An operational support aircraft routing model

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Abstract

The Canadian Forces is seeking to establish permanent and temporary operational support hubs at strategic locations around the globe to improve its logistics support effectiveness and responsiveness for deployed operations. These hubs will be used for cross-loading between modes of transportation as well as for the pre-positioning of non-perishable supplies (i.e., various non-perishable supplies could be procured at local area and positioned at hubs for future delivery) to reduce transportation costs and to improve the speed of delivery. This paper presents a mathematical model developed to address aircraft routing problems associated with the Canadian Operational Support Command hub-based support concept. The model could be used for planning sustainment flights to determine cost (or time) effective aircraft routes for the movement of cargo and supplies from various support hubs to a theatre of operation. The model is formulated as a vehicle routing problem and is implemented using mixed integer nonlinear programming. An example using historical deployment and sustainment data is presented to illustrate the methodology.

Résumé

Les Forces canadiennes (FC) ont l’intention d’établir des centres de soutien opérationnel permanents et temporaires à des endroits stratégiques à travers le monde, pour améliorer l’efficacité et la réactivité du soutien logistique des missions à l’étranger. Ces centres de soutien seront utilisés pour le transbordement entre divers moyens de transport et le prépositionnement de fournitures non périssables (diverses fournitures non périssables seront achetées localement et conservées dans les centres de soutien en vue d’une utilisation future), afin de réduire les frais de transport et d’améliorer la vitesse de livraison. Ce document présente un modèle mathématique qui a été développé pour faire face aux problèmes de routage des aéronefs liés au concept de « centre de soutien opérationnel » du Commandement du soutien opérationnel du Canada (COMSOCAN). Ce modèle pourra être utilisé pour planifier les vols de soutien, et pour déterminer les itinéraires optimaux (en termes de coût ou de durée) pour l’acheminement de matériel et de fournitures depuis divers centres de soutien jusqu’au théâtre d’opérations. Ce modèle mathématique est formulé comme un problème de routage de véhicules, et il est mis en œuvre au moyen de la programmation non linéaire en numération mixte. Un exemple faisant appel à des données historiques sur le déploiement et le soutien des troupes est présenté pour illustrer cette méthodologie.

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Executive summary

Background

The Canadian Forces (CF) is seeking to establish permanent and temporary operational support hubs at strategic locations around the globe to improve its logistics support effectiveness and responsiveness for deployed operations. These hubs would be used for cross-loading between modes of transportation, for the pre-positioning of non-perishable supplies, as refuelling stops, as staging bases during troop rotations, as well as for other associated purposes. Pre-positioned supplies can either be shipped in advance from Canada or purchased locally at hub areas to reduce transportation costs. In addition to material pre-positioning, contracts can be established within the local area for troop accommodation, equipment repair, and aircraft maintenance services.

The potential for improving the responsiveness and effectiveness of the logistics distribution can be illustrated through a few practical cases. Local procurement of materiel to avoid air shipment from Canada and savings from backhaul costs to Canada for materiel that can be repaired at hubs are clearly cost avoiding examples that improve efficiency. Likewise, additional support benefits may be incurred using sealift in cases where materiel must be shipped in advance from Canada because it cannot be locally purchased or repaired at a hub. Such materiel can be received and temporarily stored at the hub awaiting the availability of lift for onward movement into theatre. In addition, surplus or damaged materiel could be stored at hubs based on the availability of space until lift opportunities become available.

Operational research request

To implement the hub-based support concept, the Canadian Operational Support Command (CANOSCOM) has identified a set of potential locations around the globe and has requested operational research studies to facilitate better decisions concerning the selection of appropriate hubs and to optimize the use of the support hubs during sustainment operations. A first study was conducted to examine the effectiveness of the hub-based support approach and to determine optimal hub locations. Following the study, CANOSCOM has submitted a report to the Chief of Defence Staff (CDS) concerning future recommendations for establishment of a global network of such hubs. Recently, the first hub at Spangdahlem, Germany was formally opened by Commander CANOSCOM on a three year proof-of-concept basis.

This study is a further examination of the hub-based support concept to address the aircraft routing problem during sustainment operations. The aircraft routing problem consists of finding optimal aircraft routes in a network of operational support hubs for the movement of cargo and supplies to a theatre of operation in order to minimize the lift cost (or time).

