On mathematical relationships between lidar integrated backscattered light and integrated depolarization ratios for linear and circular polarization for water droplets

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ABSTRACT

Measurements of the depolarization ratio of water droplets were performed to study the relationship between layer integrated depolarization and layer integrated backscattered light for linear and circular polarization illumination. Since those particles are spherical, the depolarization of the signal is attributed to multiple scattering effects. The experimental data reported in this article support Hu relationship between the single scattering fraction A_s and the linear accumulated depolarization ratio. For circular polarization, a modified Hu relationship is established and it is shown that the use of the accumulated depolarization parameter instead of the accumulated depolarization ratio allows harmonization of the linear and circular polarization measurements into a simple mathematical expression.

Keywords: Lidar, depolarization ratio, depolarization parameter, multiple scattering.

1. INTRODUCTION

Hu et al. [1-2] have developed a relationship between the single scattering fraction and linear accumulated depolarization ratio. This relationship holds for water clouds with a wide range of extinction coefficients, mean droplet sizes, and droplet size distribution widths. The relationship also holds for various instruments fields-of-view (FOV) and for broken cloud fields. The usefulness of this relationship has been demonstrated for the CALIPSO space lidar calibration and data analyses [3]. The relationship between single scattering fraction and linear depolarization ratio is called in this paper the "Hu relationship".
Hu relationship is empirical and based mainly on Monte Carlo simulations with space lidar parameters, where the footprint of the laser is larger than the cloud scattering mean free path. We are interested to see if Hu relationship also holds for ground lidar parameters, where the footprint of the laser is significantly smaller than the cloud scattering mean free path. In addition, the Hu relationship has been established for linear polarization illumination. Over the years, the interest for circular polarization has grown because its use could present some advantages over the use of linear polarization illumination [4-7]. In that context, it is important to determine if the Hu relationship still holds for circular polarization.

Furthermore, Flynn et al [7] and Gimmestad [8] have combined lidar depolarization theory with radiative transfer theory. They suggested the use of the depolarization parameter (d) to measure 'the propensity of the scattering medium to depolarize the incident polarization'. Flynn pointed out that 'for single scattering on particles having a plane of symmetry or random orientation along the line of sight, we benefit from substantial cancellation of matrix elements based on symmetry arguments.' The scattering matrix has only one free parameter, d, and can be used to describe the depolarization. The same d is applied to both linear and circular polarizations. In this paper, we consider that, for a given FOV, what is measured is the single scattering signal added to the mean multiple scattering signal. We assume the same argument based on symmetry can be used to define a new diagonal Mueller matrix where the parameter d_{acc} would be

FOV dependent. Based on this assumption, relationship between single scattering fraction and accumulated depolarization parameter, d_{acc}, for water droplets are presented.

2. THEORY

Backscattering from spherical droplets occurring at exactly 180° does not depolarize the incident light [9]. Neither does scattering in the forward direction significantly depolarize the light [10]. It is the scattering occurring at angles slightly off 180° that presents the highest depolarization ratio. For example, 10 µm water droplets have a linear depolarization ratio of 0.6 at 178°. Lidar measurements of water clouds show strong depolarization signals for FOVs larger than a few mrad simply because multiply scattered light returns into the lidar FOV after suffering a final backscattering near 180°.

Hu et al. have established an empirical relationship between the integrated single scattering lidar signal, the total lidar signal (including multiple scattering contributions) and the accumulated linear depolarization ratio. To do so they have defined two scattering parameters:

- The accumulated single scattering fraction A_s defined as the ratio of the integrated, range-corrected, single scattering signal over the integrated, range-corrected, total (single + multiple) scattering signal.
- The accumulated depolarization ratio $\delta_{acc,lin}$ defined as the ratio of the integrated, range-corrected, perpendicularly polarized signal over the integrated, range-corrected, parallel polarized scattered signal. The subscript *lin* indicates that linear polarization is being considered

We define $P_{//T}(z',\theta)$ and $P_{//s}(z')$ as the lidar returns parallel to the transmitted laser polarization while $P_{\perp T}(z',\theta)$ and $P_{\perp s}(z')$ are the lidar returns perpendicular to the transmitted laser polarization. The subscript "*T*", "*s*" and "*m*" refer respectively to total, single and multiple scattering signals, z' is the range and θ the field-of-view (FOV). For spherical particles and under ideal experimental conditions, $P_{\perp s}(z')$ are equal to zero and the single scattering signals show no FOV, θ, dependence.

