



Defence Research and  
Development Canada

Recherche et développement  
pour la défense Canada



# **Time-frequency-based detection of fast maneuvering targets using high-frequency surface-wave radar**

T. Thayaparan, L. Stankovic and M. Dakovic

**Defence R&D Canada – Ottawa**

**Canada**

Technical Memorandum  
DRDC Ottawa TM 2009-282  
February 2010



# **Time-frequency-based detection of fast maneuvering targets using high-frequency surface-wave radar**

T. Thayaparan  
Defence R&D Canada – Ottawa

L. Stankovic, M. Dakovic  
University of Montenegro

**Defence R&D Canada – Ottawa**

Technical Memorandum

DRDC Ottawa TM 2009-282

February 2010

Principal Author

*Original signed by T. Thayaparan*

---

T. Thayaparan

Approved by

*Original signed by Caroline Wilcox*

---

Caroline Wilcox  
Defence Scientist/RAST Section

Approved for release by

*Original signed by Brian Eatock*

---

Brian Eatock  
Chair/Document Review Panel

© Her Majesty the Queen in Right of Canada as represented by the Minister of National Defence, 2010

© Sa Majesté la Reine (en droit du Canada), telle que représentée par le ministre de la Défense nationale, 2010

# Abstract

---

The time-frequency representation is a powerful tool for the analysis of non-stationary signals. In the past decades, time-frequency representations have been primarily devoted to analysis tasks in the sense that they were introduced so as to depict the time-frequency structure of time-varying signals and non-stationary processes in the time-frequency plane. However, besides this approach, there has been also a permanent interest for tackling decision problems by means of time-frequency representations. In this report, we present a time-frequency-based detection scheme for the high-frequency surface-wave radar (HFSWR) for the detection of maneuvering air targets in the presence of strong sea-clutter. The performance of the proposed method is evaluated using both synthetic and experimental data. In addition, the proposed time-frequency detection scheme is examined in detail with different signal-to-noise ratio and various examples are considered. The time-frequency-based detection method is then compared with the Fourier-based detector. Results clearly demonstrate that the time-frequency-based detector can significantly improve the detection performance of the HFSWR and add considerable physical insight over what can be achieved by conventional Fourier-based detector currently used by HFSWRs. These results distinctly suggest that the Fourier-based detector is optimal for stationary signals, whereas the Time-Frequency-based detector is optimal for non-stationary signals.

# Résumé

---

La représentation temps fréquence est un puissant outil pour l'analyse de signaux non stationnaires. Dans les dernières décennies, les représentations temps fréquence ont été utilisées principalement pour des tâches d'analyse en ce sens qu'elles ont été adoptées pour décrire la structure temps fréquence de signaux variables dans le temps et de processus non stationnaires dans le plan temps fréquence. Toutefois, mise à part cette approche, on s'est intéressé en permanence à la résolution des problèmes de décision au moyen de représentations temps fréquence. Dans le présent rapport, nous présentons un modèle de détection temps fréquence pour le radar haute fréquence à ondes de surface (HFSWR) aux fins de la détection de cibles aériennes manoeuvrant en présence d'un important retour de mer (clutter). L'efficacité de la méthode proposée est évaluée à l'aide de données à la fois synthétiques et expérimentales. En outre, le modèle de détection temps fréquence proposé est examiné en détail avec différents rapports signal/bruit, et divers exemples sont étudiés. La méthode de détection temps fréquence est ensuite comparée avec le détecteur à transformée de Fourier. Les résultats montrent clairement que le détecteur temps fréquence peut améliorer sensiblement les performances de détection du HFSWR, et permet de comprendre beaucoup mieux les phénomènes physiques par comparaison à ce que permettent les

méthodes classiques de Fourier utilisées actuellement par les HFSWR. Ces résultats indiquent nettement que le détecteur à transformée de Fourier est optimal pour les signaux stationnaires, tandis que le détecteur temps fréquence est optimal pour les signaux non stationnaires.

# Executive summary

---

## Time-frequency-based detection of fast maneuvering targets using high-frequency surface-wave radar

T. Thayaparan, L. Stankovic, M. Dakovic; DRDC Ottawa TM 2009-282; Defence R&D Canada – Ottawa; February 2010.

**Background:** The detection of an unknown deterministic signal in a high noise environment is of crucial interest in many real-world applications. In the case of a stationary signal, for example, a sinusoidal signal with constant frequency, the Fourier transform (FT) method concentrates all the signal energy in one frequency point while the noise is uniformly distributed over all frequencies. Thus, it is easy to conclude that the FT-based detection method provides the optimal detection in the case of stationary signal. However, for non-stationary signals, i.e., when the frequency content of a signal changes over time, the spectral content of such signals becomes time-varying, and thus the FT-based detector will not provide the optimal result. The time-frequency formulation of the FT, that is, by using a window in the time domain, the short time Fourier transform (STFT) has the same advantages and drawbacks similar to FT. Therefore, there is a need for more sophisticated time-frequency tools for the analysis of highly non-stationary signals. In this report, we present a novel time-frequency-based detection scheme for the high-frequency surface-wave radar (HFSWR) for the detection of maneuvering air targets in the presence of strong sea-clutter.

**Results:** We present a time-frequency-based detection scheme applied to high-frequency surface-wave radar (HFSWR) data for the detection of maneuvering air targets in the presence of strong sea-clutter. The performance of the proposed method is evaluated using both synthetic and experimental data. In addition, the proposed time-frequency detection scheme is examined in detail with different signal-to-noise ratio and various examples are considered. The time-frequency-based detection method is then compared with the Fourier-based detector. Results clearly demonstrate that the time-frequency-based detector can significantly improve the detection performance of the HFSWR and add considerable physical insight over what can be achieved by conventional Fourier-based detector currently used by HFSWRs. These results distinctly suggest that the Fourier-based detector is optimal for stationary signals, whereas the Time-Frequency-based detector is optimal for non-stationary signals.

**Significance:** The results clearly show that the proposed time-frequency detector outperforms the Fourier-based detector in terms of good detection and false alarm rates for non-stationary signals. The method presented here is not restricted to

this particular application, but it can also be applied in various other settings of non-stationary signal analysis and filtering. More generally, it is believed that the time-frequency formulation of optimum detection can provide new hints for handling open problems in a comprehensive way. The proposed time-frequency-based detector can be evaluated against the existing real-time CFAR detector for the operational and future HFSWR development.



# Sommaire

---

## Time-frequency-based detection of fast maneuvering targets using high-frequency surface-wave radar

T. Thayaparan, L. Stankovic, M. Dakovic ; DRDC Ottawa TM 2009-282 ; R & D pour la défense Canada – Ottawa ; février 2010.

**Contexte :** La détection de signaux déterministes inconnus dans un environnement très bruyant est d'une importance cruciale pour de nombreuses applications dans le monde réel. Dans le cas d'un signal stationnaire, par exemple, un signal sinusoïdal de fréquence constante, la méthode à transformée de Fourier (TF) concentre toute l'énergie du signal en un point de fréquence, tandis que le bruit est uniformément réparti sur toutes les fréquences. Ainsi, il est aisé de conclure que la méthode à transformée de Fourier (TF) assure une détection optimale dans le cas d'un signal stationnaire. Toutefois, dans le cas de signaux non stationnaires, c. à d. lorsque le contenu fréquentiel d'un signal varie dans le temps, le contenu spectral de ces signaux est variable dans le temps, de sorte que les performances du détecteur à transformée de Fourier ne sont pas optimales. La formulation temps fréquence de la TF, c. à d. l'utilisation d'une fenêtre dans le domaine temporel (transformée de Fourier fenêtrée) offre les mêmes avantages et inconvénients que la TF. Par conséquent, des outils temps fréquence plus perfectionnés sont nécessaires pour l'analyse de signaux fortement non stationnaires. Dans le présent rapport, nous présentons un nouveau modèle de détection temps fréquence pour le radar haute fréquence à ondes de surface (HFSWR) aux fins de la détection de cibles aériennes évolutives en présence d'un important retour de mer (clutter).

**Résultats :** Nous présentons un modèle de détection temps fréquence appliqué au radar haute fréquence à ondes de surface (HFSWR) aux fins de la détection de cibles aériennes évolutives en présence d'un important retour de mer (clutter). L'efficacité de la méthode proposée est évaluée à l'aide de données à la fois synthétiques et expérimentales. En outre, le modèle de détection temps fréquence proposé est examiné en détail avec différents rapports signal/bruit, et divers exemples sont étudiés. La méthode de détection temps fréquence est ensuite comparée avec le détecteur à transformée de Fourier. Les résultats montrent clairement que le détecteur temps fréquence peut améliorer sensiblement les performances de détection du HFSWR, et permet de comprendre beaucoup mieux les phénomènes physiques par comparaison à ce que permettent les méthodes classiques de Fourier utilisées actuellement par les HFSWR. Ces résultats indiquent nettement que le détecteur à transformée de Fourier est optimal pour les signaux stationnaires, tandis que le détecteur temps fréquence est optimal pour les signaux non stationnaires.

**Portée :** Les résultats montrent clairement que le détecteur temps fréquence proposé surpasse le détecteur à transformée de Fourier du point de vue de l'efficacité de détection et des taux de fausses alarmes dans le cas de signaux non stationnaires. La méthode présentée ici n'est pas limitée à cette application particulière, mais peut également s'appliquer dans divers autres contextes d'analyse et de filtrage de signaux non stationnaires. Plus généralement, on estime que la formulation temps fréquence de la détection optimale peut aider à mieux comprendre comment traiter des problèmes ouverts de manière exhaustive. Le détecteur temps fréquence proposé pourra être évalué par rapport au présent détecteur TFAC en temps réel aux fins du développement du HFSWR.