Methodology

In this paper, an operational support aircraft routing model was developed to determine optimal aircraft routes for the movement of cargo and supplies from various support hubs to a given theatre of operation. The model was formulated as a vehicle routing problem and was implemented as a mixed integer nonlinear program. A variable aircraft speed variant of the model was also formulated and examined. The model considers only the delivery of supplies from the support hubs to a theatre of operation and does not include backhauls to Canada or the support hubs.

Conclusions

This paper addressed a complex military aircraft routing problem related to the CANOSCOM hub-based operational support concept. The model developed in this study could be used by CANOSCOM to plan sustainment flights for deployed operations, to determine optimal airlift routes for supporting domestic operations or exercises, to conduct movement options and 'What-If" scenario (e.g., closing a hub) analysis, and to further study the impact of various logistics parameters and constraints (e.g., hub capacity, hub location, operational demand, etc.) on the operational support performance. The study provides a sound basis to inform decision-making about the effective use of a network of support hubs for sustaining deployed operations.

Recommendations

As a result of this study, a number of recommendations for the decision makers appear to logically follow:

1. To effectively use the hub-based support concept, CANOSCOM should consider the operational support aircraft routing model for planning future sustainment flights. Movement planners and supply Subject Matter Experts (SMEs) should provide data required to implement the methodology.

2. When using the aircraft routing model, the stock levels at hubs should be evaluated accurately during each sustainment period, as they could affect the optimal aircraft routing solution.

3. While the model is formulated as a pickup problem, it could easily be adapted to address delivery problems such as the movement of materiel during force repatriation. It is recommended that CANOSCOM consider the model to plan movement options for the drawdown of Joint Task Force Afghanistan. As such, SMEs should identify the materiel that should be shipped back to Canada and the materiel that should be post-positioned at the support hubs (e.g., Camp Mirage, Spangdahlem) in order to facilitate the route planning and optimization.

4. Finally, further analysis should be conducted to address other aspects of the operational support aircraft routing problem, including movement of personnel; priority of the items; aircraft maintenance; crew change requirements; stochastic demand of the items; stock availability at the hubs, and backhaul problems.

Sommaire

Contexte

Les Forces canadiennes (FC) ont l'intention d'établir des centres de soutien opérationnel permanents et temporaires à des endroits stratégiques à travers le monde, pour améliorer l'efficacité et la réactivité du soutien logistique des missions à l'étranger. Ces centres de soutien seront utilisés pour le transbordement entre divers moyens de transport, pour le prépositionnement de fournitures non périssables, pour le ravitaillement en carburant des aéronefs, pour le rassemblement des troupes lors des rotations, et pour d'autres activités connexes. Les articles prépositionnés pourront être expédiés à l'avance depuis le Canada, ou achetés sur place pour réduire les frais de transport. Outre le prépositionnement du matériel, des contrats pourront être signés avec des entreprises locales pour l'hébergement des troupes, la réparation des équipements et la maintenance des aéronefs.

Les possibilités d'amélioration de l'efficacité et de la réactivité du soutien logistique peuvent être illustrées par quelques exemples pratiques. L'achat sur place de certains articles, pour éviter le coût du transport aérien à partir du Canada, et la réparation du matériel dans les centres de soutien, pour éviter d'avoir à les ramener au Canada, sont des exemples évidents d'économies et de gains d'efficacité. Par ailleurs, d'autres économies pourront être réalisées grâce au transport par mer, dans le cas du matériel qui doit être expédié à l'avance à partir du Canada parce qu'il ne peut pas être acheté sur place ou réparé dans un centre de soutien. Ce matériel pourra être entreposé temporairement dans un centre de soutien en attendant qu'un avion devienne disponible pour l'expédier sur le théâtre d'opérations. Enfin, les équipements excédentaires ou endommagés pourront être retirés du théâtre d'opérations et entreposés dans les centres de soutien, selon l'espace disponible, jusqu'à ce qu'il soit possible de les ramener au Canada.

Demande d'études de recherche opérationnelle

Pour mettre en œuvre le concept de « centre de soutien opérationnel », le Commandement du soutien opérationnel du Canada (COMSOCAN) a identifié une série d'emplacements potentiels pour les centres de soutien à travers le monde, et il a demandé que l'on procède à des études de recherche opérationnelle pour faciliter la prise de décision lorsque les emplacements seront choisis, et pour optimiser l'utilisation des centres de soutien opérationnel pendant les opérations de soutien. Une première étude a été effectuée pour examiner l'efficacité du concept de « centre de soutien opérationnel », et pour déterminer l'emplacement optimal des centres de soutiens. Après cette étude, le COMSOCAN a présenté au Chef d'état-major de la Défense (CEMD) un rapport contenant des recommandations sur l'établissement d'un réseau mondial de centres de soutien opérationnel. Récemment, le premier de ces centres de soutien, à Spangdahlem (Allemagne), a été inauguré officiellement par le Commandant du COMSOCAN, pour une période de trois ans afin de valider le concept.

