} .6826 & .7396 \\ .7396 & 2.5725 \end{bmatrix}$	$\begin{bmatrix} .6249 & .5018 \\ .5018 & 1.0539 \end{bmatrix}$	$\begin{bmatrix} .6129 & .4526 \\ .4526 & .7626 \end{bmatrix}$
C	$\begin{bmatrix} .7143 & .8571 \\ .8571 & 2.3851 \end{bmatrix}$	$\begin{bmatrix} .6364 & .5455 \\ .5455 & 1.0655 \end{bmatrix}$	$\begin{bmatrix} .6190 & .4762 \\ .4762 & .7754 \end{bmatrix}$

Table 4: Covariance matrices for different process noise (1-step lag scenario)

For further comparison, the actual Mean Square Error (MSE) was also computed through 10000 Monte Carlo runs. The results are summarized in Table 6, where, as a further demonstration of their optimality, the MSE of algorithm FPFD and that of algorithm A are equal to the MSE yielded by in-sequence measurements reprocessing.

[4]Note that algorithm Z_l presented in [25, 17] also provides an optimal solution for the general l-step lag case. However, its storage requirements is high compared to the other sub-optimal OOSM algorithm (see Table 3)

q	4		1		0.5	
In-Seq	3.2550		1.6787		1.3754	
Discard	3.4545	(6.1%)	1.9079	(13.7%)	1.6250	(18.4%)
FPFD	3.2550	(0%)	1.6787	(0%)	1.3754	(0%)
A	3.2550	(0%)	1.6787	(0%)	1.3754	(0%)
B	3.2551	(.003%)	1.6787	(0%)	1.3754	(0%)
C	3.0994	(-4.8%)	1.7019	(1.4%)	1.3944	(1.4%)

Table 5: Trace of covariance matrices and relative deviation with respect to the optimal (1-step lag scenario)

q	4	1	0.5
In-Seq	$\begin{bmatrix} .6895 & .7684 \\ .7684 & 2.5937 \end{bmatrix}$	$\begin{bmatrix} .6274 & .5327 \\ .5327 & 1.1257 \end{bmatrix}$	$\begin{bmatrix} .6192 & .4704 \\ .4704 & .8063 \end{bmatrix}$
Discard	$\begin{bmatrix} .8571 & .6988 \\ .6988 & 2.6211 \end{bmatrix}$	$\begin{bmatrix} .8494 & .5868 \\ .5868 & 1.1389 \end{bmatrix}$	$\begin{bmatrix} .8565 & .5416 \\ .5416 & .8273 \end{bmatrix}$
FPFD	$\begin{bmatrix} .6895 & .7684 \\ .7684 & 2.5937 \end{bmatrix}$	$\begin{bmatrix} .6274 & .5327 \\ .5327 & 1.1257 \end{bmatrix}$	$\begin{bmatrix} .6192 & .4704 \\ .4704 & .8063 \end{bmatrix}$
A	$\begin{bmatrix} .6895 & .7684 \\ .7684 & 2.5937 \end{bmatrix}$	$\begin{bmatrix} .6274 & .5327 \\ .5327 & 1.1257 \end{bmatrix}$	$\begin{bmatrix} .6192 & .4704 \\ .4704 & .8063 \end{bmatrix}$

Table 6: MSE for 10000 Monte Carlo runs (1-step lag scenario)

2. **Cost** — In terms of track quality, the FPFD method was proved, based on the above results, to be as optimal as method A. Algorithms FPFD and A provided a similar performance, both in terms of MSE and estimation error covariance estimation. For the cost associated with data storage, and for $l_{max} = 1$, equations (47) to (49) lead to identical requirements for algorithms FPFD and A (*i.e.*, $n^2 + 3n + 2$) and less storage requirement for algorithm B (*i.e.*, $n^2 + 2n + 2$). The counterpart of this lower cost for algorithm B is its lower performance in terms of tracking quality, as shown in Tables 4 to 6.

