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Narrowband Adaptive Antennas – Basic Concepts

Mathieu Caillet

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Mathieu Caillet

Prepared by:

Royal Military College of Canada
PO Box 17000, Station Forces
Kingston, Ontario, Canada K7K 7B4

Project Manager: Mike Vinnins
Contract Number: A1410FE255
Contract Scientific Authority: Michel Clénet

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Scientific authority

Original signed by Michel Clénet

Michel Clénet

Approved by

Original signed by Mike Vinnins

Mike Vinnins
A/SH CNEW

Approved for release by

Original signed by Brian Eatock

Brian Eatock
A/Chairman DRP

Abstract

The aim of this report is to address basic concepts for narrowband adaptive antennas, also called smart antennas. This report begins with the introduction of a signal model for the study of adaptive antennas. Then, adaptive array architecture is discussed. Moreover, descriptions of performance criteria, capabilities and limitations are presented. To conclude, various narrowband algorithms are highlighted.

Résumé

L'objet de ce rapport est de présenter les concepts fondamentaux des antennes adaptatives (ou antennes intelligentes), en se restreignant au cas bande (de fréquence) étroite. Dans la première partie, un modèle de signal est introduit pour l'étude des antennes adaptatives. Puis, l'architecture d'une antenne adaptative est détaillée. En outre, les critères de performance, les possibilités et limites sont donnés. Pour conclure, quelques algorithmes pour les bandes étroites sont passés en revue.

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Executive summary

Narrowband Adaptive Antennas – Basic Concepts

Mathieu Caillet; DRDC Ottawa CR 2007-165; Defence R&D Canada - Ottawa; August 2007.

Background: Due to the increasing demand for wireless technology in the early 1990's, interference has increased. Furthermore, jamming systems have become more and more smart. Adaptive antennas present an interesting solution to overcome jamming or interference in communication systems. Research and development efforts in this area have been revived by the increasing number of studies on anti-jamming receivers.
In this report, a review of existing adaptive techniques and associated algorithms has been undertaken.

Five possible architectures through which adaptive beamforming can be carried out are summarized. Advantages and drawbacks are given for each architecture in terms of complexity. This allows to choose an architecture by minimizing the cost.
An example is proposed to understand the primitive way to cancel an interference. Through a 2-element array, the case of a desired signal and an interference is addressed.

Several different performance measures can be adopted to govern the operation of the adaptive processor that adjusts the weighting for each of the sensor element outputs. Several performance criteria are present in the literature. In this report, two fundamental adaptive arrays are studied in detail: the Applebaum and the LMS arrays. Capabilities and limitations are highlighted, and examples have been implemented with Matlab to illustrate notes and conclusions. Thus, one can decide which performance measure is appropriate to a specific application.

The adaptive control algorithm that is selected to adjust the array beam pattern is highly important since this choice directly influences both the speed of the array transient response and the complexity of the circuitry required to implement the algorithm. Among many algorithms that can be found in the literature, basic ones have been selected and are presented in this report. Here again, examples have been implemented with Matlab to illustrate advantages and drawbacks in term of convergence speed, load computing and ease of development.
Some implementation issues relating to real versus complex implementation are discussed.

Future work: The presentation of these concepts will allow the study of an adaptive antenna for anti-jamming system. For this purpose, the implementation of some adaptive algorithms in PAASoM, a Phased Array Simulation Tool, could be very interesting to design adaptive antennas and observe the performances versus the antenna element positions

and the elementary pattern. In this manner, the efficiency of the control algorithm and the beamforming network parts of the adaptive array could be tested in terms of speed convergence, pointing accuracy and null depths.

Sommaire

Narrowband Adaptive Antennas – Basic Concepts

Mathieu Caillet; DRDC Ottawa CR 2007-165; R & D pour la défense Canada - Ottawa; août 2007.

Contexte: Du fait des besoins croissants dans le domaine des technologies sans fil au début des anneés 90, les interférences se sont intensifiées. De plus, les systèmes de brouillage deviennent de plus en plus intelligents. Les antennes adaptatives présentent une solution intéressante pour remédier aux brouillages ou interférences dans les systèmes de communication. Les efforts en terme de recherche et developpement dans ce domaine ont été ravivés du fait d'un nombre important d'études sur les récepteurs à réjection de brouilleurs.
Dans ce rapport, une analyse des techniques utiliées pour les systèmes adaptatifs et les algorithmes associés est entreprise.

Cinq architectures possibles pour lesquelles la formation de faisceaux adaptatifs est effectuée ont été identifiées. Les avantages et inconvénients sont donnés pour chaque architecture en terme de complexité. Cela permet la sélection d'une architecture en minimisant son coût.
Un exemple est proposé pour permettre la compréhension du principe fondamental pour l'annulation d'une interférence. Basé sur un réseau à deux éléments, le cas d'un signal utile et d'une interférence est proposé.

Plusieurs mesures de performance peuvent être adoptées pour contrôler les opérations du processeur adaptatif qui corrige la pondération de chacun des capteurs. Divers critères de performance sont présents dans la littérature. Deux réseaux adaptatifs fondamentaux sont étudiés ici en détails : les réseaux Applebaum et LMS. Les possibilités et limitations sont soulignées, et des exemples d'implémentation avec Matlab illustrent les remarques et conclusions. Ainsi, chacun est à même de décider quelle mesure de performance est adaptée à une application spécifique.

L'algorithme de contrôle adaptatif sélectionné pour contrôler le faisceau du réseau est très important puisque ce choix influe directement sur la durée du régime transitoire ainsi que sur la complexité du circuit nécessaire à l'implémentation de l'algorithme. Parmis les algorithmes présents dans la littérature, les algorithmes fondamentaux ont été sélectionnés et sont présentés dans ce rapport. Ici encore, des exemples ont été implémentés avec Matlab pour mettre en évidence les avantages et les inconvénients en terme de rapidité de convergence, charge de calcul du traitement et facilité de developpement.
Les principaux problèmes d'implémentation liés au choix d'une implémentation dans le domaine réel ou complexe sont discutés ici.

Perspectives: Les concepts présentés ici vont permettre l'étude d'une antenne adaptative pour un système à réjection de brouilleurs. Pour cela, l'implémentation de certains algorithmes adaptatifs dans PAASoM, un outil d'analyse des réseaux phasés, pourrait s'avérer très interessante pour la conception d'antennes adaptatives ainsi que pour la comparaison des performances du système en fonction de la position des éléments rayonnants et du diagramme d'un element.

Table of contents

List of figures

Notation

d	Element spacing
$d(t)$	Reference signal
m	Number of interferences
p	Number of nulls
w	Weight associated to an antenna element
$\mathbf{w}$	Weights' vector
$\bar{\mathbf{w}}$	Steady state weight vector
x	Signal induced on an antenna element
$\hat{x}(t)$	Hilbert transform of a real signal $x(t)$
$\mathbf{x}$	Signals' vector
y	Array output
A_d	Desired signal amplitude
A_i	Interference amplitude
I	Identity matrix
L	Number of antenna elements
P	Array output power
S	Correlation vector between $d(t)$ and signals from the antenna elements
U	Steering vector
ε	Error signal
ϕ	Array correlation matrix
γ	Step size (LMS algorithm)
λ	Free space wavelength
θ	Angle of arrival (AoA)
σ^2	Thermal noise power from each antenna element
ξ_d	Input desired signal to noise ratio
ξ_i	Input interference to noise ratio

1 Introduction

The demand for wireless technology and thus society's dependence on cellular systems has grown at a staggering rate [9]. This has resulted in decreasing the quality of services due to interferences growing. One practical solution to this ever expanding problem is the use of adaptive antennas or smart antenna systems [6, 1]. Currently, the popularity of smart antennas has increased due to the advancement in powerful and low cost digital signal processors and complex signal processing techniques [2].

As radar and communication traffic augments, the suppression of interference becomes more important in all applications. Noise signals may consist of deliberate electronic countermeasures (ECM), nonhostile RF interference (RFI), clutter scatterer returns, and natural noise sources. The Signal to Noise Ratio (SNR) degradation may be further aggravated by antenna motion, poor siting conditions, multipath ray effects, and a constantly changing interference environment.

Adaptive antenna systems date from the 1950s. In 1956, Altman and Sichak [10] proposed the use of phase-lock loops (PLL) for combining the signals from different receiving antennas in a diversity system. In the 1960s PLL arrays were extensively studied [11].
A PLL array operates by aligning the phase of the signal from each element with that of a reference signal, before summing the signals to produce the array output. This array is adaptive because the antenna pattern is controlled by the incoming signal direction. The array automatically forms a beam that tracks the signal.
The PLL array has a serious drawback: it is vulnerable to interference. Because the phase-lock loops can track only one signal at a time, the array is easily confused if more than one signal is received. If an interference signal arrives that is stronger than the desired signal, it can easily capture the beam of the antenna [12].
Because PLL arrays are vulnerable to interference and jamming, newer types of adaptive arrays had been favorized, such as the LMS and Applebaum arrays that will be discussed in section 4. These arrays not only can track a desired signal and maximize array output signal-to-noise ratio, they can null interference(s) as well.

Organisation of the report

The aim of this report is to address basic concepts for narrowband adaptive antennas. For this purpose, section 2 is dedicated to a signal model for the study of adaptive antennas. Adaptive array architecture is discussed in section 3. Descriptions of performance criteria, capabilities and limitations are presented in section 4. In section 5, the various narrowband algorithms are considered.

2 Mathematic formulation

In this section, a signal model is presented for the study of adaptive antennas. The formulation is given for antenna arrays and steering vector, and the degrees of freedom of an array is reviewed.

2.1 Array

An antenna array system consisting of L antenna elements receive desired signals and interferences. Signals from each element (both desired ones and interferences) are multiplied by a complex weight and summed to form the array output (Fig. 1). The figure does not show components such as preamplifiers, bandpass filters, and so on. It follows from the figure that an expression for the array output is given by

$$y(t) = \sum_{l=1}^{L} {w_l}^* \, x_l(t) \tag{1}$$

where $*$ denotes the complex conjugate.
Using vector notation, the weights of the array system are denoted by

$$\mathbf{w} = [w_1 \; w_2 \; \ldots \; w_L]^T \tag{2}$$

and signals induced on all elements by

$$\mathbf{x}(t) = [x_1(t) \; x_2(t) \; \ldots \; x_L(t)]^T \; . \tag{3}$$

The output of the array system becomes

$$y(t) = \mathbf{w}^H \; \mathbf{x}(t) \tag{4}$$

where superscripts T and H denote transposition and the complex conjugate transposition of a vector or matrix respectively.

The output power of the array at any time t is given by the magnitude square of the array output, that is,

$$P(t) = |y(t)|^2 = y(t) \; y^*(t) \tag{5}$$

If the components of $\mathbf{x}(t)$ can be modeled as zero-mean stationary processes, then for a given $\mathbf{w}$ the mean output power of the array system is obtained by taking conditional expectation over $\mathbf{x}(t)$. Substituting for y(t) from eq. (4), the output power becomes

$$P(t) = \mathbf{w}^H \; \mathbf{x}(t) \; \mathbf{x}^H(t) \; \mathbf{w} = \mathbf{w}^H \; \phi \; \mathbf{w} \tag{6}$$

where $\phi = E[\mathbf{x}(t) \; \mathbf{x}^H(t)]$ is the array correlation matrix, with $E[.]$ the expectation operator. Elements of this matrix denote the correlation between various elements. For example, ϕ_{ij} denotes the correlation between the ith and the jth element of the array.

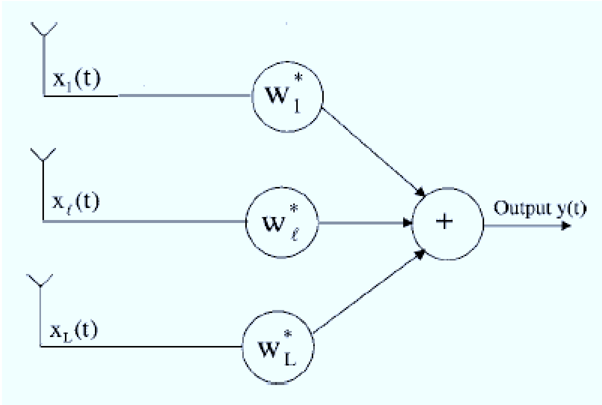


Figure 1: *Antenna array system - [1]*

2.2 Steering vector

A basic array is composed of L isotropic elements set in a linear way (Fig. 2). Under far field conditions, a directional signal incident on the array from angle θ ideally produces time delay on each element with respect to the reference element. The signal induced on the k*th* element is normally expressed in complex notation as

$$x_k(\theta) = \exp(-j\, 2\pi\, (k-1)\, (d/\lambda)\, \sin\theta) \tag{7}$$

where d is the element spacing and λ the free space wavelength.

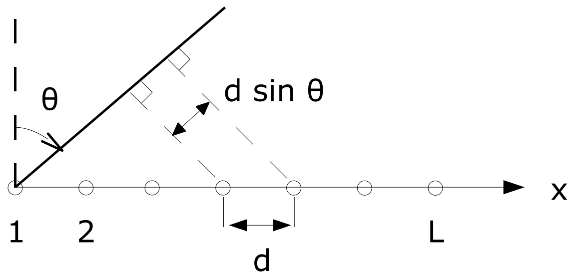


Figure 2: *Linear array with element spacing d.*

A steering vector is an L-dimensional complex vector containing responses of all L elements of the array to a narrowband source of unit power. If U denotes the steering vector and for a linear array of identical elements, one can write

$$U(\theta) = \left[\ 1 \quad \exp(-j\, 2\pi\, (d/\lambda)\, \sin\theta) \quad \ldots \quad \exp(-j\, 2\pi\, (L-1)\, (d/\lambda)\, \sin\theta)\ \right]. \tag{8}$$

According to eq. (4), the response with weight vector $\mathbf{w}$ toward an incident signal is given by

$$y(t) = \mathbf{w}^H\, U(\theta) \tag{9}$$

As the response of the array varies according to direction, a steering vector is associated with each incident signal. For a linear array of equally spaced elements with element spacing greater than half wavelength, the steering vector for every direction is unique.

For an array of identical elements, each component of this vector has unit magnitude. The phase of its k*th* component is equal to the phase difference between signals induced on the k*th* element and the reference element due to the signal associated with the steering vector. As each component of this vector denotes the phase delay caused by the spatial position of the corresponding element of the array, this vector is also known as the space vector.

2.3 Degrees of freedom

A limitation a designer must be aware of is that an L-element array has only $L-1$ degrees of freedom in its pattern. Thus there is a limit to the number of constraints an array pattern can meet at one time.

One way to consider this is to write the array factor [6]. From eq. (7) and (9), one can obtain

$$y(\theta) = w_1 + w_2\, \Psi_2(\theta) + \ldots + w_L\, \Psi_L(\theta) \tag{10}$$

where $\Psi_i = exp(-j\ 2\pi\ (i-1)d/\lambda\ sin(\theta)),\ i = 2\ldots L$ and $w_k,\ k = 1\ldots L$ is the weight associated to each antenna element.

The function $y(\theta)$ is the voltage pattern of the array for any given set of w_j.