# Table of contents

---

Abstract . . . . .	i
Résumé . . . . .	i
Executive summary . . . . .	iii
Sommaire . . . . .	v
Table of contents . . . . .	vii
List of figures . . . . .	viii
1 Introduction . . . . .	1
2 Signal Detection by using Fourier Transform . . . . .	3
2.1 Parametric Processing Extension of the Fourier Transform . . . . .	5
3 Signal Detection by using Time-Frequency Analysis . . . . .	9
3.1 Time-frequency Basics . . . . .	9
3.2 Time-Frequency-based Signal Detection . . . . .	10
3.3 Comparison with Fourier-Based Detector . . . . .	15
4 Application to the Real Radar Data . . . . .	16
4.1 Fourier Transform Case . . . . .	16
4.2 Time-Frequency Case . . . . .	16
4.3 Detection Examples . . . . .	17
5 Conclusion . . . . .	22
References . . . . .	23

# List of figures

---

Figure 1:	Detection example - linear FM signal. First row - paths parallel to the time axis, equivalent to the Fourier-based detector. Second row - paths parallel to the instantaneous frequency, equivalent to the time-frequency detector. First column - time-frequency representation. Second column - paths. Third column - sum of the TFR values along the paths. . . . .	7
Figure 2:	Detection example - sinusoidal FM signal. First row - paths parallel to the time axis, equivalent to the Fourier-based detector. Second row - paths used in the LFM example. Third row - paths parallel to the instantaneous frequency, equivalent to the time-frequency detector. First column - time-frequency representation. Second column - paths. Third column: sum of the TFR values along paths. . . . .	8
Figure 3:	Fourier transform (with threshold level) and time-frequency representation of the signal $s(n)$ in the cases of deterministic signal $x(n)$ existence (left) and non-existence (right). . . . .	11
Figure 4:	Threshold estimation - The best achieved values of the path sum for 1000 realizations of random signal without deterministic component. The horizontal line represents the threshold $R_0 = 9792$ obtained with $P_{FA} = 0.01$ . . . . .	14
Figure 5:	Histogram of the number of analyzed paths for 1000 realizations. The average number of analyzed paths is 12. . . . .	14
Figure 6:	Probability of the signal non-detection for the case of FT-based detector and the SM-based detector for different values of the parameter $L$ . . . . .	15
Figure 7:	Fourier transform of the analyzed signals with SNR=-2dB. . . . .	18
Figure 8:	Estimated probability of target non-detection versus signal-to-noise ratio for non-stationary cases using both Fourier and time-frequency based detectors. . . . .	19
Figure 9:	Signal 1 (nonstationary target velocity) with SNR=-8 dB. . . . .	20
Figure 10:	Signal 6 (stationary target velocity) with SNR=-8 dB. . . . .	21

# 1 Introduction

---

Traditionally, radar signals have been analyzed in either the time or the frequency domain. The Fourier Transform (FT) is at the heart of a wide range of techniques that are generally used in radar data analysis and processing. However, the change of frequency content with time is one of the main features we generally observe in radar data. Because of this change of frequency content with time, radar signals belong to the class of non-stationary signals. The analysis of non-stationary signals requires techniques that extend the notion of a global frequency spectrum to a local frequency description. Joint time-frequency analysis using time-frequency or wavelet transforms has improved the analysis of non-stationary signals by revealing time-varying information embedded in signals [1, 2].

During the past ten years, time-frequency analysis has been a major area of research in radar signal processing. One of the main challenges in radar detection is the unknown nature of the target's motion. The commonly used technique for radar detection is a Fourier-based approach, which assumes time invariance of the Doppler frequency. However, in real-world radar detection scenarios, when a target exhibits complex motion such as acceleration or maneuvering, standard Fourier-based methods fail to reveal a complete picture of the temporal localization of a signal's spectral components. Time-frequency representations extend the fundamental concept of spectrum to non-stationary signals and facilitate a time-varying spectral analysis by representing signal characteristics jointly in terms of time and frequency [1]-[6].

Although time-frequency-based techniques have shown a lot of promise in non-stationary signal processing, the use and development of new time-frequency representations (TFRs) has been primarily geared toward exploratory data analysis, that is, using time-frequency representations to provide a visual display of the time-varying spectral energy in the signal and then using this qualitative information as a starting point in further analysis/processing. Moreover, many techniques have been developed for deterministic, noise-free signal analysis. On the other hand, many applications in real-world realistic scenarios involve noisy or random signals and often require detection, estimation, or classification of certain non-stationary signal characteristics. The recent advance in time-frequency analysis has been the development of promising techniques that go beyond exploratory data analysis and enable time-frequency representations to be fully exploited in real applications. These techniques include optimal non-stationary spectral detection and estimation using time-frequency representations and optimal quadratic detection using bilinear time-frequency representations and time-scale representations [7]-[17]. Such time-frequency-based techniques have shown encouraging results to real problems involving non-stationary signals.

The detection of an unknown deterministic signal in a high noise environment is of crucial interest in many real-world applications. In the case of a stationary signal, for

example, a sinusoidal signal with constant frequency, the FT method concentrates all the signal energy in one frequency point, while the noise is uniformly distributed over all frequencies. Thus, it is easy to conclude that the FT-based detection method provides the optimal detection in the case of stationary signal. However, for non-stationary signals, i.e., when the frequency content of a signal changes over time, the spectral content of such signals becomes time-varying, and thus the FT-based detector will not provide the optimal result. The time-frequency formulation of the FT, that is, by using a window in the time domain, the short time Fourier transform (STFT) has the same advantages and drawbacks similar to FT. Therefore, there is a need for more sophisticated time-frequency tools for the analysis of highly non-stationary signals [1]-[6]. In this report, we present a novel time-frequency-based detection scheme for the high-frequency surface-wave radar (HFSWR) for the detection of maneuvering air targets in the presence of strong sea-clutter. The basic idea is to use a method that produces highly-concentrated energy of the desired signal around the instantaneous frequency (IF) and then applies the integration along the IF line. In the case of high noise, an algorithm for finding the possible IF paths is proposed. In this way, the detection performance will be as high as in the case of constant frequency estimation using the FT method. The time-frequency-based detection method is compared with the FT-based detector. The proposed method is then applied to the real radar signals with additive noise.

The report is organized in the following manner. The signal detection via the FT is reviewed in Section 2. Section 3 deals with time-frequency tools in detection, including the definition of the algorithm for path finding. The demonstration of the proposed method, its comparison to the FT-based detector, and the application to the real radar data are given in Section 4. In Section 5, conclusions are given.

## 2 Signal Detection by using Fourier Transform

---

Let us consider a deterministic signal:

$$x(t) = Ae^{j\omega_0 n},$$

where  $\omega_0 = 2\pi k_0/N$  is the discrete frequency and  $k_0$  is an integer such that  $|\omega_0| < \pi$  holds. Suppose that there are  $N$  samples of the discrete signal,  $s(n)$ , available. There are two possible cases:

$$s(n) = s_x(n) = x(n) + \varepsilon(n), \quad (1)$$

$$s(n) = s_\varepsilon(n) = \varepsilon(n). \quad (2)$$

In equations (1) and (2),  $\varepsilon(n)$  is complex zero mean Gaussian white noise with independent real and imaginary parts, where the total variance is  $\sigma_\varepsilon^2$ . The discrete Fourier transform of the signal  $s(n)$  is [21]:

$$S(k) = \xi NA\delta(k - k_0) + \varepsilon_F(k), \quad (3)$$

where  $\varepsilon_F(k)$  is also complex zero mean Gaussian white noise, but with variance  $\sigma_F^2 = N\sigma_\varepsilon^2$ . The parameter  $\xi$  in equation (3) is equal to 1 if the signal  $s(n)$  is of the form (1) and 0 if the signal  $s(n)$  is of the form (2). The spectrum of the signal  $s(n)$  is:

$$\begin{aligned} |S(k)|^2 &= S(k)S^*(k) = \\ &= \xi(N^2|A|^2 + NA\varepsilon_F^*(k) + NA^*\varepsilon_F(k))\delta(k - k_0) + |\varepsilon_F(k)|^2, \end{aligned} \quad (4)$$

and its expected value is:

$$E[|S(k)|^2] = \begin{cases} N^2|A|^2\delta(k - k_0) + E[|\varepsilon_F(k)|^2], & \text{for } \xi = 1, \\ E[|\varepsilon_F(k)|^2], & \text{for } \xi = 0. \end{cases} \quad (5)$$

The existence of the deterministic signal,  $\xi$ , is estimated as:

$$\hat{\xi} = \begin{cases} 1, & \text{for } \max[|S(k)|^2] > R_{S^2}, \\ 0, & \text{for } \max[|S(k)|^2] \leq R_{S^2}, \end{cases} \quad (6)$$

where  $R_{S^2}$  is the threshold level. Let us consider four possible cases [21]:

1. The deterministic component  $x(n)$  exists in the signal  $s(n)$ ,  $\xi = 1$ . The detection procedure gives  $\max[|S(k)|^2] > R_{S^2}$ , namely  $\hat{\xi} = 1$ .
2. The deterministic component  $x(n)$  exists in the signal  $s(n)$ ,  $\xi = 1$ . The detection procedure gives  $\max[|S(k)|^2] \leq R_{S^2}$ , namely  $\hat{\xi} = 0$ . In this case, the existing signal is not detected by the detector.

3. The deterministic component  $x(n)$  does not exist in the signal  $s(n)$ ,  $\xi = 0$ . The detection procedure gives  $\max[|S(k)|^2] \leq R_{S^2}$ , namely  $\hat{\xi} = 0$ .
4. The deterministic component  $x(n)$  does not exist in the signal  $s(n)$ ,  $\xi = 0$ . The detection procedure gives  $\max[|S(k)|^2] > R_{S^2}$ , namely  $\hat{\xi} = 1$ . In this case, the detector causes a false alarm.