6.2 Multiple-step lag scenario

A dynamic system, identical to the one used in [3], is considered with three different OOSM lags $l = 1, 2, 3$. The system behavior is similar to the 1-step lag case, where the dynamics are given by (52) and (53), except that here both position and velocity are measured. The measurement equation is then

$$\boldsymbol{z}(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{x}(k) + \boldsymbol{w}(k) \tag{57}$$

where $\boldsymbol{w}(k)$ has an error covariance matrix

$$\boldsymbol{R} = \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix}. \tag{58}$$

The filter is initiated at $t_0 = 0$, with

$$\hat{\boldsymbol{x}}(0|0) = \boldsymbol{z}(0), \quad \boldsymbol{P}(0|0) = \boldsymbol{R} \tag{59}$$

and ends up at $t_4 = 4s$. The three OOSM lags $l = 1, 2, 3$ correspond to times $\tau = 1.5s, 2.5s$, and $3.5s$, as illustrated in Figure 6.

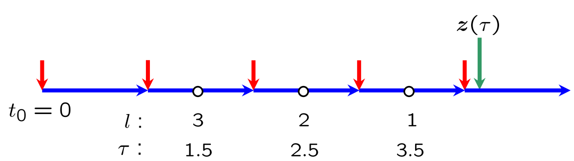

Figure 6: OOSM with three different lags $l = 1, 2, 3$

The in-sequence measurements reprocessing method, algorithm Al1, algorithm[5] Bl1, the OOSM discard solution, and algorithm FPFD are all compared. Algorithms Al1 and Bl1 are the l-step lag extensions to the 1-step lag algorithms A and B presented in [2], respectively. They both use an equivalent measurement concept originally presented in [6].

[5] Bl1 is the one-step equivalent measurement version of the "M-algorithm" defined in [16].

Lag	1	2	3
In-Seq	$\begin{bmatrix} .2287 & .0225 \\ .0225 & .0759 \end{bmatrix}$	$\begin{bmatrix} .2597 & .0381 \\ .0381 & .0832 \end{bmatrix}$	$\begin{bmatrix} .2854 & .0387 \\ .0387 & .0833 \end{bmatrix}$
FPFD	$\begin{bmatrix} .2287 & .0225 \\ .0225 & .0759 \end{bmatrix}$	$\begin{bmatrix} .2563 & .0372 \\ .0372 & .0827 \end{bmatrix}$	$\begin{bmatrix} .2906 & .0403 \\ .0403 & .0827 \end{bmatrix}$
Al1	$\begin{bmatrix} .2287 & .0225 \\ .0225 & .0759 \end{bmatrix}$	$\begin{bmatrix} .2563 & .0372 \\ .0372 & .0827 \end{bmatrix}$	$\begin{bmatrix} .2906 & .0403 \\ .0403 & .0827 \end{bmatrix}$
Bl1	$\begin{bmatrix} .2330 & .0254 \\ .0254 & .0779 \end{bmatrix}$	$\begin{bmatrix} .2667 & .0389 \\ .0389 & .0830 \end{bmatrix}$	$\begin{bmatrix} .2955 & .0403 \\ .0403 & .0828 \end{bmatrix}$
Discard		$\begin{bmatrix} .3142 & .0370 \\ .0370 & .0834 \end{bmatrix}$	

Table 7: Covariance matrices for different lag values (l-step lag scenario), $q = 0.5$

1. **Performance** — As shown in Tables 7 and 8, for the l-step lag case ($l > 1$), algorithm FPFD has a performance equal to algorithm Al1. According to the estimated covariance matrix $\hat{P}(k|k,\tau)$, the performance of algorithm FPFD does degrade as the number of step-lag increases, compared to the in-sequence reprocessing of the measurements. The difference is represented in Table 8, where the trace of $\hat{P}(k|k,\tau)$ is presented for the different algorithms, along with the trace relative deviation compared to the optimal in-sequence measurements reprocessing. This degradation is due to the dependence issue between the process noise and the state (discussed in Chapter 5).

Lag	1		2		3	
In-Seq	.3046		.3429		.3687	
Discard	.3976	(30.5%)	.3976	(16.0%)	.3976	(7.8%)
FPFD	.3046	(0%)	.3390	(-1.1%)	.3733	(1.2%)
Al1	.3046	(0%)	.3390	(-1.1%)	.3733	(1.2%)
Bl1	.3109	(2%)	.3497	(2%)	.3783	(2.6%)

Table 8: Trace of covariance matrices and relative deviation with respect to the optimality (l-step lag scenario), $q = 0.5$

Table 9 presents the MSE of each algorithm for the l-step lag cases. MSE for algorithms FPFD and Al1 are close to the MSE provided by the in-sequence measurement reprocessing method. Algorithm Bl1 is also relatively close to the optimal method. Note that the performance of algorithms Al1 and Bl1 was discussed thoroughly in [3].