Ce document présente une nouvelle étude du concept de « centre de soutien opérationnel » qui s'intéresse au problème du routage des aéronefs pendant les opérations de soutien. Il s'agit de trouver les itinéraires optimaux, dans un réseau de centres de soutien opérationnel, pour l'acheminement du matériel et des fournitures jusqu'à un théâtre d'opérations donné, afin de réduire au minimum les frais (ou le temps) de transport.

Méthodologie

Dans ce document, un modèle de routage des aéronefs de soutien opérationnel a été développé. Il s'agissait de déterminer les itinéraires optimaux pour le transport du matériel depuis divers centres de soutien jusqu'à un théâtre d'opérations donné. Ce modèle mathématique est formulé comme un problème de routage de véhicules, et il est mis en œuvre au moyen de la programmation non linéaire en numération mixte. Une variante du modèle fondée sur des vitesses de transport variables a également été proposée et examinée. Le modèle porte uniquement sur le transport du matériel depuis les centres de soutien jusqu'au théâtre d'opérations, et il ne s'intéresse pas au matériel ramené au Canada ou aux centres de soutien.

Conclusions

L'étude décrite dans ce document s'intéresse à un problème complexe de routage d'aéronefs militaires qui est lié au concept de « centre de soutien opérationnel » du COMSOCAN. Le modèle mathématique proposé pourra être utilisé par le COMSOCAN pour planifier les vols de soutien des opérations à l'étranger, pour déterminer les itinéraires optimaux pour le soutien des opérations ou des exercices au Canada, pour analyser les options relatives à l'acheminement du matériel (ex. : fermeture d'un centre de soutien), et pour étudier plus en détail l'impact de divers paramètres et de diverses contraintes logistiques (ex. : capacité et emplacement des centres de soutien, demande opérationnelle, etc.) sur l'efficacité du soutien opérationnel. L'étude fournit des données solides qui aideront les décideurs à utiliser efficacement un réseau de centres de soutien opérationnel à l'appui des missions à l'étranger.

Recommandations

Un certain nombre de recommandations pour les décideurs semblent découler logiquement de cette étude :

1. Pour mettre en œuvre le concept de « centre de soutien opérationnel », le COMSOCAN devrait songer à utiliser le modèle de routage d'aéronefs dans la planification des futurs vols de soutien. Les planificateurs des mouvements aériens et les experts en la matière devraient fournir les données requises pour l'application de cette méthodologie.

2. Lors de l'utilisation du modèle de routage d'aéronefs, il faudrait que le niveau des stocks à chacun des centres de soutien soit évalué avec précision, car il peut avoir un impact sur la solution optimale de routage des aéronefs.

3. Bien que le modèle soit formulé comme un problème de ramassage, il pourrait facilement être adapté pour faire face aux problèmes de livraison comme l'acheminement du matériel pendant le rapatriement des troupes. Il est recommandé que le COMSOCAN envisage d'utiliser le modèle pour planifier les options de mouvement du matériel lors du retrait de la Force opérationnelle interarmées en Afghanistan. Les experts devraient déterminer quels articles doivent être ramenés au Canada, et quels articles doivent être postpositionnés dans les centres de soutien (ex. : Camp Mirage, Spangdahlem) pour faciliter la planification et l'optimisation des itinéraires.

4. Enfin, d'autres analyses devraient être effectuées pour examiner d'autres aspects du problème de routage des aéronefs de soutien opérationnel, y compris les mouvements de personnel, la priorité des divers articles, la maintenance des aéronefs, le remplacement des équipages, la demande de matériel (variable aléatoire), les stocks disponibles à chacun des centres de soutien, et les problèmes liés aux voyages de retour.

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1. Introduction

This paper presents a mathematical model developed to address aircraft routing problems associated with the Canadian Operational Support Command (CANOSCOM) hub-based support concept. The model could be used for planning sustainment flights to determine cost (or time) effective aircraft routes for the movement of cargo and supplies from various support hubs to a theatre of operation.