The accumulated single scattering fraction, As, is defined as

$$A_s \equiv \frac{I_s(z)}{I_T(z,\theta)}. \quad (1)$$

Where

$$I_s(z) = \int_{z_a}^{z} (P_{//s}(z') + P_{\perp s}(z'))z'^2\, dz', \quad (2)$$

and

$$I_T(z,\theta) = \int_{z_a}^{z} (P_{//T}(z',\theta) + P_{\perp T}(z',\theta))z'^2\, dz'. \quad (3)$$

The accumulated depolarization ratio $\delta_{acc,lin}(z,\theta)$ is defined as:

$$\delta_{acc,lin}(z,\theta) = \frac{\int_{z_a}^{z} P_{\perp T}(z',\theta)z'^2 dz'}{\int_{z_a}^{z} P_{//T}(z',\theta)z'^2 dz'}, \quad (4)$$

with $P_{//T}(z',\theta) = P_{//s}(z') + P_{//m}(z,\theta')$ and $P_{\perp T}(z',\theta) = P_{\perp s}(z') + P_{\perp m}(z',\theta)$.

Using the definition above, Hu et al. have established an empirical relationship between the layer accumulated depolarization ratio and layer integrated single scattering fraction. The relation is FOV dependent and can be written as

$$A_s(\theta) \equiv \frac{I_s(z)}{I_T(z,\theta)} = \left(\frac{1-\delta_{acc,lin}(z,\theta)}{1+\delta_{acc,lin}(z,\theta)}\right)^2. \tag{5}$$

In this way, the measurement of the accumulated depolarization ratio allows correction of the lidar signal for the multiple scattering effects that distort it via: $I_s(z) = A_s(\theta)I_T(z,\theta)$.

3. MODIFIED HU RELATIONSHIP FOR CIRCULAR POLARIZATION AND FOR DEPOLARIZATION PARAMETER, *d*

Roy and Roy [10] have established a relationship between the linear and circular depolarization ratios under multiple-scattering conditions for spherical particles. The relationship is FOV dependent:

$$\delta_{acc,cir}(z,\theta) = \frac{2\delta_{acc,lin}(z,\theta)}{1-\delta_{acc,lin}(z,\theta)}. \tag{6}$$

By substituting Eq. 6 into Eq. 5, it is easy to get the relationship for water droplets between single scattering fraction and accumulated circular depolarization ratio:

$$A_s(z,\theta) = \left(\frac{1}{1+\delta_{acc,cir}(z,\theta)}\right)^2. \tag{7}$$

Also, Flynn [7] pointed out that for single scattering on particles having a plane of symmetry or random orientation along the line of sight, we benefit from substantial cancellation of matrix elements based on symmetry arguments. The scattering matrix has only one free parameter, *d*, the depolarization parameter. The general form of the Mueller matrix that describes a partially depolarizing backward scattering process is:

$$M_{atm} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-d & 0 & 0 \\ 0 & 0 & d-1 & 0 \\ 0 & 0 & 0 & 2d-1 \end{pmatrix}.$$

The depolarization parameter, *d*, is defined as[10]:

$$d = \frac{2\delta_{lin}}{1+\delta_{lin}} = \frac{\delta_{cir}}{1+\delta_{cir}}. \tag{8}$$

Eq. (8) considers only backscattering occurring at exactly 180°.