The first and third cases represent correct detection, while the second and fourth cases represent wrong detection scenarios. The probability of the second case is denoted by  $P_{ND}$  and the probability of the fourth case is denoted by  $P_{FA}$ . We can now observe the behavior of the probabilities  $P_{ND}$  and  $P_{FA}$  with respect to the chosen threshold level  $R_{S^2}$ . By increasing the level of  $R_{S^2}$ , the probability  $P_{FA}$  decreases, while the probability  $P_{ND}$  increases, and vice versa. The probability  $P_{FA}$  can be determined by analyzing the statistical properties of the noise, while the magnitude of the signal  $x(n)$  must be known in order to determine the probability  $P_{ND}$ . The common method for determining the threshold level  $R_{S^2}$  is a constant false alarm rate (CFAR) method, where the probability  $P_{FA}$  is kept constant. In the analyzed case we have:

$$P_{FA} = P[|\varepsilon_F(k)|^2 > R_{S^2}, \text{ for at least one } k] \quad (7)$$

$$= 1 - P[|\varepsilon_F(k)|^2 \leq R_{S^2}, \text{ for every } k] \quad (8)$$

$$= 1 - \prod_{k=0}^{N-1} P[|\varepsilon_F(k)|^2 \leq R_{S^2}]. \quad (9)$$

where  $P[\cdot]$  denotes probability of event  $[\cdot]$ .

It is known that the square absolute value of random variable with Gaussian probability distribution is a random variable with Rayleigh probability distribution, i.e.:

$$P[|\varepsilon_F(k)|^2 < R_{S^2}] = 1 - \exp\left(-\frac{R_{S^2}}{\sigma_F^2}\right) = 1 - \exp\left(-\frac{R_{S^2}}{N\sigma_\varepsilon^2}\right), \quad (10)$$

whereby we have:

$$P_{FA} = 1 - \left(1 - \exp\left(-\frac{R_{S^2}}{N\sigma_\varepsilon^2}\right)\right)^N. \quad (11)$$

Now we can determine the threshold level  $R_{S^2}$ , which depends on the probability  $P_{FA}$ :

$$R_{S^2} = -\ln(1 - \sqrt[N]{1 - P_{FA}})N\sigma_\varepsilon^2. \quad (12)$$

For large  $N$  and small  $P_{FA}$ , the above equation can be approximated as:

$$R_{S^2} = -\ln(P_{FA}/N)N\sigma_\varepsilon^2. \quad (13)$$

To determine the threshold level, the noise variance must be known. This variance can be estimated by using the data samples of the signal  $s(n)$  and the relation:

$$\sigma_\varepsilon^2 \cong 1.1 \left( \text{median}_{1 \leq i < N} (|\text{Re}[s(i) - s(i-1)]|)^2 + \text{median}_{1 \leq i < N} (|\text{Im}[s(i) - s(i-1)]|)^2 \right). \quad (14)$$



In many cases, the discrete frequency of the deterministic signal does not satisfy the relation  $\omega_0 = 2\pi k_0/N$ , where  $k_0$  is an integer. In these cases, when  $\omega_0 \neq 2\pi k_0/N$ , the detection result can be improved (probability  $P_{ND}$  decreased) by zero padding before the Fourier transform calculation.

If the deterministic signal is non-stationary,  $x(n)$  can be written as:

$$x(n) = A(t)e^{j\varphi(t)}, \quad (15)$$

where  $\varphi(t)$  is a nonlinear function. In this scenario, the Fourier-based detector is not the optimal one. When the signals are non-stationary, the detection capability of the Fourier-based detector is limited. In these cases, the detection problem can be solved in a better way by using the time-frequency analysis of the signal  $s(n)$ . Before we start the time-frequency formulation, we will introduce an intermediate step, the parametric processing of non-stationary signals.

## 2.1 Parametric Processing Extension of the Fourier Transform

A signal of the form  $x(n) = A(t)e^{j\varphi(t)}$  can be processed by using a parametric form of the Fourier transform:

$$X(\omega) = \int_{-\infty}^{\infty} A(t)e^{j\varphi(t)} e^{-j\psi(t;a_0,a_1,\dots,a_N)} e^{-j\omega t} dt, \quad (16)$$

where  $\psi(t; a_0, a_1, \dots, a_N)$  is expressed as a function with  $N$  parameters. If we are able to match the form of  $\varphi(t)$  with  $\psi(t; a_0, a_1, \dots, a_N)$  and find the parameters  $a_0, a_1, \dots, a_N$ , such that  $\varphi(t) = \psi(t; a_0, a_1, \dots, a_N)$  up to a linear phase term (constant frequency), then  $e^{j\varphi(t)} e^{-j\psi(t;a_0,a_1,\dots,a_N)}$  would be a pure sinusoid and all the conclusions for the Fourier transform would be satisfied [19].

In order to illustrate this method, which will lead to a TFR based signal detection, consider the short time Fourier transform

$$STFT(t, \omega) = \int_{-\infty}^{\infty} w(\tau)x(t + \tau)e^{-j\omega\tau} d\tau, \quad (17)$$

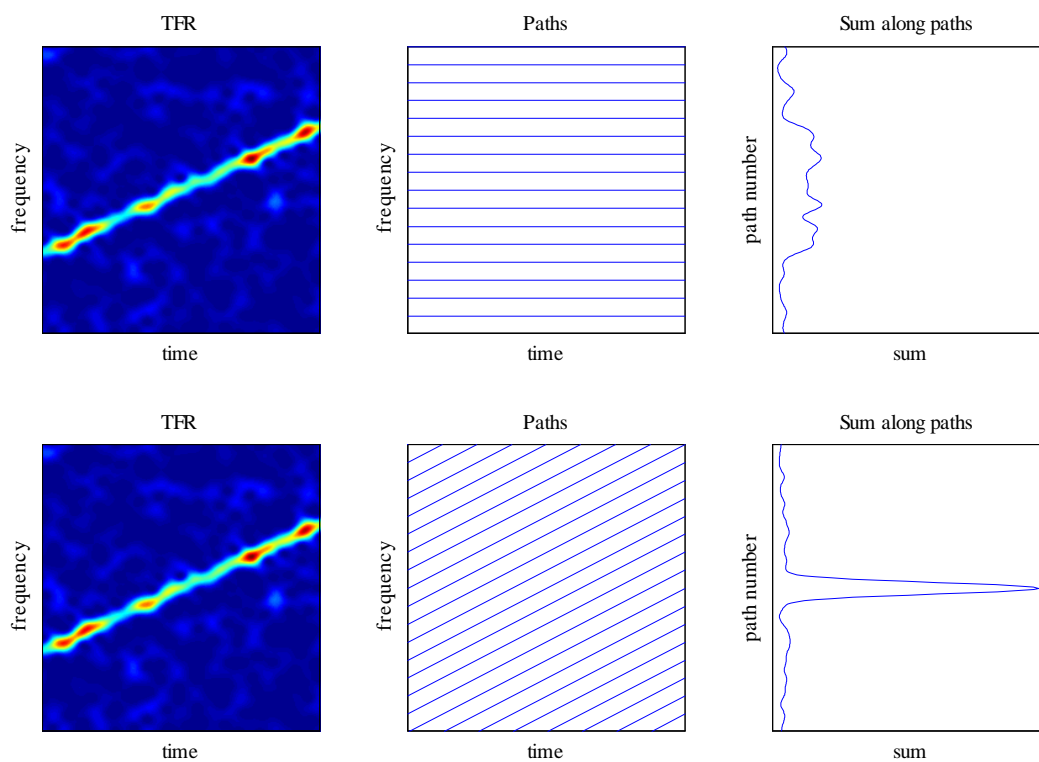
with Hanning window  $w(\tau)$  of discrete length 32 and 256 samples along the frequency axis. The spectrogram, the squared module of the STFT, will be used here as the time-frequency representation. Note that the dimension of the TFR in this case is  $256 - 32 + 1 = 225$  time instants and 256 frequency instants. We will consider

two signals: a signal with linear frequency modulation, and a signal with sinusoidal frequency modulation. The signal-to-noise ratio is 0 dB.

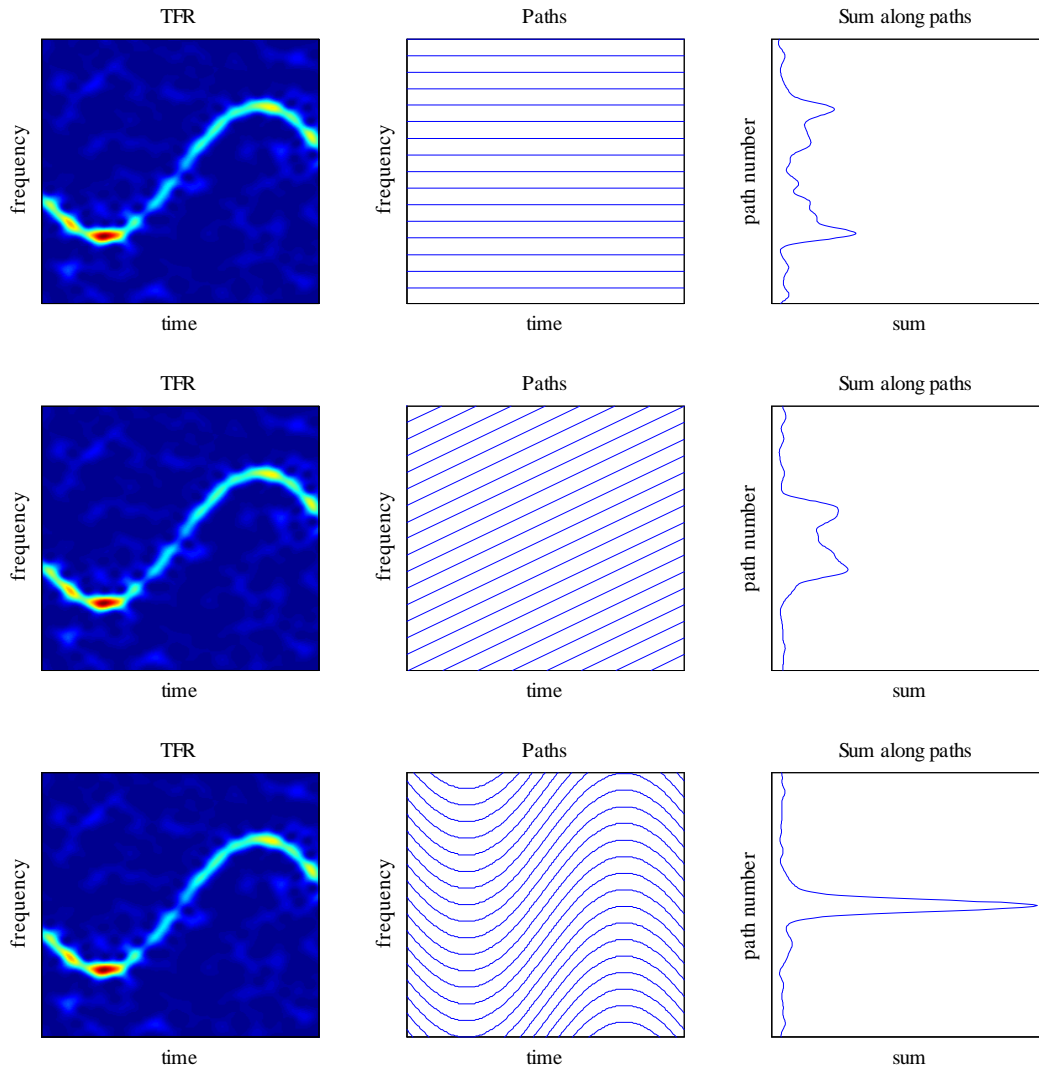
Figures 1 and 2 illustrate the basic principle of the time-frequency-based detection. Subplots in the first row in Figure 1 present the time-frequency representation of the analyzed linearly frequency modulated signal, illustrating the paths in the time-frequency plane and the summation of TFR values along the paths. In this case, the paths are parallel to the time axis and the summation along the paths is proportional to the Fourier transform of the analyzed signal due to the marginal properties of the time-frequency representation. The second row in Figure 1 presents the summation along the paths adjusted to the signal's instantaneous frequency, by determining the parameters of the linear FM (LFM) signal. The maximum value of the summation along the paths is higher in the second case. This distinctly demonstrates that the detection of these types of signals are simple and straight-forward using this approach.