One of the weights can be factored out of the expression for $y(\theta)$. For example, if w_1 is factored out,

$$f(\theta) = w_1 \left[1 + \frac{w_2}{w_1}\, \Psi_2(\theta) + \ldots + \frac{w_L}{w_1}\, \Psi_L(\theta) \right] . \tag{11}$$

In this form, the dependence of $f(\theta)$ on θ is contained in the bracketed term. The factor w_1 has no effect on the shape of the pattern, it merely controls the overall amplitude and phase of the entire pattern. Since there are $L-1$ coefficients $(w_2/w_1, \ldots, w_L/w_1)$ in the bracketed term, it means that there are $L-1$ degrees of freedom. This corresponds to the fact that the voltage pattern of an L-element array has $L-1$ nulls.

3 Adaptive Array Architecture

An adaptive array is a system consisting of an array of sensor elements and a real-time adaptive signal receiver-processor that automatically adjusts the array beam sensitivity pattern so that a measure of the quality of the array performance is improved (Fig. 3).

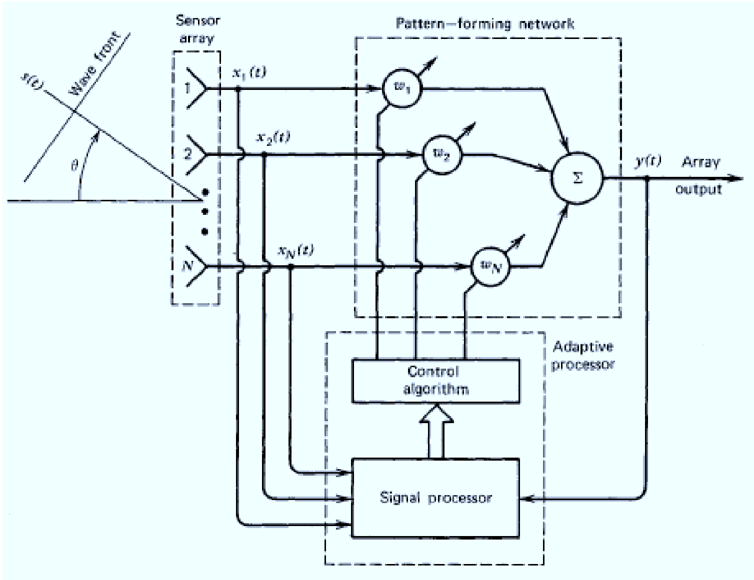


Figure 3: *Adaptive antenna general architecture - [2]*

The output of each of the *L* elements is directed to the pattern-forming network, where the output of each sensor element is first multiplied by a complex weight (having both amplitude and phase) and then summed with all other weighted sensor element outputs to form the overall adaptive array output signal. The problem facing the adaptive processor is to select the various complex weights w_k in the pattern-forming network.

The exact structure of the adaptive processor unit is critically dependent on the degree of detailed information about the operational signal environment that is available to the array. As the amount of *a priori* knowledge (e.g., desired signal location, jammer power levels, etc) concerning the signal environment decreases, the adaptive control algorithm selected for the adaptive processor unit becomes critical to a successful design.

The weights in an adaptive array may be controlled by any one of various algorithms. The "best" algorithm for a given application is chosen on the basis of several factors including the signal structures, the *a priori* information available to the adaptive processor, the performance characteristics to be optimized, the required speed of response of the processor, the allowable circuit complexity, any device or other technological limitations, and cost effectiveness. In some cases adaptation algorithms can be selected according to the kind of signal information available to the receiver as in the following when:

- The desired signal to be received by the array is known.
- The desired signal to be received is unknown, but its direction of arrival with respect to the array is known.
- The desired signal to be received is unknown and its direction of arrival with respect to the array is unknown, but the desired signal power level is known.
- No signal information is available at the outset, but as array operation progresses such information must be "learned" by the adaptive processor.

3.1 Architectures for adaptive beamforming in array antennas

T. Otiga presents five possible architectures through which adaptive beamforming can be carried out [3]. They are called digital beamforming (DBF), local beamforming (LBF), microwave beamforming (MBF), aerial beamforming (ABF), and optical beamforming (OBF). The different topologies and features of DBF, LBF, MBF and ABF with L antenna elements are presented in Figure 4 and Table 1. ABF is most of the time implemented using switched parasitic elements [13]. For OBF, the architecture is exclusively powerful in systems where the radiating elements must be located a long distance away from the beamforming network (radio-on-fiber for example).

3.2 A simple example: two omnidirectional-element array

The possibility of steering and modifying the array directional pattern to enhance desired signal reception and simultaneously suppress interference signals by complex weight selection can be illustrated by considering two omnidirectional-element array (Figure 5). A desired signal $p(t)$ arrives from the normal direction $\theta_d = 0 \; deg$, and an interference signal $I(t)$ arrives from the angle $\theta_i = \pi/6 \; rad$. For simplicity, both the interference signal and the desired pilot signal are assumed to have the same frequency f_0. Furthermore, assume that at the point exactly midway between the array elements the desired signal and the interference are in phase (this assumption is not required but simplifies the development).

The complex weights can be adjusted to enhance the reception of $p(t)$ while rejecting $I(t)$.The array output due to the desired signal is

$$P \; e^{j\omega_0 t} \; [(w_1 + w_3) + j.(w_2 + w_4)]. \tag{12}$$

For the output signal of (12) to be equal to $p(t) = P \; e^{j\omega_0 t}$, it is necessary that

$$\begin{cases} w_1 + w_3 = 1 \\ w_2 + w_4 = 0. \end{cases} \tag{13}$$

	DBF	MBF	LBF	ABF
Complex weighting	Digital	RF stage	LO stage	Aerial
Vector weighting	Digital	RF stage	LO stage	Spatial
Advantages	Digital processing only	Only 1 freq. converter Dynamic range L times wider than DBF	Weighting by phase control only Free from gain deviation in each branch	Size Electromagnetic coupling
Drawbacks	Low noise ampli., freq. and A/D converters on each branch Cost (linked to the number of antenna el.)	Weighting devices operative at RF needed	L mixers needed High LO power (to compensate the phase shifters insertion loss) Twice no. of branches needed to offer same performances as MBF	Twice no. of branches needed to offer same performances as MBF

Table 1: *Features of beamforming networks topologies - [3]*

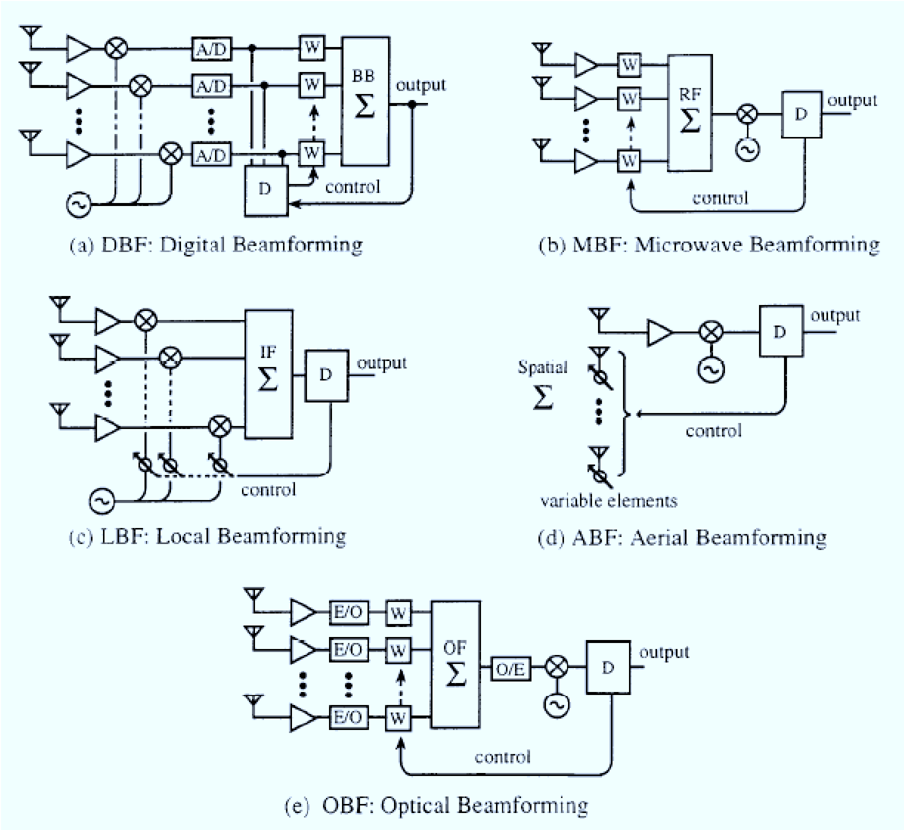


Figure 4: *Adaptive beamforming architectures - [3]*

The incident interfering noise signal exhibits a phase lead with respect to the array midpoint when impinging on the element with complex weight $w_3 + j.w_4$ of value $2\pi.(1/4).\sin(\pi/6) = \pi/4$ and a phase lag when striking the other element of value $-\pi/4$. Consequently the array output due to the incident noise is given by

$$N\ e^{j(\omega_0 t - \pi/4)}\ [w_1 + j.w_2] + N\ e^{j(\omega_0 t + \pi/4)}\ [w_3 + j.w_4]. \tag{14}$$

So for the array noise response to be zero, it is necessary that

$$\begin{cases} w_1 + w_2 + w_3 - w_4 = 0 \\ -w_1 + w_2 + w_3 + w_4 = 0. \end{cases} \tag{15}$$

Solving (13) and (15) simultaneously then yields:

$$w1 = 1/2,\ w2 = -1/2,\ w3 = 1/2,\ w4 = 1/2 \tag{16}$$

With the above weights (0.707 ^{135deg} on el. 1, and 0.707 ^{45deg} on el. 2), the array will accept the desired signal while simultaneously rejecting the interference (Figure 6).

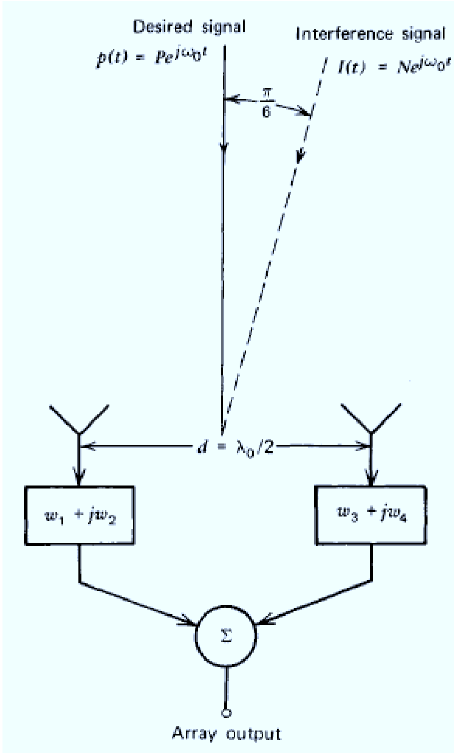


Figure 5: *Two-element array for interference suppression example - [4]*

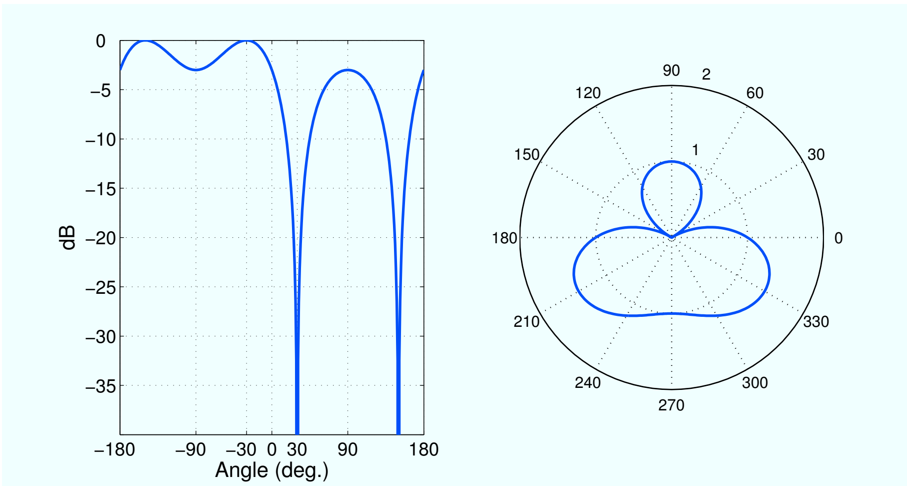


Figure 6: *Radiation pattern of a two weighted omnidirectional-element array*

While the above computation for complex weight selection will yield an array directional pattern that achieves the desired system objectives, it is not a very practical way of approaching adaptive array problems. The development of such a practical adaptive array processor is undertaken in section 4.

4 Performance criteria, capabilities and limitations

Several different performance measures can be adopted to govern the operation of the adaptive processor that adjusts the weighting for each of the sensor element outputs. Four popular performance measures are the following [2]:

1. Mean square error (MSE) criterion,
2. Signal to noise ratio (SNR) criterion,
3. Maximum likelihood (ML) criterion,
4. Minimum noise variance (MV) criterion.

Here, two optimization criteria will be reviewed:

- Maximum Signal-to-Interference-plus-Noise Ration (SINR) at the array output (Applebaum array),
- Minimum Mean Square Error (MMSE) between the actual array output and the ideal array output (Least Mean Square – LMS – Array).

It should be pointed out that conventional characteristics of antennas (e.g. gain, half power beamwidth, side-lobe levels) are not useful in the adaptive array problem. For example, side-lobe levels do not need to be low because an adaptive array places a pattern null in the interference direction. The pattern of an adaptive array will frequently have high side-lobes, but only in directions from which no interference comes. For that reason, high sidelobes are of little consequence.

4.1 Null steering beamformer

The null steering beamformer is used to form the main lobe in the look direction and cancel plane wave(s) arriving from known direction(s). Thus, null(s) are produced in the response pattern in the non desired signals direction(s).

A beam with unity response in the desired direction and p nulls in interference directions may be formed by adjusting the beamformer weights shown in Figure 1 using suitable constraints [1].

For a linear array, $x_{i\,1} \ldots x_{i\,L}$, $i = 1 \ldots L$, are obtained from the phasor formula (eq. (7)):

$$x_{i\,k} = exp(j\,2\pi\,(k-1)\,(d/\lambda)\,sin(\theta_i)), k = 1 \ldots L, \tag{17}$$

where d is the spacing element and θ_i is the angle of arrival of the i*th* signal.

The array output is $y = \mathbf{w}^H\ \mathbf{x}$ and should be equal to unity for the desired direction. For interference directions, $y = 0$. The number of antenna elements is assumed to be L, thus $L-1$ nulls are possible. When the number of interferences is $L-1$, this approach leads to solve L equations with L unknowns:

$$\begin{bmatrix} x_{0\,1} & \dots & x_{0\,l} & \dots & x_{0\,L} \\ \vdots & & \vdots & & \vdots \\ x_{l\,1} & \dots & x_{l\,l} & \dots & x_{l\,L} \\ \vdots & & \vdots & & \vdots \\ x_{L\,1} & \dots & x_{L\,l} & \dots & x_{L\,L} \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ \vdots \\ w_l \\ \vdots \\ w_L \end{bmatrix} = A\ \mathbf{w} = e, \tag{18}$$

where $e = [1\dots0\dots0]^T$.