Lag	**2**	**3**
In-Seq	$\begin{bmatrix} .2564 & .0371 \\ .0371 & .0813 \end{bmatrix}$	$\begin{bmatrix} .2835 & .0404 \\ .0404 & .0845 \end{bmatrix}$
Discard	$\begin{bmatrix} .3110 & .0356 \\ .0356 & .0814 \end{bmatrix}$	$\begin{bmatrix} .3120 & .0382 \\ .0382 & .0847 \end{bmatrix}$
FPFD	$\begin{bmatrix} .2592 & .0380 \\ .0380 & .0817 \end{bmatrix}$	$\begin{bmatrix} .2887 & .0408 \\ .0408 & .0848 \end{bmatrix}$
Al1	$\begin{bmatrix} .2592 & .0380 \\ .0380 & .0817 \end{bmatrix}$	$\begin{bmatrix} .2887 & .0408 \\ .0408 & .0848 \end{bmatrix}$
Bl1	$\begin{bmatrix} .2613 & .0375 \\ .0375 & .0815 \end{bmatrix}$	$\begin{bmatrix} .2894 & .0403 \\ .0403 & .0847 \end{bmatrix}$

Table 9: MSE for different lag values (l-step lag scenario) and for 10000 Monte Carlo runs, $q = 0.5$

2. **Cost** — According to equations (47) to (49), the FPFD method has a storage requirement that is equal to the one of algorithm Al1 only for $l_{max} = 1$ (1-step lag situation). For l-step lag scenarios, where $l_{max} > 1$, algorithm FPFD requires less storage space than algorithm Al1. This is also the case with algorithm Bl1. As shown previously, algorithm Bl1 requires less storage for the 1-step lag case ($l_{max} = 1$). However, its storage requirements are larger for multiple-step lag OOSMs ($l_{max} > 1$).

6.3 2D nonlinear measurement model and sensor communication delays

The following example is drawn from the practical examples in [3, 17]. It aims at comparing the FPFD method against other OOSM algorithms when nonlinear measurements conversion are considered. As in [3, 17], a target is tracked using two GMTI sensors. The target motion follows the constant velocity model in two dimensions with process noise spectral density $q = 1m^2/s^3$. The two GMTI sensors have nearly orthogonal lines-of-sight and both have a slant range of about 100 km from the target. Each sensor observation is in polar coordinates and includes range (r), azimuth (θ) and range rate ($\dot{r}$). The related standard deviations are 10 m, 1 $mrad$ and 1 m/s, respectively. Since the bias significance factor is below 0.4 ($r\sigma_\theta^2/\sigma_r \approx 0.001$), the measurements are converted into Cartesian coordinates using the conventional coordinate transformation [4, 7]. Three scenarios are simulated. Scenario 1-SL, 3-SL and 5-SL see sensor 1 have its last measurement delayed with one lag,

Scenario 1-SL	Sensor ID	1	2	1	2	1	2	1	2	1	2	2	1
	Time Stamp (s)	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	27.5	25
Scenario 3-SL	Sensor ID	1	2	1	2	1	2	1	2	2	1	2	1
	Time Stamp (s)	0	2.5	5	7.5	10	12.5	15	17.5	22.5	25	27.5	20
Scenario 5-SL	Sensor ID	1	2	1	2	1	2	2	1	2	1	2	1
	Time Stamp (s)	0	2.5	5	7.5	10	12.5	17.5	20	22.5	25	27.5	15

Table 10: Communication delays in a 2 GMTI radar network. 1-step lag (1-sl), 3-step lag (3-sl) and 5-step lag (5-sl) OOSM scenarios.