We consider that, for a given FOV, what is measured is the mean multiple scattering signal plus the single scattering signal. We assume that the same argument based on symmetry can be used to define a new diagonal Mueller matrix wherein the parameter d_{acc} replacing d would be FOV dependent. If this hypothesis is right, transformation of FOV dependent linear and circular depolarization ratios into a generalized FOV dependent depolarization parameter will make those data superpose perfectly. The generalization of Eq. 8 by considering multiple scattering contained in a given FOV, θ, is

$$d_{acc}(z,\theta) = \frac{2\delta_{acc,lin}(z,\theta)}{1+\delta_{acc,lin}(z,\theta)} = \frac{\delta_{acc,cir}(z,\theta)}{1+\delta_{acc,cir}(z,\theta)} \tag{9}$$

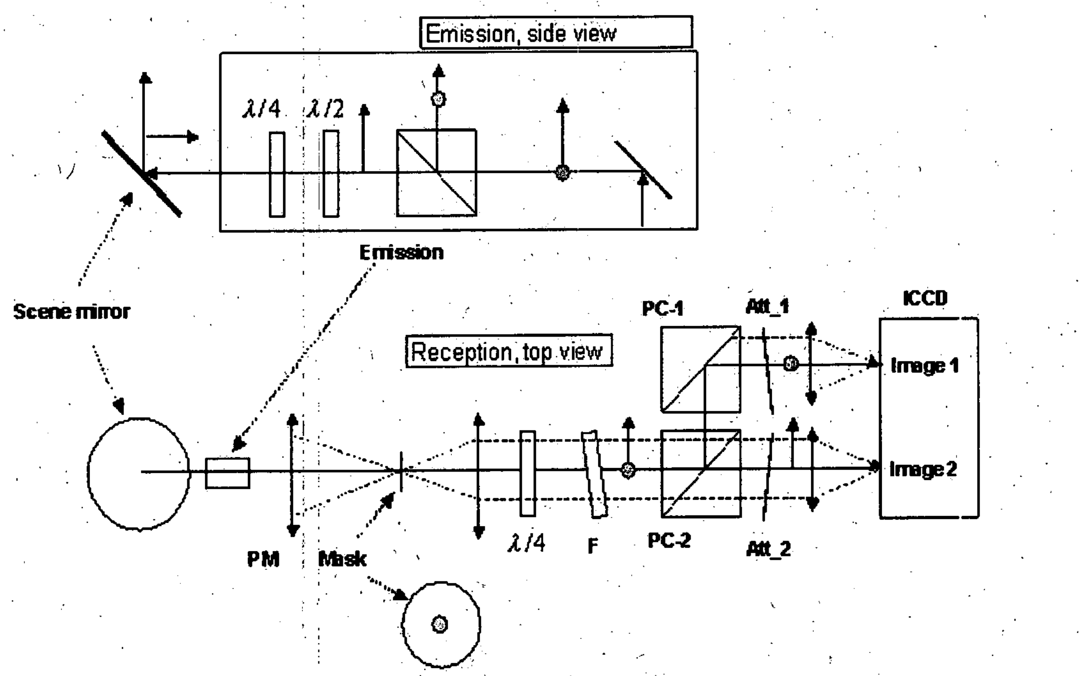
where d_{acc} is called layer accumulated depolarization parameter and is FOV dependent.

With equations 5, 7 and 9, we easily find as suggested by Gimmestad [8]:

$$A_S(\theta) = (1-d_{acc}(z,\theta))^2. \tag{10}$$

4. EXPERIMENTAL VALIDATION OF THE CONCEPT

Hu and modified Hu relationships (Equations 5 and 7) were verified with ground lidar measurements on water droplets with size ranging from 10 to 20 μm. The experimental conditions are described in [10]. In short, the measurements have been performed using a dual polarization multiple-field-of-view (MFOV) lidar. The MFOV lidar measurements were made in a 22-m long aerosol chamber located 105 m from the lidar. A transmissometer was used to measure the clouds optical depth. The MFOV lidar operated at 532 nm. It consisted of a telescope coupled to a gated intensified CCD camera (G-ICCD) synchronized with a doubled Nd-YAG laser polarized linearly or circularly. Figure 1 illustrates the dual polarization imaging lidar (see [10] for details). In short, the emission module uses a polarized cube beam splitter to obtain a linear polarization purity of 1/500. It is followed by $\lambda/2$ and $\lambda/4$ waveplates with their optical axis parallel to the incident polarization beam. The laser beam is pointed at the desired location using a scene mirror and a scanner (not shown on the figure). The camera recorded two images simultaneously, one for each polarization, and for each preset distances. The calculated encircled energy for a given FOV provides the lidar signal for a given distance. The delays applied to the G-ICCD camera were set to study the evolution of the lidar return as a function of the penetration depth into the aerosol chamber for five different ranges. To allow the collection of scattered light from a volume located at a specific range from the lidar and extending over a specific penetration depth, two acquisition parameters of the ranged-gated camera had to be adequately chosen: the camera delay and the camera gate width.