Figure 2 illustrates the time-frequency-based detection of a sinusoidally frequency modulated deterministic signal. The first row presents the summation along the paths equivalent to the Fourier transform, the second row presents the summation along the paths adjusted to the LFM signal from the previous figure, and the third row presents the summation along the paths adjusted to the instantaneous frequency and parameters of the analyzed signal. The maximum value is obtained when the path coincides with signal's instantaneous frequency. The proposed detection method is based on searching the best path (with the maximum summation of the TFR values along the path).

The above example shows that the detection of non-stationary signal can only be considered by Fourier transform tools if we know the signal form and we are able to adjust the signal parameters to the instantaneous frequency. However, in practice the signal form is not known *a priori* and the parametric approach to these types of applications is quite limited. We will next show that similar principles can be used in non-parametric formulation of the detection, without using any *a priori* knowledge about the signal form.



**Figure 1:** Detection example - linear FM signal. First row - paths parallel to the time axis, equivalent to the Fourier-based detector. Second row - paths parallel to the instantaneous frequency, equivalent to the time-frequency detector. First column - time-frequency representation. Second column - paths. Third column - sum of the TFR values along the paths.



**Figure 2:** Detection example - sinusoidal FM signal. First row - paths parallel to the time axis, equivalent to the Fourier-based detector. Second row - paths used in the LFM example. Third row - paths parallel to the instantaneous frequency, equivalent to the time-frequency detector. First column - time-frequency representation. Second column - paths. Third column: sum of the TFR values along paths.

# 3 Signal Detection by using Time-Frequency Analysis

---

## 3.1 Time-frequency Basics

For signals whose spectral content varies over time, time-frequency distributions are introduced in order to improve concentration. The basic quadratic time-frequency distribution is the Wigner distribution (WD):

$$WD(t, \omega) = \int_{-\infty}^{\infty} w\left(\frac{\tau}{2}\right)w\left(-\frac{\tau}{2}\right)x\left(t + \frac{\tau}{2}\right)x^*\left(t - \frac{\tau}{2}\right)e^{-j\omega\tau} d\tau. \quad (18)$$

The Wigner distribution has the best concentration among quadratic distributions [1, 18]. However it can not be used in practice due to very pronounced cross-terms and other interferences.

A method which is based on the idea of preserving auto-terms, as in the Wigner distribution, with elimination or significant reduction of the cross-terms, is introduced as the S-method (SM). It has been derived based on the relationship between the STFT and the pseudo Wigner distribution:

$$WD(t, \omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} STFT(t, \omega + \theta)STFT^*(t, \omega - \theta)d\theta. \quad (19)$$

This relation has led to the definition of a time-frequency distribution [20]:

$$SM(t, \omega) = \frac{1}{\pi} \int_{\theta} P(\theta)STFT(t, \omega + \theta)STFT^*(t, \omega - \theta)d\theta, \quad (20)$$

where  $P(\theta)$  is a finite frequency domain window (we also assume rectangular form),  $P(\theta) = 0$  for  $|\theta| > L_P$ . Two special cases are along the most widely used distributions: the spectrogram  $P(\theta) = \pi\delta(\theta)$  and the pseudo Wigner distribution  $P(\theta) = 1$ . Distribution obtained in this way, referred to as the S-method, belongs to the Cohen class. A kernel function of the S-method is given by  $c(\theta, \tau) = P(\theta/2) *_{\theta} A_{ww}(\theta, \tau)/2\pi$ , where  $A_{ww}(\theta, \tau)$  is the ambiguity function of  $w(\tau)$ . It is generally a function of infinite duration in  $\theta$  and non-separable in  $\theta$  and  $\tau$ .

The S-method can produce the representation of a multicomponent signal such that the distribution of each component is its WD, avoiding cross-terms, if the STFTs of the components do not overlap in the time-frequency plane. For  $x(t) = \sum_{m=1}^M x_m(t)$ ,

the S-method has the form:

$$SM(t, \omega) = WD_{at}(\omega, t) = \sum_{m=1}^M \int_{-\infty}^{\infty} w_e(\tau) x_m(t + \frac{\tau}{2}) x_m^*(t - \frac{\tau}{2}) e^{-j\omega\tau} d\tau, \quad (21)$$

where  $w_e(\tau) = w(\tau/2)w(-\tau/2)$  is an equivalent window and  $WD_{at}(\omega, t)$  denotes the sum of the pseudo Wigner distributions of the individual signal components (without cross-components terms).

The discrete form of the S-method reads:

$$SM(n, k) = \sum_{i=-L_d}^{L_d} P(i) STFT(n, k+i) STFT^*(n, k-i) \quad (22)$$

$$SM(n, k) = |STFT(n, k)|^2 + 2 \operatorname{Re} \left[ \sum_{i=1}^{L_d} P(i) STFT(n, k+i) STFT^*(n, k-i) \right], \quad (23)$$

where the terms in the summation improve the quality of the spectrogram  $|STFT(n, k)|^2$  towards that of the Wigner distribution.

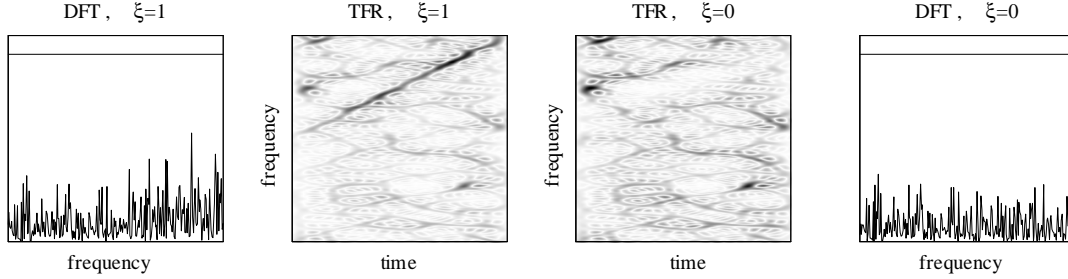
## 3.2 Time-Frequency-based Signal Detection

Let us consider a linearly frequency modulated signal of the form:

$$x(n) = Ae^{jan^2}. \quad (24)$$

Let  $S(k, n)$  denote the time-frequency representation of the signal  $s(n)$ . Figure 3 shows the Fourier transform of the signal  $s(n)$  for the cases  $\xi = 1$  (first plot from the left) and  $\xi = 0$  (last plot). The time-frequency representation of the signal  $s(n)$  for both cases are also shown in Figure 3. The S-method with rectangular window and  $L = 12$  is used as the time-frequency representation of the signal [3]. The number of samples is  $N = 256$ , the noise variance is  $\sigma_\varepsilon^2 = 4$ , and the signal amplitude  $A = 1$ , so that the signal-to-noise ratio is approximately  $-6$  dB. The reference level of the Fourier transform-based detector is calculated with the false alarm probability  $P_{FA} = 0.01$ , and is shown in the first and last plots. The same realization of the noise is used for all four plots. Figure 3 clearly shows the limited ability of the Fourier transform-based detector. On the other hand, it is easy to see if a deterministic component  $x(n)$  exists in the time-frequency representation of the signal  $s(n)$ .

This example shows that the time-frequency analysis can be used for the non-stationary signal detection in the presence of a strong noise. The basic problem at hand is to automate the signal decision procedure if the analyzed TFR signal contains a



**Figure 3:** Fourier transform (with threshold level) and time-frequency representation of the signal  $s(n)$  in the cases of deterministic signal  $x(n)$  existence (left) and non-existence (right).

deterministic component. The algorithm for the signal detection in the arbitrary time-frequency representation is described below.