The solution of this equations system is:

$$\mathbf{w} = A^{-1}\ e\ . \tag{19}$$

When the number of interferences is less than $L-1$ and equal to p, the number of antenna elements needed is $p+1$. So, the weights associated to the unused elements are set to zero. Godara gives a suitable formula in this case [1]:

$$\mathbf{w} = e^T\ A^H\ (AA^H)^{-1}\ . \tag{20}$$

Application and implementation with Matlab

The simplest case is a 3-element array because it is possible to test $L-1$ and $L-2$ interferences' cases. In the following lines, a few examples are described with $L = 3$ and some conclusions are given from them. In these examples, the desired signal is called D and the i*th* interferences I_i. The spacing element d is fixed to 0.5 λ.

Two non desired signals ($p = 2$)

Example 1: $\theta_{D} = 0\ deg$, $\theta_{I_1} = 60\ deg$, $\theta_{I_2} = -60\ deg$

The computed weights (amplitude, phase in deg.) have zero phases because the signals scenario is "symmetric":
$w_1(A,\varphi) = (0.2614, 0)$, $w_2(A,\varphi) = (0.4772, 0)$, $w_3(A,\varphi) = (0.2614, 0)$.

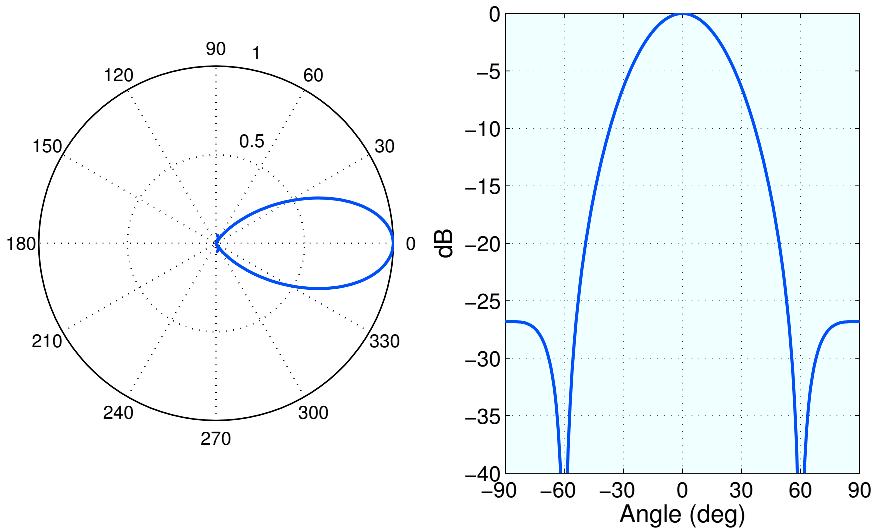


Figure 7: *Adapted pattern of a linear array of 3 isotropic elements, 2 interference.* $\theta_D = 0\ deg$, $\theta_{I_1} = 60\ deg$, $\theta_{I_2} = -60\ deg$

The corresponding pattern with omnidirectional sources is presented on Figure 7. The pattern is symmetric around 0 degree and has nulls at $\theta = \pm\ 60$ degree.

Example 2: $\theta_D = 0\ deg$, $\theta_{I_1} = 30\ deg$, $\theta_{I_2} = -60\ deg$

In respect to the early case, the situation has been here unbalanced by changing one of the interference angle of arrival (AoA). The computed weights are now:
$w_1(A,\varphi) = (0.3615, -32.9423)$, $w_2(A,\varphi) = (0.3932, 0.0000)$, $w_3(A,\varphi) = (0.3615, 32.9423)$.
The obtained pattern is given on Figure 8 . It can be noted that the look direction of the main lobe is a little bit different of the desired one. But when looking at the polar plot, the condition $y = 1$ is verified for $\theta_D = 0\ deg$. Nulls appear at the right angles.

Example 3: $\theta_D = 70\ deg$, $\theta_{I_1} = 30\ deg$, $\theta_{I_2} = -60\ deg$

This configuration is an amplified situation of the early case (example 2). The desired look direction is 70 deg and the interferences' AoA are still the same than previously. The computed weights are:
$w_1(A,\varphi) = (1.3061, -22.0870)$, $w_2(A,\varphi) = (1.4205, 10.8553)$, $w_3(A,\varphi) = (1.3061, 43.7976)$.
The obtained pattern is showed on Figure 9. The shift of the look direction is very similar to the previous case and the array output $y = 1$ is obtained in a side-lobe.

One non desired signal (p = 1)

Example 4: $\theta_D = 0\ deg$, $\theta_I = 45\ deg$

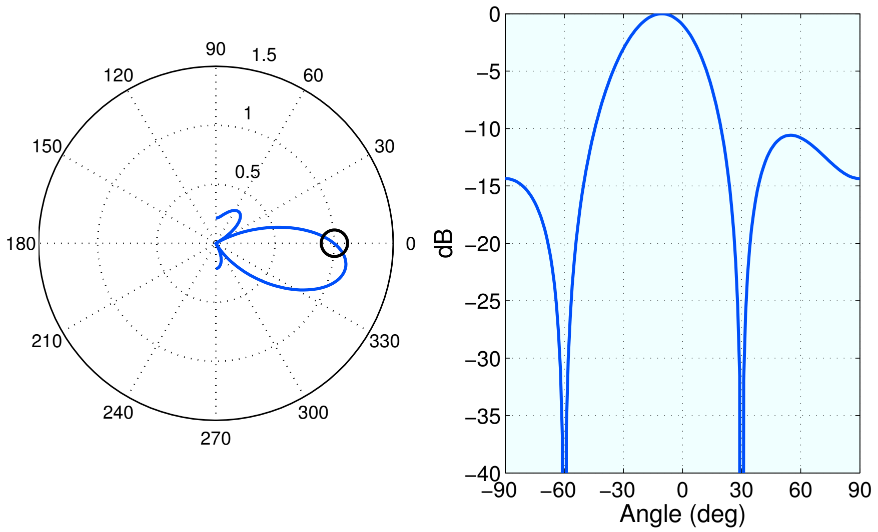


Figure 8: *Adapted pattern of a linear array of 3 isotropic elements, 2 interference.* $\theta_D = 0\ deg$, $\theta_{I_1} = 30\ deg$, $\theta_{I_2} = -60\ deg$

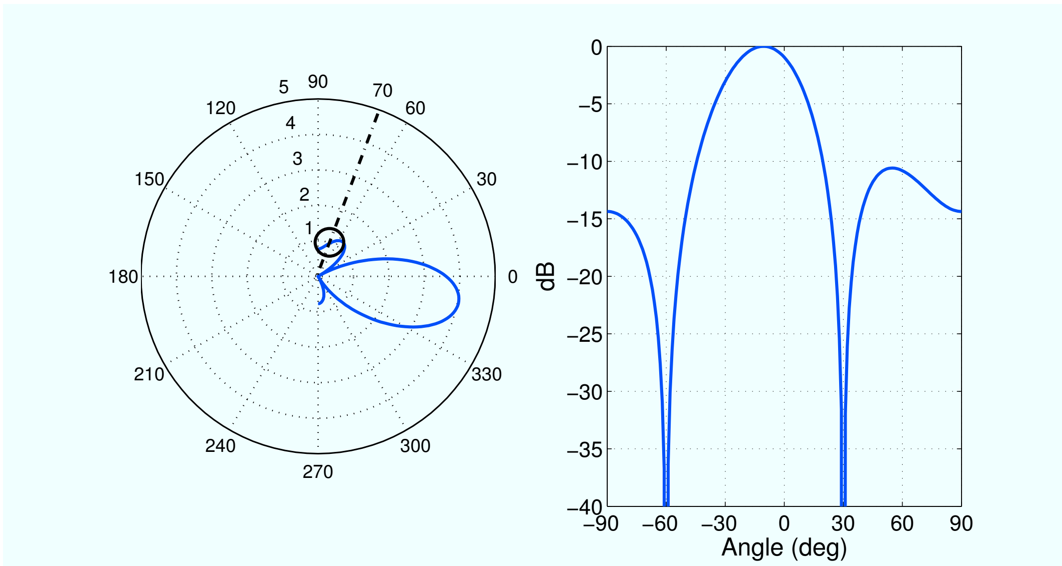


Figure 9: *Adapted pattern of a linear array of 3 isotropic elements, 2 interference.* $\theta_D = 70\ deg$, $\theta_{I_1} = 30\ deg$, $\theta_{I_2} = -60\ deg$

Here, only two elements are needed because only one interference is present. So, one of the weights will be zero. The two others are computed to form a null at 45 deg.:
$w1(A,\varphi) = (0.5580, -26.3604)$, $w2(A,\varphi) = (0.5580, 26.3604)$, $w3(A,\varphi) = (0,0)$.
The pattern computed with these weights (Figure 10) is similar to the previous one in the meaning that the main lobe is different from the desired look direction.

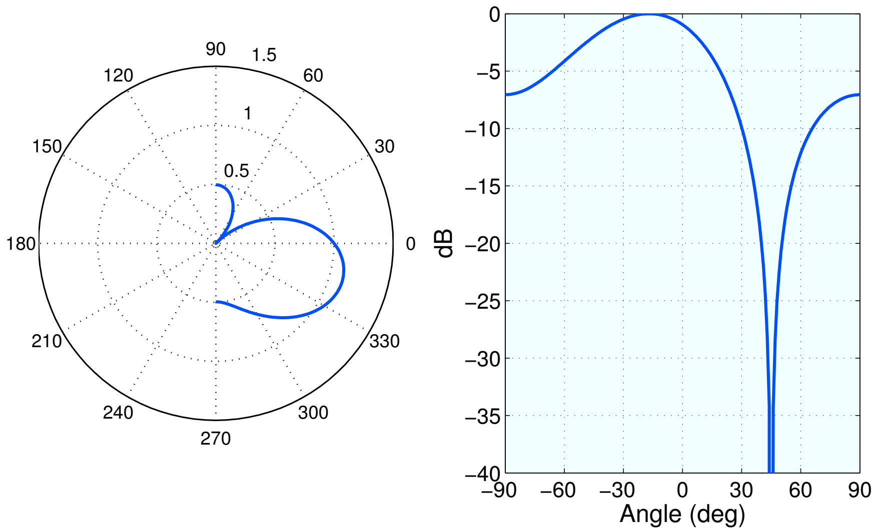


Figure 10: *Adapted pattern of a linear array of 3 isotropic elements, 1 interference.* $\theta_D = 0\ deg$, $\theta_I = 45\ deg$

From these four cases, it is possible to conclude that the shape of the pattern is mainly due to nulls constraints. For the look direction, more constraints are needed. The null steering beamformer has been presented by Godara as a solution that is not able to minimize the uncorrelated noise at the array output [1]. Indeed, the SNR is not always maximum, especially when the look direction angle is important.

4.2 Optimal beamformer (Applebaum adaptive array)

The null steering scheme described in the previous section requires knowledge at the same time of the directions of interference sources and those of the desired signal. The beamformer using the weights estimated by this scheme does not maximize the output SNR [1]. The optimal beamforming method, also known as the minimum variance distortionless response (MVDR) beamformer, overcomes these limitations and maximizes the output SNR in the absence of errors. It should be noted that the optimal beamformer does not require knowledge of directions and power levels of interferences as well as the level of the background noise power to maximize the output SNR. It only requires the direction of the desired signal.

The steady-state complex weight vector $\bar{\mathbf{w}}$ which maximize the output SNR is given by

$$\bar{\mathbf{w}} = \phi^{-1}.\, U_s \tag{21}$$

where $\phi = E(\mathbf{x}^*\ \mathbf{x}^T)$ is the covariance matrix of the signals incident on the array and U_s is the steering vector for the direction of the desired signal. E[.] denotes the expectation operator. The proof of equation (21) is given in [14]. Figure 11 shows the typical feedback loop for a steered beam adaptive array.

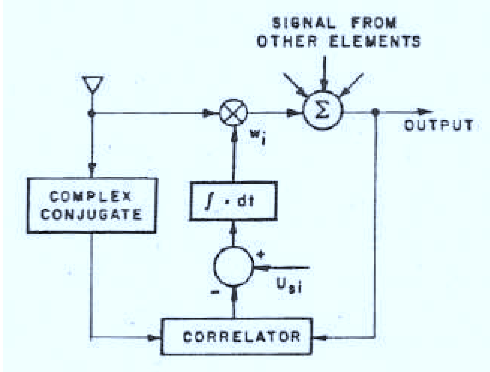


Figure 11: *Typical Applebaum adaptive array feedback loop - [5]*

Some performance characteristics and limitations that occur with the Applebaum array are now discussed.

4.2.1 Power inversion effect

The ideal environment of the adaptive array contains $m+1$ Continuous Wave (CW) signals (one desired signal and m jammers) which are incident on the array. The thermal noise voltages from the array elements are assumed Gaussian with zero means and are uncorrelated with each other. The carrier phases of the CW signals are uniformly distributed on $(0, 2\pi)$ and are statistically independent of each other and of the thermal noise voltages. Under these conditions, the covariance matrix ϕ given by

$$\phi = \sigma^2\, I + A_d^2.U_d{}^*.U_d{}^T + \sum_{k=1}^{m} A_i^2.U_{ik}{}^*.U_{ik}{}^T \qquad (22)$$

where σ^2 is the thermal noise power from each element, I is an identity matrix, A_d is the input desired signal amplitude, A_{ik} is the k*th* jammer amplitude, U_d and U_{ik} are, respectively, the desired signal and k*th* jammer vectors (containing complex values at each element). * denotes complex conjugate and superscript T denotes transpose.

In the absence of all jammers, (22) becomes

$$\phi = \sigma^2(I + \xi_d.U_d{}^*.U_d{}^T) \qquad (23)$$

where $\xi_d = A_d^2/\sigma^2$ is the input signal to thermal noise ratio.

Figure 12 shows the pattern of a linear array of seven isotropic elements. The spacing between the elements is a half wavelength. Jammers are assumed to be absent and the desired signal is incident from broadside. This example is given by Gupta in [5].

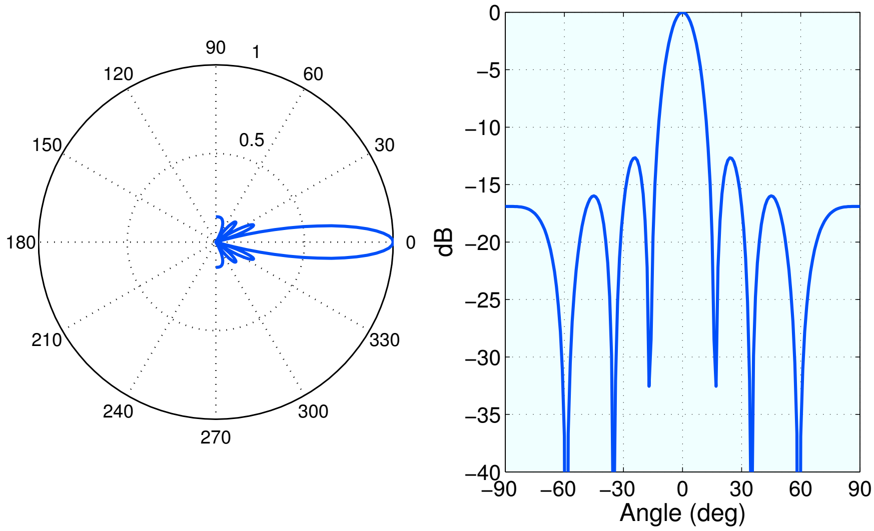


Figure 12: *Adapted pattern of a linear array of 7 isotropic elements;* $d = 0.5\ \lambda$, $\theta_d = 0\ deg$. *No jammer. - From [5]*

Here, weights have been computed and output power calculated with the formula [7]:

$$P_d = A_d^2 . \left| \mathbf{w}^T . U_d{}^* \right|^2 \ / \ 2 \tag{24}$$

Figure 13 presents the output signal power as a function of the input SNR (ξ_d). In [5] (eq. (7)), the theoretical array output power written using (21) and (23), and the curve obtained in Fig. 13 is the same as those obtained by Gupta from the theoretical output power. For $\xi_d > -8\ dB$, the output signal power decreases with an increase in the input SNR. This phenomena is called the power inversion effect. Indeed, the Applebaum array computes weights in order to have the optimal signal to noise ratio. It does so by nulling the strong signal in favor of the weak one. When an interference stronger than the desired signal is present, this behavior is wished. But when the desired signal power is greater than the thermal noise one, the desired signal is nulled in favor of the noise. From [5] (eq. (7)), the author showed that the output desired signal power is inversely proportional to the input desired signal power, which is undesirable.