PM: 20 cm parabolic mirror, F: 532 nm inteference filter, PC_1 and PC_2 polarization cube beam splitter, Att_1 and Att_2: attenuators

Fig. 1. Linear and circular polarization lidar set-up.

Figure 2 shows a scattering volume which has been separated into 5 sections. The separation is not a physical one. It illustrates the fact that, by an appropriate selection of the camera gating parameters (delay and width), light returning from the scatters included between the ranges $ct_1/2$ and $c(t_1+\Delta T)/2$ (section 1) or $c(t_1+2\Delta T)/2$ and $c(t_1+3\Delta T)/2$ (section 3) can be collected once at a time. One example of polarization images recorded with the ICCD camera is shown at the bottom of the Fig. 2. The measurements have been performed on water droplets. Each of the images covered a 10 mrad full angle FOV and the first backscattering order corresponding to the center part of the images has been attenuated by a factor 100 with a mask covering 1.3 mrad in order to enhance the multiple scattering effects.

The data using fixed-gate method is reported here, i.e., the camera gate width was fixed and the window was slid in order to sequentially record lidar signals from each cell of the scattering volume. The camera dynamic range was optimized by adjusting the number of laser shots accumulated on the chip before its reading. Accumulating more laser pulses on the CCD chip before its reading allowed to better observe the multiple scattering effects.

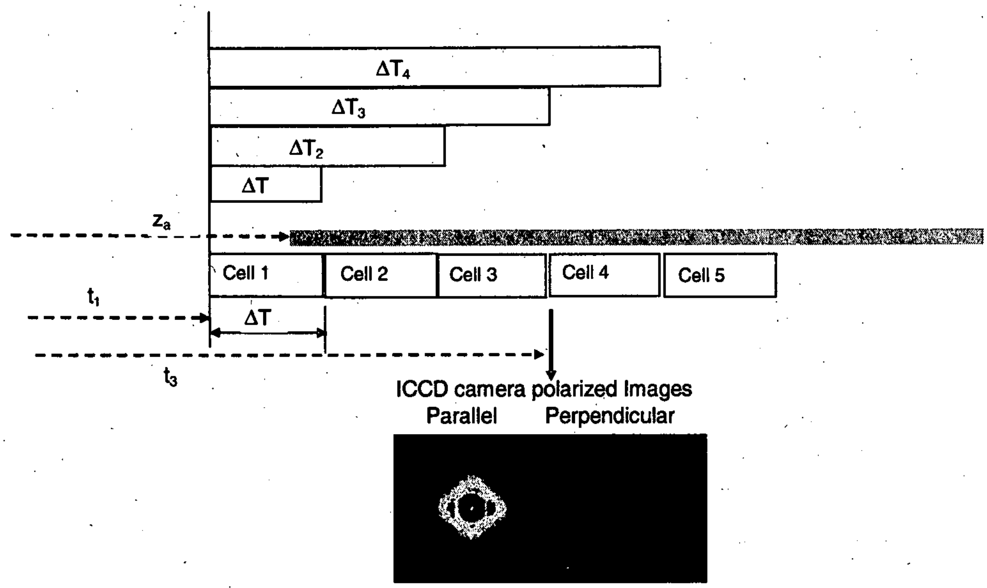


Fig. 2 : MFOV lidar measurements with an ICCD camera with variable (top) and fixed (bottom) gate method. The parameter t_i corresponds to the camera delay before acquisition. The parameter ΔT corresponds to the camera gate width or depth of acquisition. The images correspond to a measurement obtained on a water droplet cloud at the beginning of the cloud (Cell 2) with the dual channel polarization ICCD lidar for a linear polarization illumination.