	Trace($\boldsymbol{P}$)			Trace(MSE)		
Scenario	$1l$	$3l$	$5l$	$1l$	$3l$	$5l$
FPFD	252.39	249.24	253.62	251.63	259.68	260.03
Al1	252.39	249.24	253.62	251.63	259.68	260.03

Table 11: Traces of the average filter covariance matrix (Trace($\boldsymbol{P}$)) and of the actual MSE matrix (Trace(MSE)) for the OOSM algorithms Al1 and FPFD. Results are obtained from 1000 Monte Carlo runs

three lags and five lags, respectively. The lists of the measurements sent to the central tracker by the two GMTI sensors are presented in Table 10 for the three scenarios. The initial target state is [70000 m, 70000 m, 60 m/s, 20 m/s]. Track initialization is made according to the two-point initialization technique [5].

Table 11 shows the trace of the average filter covariance matrix and the trace of the actual MSE matrix for the OOSM algorithms Al1 and FPFD. Clearly, for the three scenarios, the results are the same whether algorithm Al1 or algorithm FPFD is used. Table 12 shows the normalized estimation error squared (NEES) [5] for the 4-dimensional state based on 1000 runs. Both algorithms Al1 and FPFD result in a NEES of 3.87 in scenario $1l$, 4.00 in scenario $3l$ and 4.14 in scenario $5l$. The NEES for algorithms Al1, Bl1 and FPFD all lie within the two-sided 95% confidence bounds based on the χ^2_{4000} distribution (3.8261, 4.1767) [5]. Therefore, algorithms Al1, Bl1 and FPFD are statistically consistent for the three scenarios.

The aim of this practical example is to show that algorithm FPFD has the same performance as algorithm Al1 in a practical OOSM example that involves nonlinear measurement conversions. The results presented in Tables 11 and 12 are conclusive, since the measured performance of algorithm FPFD is identical to the measured performance of algorithm Al1.

Moreover, Table 13 shows the CPU times of algorithms Al1, Bl1 and FPFD for 1000 Monte Carlo runs. Although the measured CPU times represent only imprecise approximations of

Scenario	$1l$	$3l$	$5l$
In-Seq	3.87	3.97	4.01
FPFD	3.87	4.00	4.14
Al1	3.87	4.00	4.14
Bl1	3.87	4.01	4.08

Table 12: Normalized Estimation Error Squared (NEES) at last update time for different OOSM algorithms. 1-step lag (1-SL), 3-step lag (3-SL) and 5-step lag (5-SL) OOSM scenarios. Results are obtained from 1000 Monte Carlo runs.

Lag	1	3	5
FPFD	0.71	0.67	0.73
Al1	0.82	0.84	0.82
Bl1	0.75	0.68	0.69

Table 13: CPU times (s) for 1000 Monte Carlo runs

the computational complexity of the algorithms, they are used here for comparison purposes. The measured CPU times of algorithm FPFD are comparable to those of algorithm Bl1, which are lower than those of algorithm Al1 in all of the three lag cases shown in Table 13.

Finally, Tables 14 and 15 show the performance of algorithm FPFD according to the storage interval B defined in (51) and for the 1-step lag scenario described in Table 10 (Scenario 1-SL). The NEES obtained with algorithm FPFD and shown in Table 14 are 3.99, 4.01 and 4.02 for $B = 1$, $B = 4$ and $B = 6$, respectively. Therefore, the NEES increases slightly as B augments. Recall that the corresponding two-sided 95% confidence bounds for 1000 Monte Carlo are $[3.8261, 4.1767]$. The traces of the actual MSE matrices shown in Table 15 also increase slightly as B augments, while the traces of the average filter covariance matrices decrease as B augments. Note that for $B > 1$ algorithm FPFD loses its optimality compared to the in-sequence measurements reprocessing method. Hence, we have $\boldsymbol{e} \neq 0$ in (45) when $B > 1$.

B	1	4	6
FPFD	3.99	4.01	4.02
In-Seq		3.99	

Table 14: NEES for algorithm FPFD at last update time, for the 1-step lag OOSM case (Scenario 1-SL) and in terms of the storage interval B. Comparison with the In-sequence reprocessing method (In-Seq) from 1000 Monte Carlo runs.