Let us consider the time-frequency representation,  $S(k, n)$ , of the signal  $s(n)$ , where  $k = 0, 1, \dots, M - 1$  and  $n = 0, 1, \dots, N - 1$ . Assume that the instantaneous frequency of a deterministic signal  $x(n)$  is a continuous function. We define the path in the time-frequency plane as an array of  $n$  frequency indices  $\pi(n)$ , with  $0 \leq \pi(n) < N$  for every  $n$ . We then observe the ensemble of such paths that have the property  $|\pi(n) - \pi(n - 1)| \leq D$  for some specified value  $D$  and for  $n = 1, 2, \dots, N - 1$ . The value of  $D$  is then to be the maximum allowed frequency index change for two consecutive time instants, or frequency step. We then observe one path  $\pi_m(n) \in \Pi_D$  and sum the time-frequency representation values along the observed path. That is:

$$J_m = \sum_{n=0}^{N-1} S(\pi_m(n), n). \quad (25)$$

Denote the maximum of the observed sum over the ensemble  $\Pi_D$  as:

$$J_{\max} = \max_{\pi_m \in \Pi_D} J_m = \sum_{n=0}^{N-1} S(\pi_{\max}(n), n), \quad (26)$$

where  $\pi_{\max}$  is the best path. The quantity defined in this way represents a reliable criteria for determining the deterministic component  $x(n)$  that exists in the time-frequency representation of the signal  $S(k, n)$ . Namely, if  $J_{\max} > R_J$  holds, where  $R_J$  denotes the threshold level, it can be concluded that the deterministic component exists in the signal  $s(n)$ ; in other words,  $\hat{\xi} = 1$ .

The basic problem with this type of detector realization is the threshold level  $R_J$  determination. In the case of the second order time-frequency representation, it

is logical to assume that the level  $R_J$  is proportional to  $\sigma_\varepsilon^2$ . For a specific time-frequency representation and for a chosen probability  $P_{FA}$ , it is sufficient to determine experimentally the threshold level  $R_0$  when the false alarm probability is equal to  $P_{FA}$  for a noise variance  $\sigma_\varepsilon^2 = 1$ . The threshold level for non-unity noise variance can be calculated as  $R_J = R_0\sigma_\varepsilon^2$ . The determination of  $R_0$  in this way, demands the processing of many noise realizations if  $P_{FA}$  is significantly small, but this procedure should be used only once for a given time-frequency representation and for a given  $P_{FA}$ . The algorithm for the threshold level  $R_0$  determination is described below:

1. Choose the time-frequency representation  $TFR[\cdot]$ , probability of false alarm  $P_{FA}$ , and the maximum allowed frequency step  $D$ .
2. For  $i = 1, 2, \dots, M_i$ :
  - (a) Take a realization of noise only signal  $x(n)$  with unity noise variance  $\sigma_\varepsilon^2 = 1$ .
  - (b) Calculate the time-frequency representation  $S(k, n) = TFR[x(n)]$ .
  - (c) Find the best possible path,  $\pi_{\max}(n)$ , and calculate:

$$J_{\max}(i) = \sum_{n=0}^{N-1} S(\pi_{\max}(n), n). \quad (27)$$

3. Calculate the threshold level  $R_0$  such that in  $M_i P_{FA}$  iterations we obtain  $J_{\max}(i) > R_0$  (and  $J_{\max}(i) < R_0$  in the remaining  $M_i(1 - P_{FA})$  iterations).

The second problem is specifying the number of path ensembles  $\Pi_D$  and the determination of the best path. In order to decrease the total number of the paths (of the order  $N^M$ ), we can apply the following approach:

1. For each time index  $n$ , find the maximum  $S_{\max}(n)$  and the position of the maximum  $k_{\max}(n)$

$$S_{\max}(n) = \max_k S(k, n), \quad (28)$$

$$k_{\max}(n) = \arg \max_k S(k, n). \quad (29)$$

Assume that the best path  $\pi_{\max}(n)$  passes through at least one of the selected maximums.

2. For each time index  $t \in \{1, 2, \dots, N\}$ , form the path  $\pi_t(n)$  starting at the point  $(t, k_{\max}(t))$  in the time-frequency plane.
  - (a) Put point  $(t, k_{\max}(t))$  into the path:  $\pi_t(t) = k_{\max}(t)$ .



(b) For  $p = t + 1, t + 2, \dots, N$ , the path point is:

$$\pi_t(p) = \arg \max_{k \in K} S(k, p), \quad (30)$$

where set  $K$  includes frequency points  $K = \{k | \pi_t(p - 1) - D \leq k \leq \pi_t(p - 1) + D\}$ . Note that this procedure limits the frequency step between two consecutive time instants to  $D$ .

(c) For  $p = t - 1, t - 2, \dots, 1$ , the path point is:

$$\pi_t(p) = \arg \max_{k \in K} S(k, p), \quad (31)$$

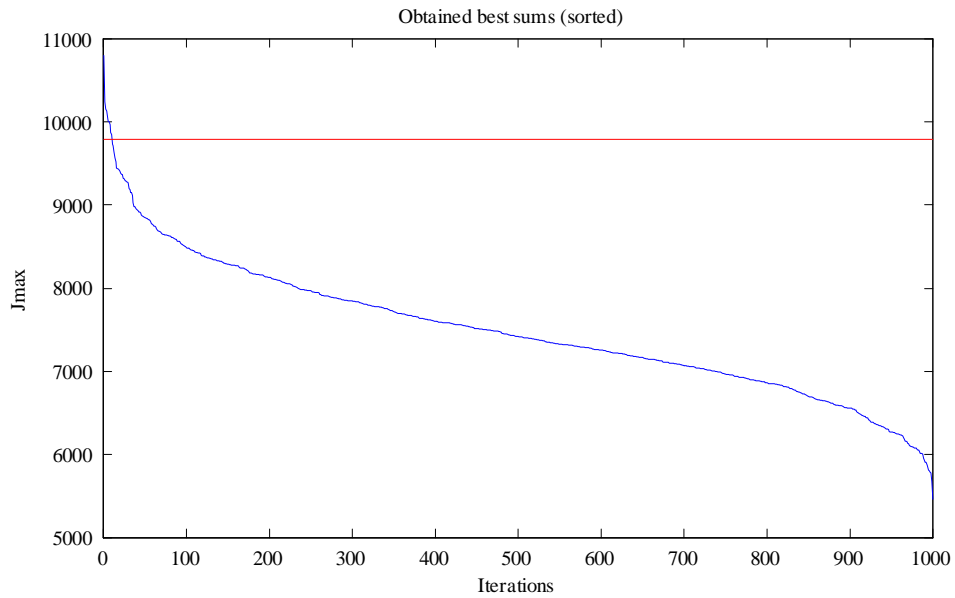
where  $K = \{k | \pi_t(p + 1) - D \leq k \leq \pi_t(p + 1) + D\}$ .

3. Calculate the summation  $J_t(i) = \sum_{n=0}^{N-1} S(\pi_t(n), n)$ .
4. The best path is the path with maximum  $J_t(i)$ .

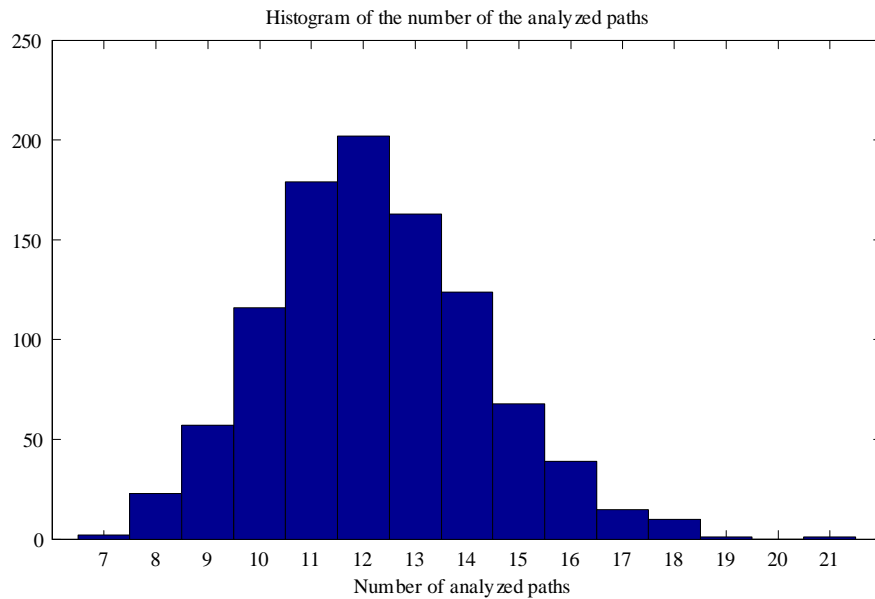
Note that the number of analyzed paths in this procedure is equal to  $N$ . The number of paths can be further decreased if, in step 2 of the previous procedure, we perform the path forming only if the starting point  $(t, k_{\max}(t))$  is not included in any of the previous analyzed paths  $\pi_{t-1}(n), \pi_{t-2}(n), \dots, \pi_1(n)$ . This can be done because if the point  $(t, k_{\max}(t))$  belongs to the path  $\pi_p(n)$  for some  $p$ , then the path  $\pi_t(n)$  coincides with the path  $\pi_p(n)$ . Also note that in step 2 of the previous procedure, we can process the time instants  $t$  in an arbitrary order. We suggest that the value of  $S_{\max}(t)$  determines the order of processing the time instants. That is, we process the time instant with the highest  $S_{\max}(t)$  first and the time instant with the smallest  $S_{\max}(t)$  should be the last one processed. This re-ordering can not change the best path, but can change the total number of analyzed paths.

Figure 4 presents the threshold estimation for the spectrogram with a 32-point Hanning window. It should be noted that  $J_{\max}$  in Figure 4 is plotted in descending order. The signal length is 256 samples and STFT is calculated over 256 frequency bins. The probability of the false alarm is  $P_{FA} = 0.01$ . The threshold is estimated by analyzing the best paths in the noise-only case with  $\sigma_\varepsilon^2 = 1$ . The number of realizations is  $M_i = 1000$ . The threshold is determined according to the number of expected false alarms,  $M_i \cdot P_{FA} = 10$ . In this way, we can implement the desired probability of false alarms.