1.1 Background

The Canadian Forces (CF) is seeking to establish a network of permanent and temporary operational support hubs at strategic locations around the globe to address some of its logistics support issues for deployed operations. These hubs would be used for cross-loading between modes of transportation, for the pre-positioning of non-perishable supplies, as refuelling stops, as staging bases during troop rotations, as well as for other associated purposes. Pre-positioned supplies can either be shipped in advance from Canada or purchased locally at hub areas to reduce transportation costs. Typical items that can be stored at hubs include subsistence items (e.g., food and water), fuel (e.g., petroleum, oil, and lubricants), spare parts, medical supplies, and construction material. Specific supplies (e.g., clothing) and sensitive materiel (e.g., ammunition, weapons) would be delivered directly from Canada to a theatre of operation. In addition to material pre-positioning, contracts can be established within the local area for troop accommodation, equipment repair, and aircraft maintenance services.

The potential for improving the responsiveness and effectiveness of the logistics distribution is a key factor in the consideration of a network of operational support hubs. This can be illustrated through a few practical cases. Local procurement of materiel to avoid air shipment from Canada and savings from backhaul costs to Canada for materiel that can be repaired at hubs are clearly cost avoiding examples that improve efficiency. Likewise, additional support benefits may be incurred using sealift in cases where materiel must be shipped in advance from Canada because it cannot be locally purchased or repaired at a hub. Such materiel can be received and temporarily stored at the hub awaiting the availability of lift for onward movement into theatre. In addition, surplus or damaged materiel could be moved out of theatre and stored at hubs based on the availability of space until potential lift opportunities become available.

To implement the hub-based support concept, CANOSCOM has identified a set of potential locations around the globe and has requested operational research support to facilitate better decisions concerning the selection of effective support hubs. Studies were conducted to examine various strategic lift and pre-positioning options and to analyse the approach for hub-based support [1, 2]. In particular, a model was developed in [2] to study the performance of the hub-based support concept and to determine the optimal support hub locations. These previous studies indicated that pre-positioning of supplies (through local procurement) at strategic locations would offer potential cost avoidance on sustainment lift and could be a potential strategy for improvement of the CF's support capability. Following the recommendations of these studies, a first support hub is being implemented by the CF in Spangdahlem, Germany and other hub options are under investigation. The following study is a further investigation of the hub-based support concept to analyze the aircraft routing problems during sustainment operations.

1.2 Problem description

Once established, the operational support hub network will be used for sustaining deployed operations. Sustainment involves the movement of cargo and supplies from one or multiple support hubs to a theatre of operation using a fleet of aircraft. Various classes of supply could be procured locally and stored at hubs. To effectively use the system of operational support hubs, optimal airlift routes for planning sustainment flights need to be determined. Different operational parameters and constraints (e.g., stock availability at each hub, aircraft maintenance locations, aircraft payload, crew changes, aircraft range, etc.) should be considered in the planning and scheduling of airlift routes. This study examines the Operational Support Aircraft Routing Problem (OSARP) associated with the hub-based support concept. The OSARP consists of finding the cost (or time) effective aircraft routes in a network of operational support hubs for the movement of cargo and supplies to a theatre of operation. The problem has the following features:

a. *Multi-commodity*: The OSARP involves the delivery of various commodity types known as classes of supply from the support hubs. Each hub can store several supply classes.

b. *Multi-depot*: The OSARP involves the movement of materiel from multiple locations or depots (i.e., operational support hubs) and requires inter-depot routes. The problem is called a multi-depot routing problem. All aircrafts are based at the main hub in Canada.

c. *Multi-trip*: With limited transport assets, the movement of supplies would be conducted with one or two aircrafts performing multiple trips between the hubs and the theatre. This is known as a multi-trip routing or a split delivery problem. For crew change and maintenance requirements, all aircrafts should return to their main base in Canada after a sustainment period is completed.

d. *Capacitated*: An aircraft has a limited capacity. The aircraft capacity can be defined by the maximum cargo payload (i.e., weight capacity) and the maximum number of pallets (i.e., volume capacity). The aircraft has also a maximum range (i.e., maximum travel distance before refuelling).

1.3 Objective

The purpose of this paper is to document the development and formulation of a mathematical model to address aircraft routing problems associated with the hub-based support concept. This model would be used for planning cost/time effective aircraft routes for sustainment flights.