4.2.2 Presence of jammers

When a jammer is present, a null is steered in its direction. Figure 14 shows a 24-degree-AoA jammer, which corresponds to the closest sidelobe peak from the main beam of the seven-element array with no jammer. The input interference to noise ratio is $\xi_i = A_i{}^2/\sigma^2 = 20\ dB$.
The closer the separation between AoA of the desired and interference signal(s), the larger

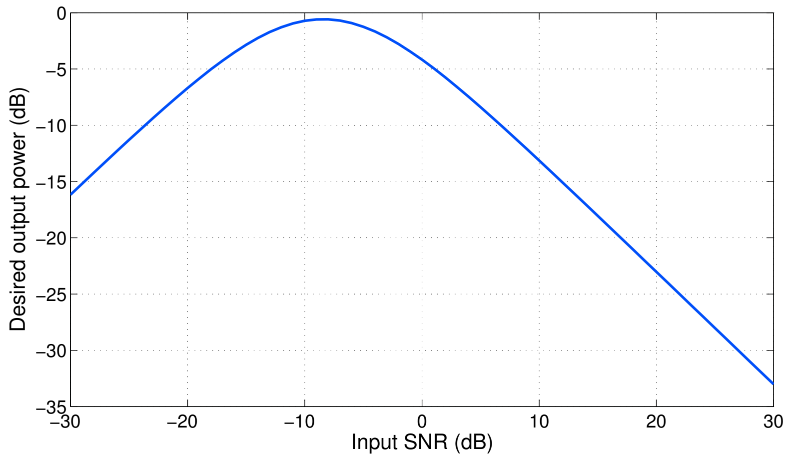


Figure 13: *Output signal power of a linear array of 7 isotropic antenna elements versus the input SNR.* $d = 0.5\ \lambda$, $\theta_d = 0\ deg$. *No jammer. - From [5]*

the SNR drops (Fig. 7 of [5]). This deterioration is as important as the number of elements is low. The Signal to Interference plus Noise Ratio (SINR) is linear versus the input SNR.

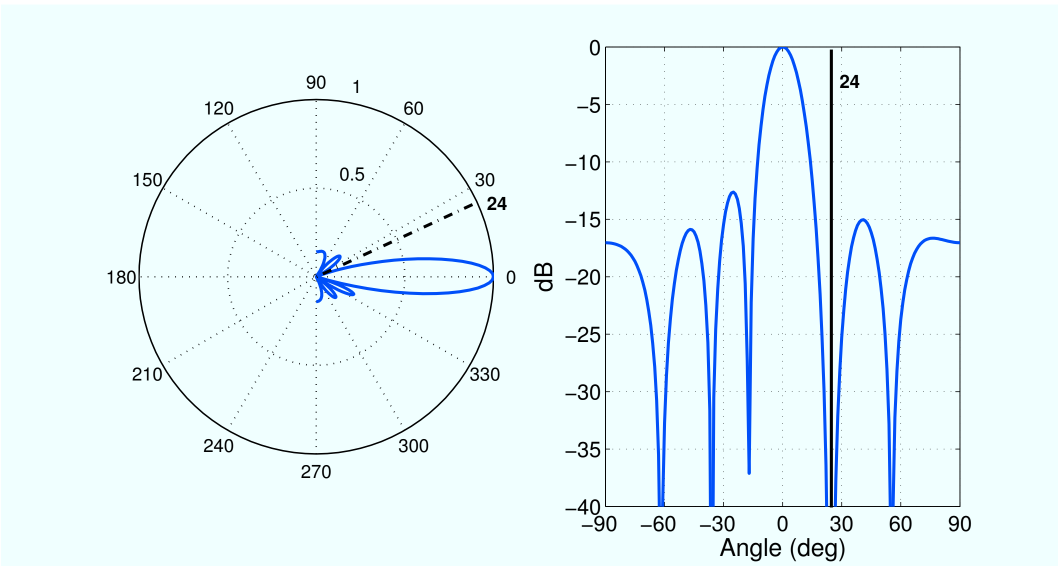
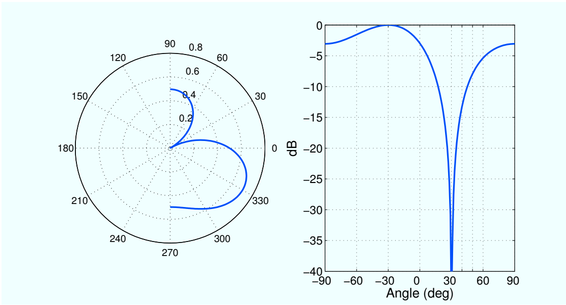
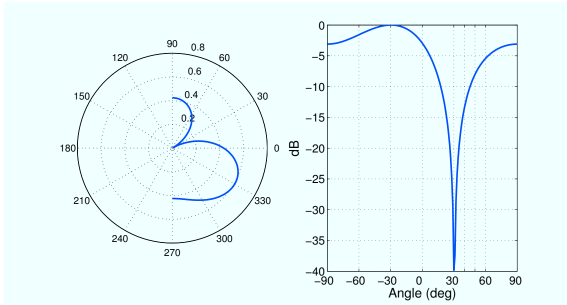


Figure 14: *Adapted pattern of a linear array of 7 isotropic elements;* $d = 0.5\ \lambda$, $\theta_d = 0\ deg$. *One jammer,* $\xi_i = 20\ dB$, *AoA* $= 24\ deg$.

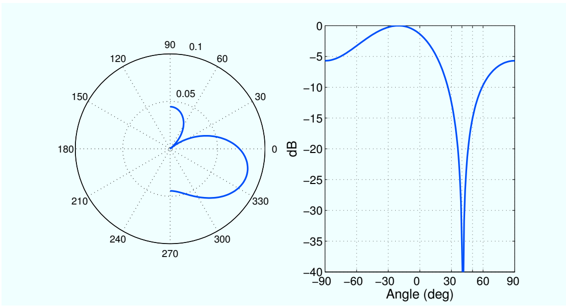
One can wonder what would happen if the number of interferences is greater than the degree of freedom, i.e the jammers are above the number of elements minus one ? Compton presents theoretical results and an example in [6] and his conclusions are verified here with a 2-element array. This array has only one degree of freedom and can steer its only available null in the direction of one jammer. To show the influence of an added jammer, it is possible to plot different patterns vs. the power of this second interference. Here, the AoA of the desired signal is $\theta_d = 0\ deg$, its SNR is 0 dB. The AoA of the two jammers are $\theta_{i_1} = 30\ deg$ and $\theta_{i_2} = 50\ deg$ with respective interference to noise ratios $\xi_{i_1} = 20\ dB$ and ξ_{i_2} is varying. The patterns for ξ_{i_2} equal -30, 0, 20 and 40 dB are presented on Fig. 15.



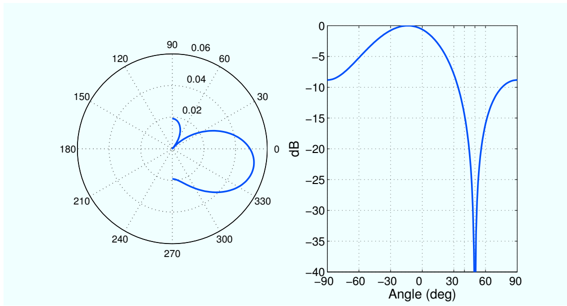
(a) $w_1(A,\varphi) = (0.3526, -44.7146)$,
$w_2(A,\varphi) = (0.3526, 44.7146)$



(b) $w_1(A,\varphi) = (0.3032, -44.4166)$,
$w_2(A,\varphi) = (0.3032, 44.4166)$



(c) $w_1(A,\varphi) = (0.0427, -31.2611)$,
$w_2(A,\varphi) = (0.0427, 31.2611)$



(d) $w_1(A,\varphi) = (0.0263, -21.2299)$,
$w_2(A,\varphi) = (0.0263, 21.2299)$

Figure 15: *Patterns of a linear array of 2 isotropic elements for several ξ_{i_2}. $d = 0.5\ \lambda$, 2 jammers. $\theta_d = 0\ deg$, $\theta_{i_1} = 30\ deg$, $\theta_{i_2} = 50\ deg$, $SNR = 0$, $\xi_{i_1} = +20\ dB$. (a) $\xi_{i_2} = -30\ dB$; (b) $\xi_{i_2} = 0\ dB$; (c) $\xi_{i_2} = +20\ dB$; (d) $\xi_{i_2} = +40\ dB$. Steady-state weight values are given under each pattern (amplitude, phase).*

When ξ_{i_2} is very small, the second interference signal has little effect, so the situation is the same as with one interference signal. With $\xi_{i_2} = -30\ dB$, the second interference is virtually not present. As ξ_{i_2} increases, the weights drop to very small values. This phenomena is due to the fact that there are too many conditions for the array to meet. The only way for the array to reduce the power of interferences is to turn off the weights. In this case, the result is a further sharp drop in SINR (Fig. 16).
When ξ_{i_2} is equal to ξ_{i_1} (Fig. 15(c)), a null is steered close to an angle half way between $\theta_{i_1} = 30\ deg$ and $\theta_{i_2} = 50\ deg$. As ξ_{i_2} grows, the pattern null moves over to θ_{i_2}. Finally, one can note that the maximum gain does not match with the AoA of the desired signal (θ_d). As in the previous note entitled "Null steering beamforming", no constraints are defined and what happens is $\mathbf{w}^T.U_s = SNR/2$ when one interference is present.

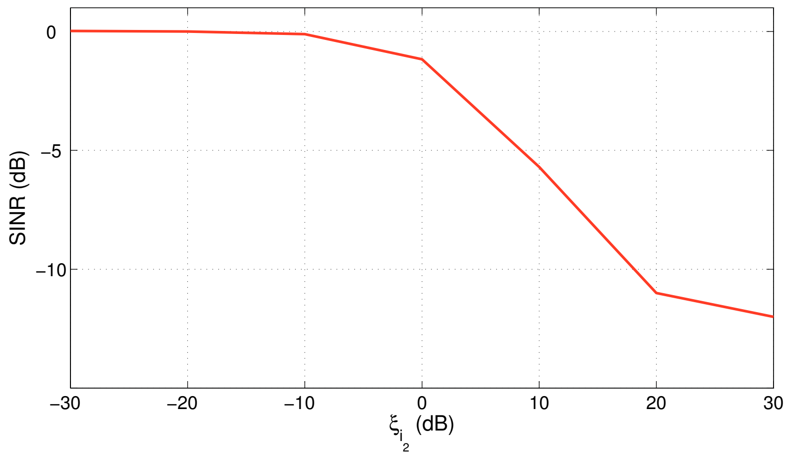


Figure 16: *The effect of* ξ_{i_2} *on SINR for a linear array of 2 isotropic elements.* $d = 0.5\ \lambda$, *2 jammers.* $\theta_d = 0\ deg$, $\theta_{i_1} = 30\ deg$, $\theta_{i_2} = 50\ deg$, $SNR = 0$, $\xi_{i_1} = +20\ dB$. *- From [6]*

Adaptive arrays often operate with more interfering signals incident on them than they have degrees of freedom. However, if some of these signals are weak, the situation is not necessarily disastrous. How well the array performs depends on the power of the interfering signals, their arrival angles, and the array element configuration [6].

4.2.3 Multiple desired signal

In [7], the author addressed theoretical properties and most favorable choices for multiple simultaneous desired signals receptions. It is shown that the performance of a steered beam adaptive array depends upon the range of input signal strengths and the choice of the steering vector.

As mentioned before, the steady-state weight vector of the adaptive array is given by eq. (21). If the array is used in a multiple desired signal environment, the steering vector should be chosen such that major lobes are steered in all the desired signal directions. Thus, for multiple desired signals

$$U_s = \sum_{l=1}^{n} a_l\ U_{d\ l}{}^* \tag{25}$$

where $U_{d\ l}$ is the lth desired signal vector, a_l is a weighting factor, and n is the total number of incident desired signals. For $a_1 = a_2 = \ldots = a_n$, the pattern has multiple beams with an independent beam in each desired signal direction.

The stronger the desired signal, the larger the factor $a_l\ U_{d\ l}{}^*$ will be. Because of the power inversion effect, this in turn will result in more suppression of the beam in the direction of the desired signal. Thus, the choice of identical a_l will suppress the lobes in strong signal directions more than in weak signal directions. Fig. 17 shows the adapted pattern of a linear array of 10 isotropic elements for a signal environment consisting of two desired signals. This example is given by Gupta in [7]. One of the signals is incident from $\theta_d = -30$ *deg* and has an input SNR of 10 dB. The other signal is incident from $\theta_d = 30$ *deg* and has an input SNR of 0 dB. The two signals are assumed to be of the same frequency. There is no jammer present and the spacing between the array elements is a half wavelength. One can note that the lobe in the stronger signal direction, i.e., $\theta_d = -30$ *deg*, has been suppressed while the array maintains the lobe along $\theta_d = 30$ *deg*.