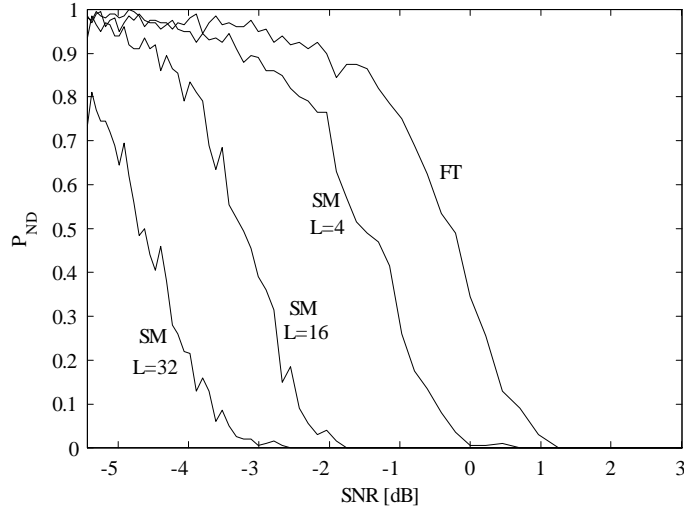
Figure 5 presents the histogram of the number of analyzed paths in the procedure for searching the best path in the noise-only case. The mean number of the analyzed paths per realization is 12. An illustrative example of the path formation is presented in Annex A.



**Figure 4:** Threshold estimation - The best achieved values of the path sum for 1000 realizations of random signal without deterministic component. The horizontal line represents the threshold  $R_0 = 9792$  obtained with  $P_{FA} = 0.01$ .



**Figure 5:** Histogram of the number of analyzed paths for 1000 realizations. The average number of analyzed paths is 12.



**Figure 6:** Probability of the signal non-detection for the case of FT-based detector and the SM-based detector for different values of the parameter  $L$ .

### 3.3 Comparison with Fourier-Based Detector

The performance of time-Frequency-based signal detectors and the comparison with the Fourier-based detector are shown in Figure 6. The case of a linearly frequency modulated signal is considered:

$$x(n) = e^{j\frac{\pi}{128}n^2}, \quad (32)$$

for  $-128 \leq n \leq 127$  in the presence of additive Gaussian white noise. For each signal-to-noise ratio (SNR), 200 realizations are observed and the detection is performed by using the Fourier transform and the S-method with  $L = 4$ ,  $L = 16$  and  $L = 32$ . The dependency of the probability  $P_{ND}$  on signal-to-noise ratio is shown in Figure 6. As we expect, the S-method with large enough  $L$  is a good signal detector, even if the signal-to-noise ratio is small. The false alarm probability for all analyzed detectors is  $P_{FA} = 0.0027$ .

## 4 Application to the Real Radar Data

---

Let us now consider the detection of target signals in experimental high-frequency surface-wave radar systems. Suppose that the target velocity is high enough so that the sea-clutter can be removed by high-pass filtering.

### 4.1 Fourier Transform Case

For stationary targets, the Fourier transform is the optimal detector. Let us now consider a single range cell with the signal representing that range cell denoted by  $x(n)$ . The detection algorithm in the case of a constant false alarm rate is:

1. Choose the probability of false alarms  $P_{FA}$ .
2. Estimate the noise variance for the considered signal  $\sigma_\varepsilon^2$ . A good estimation can be obtained as:

$$\sigma_\varepsilon^2 \approx 1.1 \left( \operatorname{median}_{1 \leq i < N} (|\operatorname{Re}[x(i) - x(i-1)]|)^2 + \operatorname{median}_{1 \leq i < N} (|\operatorname{Im}[x(i) - x(i-1)]|)^2 \right). \quad (33)$$

3. Calculate the reference level  $R_{FT} = -\ln(P_{FA}/N)N\sigma_\varepsilon^2$ , where  $N$  is the signal length.
4. Calculate the discrete Fourier transform of the signal,  $X(k) = \operatorname{DFT}_N[x(n)]$ .
5. If there exists  $k$  such that  $|X(k)|^2 > R_{FT}$ , then we make a decision that a target signal exists.

### 4.2 Time-Frequency Case

The algorithm mentioned above is optimal in the case of stationary signals. When the target velocity changes in the considered time interval, the target's signal becomes non-stationary, and the Fourier transform is no longer an optimal detector. In these cases, we can use the time-frequency-based detector. Now, the detection algorithm is:

1. Choose the probability of the false alarm,  $P_{FA}$ .
2. Choose the time-frequency representation.
3. Consider the noise-only case with unit variance. Estimate the reference level  $R_0$ , so that the criterion  $J_{\max} > R_0$  gives the false alarm rate as chosen in step 1.

4. Estimate the noise variance,  $\sigma_\varepsilon^2$ .
5. Calculate the time frequency representation of the analyzed signal.
6. Find  $J_{\max}$  and compare it with the reference level  $R_J = R_0\sigma_\varepsilon^2$ . If  $J_{\max} > R_J$ , we can make a decision that the target signal exists in the considered range cell.

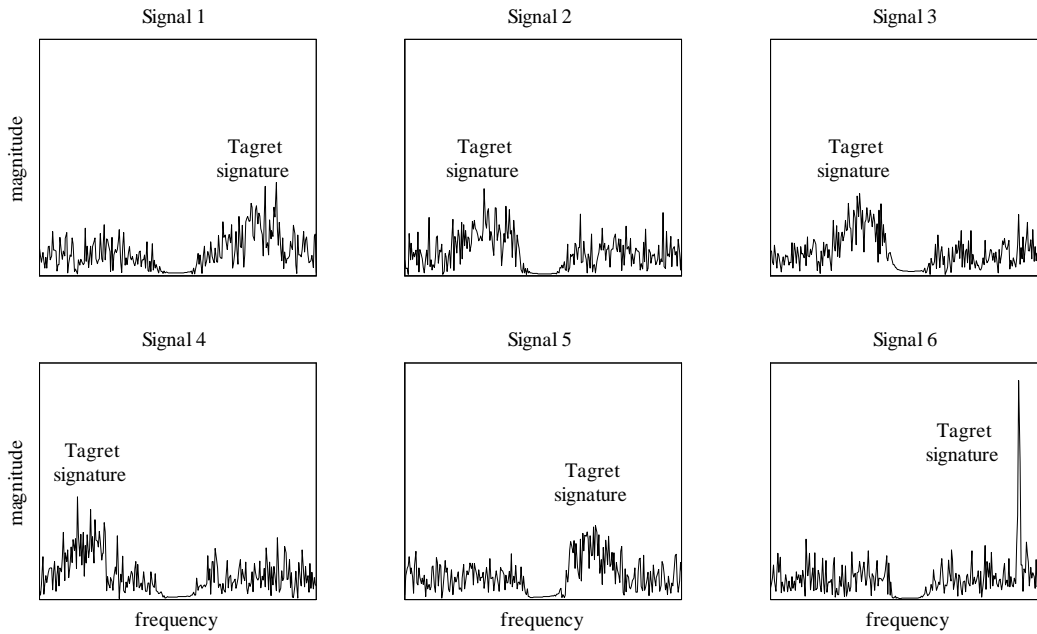
Note that steps 1-3 should be performed only once, so the time-consuming step 3 does not slow down the detection process.

### 4.3 Detection Examples

The signals in the following comparisons are experimental aircraft data collected by a HFSWR, which used a 10-element linear receiving antenna array. The data was collected with a target present. The radar carrier frequency is 5.672 MHz and the pulse repetition frequency is 9.17762 Hz. There are 6 trials and each trial corresponds to a block of 256 pulses. The detailed description of the radar is given by [3]. We consider five non-stationary cases (signals 1-5) and one stationary case (signal 6). The signals are highpass filtered in order to remove strong sea clutter. Figure 7 presents the Fourier transform of the filtered analyzed signals with SNR = -2 dB. The S-method is chosen as the time-frequency representation with large  $L$  ( $L = 64$ ). The reference level  $R_0 = 1320$  is determined according to the previously described procedure. The false alarm probability is  $P_{FA} = 0.0027$ .

In order to estimate the threshold values of the detector, we add noise to the analyzed signal so that a noisy signal can be obtained. Detection algorithms of stationary and non-stationary cases are then applied to the noisy signal. Table 1 shows the number of non-detected target signals for 100 noise realizations with varying SNR. It is obvious that the time-frequency-based approach outperforms the Fourier detection approach when the target signal is non-stationary. In the case of a stationary signal (signal 6), the proposed method is slightly worse than the Fourier-based approach. The Fourier-based detection is optimal in these cases. The estimated probability of target non-detection with varying SNR is given in Figure 8 for non-stationary cases using both the Fourier and time-frequency-based detectors. Figures 9 and 10 represent two typical realizations for non-stationary and stationary signals and its detection procedure, respectively. Both Table 1 and Figure 8 also show that the time-frequency-based detector is able to detect the non-stationary target signals correctly when the SNR is higher than -8 dB.

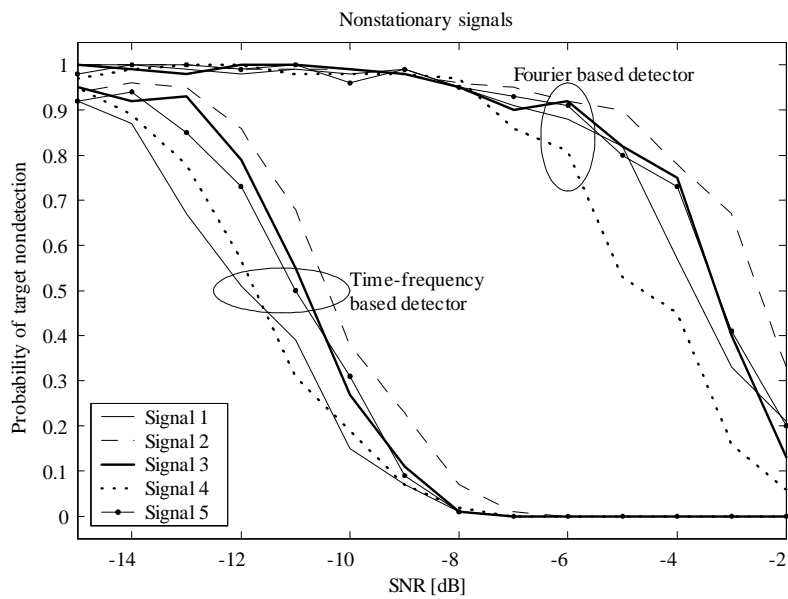
These results specifically suggest that the Fourier detector is optimal when the signals are stationary, whereas the Time-Frequency-based detector is optimal for non-stationary signals.