Ideally, all the laser beam energy should be contained within a very small FOV. Under this condition, a lidar with a FOV around 1mrad, will show a depolarization ratio close to 0.01. However, reflection of the laser beam on imperfect optics is the cause of the spreading of the laser beam. This is the principal cause of an increase of the secondary polarization signal. 90% of the laser beam energy is within 1.3 mrad and around 10% of the beam energy is spread over 10 mrad. This off-axis energy caused an increase of the secondary polarization signal. To correct the laser beam spreading in the larger FOVs and the impacts of imperfect optics, the signal from the perpendicular polarization at FOV of 1.3 mrad, $P_{T\perp}(z',\theta=1.3mrad)$ is subtracted from the total perpendicular polarization signal $P_{T\perp}(z',\theta)$ and the signal from the parallel polarization is corrected with the laser beam profile $F(\theta)$ as in equations 11 and 12.

$$I_{T\perp}(z)=\int_{z_a}^{z}(P_{T\perp}(z',\theta)-P_{T\perp}(z',\theta=1.3mrad))z'^2dz'. \qquad (11)$$

$$I_{T//}(z)=\int_{z_a}^{z}\frac{P_{T//}(z',\theta)}{F(\theta)}z'^2dz'. \qquad (12)$$

The laser beam profile was obtained empirically. Its values for FOVs of 1.3 and 9.8 mrad were set respectively to 0.93 and 1.

The measurements were performed with linear and circular polarization alternatively. Figure 3 shows the layer integrated energy as a function of the accumulated depolarization ratio for a water trial (for circular polarization illumination). Experimental data for different optical depths and penetration depths are plotted on Figure 3. The curves show that the layer integrated signals increase practically linearly as a function of the accumulated depolarization ratio. The single scattering integrated backscatterings $I_s(z)$ used in Eq. 1 correspond to the values of the layer integrated energy for depolarization ratios of zero. They are derived by linear regressions. The different data points correspond to different FOVs ranging from 2 mrad to 10 mrad. Similar curves are obtained for the linear polarization measurements, except that the accumulated depolarization ratio values are lower.

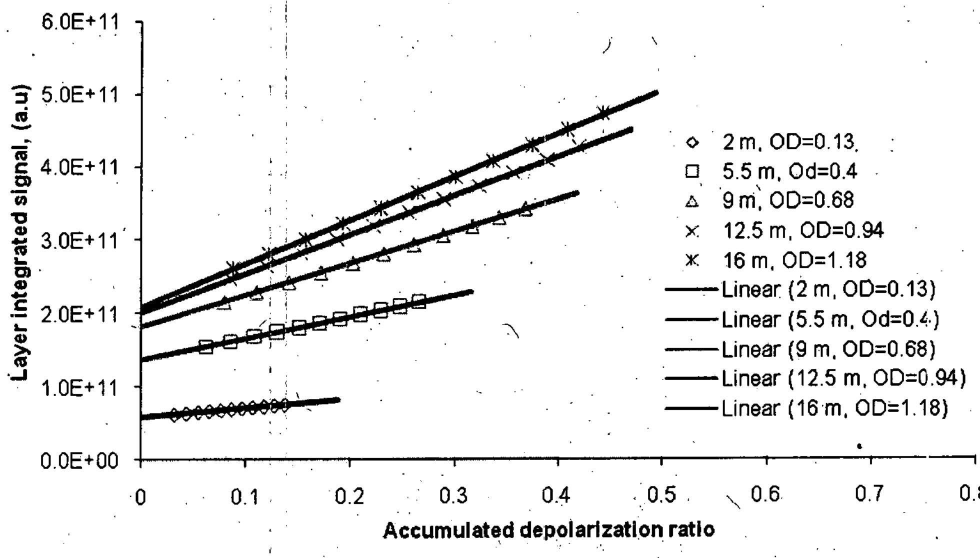


Fig. 3. Derivation of the single-scattering component ($I_s(z)$) from MFOV lidar observations through extrapolation. The measured layer integrated energy is plotted for five penetrations depth as a function of the accumulated circular depolarization ratios for a water droplets experiment. O.D means optical depth.