1.4 Literature review

The OSARP problem can be viewed as a particular case of the general Vehicle Routing Problem (VRP), which consists of designing an optimal set of routes for a fleet of vehicles in order to serve a given set of geographically distributed customers. The problem addresses some of the strategic lift issues associated with the transport of military logistics through a network of operational support hubs. In the literature, various studies have been undertaken to address logistics transport and pre-positioning issues for military deployments. Rappoport *et al.* [3] developed an airlift heuristic for the US Air Force to support strategic lift planning and movement requirements. The authors decomposed the strategic lift problem into assignment and scheduling sub-problems. The assignment sub-problem addressed the allocation of manifest requirements to planes. To simplify the problem, requirements were aggregated into bulk, oversize and outsize cargo, without going down to the item level (this simplification would be valid for the deployment lift problem but it could not be used for the OSARP as the sustainment demand is defined by item and different classes of supply should be picked up from multiple hubs). The scheduling sub-problem addressed issues associated with mission routing and aircraft sortie scheduling. Cox [28] developed an alternative hub-and-spoke combined location-routing integer linear programming prototype model for the United States Air Force's Air Mobility Command. The model was used to determine what advantages a hub-and-spoke system offers, and in which scenarios it is better-suited than the direct delivery method. The study focused on the lift requirements during deployment operations and considered sealift options in the hub-and-spoke system. As for Rappoport *et al*, requirements are aggregated to simplify the deployment lift problem. The model featured the following elements: time windows, multiple frequency servicing, aircraft basing assignments and routing, and the selection of the optimal number of local-delivery aircraft to be used. A notional transoceanic airlift problem was used to demonstrate the methodology. Granger *et al.* [4] used stochastic simulation and network approximation methods to measure the performances of strategic lift operations along a transportation network. They compared the simulation model and the network approximation model for the impact of stochastic flying times and ground times on a simplified airlift network. Based on the study, they suspect that a combination of simulation and network optimization models should yield much better performance than either one of these alone. Baker *et al.* [5] developed a time-dynamic linear programming model to analyze and optimize the US Air Force strategic lift capability. In particular, the model was developed to solve some aspects of the military deployment airlift problem. The problem involves the movement of equipment and personnel from a given origin to a given destination under time and resource constraints. The model was designed to capture various aspects of an airlift system in a large-scale military deployment, including aerial refueling, tactical aircraft shuttles, and constraints based on crew availability but it does not consider aircraft routing problems. Guéret *et al.* [6] studied the military deployment lift problem and developed an aircraft loading system for the French military operations planning. The authors introduced two-phase methods for solving the aircraft loading problem: a list-based heuristic and a loading pattern generation algorithms. They used local search heuristics to optimize the loading solution. Ghanmi and Shaw [1] conducted an analysis of some of the strategic lift for the CF and developed a simulation framework to study the effectiveness of a variety of pre-positioning options. For tactical logistics transport, Ghanmi et al. [7] examined a Multinational Intra-Theatre Distribution (MN ITD) concept for alliance or coalition operations. MN-ITD considers the creation of a MN logistics distribution centre that combines different materiel flows into a single system to improve multinational logistics distribution in a theatre of operation. They developed a discrete event simulation framework to assess the MN ITD time responsiveness.

In DRDC, different operational research studies have also been conducted to address various logistics transport problems. Taylor [8] studied the planning equations defined in Chapter 2 of CFACM 60-2603 guide [9] as they applied to airlift planning and analysis. These equations permit the determination of the airlift requirement based on physical dimensions and weight of cargo to be moved. However, the allocation of cargo to various airframes being considered in a given scenario must be made a priori with the potential for a sub-optimal overall airlift plan. Comeau [10] developed a Strategic Mobility Optimization Model for Lift Capability Analysis (STRATL). The model used a linear optimization routine to allocate cargo to various aircraft and ships under consideration. STRATL was developed for strategic lift concept development. It could be used to analyse various lift capability options and to solve fleet mix problems involving sealift and airlift assets. Dickson [11] conducted a study on the examination of the strategic lift capability of aircraft fleets with the potential for acquisition. He examined the fleet sizes (strategic lift capability) required to meet current and projected CF commitments based on representative force structures deployed to typical locations. Recently, Ghanmi [2] examined the hub-based support concept for the CF and developed performance measures to assess the effectiveness of different operational support hubs. He also developed a mathematical model to determine the optimal number and locations of hubs. The model was formulated as a discrete facility location problem and was implemented using mixed integer nonlinear programming.