	Trace($\boldsymbol{P}$)			Trace(MSE)		
B	1	4	6	1	4	6
FPFD	252.3	250.8	250.4	247.1	247.4	247.9
In-Seq		252.3			247.1	

Table 15: Traces of the average filter covariance matrix (Trace($\boldsymbol{P}$)) and of the actual MSE matrix (Trace(MSE)) for the 1-step lag OOSM case (Scenario 1-SL) and in terms of the storage interval B of algorithm FPFD. Comparison with the in-sequence measurement reprocessing method (In-Seq) from 1000 Monte Carlo runs.

7 Conclusions

In the preceding, a forward prediction and decorrelation-based method for processing out-of-sequence measurements was presented. In terms of track quality, the proposed FPFD method was proved to be optimal for the 1-step lag case. For the multiple-step lag case, the method loses its optimality compared to the in-sequence measurements reprocessing and to the optimal solution presented in [25]. Nonetheless, its results are valuable since they are equal to those obtained with some of the most recent retrodiction methods presented in the literature, while requiring less data storage. Finally, the performance of the FPFD method depends on the track storage interval B. It was shown that the performance deterioration is minor for small values of B. Moreover, when the sampling rate is fixed so that it is known when a measurement has not arrived, the time of track storage can be brought close enough to the OOSM time such that there is no performance deterioration due to the track storage time. In such conditions, the FPFD method can be a valuable choice for practical applications.

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Annex A: Dynamics model

The tracked target is assumed to be moving with constant velocity in a 2D space, where the acceleration acts as an input[6]. Cartesian coordinates are used in order to be in a linear system. The state to be estimated is composed of the target's coordinates, *viz.*, the position and the linear velocity. With the following state variable notation

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{\upsilon} = \begin{bmatrix} \boldsymbol{\upsilon}_1 \\ \boldsymbol{\upsilon}_2 \end{bmatrix} \tag{A.1}$$

the equations of such a target can be expressed as

$$\boldsymbol{x}(k+1) = \boldsymbol{F}(k, k-1)\boldsymbol{x}(k) + \boldsymbol{\Gamma}\boldsymbol{\upsilon}(k) \tag{A.2}$$

where $\boldsymbol{\upsilon}$ is a vector of random variables that reflects the unforeseeable variation of the acceleration in both directions. Note that $\boldsymbol{\Gamma}\boldsymbol{\upsilon}(k)$ is the same as $\boldsymbol{\upsilon}(k, k-1)$, which represents the cumulative effect of the process noise over the time increment $k-1, k$. The state transition matrix is given by

$$\boldsymbol{F}_k = \begin{bmatrix} 1 & 0 & h & 0 \\ 0 & 1 & 0 & h \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.3}$$

where h is the time increment. The matrix $\boldsymbol{\Gamma}$ in (A.2) depends on the model used to represent the discrete-time nature of the process noise $\boldsymbol{\upsilon}$. Two examples are given below. The first one

$$\boldsymbol{\Gamma} = \begin{bmatrix} h^2/2 & 0 \\ 0 & h^2/2 \\ h & 0 \\ 0 & h \end{bmatrix} \tag{A.4}$$

represents to the pulse model, while the following one

$$\boldsymbol{\Gamma} = \begin{bmatrix} \frac{\sqrt[3]{h^2}}{2\sqrt{3}} & \frac{\sqrt[3]{h^2}}{2} & 0 & 0 \\ 0 & 0 & \frac{\sqrt[3]{h^2}}{2\sqrt{3}} & \frac{\sqrt[3]{h^2}}{2} \\ 0 & \sqrt{h} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{h} \end{bmatrix} \tag{A.5}$$

[6]Since the acceleration may change unforeseeably, it is often modeled as a random variable, and so will be as such.