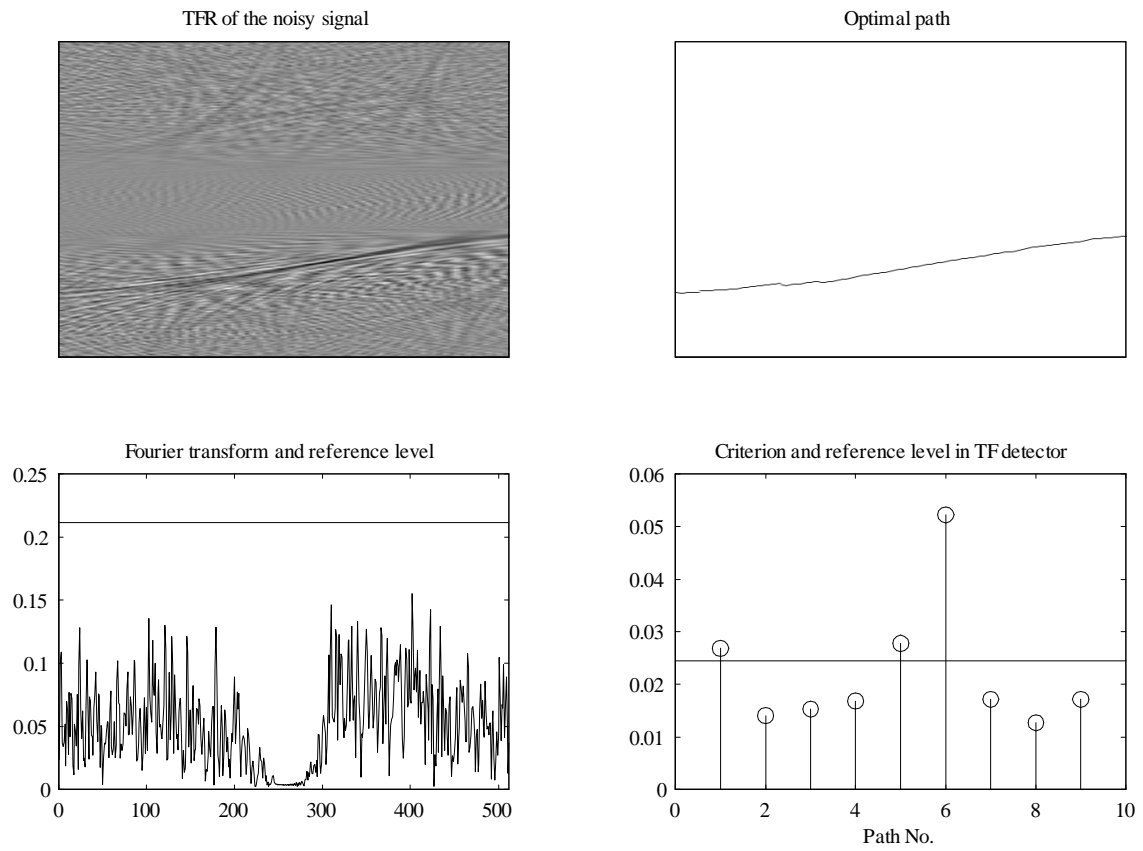
**Figure 7:** Fourier transform of the analyzed signals with  $SNR=-2dB$ .

SNR [dB]	Signal 1		Signal 2		Signal 3		Signal 4		Signal 5		Signal 6	
	FT	TF	FT	TF	FT	TF	FT	TF	FT	TF	FT	TF
-2	21	0	33	0	13	0	6	0	20	0	0	0
-3	33	0	67	0	40	0	16	0	41	0	0	0
-4	57	0	78	0	75	0	45	0	73	0	0	0
-5	82	0	90	0	82	0	53	0	80	0	0	0
-6	88	0	92	0	92	0	81	0	91	0	0	0
-7	91	0	95	1	90	0	86	0	93	0	0	1
-8	95	1	96	7	95	1	97	2	95	1	2	6
-9	99	7	98	23	98	11	98	7	99	9	7	13
-10	98	15	99	38	99	27	98	19	96	31	20	33
-11	99	39	99	68	100	55	98	31	100	50	41	55
-12	98	51	99	86	100	79	100	57	99	73	65	69
-13	99	67	100	95	98	93	100	78	100	85	79	82
-14	100	87	100	96	99	92	99	89	100	94	86	87
-15	100	92	100	94	100	95	97	95	98	92	91	95

**Table 1:** Number of undetected target signals in 100 trials for various SNR. FT - Fourier based detector, TF - time-frequency based detector.

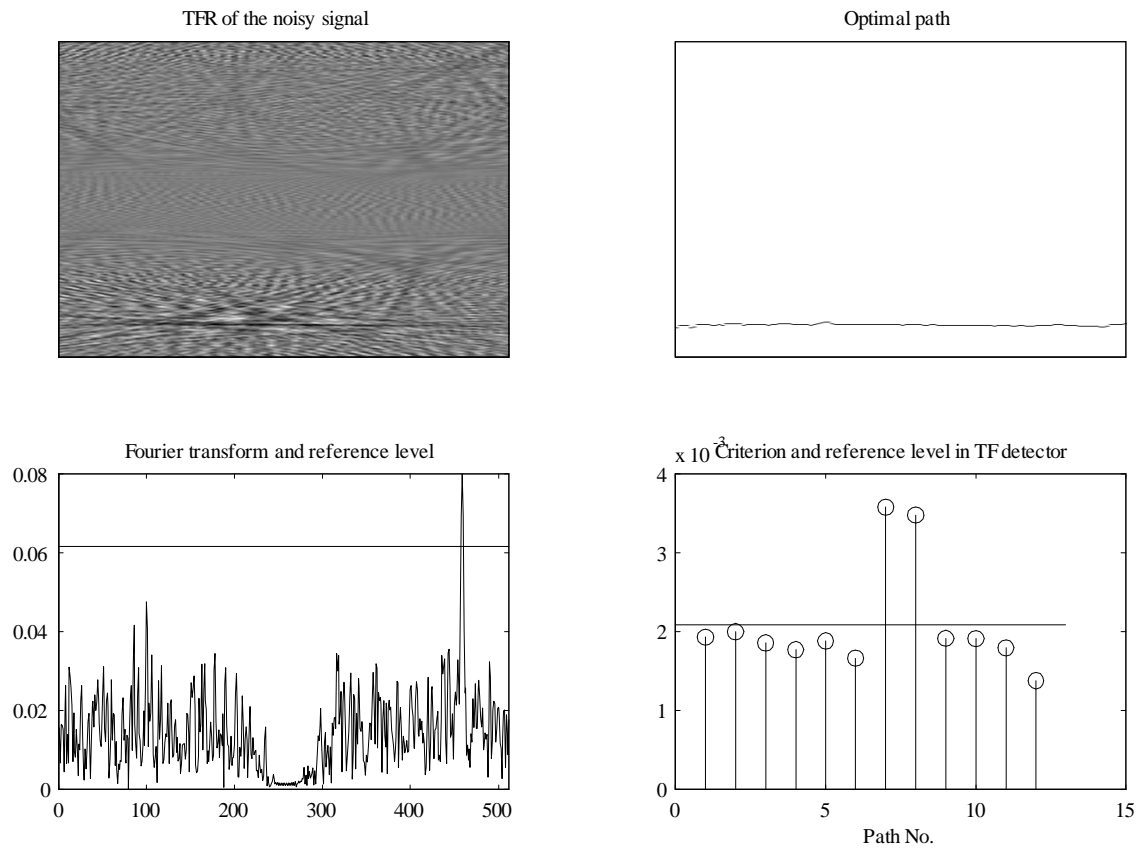


**Figure 8:** Estimated probability of target non-detection versus signal-to-noise ratio for non-stationary cases using both Fourier and time-frequency based detectors.



**Figure 9:** Signal 1 (nonstationary target velocity) with  $SNR=-8$  dB.





**Figure 10:** Signal 6 (stationary target velocity) with SNR=-8 dB.

## 5 Conclusion

---

In this report, we present a time-frequency-based detector, which yields substantial improvement in its performance over the traditional Fourier-based detector. In this method, we choose the s-method as the time-frequency tool due to its desirable properties. The proposed time-frequency based detector requires no prior knowledge of the event to be detected. The performance of the proposed method is evaluated using both simulated and experimental data. The reliability and robustness of the method is studied with respect to signal-to-noise ratio. Results demonstrate that the time-frequency-based approach provides an effective technique for detecting and analyzing maneuvering air targets in heavily cluttered regions. The proposed time-frequency-based detector approach successfully detects the maneuvering target in all cases of the experiment. When the target is stationary, the Fourier-based detector approach is optimal. However, when the target is maneuvering or non-stationary, the time-frequency-based detector approach produces reliable and robust results. Results also show that the proposed time-frequency detector outperforms the Fourier-based detector in terms of good detection and false alarm rates. The method presented here is not restricted to this particular application, but it can also be applied in various other settings of non-stationary signal analysis and filtering. More generally, it is believed that the time-frequency formulation of optimum detection can provide new hints for handling open problems in a comprehensive way. The proposed time-frequency-based detector can be evaluated against the existing real-time CFAR detector for the operational and future HFSWR development.