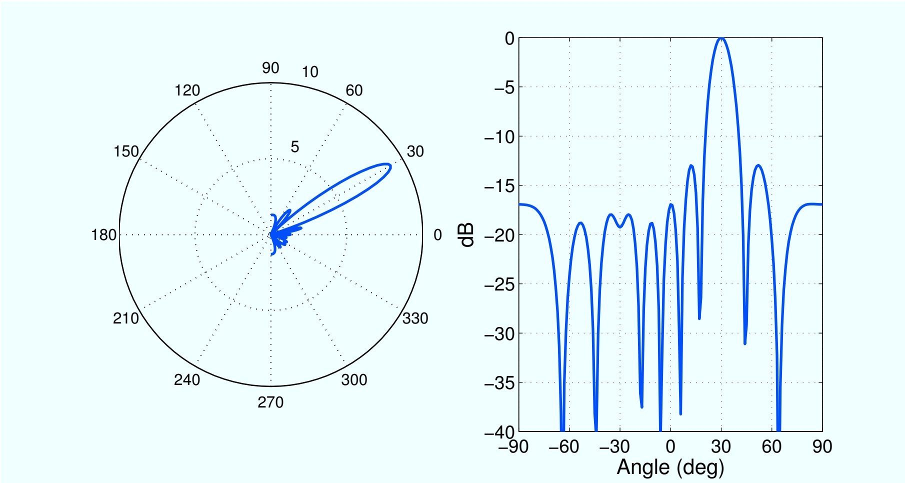


Figure 17: *Adapted pattern of a linear array of 10 isotropic elements in the presence of two signals;* $d = 0.5\ \lambda$, $\theta_{d_1} = -30$ *deg*, $\theta_{d_2} = 30$ *deg*, $\sigma^2 = 1$, $\xi_{d_1} = 10$ *dB*, $\xi_{d_2} = 0$ *dB*, $a1 = a2 = 1$. *- From [7]*

Two choices for the weighting factors a_l are proposed by Gupta [7] to improve this results. The previous knowledge of the signal strengths is required. The first choice for a_l is given by

$$a_l = \xi_{d\ l} \tag{26}$$

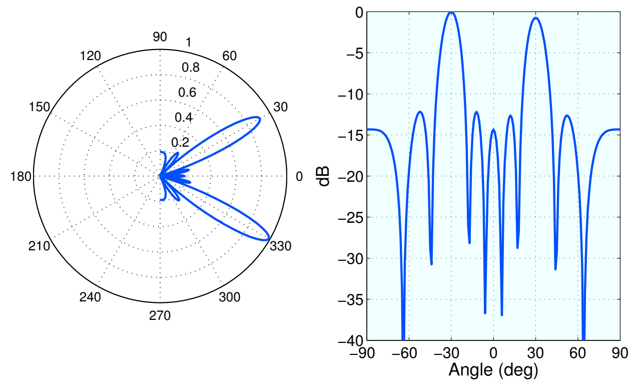
Fig. 18(a) presents the adapted pattern with a_l defined by equation (26).
The second choice for a_l is given by

$$a_l = 1 + \xi_{d\ l}.U_{d\ l}{}^T.U_{d\ l}{}^* \tag{27}$$

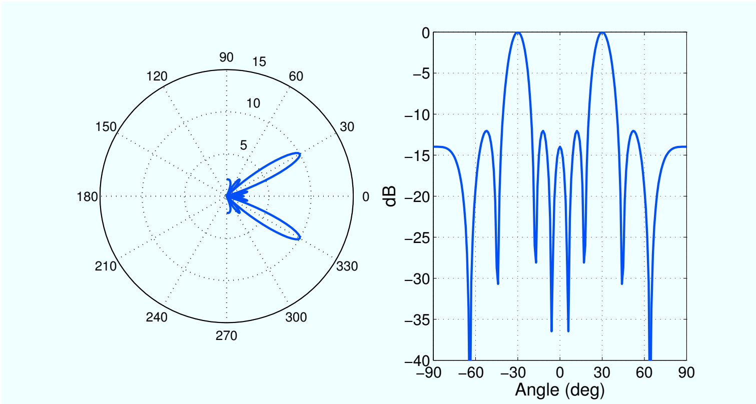
Fig. 18(b) show the obtained pattern with a_l defined by equation (27). For this choice, the weighting factors depend of the SNR as well as the steering vector of each desired signals.

Note: for an array of L isotropic elements, $U_{d\ l}{}^T.U_{d\ l}{}^* = L$ for $l = 1, 2, \ldots, n$.

For both choices, the adapted pattern has amplitude balanced beams. But, as regarding the output signal to noise, the improvement is better in the second case. Indeed, in the



(a)



(b)

Figure 18: *Adapted pattern of a linear array of 10 isotropic elements in the presence of two signals;* $d = 0.5\ \lambda$, $\theta_{d_1} = -30\ deg$, $\theta_{d_2} = 30\ deg$, $\sigma^2 = 1$, $\xi_{d_1} = 10\ dB$, $\xi_{d_2} = 0\ dB$; *(a) a1 = 10, a2 = 1; (b) a1 = 101, a2 = 11.*

first case, the output SNR degrades for weak desired signals (input $SNR < -10\ dB$). In the second case, the output SNR in proportional to the input SNR in a linear manner. In presence of jammers, results lead to the same [7].

Results obtained here are the same as those from Gupta. Complete conclusions and notes can be find in [7].

4.2.4 Pointing accuracy

Because the computation of weights needs the knowledge of the desired signal angle (eq. (21)), effects happening when an error on AoA is present have to be exposed. In [15] and [5], the problem of pointing accuracy is reviewed. When the assumed AoA of the desired signal is in error, the drop in the desired signal power is accompanied by an increase in the noise power and the output SNR will drop from its maximum value (Fig. 19). Curves are drawn for various values of input SNR. Thus, an Applebaum type steered beam adaptive array is not suitable if the exact angle of arrival of the desired signal is not known. For strong signals, the output SNR will be less than the input SNR.

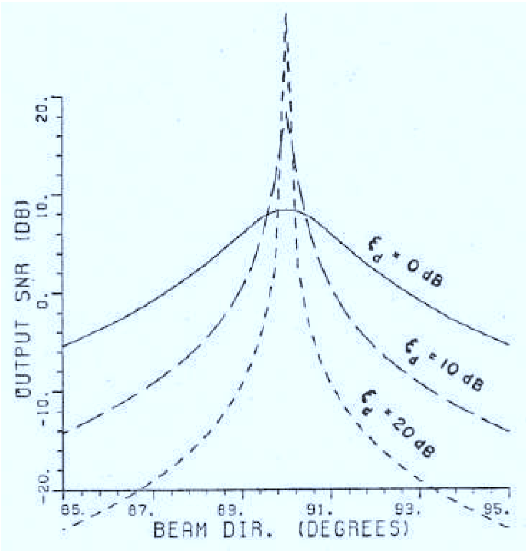


Figure 19: *Output SNR of a linear array of seven isotropic antenna elements versus the beam direction.* $\theta_d = 90$ *deg (broadside). No jammer - [5]*

4.2.5 Concluding remarks

Concepts and features have been presented for the Applebaum array. Advantages and drawbacks have been reviewed. The main advantage of the Applebaum array is its capability to steer a beam in a given direction and cancel (or attenuate) some interferences at the same time. One limitation is the fact that the SINR drops in a steep way when the desired signal angle is in error. Another limitation is the power inversion effect: when a signal is stronger than another one or thermal noise, the Applebaum array nulls it in favor of the weak signal (or noise). This effect can be used to cancel strong signal when the desired signal angle is not known in advance. The power-inversion array is discussed in section 4.3.
The Applebaum array is able to steer several beams in given directions. When the signals received have different powers and when the strength are known, it is interesting to use weight factors to improve the SINR. Two choices have been reviewed in 4.2.3 for this purpose.

4.3 Power Inversion Array

An Applebaum array may be used in another manner, in a power inversion mode [16, 8]. Power inversion refers to the fact that the Applebaum array can invert the power ratio of two received signals. The power inversion technique is useful when no detailed information is

required about the desired signal waveform or AoA. However, The SINR improvement of a power inversion array is not as good as that of the LMS or Applebaum array.
The power inversion array is in essence an Applebaum array. But to obtain the power inversion behavior, several changes are needed: the integrator in the Applebaum feedback loop is changed to a low-pass filter, and the loop gain is chosen in a special way. Moreover, the steering vector is chosen in a different way. The feedback loop shown in Fig. 20 has a transfer function for the low-pass filter given by

$$H(s) = \frac{k}{\tau s + 1} \tag{28}$$

The loop gain k allow to control the dynamic range of the array.

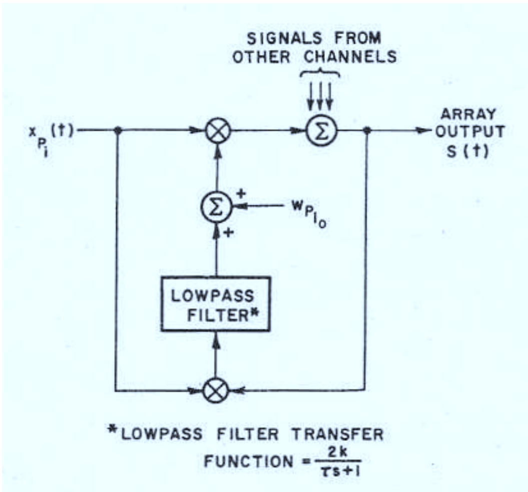


Figure 20: *Power-inversion loop - [8]*

Because the desired signal AoA is not known in advance, the steering vector is set up to obtain a pattern covering the sector of space from which the desired signal may arrive. If only one element is on and the others are off, a pattern equal to the element pattern is obtained. In such way, desired signals from any direction can be received.

The power inversion array based on the diagram in Fig. 20 satisfies the differential equation

$$\frac{d\mathbf{w}}{dt} + (I + k\,\phi) = T \tag{29}$$

where $T = [1\ 0\ \ldots\ 0]^T$, I is the identity matrix and ϕ is the correlation matrix as defined previously.

The steady-state weight vector $\bar{\mathbf{w}}$ will be

$$\bar{\mathbf{w}} = (I + k\,\phi^{-1})\,T \tag{30}$$

The loop gain k allows to control the dynamic range of the array. Fig. 21 shows the output SNR as a function of the input SNR for several values of the loop gain $K = k\ \sigma^2$. For a given K, the output SNR first increases with the rise of input SNR (in the range where the input SNR is too small to affect the weights) and then decreases (when the desired signal is nulled by the array). The problem here is to prevent the array from nulling the desired signal.

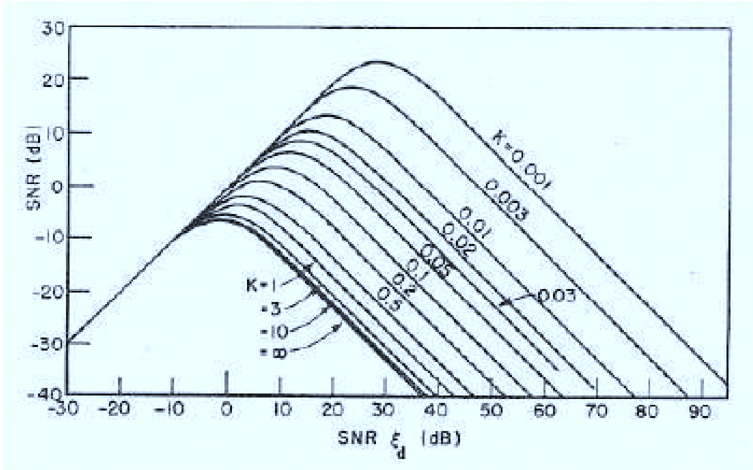


Figure 21: *Output SNR versus input SNR (no interference). - [8]*

The optimum choice of the loop gain K depends on two factors :

- the required minimum SNR out of the array (which depends on the receiver and the type of modulation used in communication systems),
- the dynamic range of signal levels that must be accommodated by the array.

For example, if the input signal level varies between 0 and 20 dB, and an output SNR of 0 dB is required, Fig. 21 shows that K cannot be larger than about 0.07. If a higher value of K is used, the output SNR will drop below 0 dB before the input SNR reaches 20 dB. A lower value of K also should not be used because this would yield poorer interference protection, as will be seen.

The ideal application of the power-inversion array is to a communication system where the output SNR can be less than 0 dB. One such application is to spread-spectrum systems, which often operate with the signal below the noise, because of the processing gain of the spread spectrum receiver. With a desired signal which strength is below noise level, the array does not try to null the desired signal. In this case, the designer has wide latitude in the choice of K, which can then be chosen for good interference suppression.

4.3.1 Interferences

In [6], the author computed typical examples with a 2-element array (0.5 λ element spacing) showing the SINR as a function of the input SNR for K = 0.01, 0.1 and 1 (Fig. 22). A desired signal is expected to arrive from broadside ($\theta_d = 0$) and an interference from 50 deg ($\theta_i = 50$). Interesting notes are gathered here.

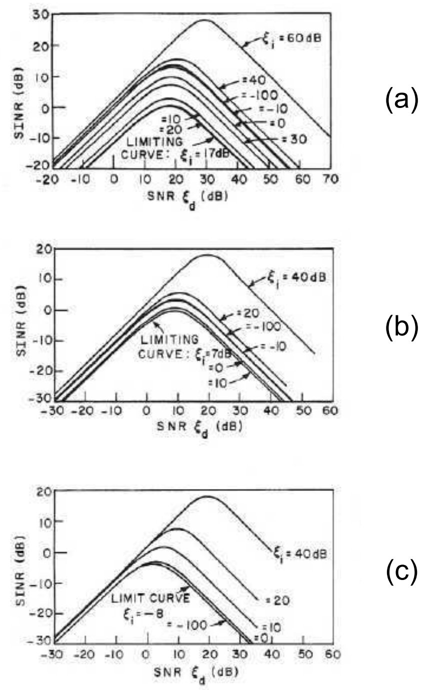


Figure 22: *Output SINR versus input SNR:* $\theta_d = 0$, $\theta_i = 50$. *(a)* $K = 0.01$ *(b)* $K = 0.1$ *(c)* $K = 1$ *- [6]*

First, for low Interference to Noise Ratio (INR), the interference is virtually not present. Second, as the interference power increases, the SINR first drops and then rises. As the loop gain K increases, there is less significant drop in the SINR for intermediate values of INR.
Third, for any given INR there is a finite range of input SNR over which the SINR is above any given value. For interference power less or greater than the desired signal power, the acceptable range for the input SNR is wider.
Fourth, it is interesting to note that for strong interference signals, the SINR can be substantially better than it would be without interference. The reason for this behavior is that, without interference, the adaptive array devotes its single null to the desired signal. However, with strong interference, the array is forced to use its null on the interference. The desired signal is then not in a null.

Finally, the array performance depends on the spatial separation of the desired and interfering signal, and on the signals' bandwidth. The antenna pattern is frequency dependent, so its response varies over the signal bandwidth. Since the pattern varies much more rapidly with frequency in the nulls than elsewhere, it is primarily interference bandwidth that affects the performances. Desired signal bandwidth has only a negligible effect.

4.3.2 Examples

Fig. 23 presents the computed pattern with a 5-element array and one strong interference at 30 deg. and a weak desired signal broadside. Some ripples occur on both sides of the interference angle. This could unbalance the levels of desired signals.