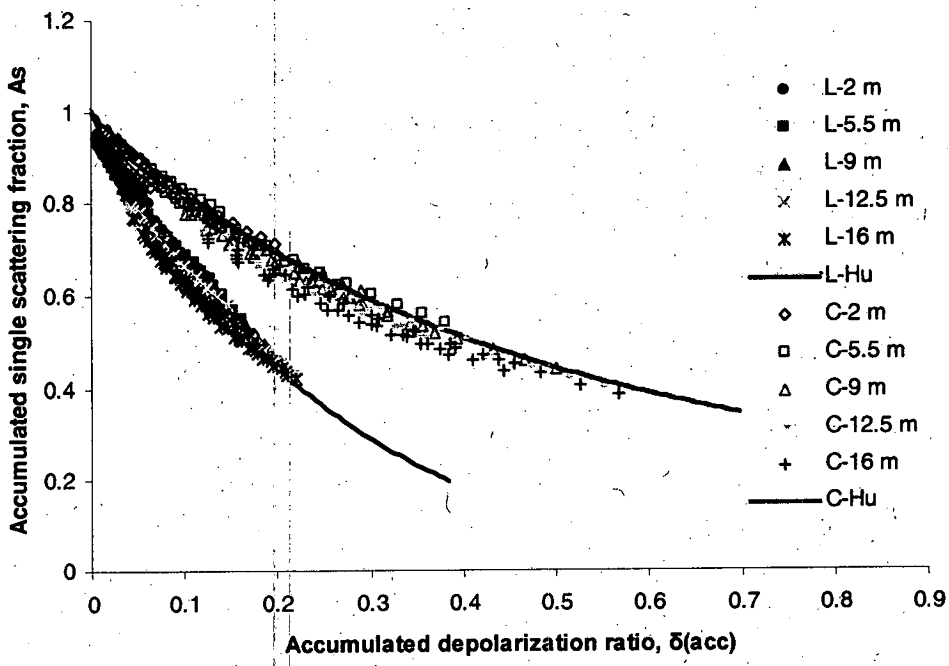


Fig.4. Measured accumulated single scattering factor, A_s as a function of the accumulated linear and circular depolarization ratios ($\delta_{acc,lin}(z,\theta)$ and $\delta_{acc,cir}(z,\theta)$) for water droplets clouds. The different penetration depths are represented with different symbols and the letters L and C represent the linear and circular polarization measurements

Figure 4 shows the accumulated single scattering factor A_s, as function of the accumulated linear and circular depolarization ratios. The experimental results agree equally well with Hu equation (Eq. 5) and Hu modified equation for circular polarization (Eq. 7). We found that the best fit relations on our experimental data between the accumulated single scattering factor, A_s, and the accumulated linear and circular depolarization ratios are respectively: $\left(\frac{1-1.061\delta_{acc,lin}(z,\theta)}{1+1.061\delta_{acc,lin}(z,\theta)}\right)^2$ and $\left(\frac{1}{1+1.137\delta_{acc,cir}(z,\theta)}\right)^2$. These correspond respectively to -3% and -5% departures on average from the Hu equation for linear depolarization ratio $\delta_{acc,lin}$ ranging from 0 to 0.25 and modified Hu equation for circular depolarization ratio $\delta_{acc,cir}$ ranging from 0 to 0.5. Taking into consideration the empirical nature of the functions and experimental errors, the agreements are considered very good.

On Figure 5, the experimental relationship between the accumulated single scattering fraction and the accumulated depolarization parameter is compared to the theoretical relation suggested by Gimmestad (Eq. 10). The transformation of linear and circular depolarization ratio into depolarization parameter, d_{acc} is obtained using Eq. 9. Considering the possible experimental errors, the data obtained from linear and circular polarization measurements superpose quasi perfectly and follow Eq. 10.