## References

---

- [1] L. Cohen, *Time-frequency analysis*, Prentice-Hall, 1995.
- [2] V.C. Chen and H. Ling, *Time-Frequency transform for radar imaging and signal analysis*, Artech House, Boston.
- [3] T. Thayaparan and S. Kennedy, "Detection of a manoeuvring air targets in sea-clutter using joint time-frequency analysis techniques," *IEE Proc.-Radar Sonar Navig.*, Vol. 151, No. 1, pp 19-30, 2004.
- [4] A. Yasotharan and T. Thayaparan, "Strengths and limitations of the Fourier method for detecting accelerating targets by pulse Doppler radar," *IEE Proc.-Radar Sonar Navig.*, Vol 149, No. 2, pp 83-88, 2002.
- [5] A. Yasotharan and T. Thayaparan, "A time-frequency method for detecting an accelerating target in sea or land clutter," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 42, Issue 4, pp. 1,289 – 1,310, 2006.
- [6] LJ. Stanković, T. Thayaparan and M. Daković, "Signal Decomposition by Using the S-Method with Application to the Analysis of HF Radar Signals in Sea-Clutter," *IEEE Transactions on Signal Processing*, Vol. 54, Issue 11, pp. 4,332 – 4,342, 2006.
- [7] A.M. Sayeed and D.L. Jones, "Optimal kernels for nonstationary spectral estimation," *IEEE Trans. Signal Processing*, Vol. 43, pp. 478-491, February 1995.
- [8] W. Kozek and K. Riedel, "Quadratic time-varying spectral estimation for underspread processes," in *Proc. IEEE Int. Symp. Time-Frequency Time-Scale Anal.*, pp. 460-463, 1994.
- [9] A. Papandreou, S. M. Kay, and G. F. Boudreaux-Bartels, "The use of hyperbolic time-frequency representations for optimum detection and parameter estimation of hyperbolic chirps," in *Proc. IEEE Int. Symp. Time-Frequency Time-Scale Anal.*, pp. 369-372, 1994.
- [10] P. Flandrin, "A time-frequency formulation of optimum detection," *IEEE Trans. Acoust., Speech, Signal Processing*, Vol. 36, No. 9, pp. 1,377–1,384, September 1988.
- [11] A.M. Sayeed and D.L. Jones, "Optimal detection using bilinear time-frequency and time-scale representations," *IEEE Trans. Signal Processing*, Vol. 43, pp. 2,872-2,883, December 1995.

- [12] A. M. Sayeed and D. L. Jones, "Optimal Quadratic detection and estimation using generalized joint signal representations," *IEEE Trans.on Signal Processing*, Vol. 44, No. 12, pp. 3,031-3,043, December 1996.
- [13] C. Richard, "Time-frequency-based detection using discrete-time discrete-frequency Wigner distributions," *IEEE Trans.on Signal Processing*, Vol. 50, No. 9, pp. 2,170-2,176, September 2002.
- [14] C. Richard and R. Lengellé, "Data-driven design and complexity control of time-frequency detectors," *Signal Process.*, vol. 77, pp. 3748, 1999.
- [15] C. Richard, "Linear redundancy of information carried by the discrete Wigner distribution," *IEEE Trans. Signal Processing*, vol. 49, pp.2536-2544, November 2001.
- [16] B.W. Gillepsie and L.E. Atlas, "Optimizing time-frequency kernels for classification," *IEEE Trans. Signal Processing*, vol. 49, pp. 4854-96, March 2001.
- [17] M. Davy, C. Doncarli, F. G. Boudreaux-Bartels, "Improved optimization of time-frequency-based signal classifiers," *IEEE Signal Processing letters*, Vol. 8, No. 2, pp. 52-56 February 2001.
- [18] LJ. Stanković: "The auto-term representation by the reduced interference distributions; The procedure for a kernel design", *IEEE Trans.on Signal Processing*, vol.44, no.6, June 1996, pp.1557-1564.
- [19] S. Barbarossa: "Analysis of multicomponent LFM signals by a combined Wigner-Hough transform", *IEEE Trans.on Signal Processing*, vol.43, no.6, June 1995, pp.1511-1515.
- [20] LJ. Stanković, "A method for time-frequency signal analysis," *IEEE Trans. Signal Processing*, vol. 42, pp. 225-229, January 1994.
- [21] S. M. Kay, "Fundamentals of statistical signal processing: detection theory," *Prentice Hall Signal Processing Series*, Prentice Hall PTR, New Jersey, USA, 1993.

## Annex A: Illustration of the path formation

We will consider here TFR with 5 time instants and 5 frequency bins. Let the values of the TFR be:

1	0	3	1	2
0	6	2	4	3
5	1	0	1	6
3	2	5	2	2
3	3	1	4	3

Let us first find the maximum value of each row. We will use these maximums as the starting points for the paths:

1	0	3	1	2
0	6	2	4	3
5	1	0	1	6
3	2	5	2	2
3	3	1	4	3

Row	3	2	4	2	3
Column	1	2	3	4	5
Maximum	5	6	5	4	6

Let us sort the maximums into decreasing order:

Row	2	3	3	4	2
Column	2	5	1	3	4
Maximum	6	6	5	5	4

and consider the highest maximum as the path's starting point. We will obtain the path:

1	0	3	1	2
0	6	2	4	3
5	1	0	1	6
3	2	5	2	2
3	3	1	4	3

with:

$$J_1 = 5 + 6 + 3 + 4 + 6 = 24$$

We will now form the binary matrix with same dimensions as the TFR and put '1' at each point along the path:

0	0	1	0	0
0	1	0	1	0
1	0	0	0	1
0	0	0	0	0
0	0	0	0	0

and take the second maximum. That is, the value 6 in the third row and fifth column of the TFR. We check that our binary matrix has 1 in the third row and fifth column. There is no need for the path formation since the path will coincide with previously analyzed path. The third maximum is 5 from the third row and first column of the TFR. The binary matrix has 1 at that place so that there is no need to analyze this path. The next maximum is 5 from the fourth row and third column. Here, the binary matrix has the value of 0, so that we should examine the new path and obtain:

1	0	3	1	2
0	6	2	4	3
5	1	0	1	6
3	2	5	2	2
3	3	1	4	3

yielding the sum

$$J_2 = 3 + 3 + 5 + 4 + 3 = 18.$$

Now we should update the binary matrix with the new path points:

0	0	1	0	0
0	1	0	1	0
1	0	0	0	1
0	0	1	0	0
1	1	0	1	1

The last maximum 4 is in the second row and third column. Again, the binary matrix has the value 1 at that point so we shall avoid this path.

The total number of analyzed paths is 2 and the best path is the first path with  $J_1 = J_{\max} = 24$ .

This page intentionally left blank.



**DOCUMENT CONTROL DATA**

(Security classification of title, body of abstract and indexing annotation must be entered when document is classified)

1. ORIGINATOR (The name and address of the organization preparing the document. Organizations for whom the document was prepared, e.g. Centre sponsoring a contractor's report, or tasking agency, are entered in section 8.)  Defence R&D Canada – Ottawa 3701 Carling Avenue, Ottawa ON K1A 0Z4, Canada		2. SECURITY CLASSIFICATION (Overall security classification of the document including special warning terms if applicable.)  UNCLASSIFIED	
3. TITLE (The complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S, C or U) in parentheses after the title.)  Time-frequency-based detection of fast maneuvering targets using high-frequency surface-wave radar			
4. AUTHORS (Last name, followed by initials – ranks, titles, etc. not to be used.)  Thayaparan, T.; L. Stankovic, M. D.			
5. DATE OF PUBLICATION (Month and year of publication of document.)  February 2010		6a. NO. OF PAGES (Total containing information. Include Annexes, Appendices, etc.)  42	6b. NO. OF REFS (Total cited in document.)  21
7. DESCRIPTIVE NOTES (The category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.)  Technical Memorandum			
8. SPONSORING ACTIVITY (The name of the department project office or laboratory sponsoring the research and development – include address.)  Defence R&D Canada – Ottawa 3701 Carling Avenue, Ottawa ON K1A 0Z4, Canada			
9a. PROJECT OR GRANT NO. (If appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant.)  11ag04		9b. CONTRACT NO. (If appropriate, the applicable number under which the document was written.)	
10a. ORIGINATOR'S DOCUMENT NUMBER (The official document number by which the document is identified by the originating activity. This number must be unique to this document.)  DRDC Ottawa TM 2009-282		10b. OTHER DOCUMENT NO(s). (Any other numbers which may be assigned this document either by the originator or by the sponsor.)	
11. DOCUMENT AVAILABILITY (Any limitations on further dissemination of the document, other than those imposed by security classification.) ( X ) Unlimited distribution ( ) Defence departments and defence contractors; further distribution only as approved ( ) Defence departments and Canadian defence contractors; further distribution only as approved ( ) Government departments and agencies; further distribution only as approved ( ) Defence departments; further distribution only as approved ( ) Other (please specify):			
12. DOCUMENT ANNOUNCEMENT (Any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availability (11). However, where further distribution (beyond the audience specified in (11)) is possible, a wider announcement audience may be selected.)			

13. ABSTRACT (A brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual.)

The time-frequency representation is a powerful tool for the analysis of non-stationary signals. In the past decades, time-frequency representations have been primarily devoted to analysis tasks in the sense that they were introduced so as to depict the time-frequency structure of time-varying signals and non-stationary processes in the time-frequency plane. However, besides this approach, there has been also a permanent interest for tackling decision problems by means of time-frequency representations. In this report, we present a time-frequency-based detection scheme for the high-frequency surface-wave radar (HFSWR) for the detection of maneuvering air targets in the presence of strong sea-clutter. The performance of the proposed method is evaluated using both synthetic and experimental data. In addition, the proposed time-frequency detection scheme is examined in detail with different signal-to-noise ratio and various examples are considered. The time-frequency-based detection method is then compared with the Fourier-based detector. Results clearly demonstrate that the time-frequency-based detector can significantly improve the detection performance of the HFSWR and add considerable physical insight over what can be achieved by conventional Fourier-based detector currently used by HFSWRs. These results distinctly suggest that the Fourier-based detector is optimal for stationary signals, whereas the Time-Frequency-based detector is optimal for non-stationary signals.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (Technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

High-Frequency Surface-Wave Radar  
S-method  
Wigner Distribution  
Short-Time Fourier Transform  
Fourier Transform  
Sea-Clutter  
Accelerating Targets



## **Defence R&D Canada**

Canada's leader in Defence  
and National Security  
Science and Technology

## **R & D pour la défense Canada**

Chef de file au Canada en matière  
de science et de technologie pour  
la défense et la sécurité nationale



[www.drdc-rddc.gc.ca](http://www.drdc-rddc.gc.ca)