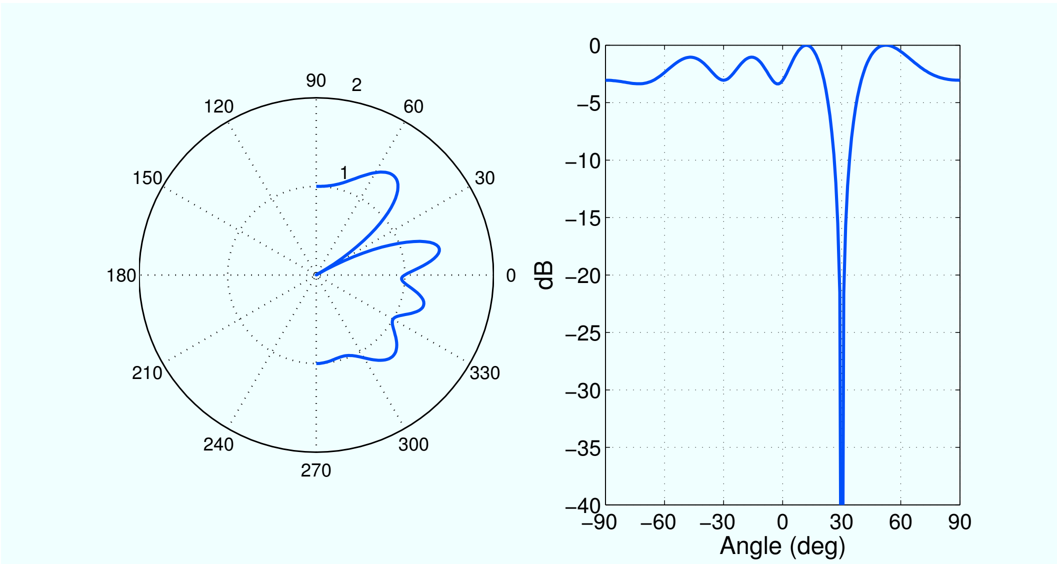


Figure 23: *Adapted pattern of a linear array of 5 isotropic elements, 1 interference.* $\theta_d = 0$ *deg*, $\theta_i = 30$ *deg*, $\xi_d = -30$ *dB*, $\xi_i = 30$ *dB, k=1.*

The phase of the 5-element computed pattern is shown in Fig. 24. One can note some ripples on both sides of the interference angle as well. For communication systems, these amplitude and phase undulations can be seen as drawbacks, especially for systems in which the phase is a fundamental reference (Radar, GPS, ...). A phase slope is present at the interference angle.

When the power of a desired signal (input SNR) is close to 0 dB, then the power inversion array tries to cancel it (Fig. 25). This shows again the advantage of using this adaptive system for input SNR signal lower than 0 dB.

Another way to receive signals when the angles of arrival are not known is to use the LMS array. This adaptive array has been proposed by Widrow in 1967 [4] and its concepts are reviewed in section 4.4.

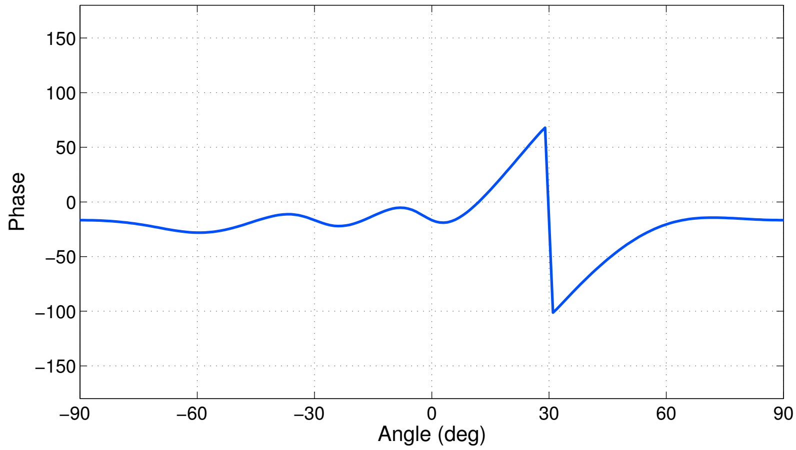


Figure 24: *Phase of a linear array of 5 isotropic elements, 1 interference.* $\theta_d = 0$ *deg*, $\theta_i = 30$ *deg*, $\xi_d = -30$ *dB*, $\xi_i = 30$ *dB, k=1.*

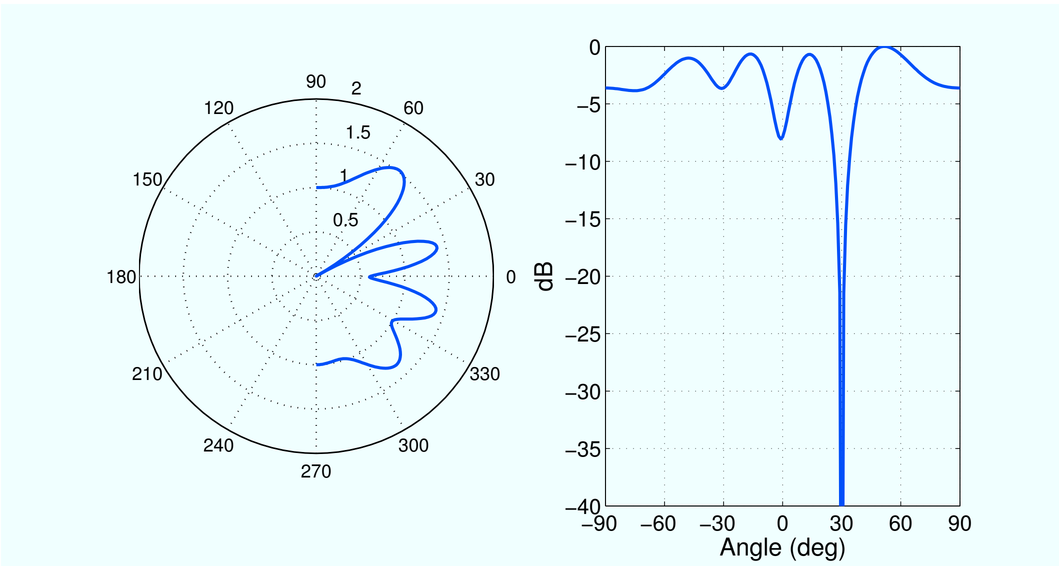


Figure 25: *Adapted pattern of a linear array of 5 isotropic elements, 1 interference.* $\theta_d = 0$ *deg*, $\theta_i = 30$ *deg*, $\xi_d = -5$ *dB*, $\xi_i = 30$ *dB, k=1.*

4.4 Optimization using reference signal (LMS adaptive array)

A narrowband beamforming structure employing a reference signal to estimate the weights of the beamformer is shown in Figure 26. The array output is subtracted from an available reference signal $d(t)$ to generate an error signal

$$\varepsilon(t) = d(t) - \mathbf{w}^H\, \mathbf{x}(t) \tag{31}$$

used to control the weights. Weights are adjusted such that the mean squared error between the array output and the reference signal is minimized [4, 6, 1].

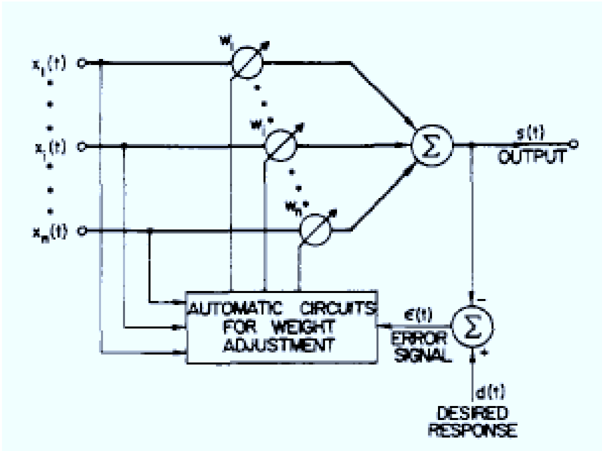


Figure 26: *An array system using reference signal. - [4]*

From (31), the mean squared error (MSE) for a given **w** is expressed by

$$E[\varepsilon^2(t)] = E[d^2(t)] - 2\mathbf{w}^H\, S + \mathbf{w}^H\, \phi\, \mathbf{w} \tag{32}$$

where $S = E[\mathbf{x}^*(t)\ d(t)]$, $\phi = E[\mathbf{x}(t)\ \mathbf{x}^H(t)]$ and E[.] denotes expectation, H conjugate transpose and, * conjugate.
The mean square error surface is a quadratic function of **w**. The quadratic form of Eq. (32) is important, because it implies that the bowl has a well-defined minimum, and only one such minimum. The MSE is minimized by setting its gradient with respect to **w** equal to zero, and the solution yields the optimal weight vector:

$$\nabla_{\mathbf{w}}(E[\varepsilon^2(t)]) = 0 \tag{33}$$

where $\nabla_{\mathbf{w}}$ denotes the gradient with respect to **w**. Since

$$\nabla_{\mathbf{w}}(E[\varepsilon^2(t)]) = -2\,S + 2\,\phi\,\mathbf{w} \tag{34}$$

the optimal weight vector is

$$\mathbf{w}_{opt} = \phi^{-1}\, S \tag{35}$$

where it is assumed that ϕ is non-singular so its inverse exist (ϕ is nonsingular whenever the element signals contain thermal noise).

Another manner to understand how use the power inversion effect to cancel interferences is based on the optimization using a reference signal. This approach is presented by Zahm in [16]. The reference signal is set to zero and the original weight vector has to be adjusted.

4.4.1 The relationship between the LMS and Applebaum array

The weights in the Applebaum array satisfy

$$\frac{d\mathbf{w}}{dt} + k\,\phi\,\mathbf{w} = k\,U_s \tag{36}$$

where U_s is the steering vector (which define the direction of the steered beam).
The weights in the LMS array satisfy

$$\frac{d\mathbf{w}}{dt} + k\,\phi\,\mathbf{w} = k\,S \tag{37}$$

where S is the correlation between a desired signal $d(t)$ and the signals from the antenna elements.

It is obvious from these two equations that if $U_s = S$, the two arrays will perform identically. Moreover, the steady-state weights in the LMS array also yield maximum SINR. Thus there is little difference between the two arrays from a mathematical standpoint. Rather, the difference between the two is more a matter of application [6]. The Applebaum array is useful when one knows the desired signal AoA in advance (otherwise, one cannot choose U_s). The Applebaum array has often been applied in radar, where the desired signal direction is dictated by the transmitting antenna pattern. Moreover, in the Applebaum array, one does not have to know the desired signal waveform. The LMS array, on the other hand, is just the opposite. Its use does not require any knowledge of desired signal AoA. As long as a reference signal correlated with the desired signal can be obtained, the array pattern will automatically track the desired signal with maximum signal-to-noise ratio. However, to obtain a reference signal, one must know a great deal about the structure of the desired signal waveform.

4.4.2 Reference signal generation

An LMS array can automatically track a desired signal if a reference signal correlated with the desired signal is provided. For most communication systems, the desired signal may arrive from any direction (or at least from any direction within some sector of space). In this case, an LMS array is the best option because the desired signal can be tracked without the knowledge of the direction. However, to use an LMS array, one must somehow obtain a reference signal. Compton gives some rules and examples in [6] for the reference signal generation in communication systems. Main conclusions are overviewed here.

Reference signal does not need to be an exact replica of the desired signal, it only needs to be correlated with the desired signal and uncorrelated with the interference. Such a reference signal is adequate because the steady-state array weights depend only on the correlation between the reference signal and the received signals. For some types of modulation used in communication systems, a reference signal may be generated directly from the array output signal with a signal processing loop (Fig. 27). To design such a loop, one must know the nominal frequency of the desired signal and the basic form of its modulation, but not the precise waveform. Unfortunately, there's no general method for designing reference loops for arbitrary types of desired signal modulation. Rather, a number of specialized loops have been developed for specific signals.

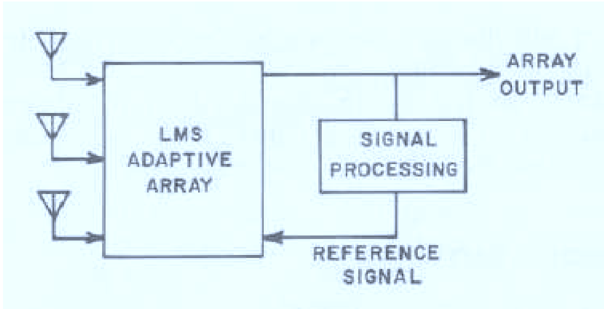


Figure 27: *A reference signal generation loop. - [6]*

The desired signal component of the reference signal must have the same carrier frequency as the array output desired signal. A frequency difference between the two signals causes the array weights to cycle at the difference frequency. In practice, the easiest way to get the reference signal on the right frequency is to pass the array output signal through a processing loop as in Fig. 27. This technique automatically yields a reference signal on the proper frequency without the designer having to know this frequency exactly.
Moreover, the desired signal component of the reference signal must have the same carrier phase as the array output desired signal. Otherwise, the array output phase cannot synchronize with the reference signal phase but instead just chases it forever. The result is again that the array weights cycle sinusoidally with time and shift the array output frequency. To avoid weight cycling, the phase shift has to be adjusted through the reference loop to the correct value at the desired-signal frequency. When the desired-signal frequency varies over some band, as in communication system with Doppler shifts, the reference loop phase shift must be aligned at the center of the band and the loop parameters chosen so the loop phase shift does not exceed 45 degrees at the band edges (phase shifts greater than 90 deg. cause the LMS loop to become unstable).
Finally, to make the array acquire the desired signal and null the interference, the reference loop should have the proper "gain" characteristics. If g_d, g_i and g_n are the gains of the reference loop to respectively the desired signal, the interference and thermal noise, these values should be:

- $|g_i| < 1$ to have the array output interference smaller and smaller, becoming null in steady state.
- Similarly, as long as $|g_n| < 1$, the thermal noise in the array output will be reduced to its minimum value.

- The situation for the desired signal is slightly more complicated. To obtain proper operation from the array, the desired signal part of the reference signal should have a fixed amplitude. For that, one technique is to use a limiter and have $|g_d| > 1$. This requirement is important for small signals for initial acquisition of the desired signal. The array feedback will make the amplitude of the desired signal increase with time until the limiter takes over and sets the reference signal amplitude.

Compton presents specific techniques in [6] for generating reference signals in communication systems for four types of modulation: spread spectrum with biphase modulation, coded amplitude modulation (AM), spread spectrum with quadriphase modulation, and binary frequency shift keying (FSK).

Two performance measures have been reviewed in this chapter to adjust the weighting for each antenna element. To implement an adaptive criteria, an algorithm has to be selected. Several algorithms are presented in section 5.

5 Algorithms

The adaptive control algorithm that is selected to adjust the array beam pattern is highly important since this choice directly influences both the speed of the array transient response and the complexity of the circuitry required to implement the algorithm. The array weights are adjusted using the available information derived from the array output, array signals, ..., to make an estimate of the optimal weights. There are many schemes and some are described in this section.

5.1 Sample Matrix Inversion (SMI)

This algorithm estimates array weights by replacing correlation matrix ϕ by its estimate [17]. An unbiased estimate of ϕ using N samples of the array signals may be obtained using a simple averaging scheme as follows:

$$\hat{\phi}(N) = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{x}(n)\, \mathbf{x}^H(n) \tag{38}$$

where $\hat{\phi}(N)$ denotes the estimated array correlation matrix using N samples, and $\mathbf{x}(n)$ denotes the array signal sample vector.
The estimate of ϕ may be updated when the new samples arrive using

$$\hat{\phi}(n+1) = \frac{n\, \hat{\phi}(n) + \mathbf{x}(n+1)\, \mathbf{x}^H(n+1)}{n+1} \tag{39}$$

It should be noted that as the number of samples grows, the matrix update approaches its true value and thus the estimated weights approaches optimal weights.