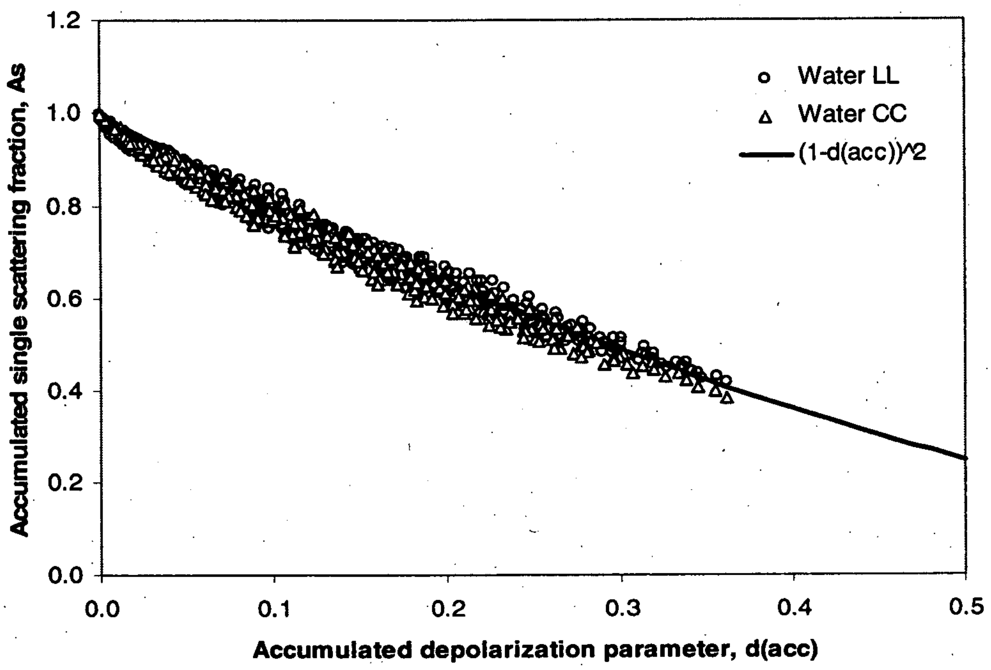


Figure 5 Measured accumulated single scattering factor, A_s, as a function of the accumulated polarization parameter (d_{acc}) for water droplet clouds. LL and CC mean linear and circular polarization measurements.

5. CONCLUSION

Measurements on water droplets under controlled environment have been performed to validate empirical relationships established between the layer integrated backscatter light and layer accumulated linear or circular depolarization ratio. The relationship between accumulated single scattering fraction and accumulated depolarization parameter was also examined based on the assumption that off-axis matrix elements are negligible and that a simple diagonal Mueller matrix can be used to describe spherical particle depolarization caused by multiple scattering. The main conclusions that can be drawn are:

- The experimental data on water droplets (significantly larger than the probing wavelength) strongly support Hu relationship between the single scattering fraction A_s and the linear accumulated depolarization ratio.
- Modified Hu relationships exist for circular polarization illumination. The mathematical relations for linear and circular depolarization light are different but are easily transformed to each other
- Mueller matrix and the accumulated depolarization parameter $d_{acc}(\theta, z)$ can be used to quantify depolarization caused by multiple scattering. Using $d_{acc}(\theta, z)$, the relationship between the single scattering fraction A_s and the $d_{acc}(\theta, z)$ is independent of whether the measurement is done with linear or circular polarization. A diagonal Mueller matrix provides good results. It seems that the averaged multiple scattering signal over the azimuth is sufficient to make off-axis matrix element negligible.

Hu type relationships appear to be robust and have a very simple mathematical form. In addition, the relation between the accumulated single scattering factor (A_s) and the layer accumulated depolarization ratio appears to be independent of the geometry of the measurements and contains information on the optical depth and thus on the extinction coefficient. This suggests that a lidar inversion technique based on Hu relationship and generalized relationship could be derived to evaluate the extinction coefficient. We are currently investigating in this new area and preliminary results are encouraging.

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