is used by the Brownian motion modeling. Note that the latter requires four random variables instead of two. If the process noises are assumed to have the same standard deviation $\boldsymbol{\delta}_v$, the pulse model results in the following process noise covariance matrix

$$\begin{aligned} \boldsymbol{Q} &= \boldsymbol{\delta}_v^2 \boldsymbol{\Gamma}\boldsymbol{\Gamma}^T && \text{(A.6)} \\ &= \boldsymbol{\delta}_v^2 \begin{bmatrix} \frac{h^4}{4} & 0 & \frac{h^3}{2} & 0 \\ 0 & \frac{h^4}{4} & 0 & \frac{h^3}{2} \\ \frac{h^3}{2} & 0 & h^2 & 0 \\ 0 & \frac{h^3}{2} & 0 & h^2 \end{bmatrix} && \text{(A.7)} \end{aligned}$$

while the covariance matrix for the Brownian motion is given by

$$\begin{aligned} \boldsymbol{Q} &= \boldsymbol{\delta}_v^2 \boldsymbol{\Gamma}\boldsymbol{\Gamma}^T && \text{(A.8)} \\ &= \boldsymbol{\delta}_v^2 \begin{bmatrix} \frac{h^3}{3} & 0 & \frac{h^2}{2} & 0 \\ 0 & \frac{h^3}{3} & 0 & \frac{h^2}{2} \\ \frac{h^2}{2} & 0 & h & 0 \\ 0 & \frac{h^2}{2} & 0 & h \end{bmatrix} && \text{(A.9)} \end{aligned}$$

Annex B: l-step lag case analysis of $\boldsymbol{P}(k)$ and $\boldsymbol{P}(k)_{\psi_l}$

It will be demonstrated that $\boldsymbol{P}(k)_{\psi} - \boldsymbol{P}(k)$ is always positive definite for the 1-step lag cases. The results will then be generalized to the l-step lag case.

B.1 1-step lag case

Let $\boldsymbol{P}_{k-2}$ be the error covariance matrix at one sampling interval before the OOSM time. In the case the measurement is available, we have:

$$\boldsymbol{P}(k-1)^{-1} = [\boldsymbol{F}\boldsymbol{P}(k-2)\boldsymbol{F}^T + \sigma_v^2\Upsilon]^{-1} + [\sigma_R^2\boldsymbol{\zeta}]^{-1} \tag{B.1}$$

$$\boldsymbol{P}(k)^{-1} = \boldsymbol{A} + [\sigma_R^2\boldsymbol{\zeta}]^{-1} \tag{B.2}$$

where

$$\boldsymbol{A} = \left[\left[[\boldsymbol{F}\boldsymbol{F}\boldsymbol{P}(k-2)\boldsymbol{F}^T\boldsymbol{F}^T + \sigma_v^2\boldsymbol{F}\Upsilon\boldsymbol{F}^T]^{-1} + [\sigma_R^2\boldsymbol{F}\boldsymbol{\zeta}\boldsymbol{F}^T]^{-1}\right]^{-1} + \sigma_v^2\Upsilon\right]^{-1} \tag{B.3}$$

In the case where the measurement at time k is missing, we have:

$$\boldsymbol{P}_{\psi_1}(k-1)^{-1} = [\boldsymbol{F}\boldsymbol{P}(k-2)\boldsymbol{F}^T + \sigma_v^2\Upsilon]^{-1} \tag{B.4}$$

$$\boldsymbol{P}_{\psi_1}(k)^{-1} = \boldsymbol{B} + [\sigma_R^2\boldsymbol{\zeta}]^{-1} \tag{B.5}$$

where

$$\boldsymbol{B} = \left[\boldsymbol{F}\boldsymbol{F}\boldsymbol{P}(k-2)\boldsymbol{F}^T\boldsymbol{F}^T + \sigma_v^2\boldsymbol{F}\Upsilon\boldsymbol{F}^T + \sigma_v^2\Upsilon\right]^{-1} \tag{B.6}$$

Then the difference between the track estimated error covariance without a measurement update at time k ($\boldsymbol{P}(k)_{\psi_1}$) and the track estimated error covariance with a measurement update at time k ($\boldsymbol{P}(k)$) is:

$$\boldsymbol{P}(k)_{\psi_1} - \boldsymbol{P}(k) = \left[\boldsymbol{A} + [\sigma_R^2\boldsymbol{\zeta}]^{-1}\right] - \left[\boldsymbol{B} + [\sigma_R^2\boldsymbol{\zeta}]^{-1}\right] \tag{B.7}$$

$$\boldsymbol{P}(k)_{\psi_1} - \boldsymbol{P}(k) = \boldsymbol{A} - \boldsymbol{B} \tag{B.8}$$

since $\boldsymbol{A} - \boldsymbol{B}$ is positive definite, then we have:

$$\boldsymbol{x}^T(\boldsymbol{A} - \boldsymbol{B})\boldsymbol{x} > 0 \tag{B.9}$$

for all nonzero vectors $\boldsymbol{x} \in \mathcal{R}^n$. Equation (B.9) can be developed into

$$\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x} - \boldsymbol{x}^T\boldsymbol{B}\boldsymbol{x} > 0 \tag{B.10}$$

$$\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x} > \boldsymbol{x}^T\boldsymbol{B}\boldsymbol{x} \tag{B.11}$$

$$\boldsymbol{x}^T\boldsymbol{A}^{-1}\boldsymbol{x} < \boldsymbol{x}^T\boldsymbol{B}^{-1}\boldsymbol{x} \tag{B.12}$$

For simplicity, let $\boldsymbol{A}$ and $\boldsymbol{B}$ be expressed as

$$\boldsymbol{A} = \left[\left[[\boldsymbol{C}]^{-1} + [\boldsymbol{D}]^{-1}\right]^{-1} + \sigma_v^2\Upsilon\right]^{-1} \tag{B.13}$$

$$\boldsymbol{B} = [\boldsymbol{C} + \sigma_v^2\Upsilon]^{-1} \tag{B.14}$$

where

$$C = FFP(k-2)F^T F^T + \sigma_\upsilon^2 F \Upsilon F^T \quad \text{(B.15)}$$

$$D = \sigma_R^2 \zeta \quad \text{(B.16)}$$

Using (B.13) and (B.14) in (B.12) yields

$$x^T \left[\left[[C]^{-1} + [D]^{-1} \right]^{-1} + \sigma_\upsilon^2 \Upsilon \right] x < x^T [C + \sigma_\upsilon^2 \Upsilon] x \quad \text{(B.17)}$$

$$x^T \left[[C]^{-1} + [D]^{-1} \right]^{-1} x + x^T \sigma_\upsilon^2 \Upsilon x < x^T C x + x^T \sigma_\upsilon^2 \Upsilon x \quad \text{(B.18)}$$

$$x^T \left[[C]^{-1} + [D]^{-1} \right]^{-1} x < x^T C x \quad \text{(B.19)}$$

$$x^T \left[[C]^{-1} + [D^{-1}] \right] x > x^T C^{-1} x \quad \text{(B.20)}$$

$$x^T C^{-1} x + x^T D^{-1} x > x^T C^{-1} x \quad \text{(B.21)}$$

$$x^T D^{-1} x > 0 \quad \text{(B.22)}$$

This is always true since D^{-1} is positive definite. Therefore, $P(k)_\psi - P(k)$ is always positive definite for the 1-step lag cases. On the contrary, the statement

$$x^T D^{-1} x \leq 0 \quad \text{(B.23)}$$

is always false. This means that a track that has a 1-step lag missing measurement will always have poorer accuracy compared to a track that does not miss the measurement.

B.2 General l-step lag case

Because the Kalman equations are recursive, the proofs that $P(k)_\psi - P(k)$ is always positive definite for the 1-step lag cases can be extended to the general l-step lag case. Consequently, in terms of track accuracy it is always better to include a measurement than discarding it. Although its impact over track accuracy may change depending on some conditions, such as the σ_υ/σ_R ratio and the measurement age, there are no conditions where it is better to discard a measurement when σ_υ, σ_R and the sampling interval stay constant throughout the entire filtering duration.

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François Rhéaume
RDDC - Valcartier
2495, boul. Pie-Xi Nord
Québec Qc, G3J 1X5 Canada

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Out-of-sequence measurements filtering using forward prediction (U)

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In target tracking applications, there may be situations where measurements from a given target arrive out of sequence at the processing center. This problem is commonly referred to as Out-of-Sequence Measurements (OOSMs). So far, most of the existing solutions to this problem are based on retrodiction, where backward prediction of the current estimated state is used to incorporate the OOSMs at appropriate time instants. This paper suggests a new method for tackling the OOSMs problem without backward prediction. Based on a forward prediction and de-correlation approach, the method has proved to compare favorably to the best retrodiction-based methods, while requiring less data storage in most cases.

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Data fusion, Tracking, Out-of-sequence, De-correlation

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