The expression of optimal weights requires the inverse of the array correlation matrix, and this process of estimating ϕ and then its inverse may be combined to update the inverse of the array correlation matrix from array signal samples using the Matrix Inversion Lemma as follows:

$$\hat{\phi}^{-1}(n) = \hat{\phi}^{-1}(n-1) - \frac{\hat{\phi}^{-1}(n-1)\, \mathbf{x}(n)\, \mathbf{x}^H(n)\, \hat{\phi}^{-1}(n-1)}{1 + \mathbf{x}^H(n)\, \hat{\phi}^{-1}(n-1)\, \mathbf{x}(n)} \tag{40}$$

The matrix is initialized as

$$\hat{\phi}^{-1}(0) = \frac{1}{\varepsilon_0} I,\ \varepsilon_0 > 0\ . \tag{41}$$

This scheme of estimating weights using the inverse update is referred to as the recursive least squares (RLS) algorithm.

5.2 Gradient-based

The gradient approach to solving the control problem posed by adaptive array weight adjustment is very popular since it is a relatively simple and generally well understood method that permits the solution of a large class of problems. The various gradient-based algorithms include the following:

- Least mean square (LMS)
- Differential steepest descent (DSD)
- Accelerated gradient (AG)

Variations of the above algorithms can be derived by introducing constraints into the adjustment rule. Other gradient-based algorithms are Perturbation, Constant Modulus and Conjugate Gradient Method.

As a basic approach, only the LMS algorithm is considered. The cases of Applebaum and LMS arrays are presented in section 5.2.1 and 5.2.2.

5.2.1 Discrete Applebaum array

By considering Fig. 11, the weight's change for one element i is given by

$$\frac{dw_i}{dt} = k\,[U_{s\,i}{}^* - x_i{}^*(t)\;y(t)] \tag{42}$$

where x_i is the signal incident on element i and y is the array output.
All signals can be sampled every Δt seconds. $x_i(n)$ and $w_i(n)$ denote the values of $x_i(t)$ and $w_i(t)$ at sample time t_n:

$$t_n = n\;\Delta t\;. \tag{43}$$

Furthermore, the derivative dw_i/dt in eq. 42 could be approximate by

$$\frac{dw_i}{dt} \cong \frac{w_i(n+1) - w_i(n)}{\Delta t}\;. \tag{44}$$

Thus the discrete version of the LMS feedback equation is

$$w_i(n+1) = w_i(n) + \gamma\, x_i{}^*(n)\;\varepsilon(n) \tag{45}$$

where $\gamma = k\;\Delta t$ is the step size [6]. For stability, γ must be restrict to the range

$$0 < \gamma < \frac{2}{P_t} \tag{46}$$

where P_t is the total received power.
The discrete equation of the Applebaum feedback loop in vector form is

$$\mathbf{w}(n+1) = \gamma\, U_s - [\gamma\, \phi - I]\, \mathbf{w}(n) \tag{47}$$

Compton gives an example in [6] and compares the computation of weights with steady-state and transient equations. These cases are studied here under the same conditions. A 2-element array is considered and no jammer are present. A desired signal is incident at $\theta_d = 50\, deg$ and ξ_d is 0 dB. With $\gamma = 0.1$ and $w_1(0) = w_2(0) = 1 - j$, the weights' evolution after 100 iterations are those given by Fig. 28. The final pattern is shown on Fig. 29.

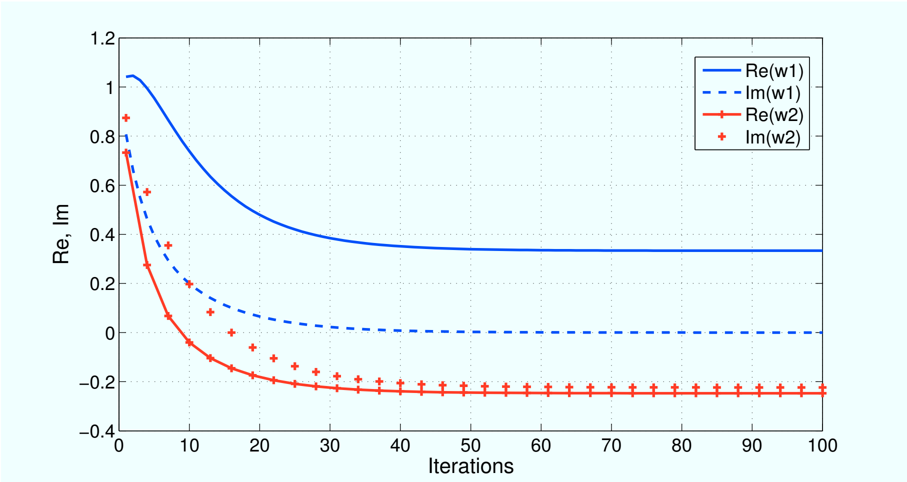


Figure 28: *Discrete weight transients.* $\theta_d = 50\ deg$, $\xi_d = 0\ dB$, *no jammer.* $\gamma = 0.1$. *- From [6]*

With $\gamma = 0.1$, the transient weight values are:

	$w_1(A,\varphi)$	$w_2(A,\varphi)$
Transient (50 it.)	(0.3396,0.4658)	(0.3271,-138.3715)
Transient (100 it.)	(0.3333,-0.0055)	(0.3333,-137.8825)
Steady-state	(0.3333,0)	(0.3333,-137.8880)

If ξ_d is now 10 dB, the step size has to be reduce and the number of iterations increased, $\gamma = 0.05$ and 200 iterations, to avoid instability. When an interference is present, the situation remains the same ($\xi_d = -20\ dB$, $\xi_i = 20\ dB$, $\gamma = 0.005$ and 1000 it.). Thus, the convergence speed is proportional to the power of signals incident on the array.

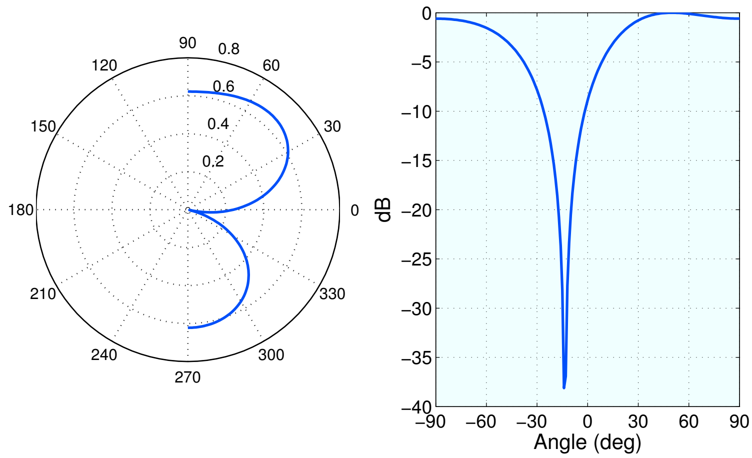


Figure 29: *Pattern of a discrete 2-element array.* $\theta_d = 50$ *deg,* $\xi_d = 0$ *dB, no jammer.* $\gamma = 0.1$, *100 iterations.*

5.2.2 LMS discrete array

In the LMS array, the weights are controlled according to a gradient algorithm. Specifically, the differential equation is

$$\frac{d\mathbf{w}}{dt} = -k\ \nabla_{\mathbf{w}}(E[\varepsilon^2(t)]) \tag{48}$$

where $k > 0$. Since the gradient on the $E[\varepsilon^2(t)]$ surface is a vector pointing in the maximum up-hill direction, and $k > 0$, this equation forces the weights to move in the steepest down-hill, or steepest-descent, direction. Moreover, this equation makes the time rate change of $\mathbf{w}$ proportional to the slope of the $E[\varepsilon^2(t)]$ surface. Since the slope of a quadratic surface increases linearly with distance from its minimum point, Eq. (48) makes the weights change rapidly when they are far from the bottom of the bowl, but only slowly when they are close to the bottom.
The differential equation could be written for the weight vector in the form [6]

$$\frac{d\mathbf{w}}{dt} + k\ \phi\ \mathbf{w} = k\ S \tag{49}$$

Equation (48) may be represented by the block diagram in Fig. 30.

The discrete equation of the LMS feedback loop in vector form is obtained from eq. 47 by replacing the steering vector U_s by S:

$$\mathbf{w}(n+1) = \gamma\, S + (I - \gamma\, \phi)\ \mathbf{w}(n) \tag{50}$$

Example: Determination of the array weights for a linear array of 5 omnidirectional elements ($\lambda/2$ element spacing).

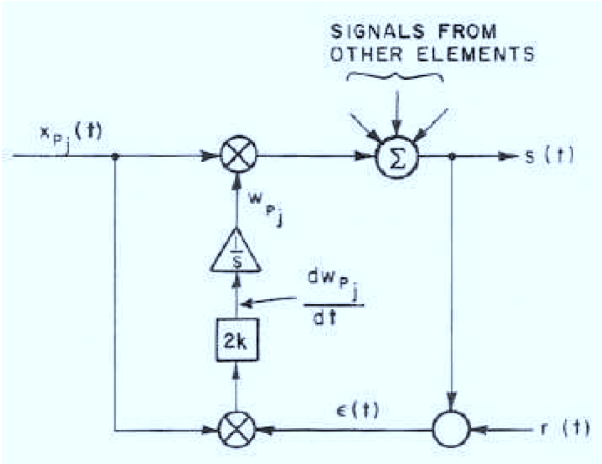


Figure 30: *The complex LMS loop - [6]*

Two kinds of signals are considered here, signals of interest $d_i(t)$ and interferences $I_i(t)$. Desired signals and interferences have the same amplitude. $d_i(t)$ is a cosine with a frequency f_0 and $I_i(t)$ is a random signal. The reference signal is a cosine with a frequency f_0. The desired and reference signals are in phase. The angles of arrival for the signals of interest are $\theta_{d_1} = 45$ *deg*. and $\theta_{d_2} = -30deg.$, and for the interferences $\theta_{I_1} = 0$ *deg*. and $\theta_{I_2} = -60 degrees$. The adapted pattern for 500 iterations and $\gamma = 0.05$ is given on Fig. 31.

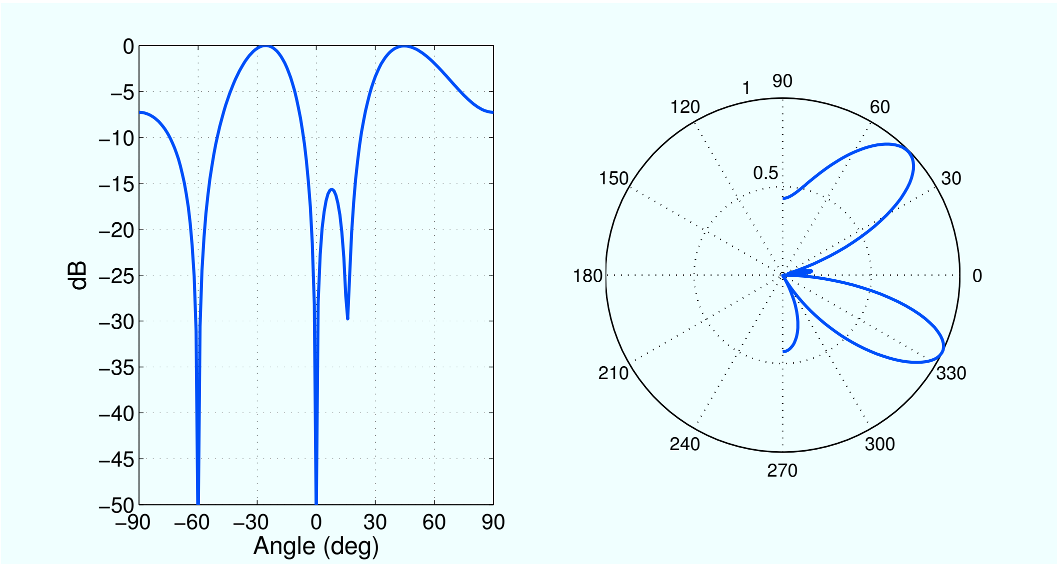


Figure 31: *Example of adapted pattern of a linear array of 5 isotropic elements, 2 interferences.* $d = 0.5\ \lambda$, $\theta_{d_1} = 45$ *deg*, $\theta_{d_2} = -30$ *deg*, $\theta_{I_1} = 0$ *deg*, $\theta_{I_2} = -60$ *deg*, *500 iterations,* $\gamma = 0.05$. *Computed with a gradient-based LMS algorithm implemented in Matlab*

The computed weights are:

$$\mathbf{w}(A,\phi) = \begin{bmatrix} 0.9872 & 132.7908 \\ 0.5263 & -23.9891 \\ 1 & 0 \\ 0.7478 & -153.8077 \\ 0.2290 & -129.4625 \end{bmatrix}$$

If the desired and reference signals are out of phase (for example quadrature), it is not possible to compute the adapted pattern. Fig. 32 presents the reference signal and the array output signal. As mentioned in section 4.4.2, if the reference signal and the desired signals haven't the same phase carrier, the array output phase cannot synchronize with the reference signal phase, and instead just chases it forever.

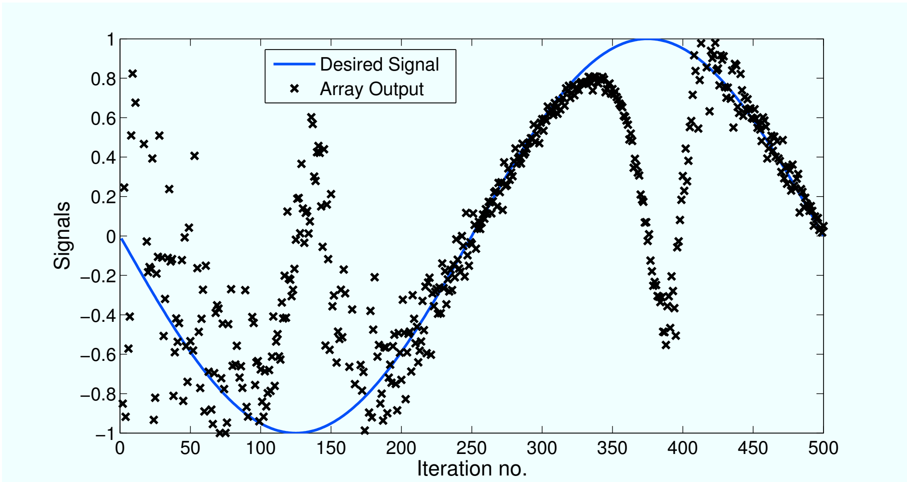


Figure 32: *Reference signal and array output vs. iteration with reference signal and desired signals in quadrature*

5.3 Random search algorithms

Gradient-based algorithms are particularly suitable for performance measures that are either quadratic or at least unimodal. For some classes of problems, the mathematical relation of the variable parameters to the performance measure is either unknown or is too complex to be useful [2]. In other problems, constraints may be placed on the variable parameters of the adaptive controller with the result that the performance surface may no longer be unimodal. When the performance surface of interest is multimodal and contains saddlepoints, then any gradient-based algorithm must be used with caution.

One way of approaching such problems is to employ a search algorithm. Algorithms belonging to this class have global search capabilities for both unimodal and multimodal

performance surfaces and work for any computable performance measure. Until recently search algorithms were used only reluctantly, principally because little effort was made to use past information in guiding the search thereby resulting in slow convergence.
Furthermore, search algorithms are not very powerful in unimodal applications. They do, however, have the advantage of being simple to implement in logical form, require little computation, are insensitive to discontinuities and exhibit a high degree of efficiency where little is known about the performance surface. The major types of search algorithms are systematic and random searches.

5.4 Real vs. complex algorithm

In this section, some implementation issues relating to real vs. complex implementation are discussed. Major conclusions from [1] are presented here.

In some cases, the input data to the weight adaption scheme are real, and in others, the data are complex (with real and imaginary parts denoting in-phase and quadrature components). In both of these cases, the weights could be updated using the real LMS algorithm or the complex LMS algorithm. The former uses real arithmetic and real variables, and updates real weights (in-phase and quadrature component are updated separately when complex data are available), whereas the complex algorithm uses complex arithmetic and variables, and weights are updated as well as implemented as complex variables similar to the treatment presented in this book. For real data using complex algorithm, you need to generate the quadrature component using the Hilbert transformer.

5.4.1 Beamformer Structures

Figure 33 shows a real beamforming system and Figure 34 shows an in-phase and quadrature (IQ) or complex beamforming system. These two beamformers are narrowband systems. The real beamforming system has a single real valued output that can be produced by using real multiplication to achieve the weighting of the array signals. The other has a complex valued output and can be produced by using the complex multiplication to achieve the weighting of the array signals.
In both case, weights are complex and are given by

$$w = w_I + j\, w_Q \tag{51}$$

Real signals are converted to in-phase and quadrature signals using a quadrature filter (QF). The output of the quadrature filter is related to its input by the Hilber transform. The relation between in phase and quadrature signals is

$$x(t) = x_I(t) + j\, x_Q(t) \tag{52}$$

where $x_Q(t) = \hat{x}_I(t)$ and $\hat{x}(t)$ denote the Hilbert transform of a real signal $x(t)$.

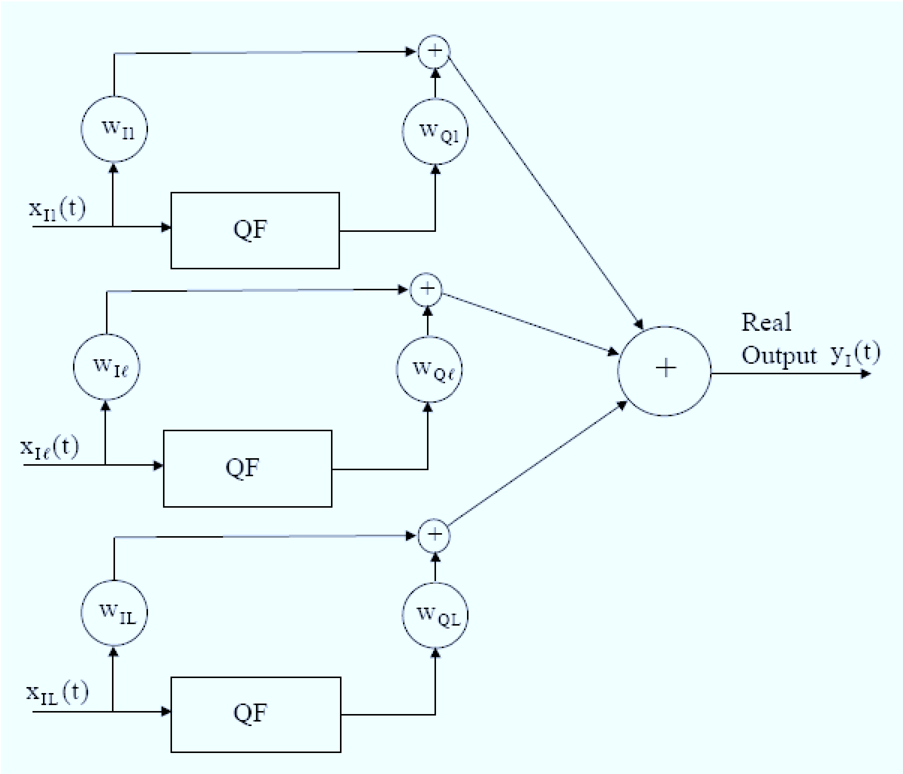


Figure 33: *Real beamforming system - [1]*

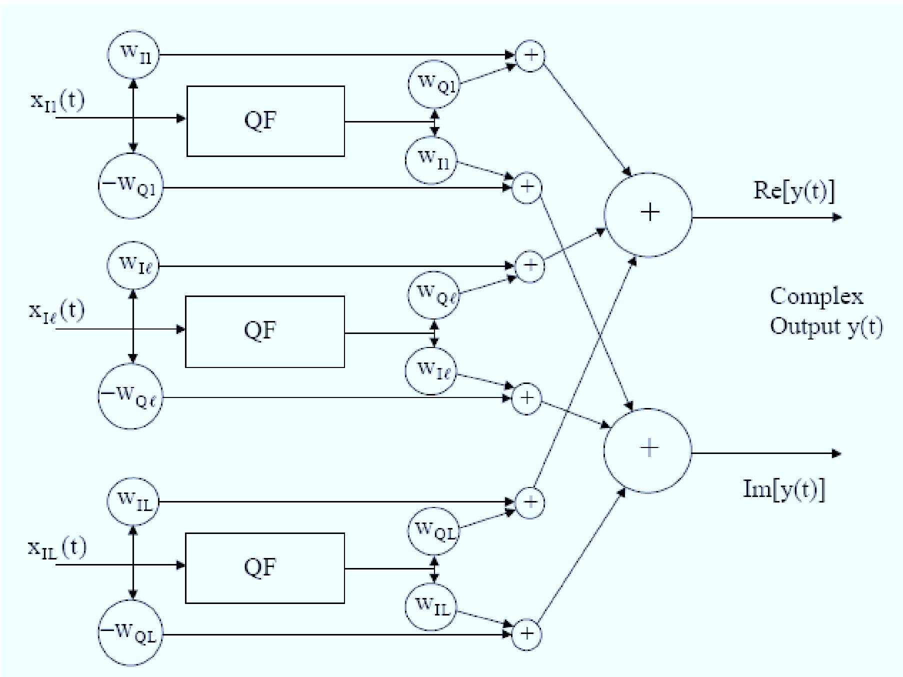
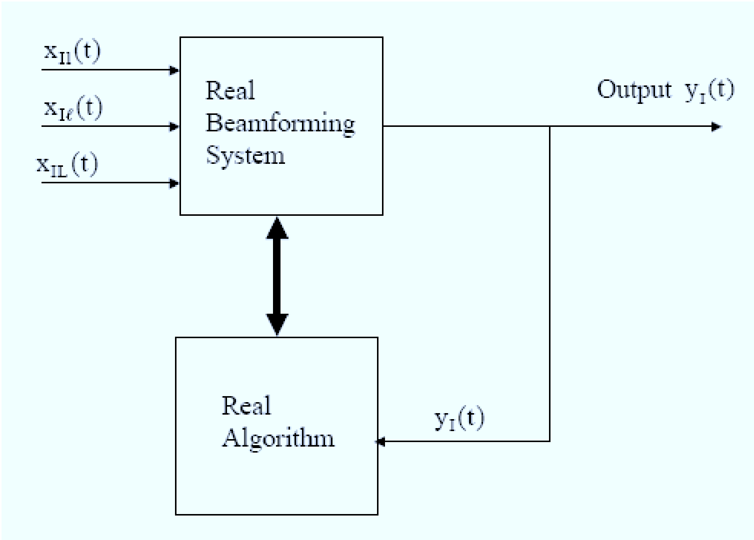


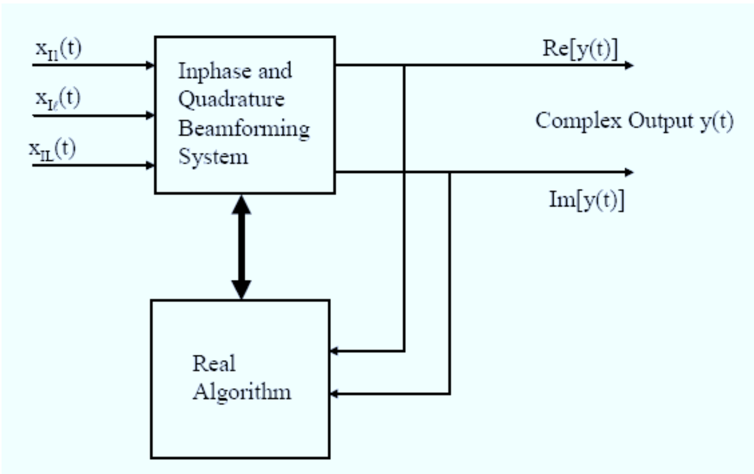
Figure 34: *In-Phase and Quadrature beamforming system - [1]*

5.4.2 Real and complex algorithms

Implementation of the real LMS algorithm for the real beamforming system is shown in Figure 35(a) and for the IQ beamforming system it is shown in Figure 35(b). In the real beamforming system, y_I represents the only out of the system. In the IQ beamforming system, it is the real part of the output. Real multiplications are used in estimating the real and imaginary parts.



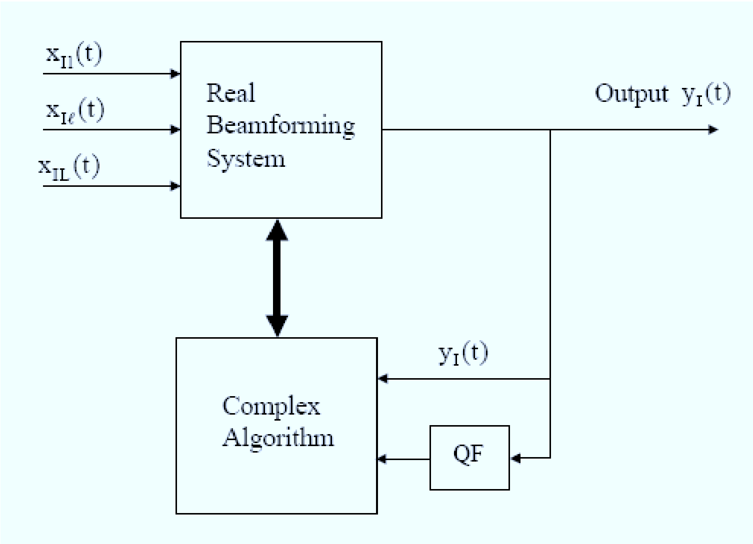
(a)



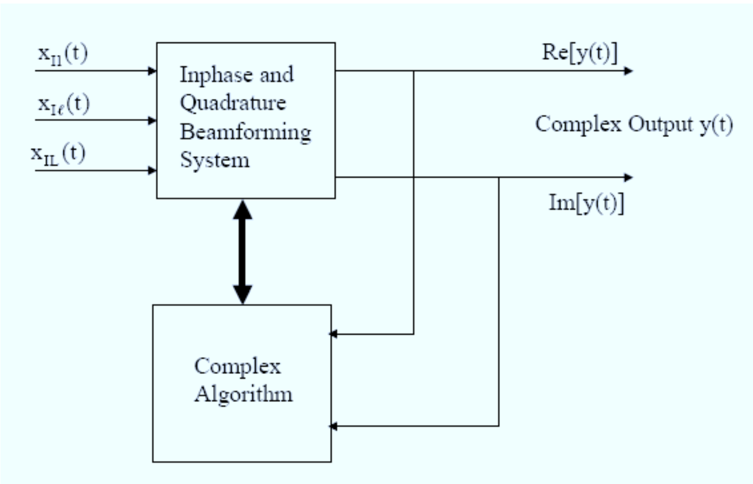
(b)

Figure 35: *(a) Real algorithm for real beamforming system. - (b) Real algorithm for IQ beamforming system. - [1]*

Implementation of the complex algorithm for the real beamforming system is shown in Figure 36(a), and for the IQ beamforming system in Figure 36(b). The signal on the input of the complex algorithm is the complex array output when the complex beamformer system is used and is equal to $y_I + j\,\hat{y}_I$ when the real beamformer is used.



(a)



(b)

Figure 36: *(a) Complex algorithm for real beamforming system. - (b) Complex algorithm for IQ beamforming system. - [1]*

Godara shows in [1] that the convergence time constant for the complex LMS algorithm is half that of the real LMS algorithm. This means that the convergence speed of the complex LMS algorithm is twice that of the real LMS algorithm.

5.5 Concluding remarks

Some adaptive algorithms have been described in this section. Monzingo and Miller have presented a characteristics' summary in [2]. Table 2 presents the main characteristics of the reviewed adaptive algorithms. The LMS algorithm seems to be a good compromise regarding convergence speed, ease of implementation and computational load.
Furthermore, the convergence speed of the complex LMS algorithm is twice that of the real LMS algorithm.

	LMS	**SMI**	**Random Search**
Algorithm philosophy	Steepest descent	Direct estimate of covariance	Trial and error
Transient response characteristic	Misadjustment vs. convergence speed trade-off is acceptable for numerous applications	Achieves the fastest convergence with most favorable misadjustment vs. convergence speed trade-off	Unfavorable misadjustment vs. convergence speed trade-off compared to LMS; accelerated steps improve speed
Algorithm strengths	Easy to implement, requiring N correlators and integrators. Tolerant of hardware errors.	Very fast convergence speed independent of eigenvalue spread.	Can be applied to any directly measurable performance index; easy to implement, with meager instrumentation and computation requirements
Algorithm weaknesses	Convergence speed sensitive to eigenvalue spread.	Requires $N \ (N+1)/2$ correlators to implement; matrix inversion requires adequate precision and $N^3/2 + N^2$ complex multiplies	Convergence speed sensitive to eigenvalue spread and the slowest of all algorithms considered

Table 2: *Operational characteristics summary of presented adaptive algorithms - [2]*

6 Conclusion and perspectives

The first objective of this report was to review some basic concepts on narrowband adaptive antennas. First, a signal model has been presented to analyze adaptive antennas. The definitions of steering vector and degrees of freedom have been given.
Moreover, the architecture of adaptive arrays have been detailed. Different adaptive beamforming systems for antenna arrays have been discussed and compared in term of complexity.
Furthermore, performance criteria, capabilities and limitations have been studied for two adaptive antennas: the Applebaum and the LMS arrays. These two arrays are the basic systems and their concepts have been explained. Some examples have been implemented using Matlab.
Finally, some algorithms which allow to process the adaptive antennas have been reviewed. Several algorithms have been presented, a lot of derivations or other ones are present in the literature.

The beamformer structure of Figure 1 discussed in this report is for narrowband signal. As the signal bandwidth increases, beamformer performance using this structure starts to deteriorate [6]. For processing broadband signals, a tap delay line structure is normally used. Alternative methods are also existing. For complete details on broadband adaptive antennas, it is possible to refer to [1] (chap. 4).

The presentation of these concepts will allow the study of an adaptive antenna for anti-jamming system. For this purpose, the implementation of some adaptive algorithms in PAASoM, a Phased Array Simulation Tool, could be very interesting to design adaptive antennas and observe the performances vs. the antenna element positions and the elementary pattern. In this manner, the efficiency of the control algorithm and the beamforming network parts of the adaptive array could be tested in term of speed convergence, pointing accuracy and null depths.

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The aim of this report is to address basic concepts for narrowband adaptive antennas, also called smart antennas. This report begins with the introduction of a signal model for the study of adaptive antennas. Then, adaptive array architecture is discussed. Moreover, descriptions of performance criteria, capabilities and limitations are presented. To conclude, various narrowband algorithms are highlighted.

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