1.5 Document Structure

The remainder of this report is organized as follows. Section 2 presents an overview of the classical VRP and discusses its solution methodologies. The VRP is used as the basis for the development of an operational support aircraft routing model. The mathematical formulation and the underlying assumptions of the model are discussed in Section 3. Section 4 presents an example application to illustrate the methodology. Concluding remarks and recommendations for future analysis and development are found in the fifth and final section.

2. Vehicle Routing Problem

This section presents an overview of the classical VRP, highlights the fundamental aspects of the problem formulation and underlying assumptions, and discusses various solution methodologies.

2.1 Overview

The VRP calls for the determination of the optimal set of routes to be performed by a fleet of vehicles to serve a given set of geographically distributed customers. This problem is one of the most widely studied combinatorial optimization problems [12], and has applications in various contexts, such as courier services, commercial distribution, air transportation, and school bus routing. Typical VRP objectives include minimization of the global transportation cost, minimization of the number of vehicles required to serve all customers, balancing of the routes for travel time and vehicle load, minimization of the penalties associated with partial service of the customers, or any weighted combination of these objectives. Typical operational constraints include the vehicle capacity, customer demand satisfaction, and delivery time requirements. The VRP has been demonstrated to be a nondeterministic polynomial-time hard (NP-hard) problem [13]. The difficulty of the problem depends on the system size (i.e., number of customers, number of depots, and transportation network) as well as the problem formulation (i.e., the constraints, objective function, and type and number of variables).

The classical VRP is the capacitated vehicle routing problem, which considers the delivery of a common commodity to a set of customers with deterministic demands from a single depot, by means of a fixed homogenous fleet of capacitated vehicles. Figure 1 depicts an example of a VRP solution using one depot, three vehicles and ten customers (i.e., nodes). Each vehicle visits a subset of customers along a specific route starting and ending at the depot. All customers' demand should be satisfied, the capacity of the vehicles should not be exceeded and the total transportation cost should be minimized.

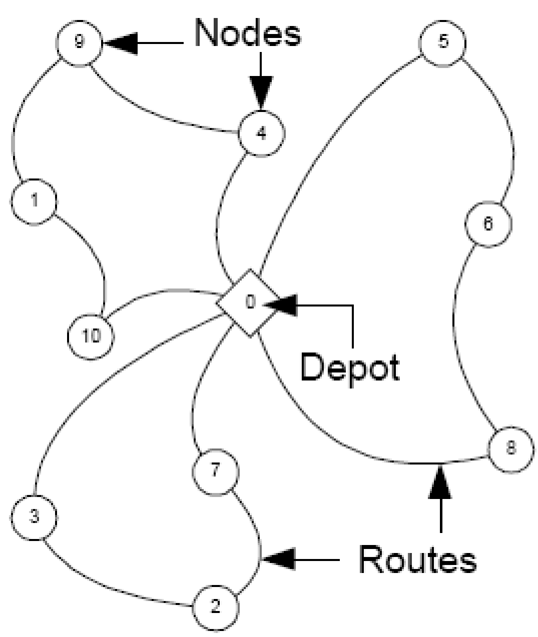

Figure 1: *Example of Vehicle Routing Problem.*

Many extensions of the classical VRP have been studied in the literature. These extensions are generally classified under various assumptions, and include constraints on the distance, time windows, pickup and delivery, and backhauls. The distance constrained VRP imposes a bound on the total distance (or time) traveled on each route, corresponding to the vehicle range. The VRP with time windows considers both routing and scheduling problem aspects. A time interval is associated with each customer, and the vehicle visiting a given customer cannot arrive after the end of the time window. The VRP with pickup and delivery problem is a further generalization of the classical VRP, in which each customer request is associated with two quantities representing the demand to be delivered and the quantity to be picked up. The VRP with backhauls considers two subsets of customers, one requiring a given quantity of product to be delivered to (known as linehaul customers) and one requiring an inbound product to be picked up at (known as backhaul customers).

In addition to the main previous VRP classes, other variants have been examined in the literature which include those known as the multi-depot, multi-commodity, multi-trip, stochastic, and split delivery problems. The multi-depot VRP considers more than one depot for the delivery of commodity to the customers whereas the multi-commodity VRP involves different commodity types for the demand. The multi-trip VRP assumes that each vehicle may perform several routes in the same planning period. The stochastic VRP addresses routing under uncertainty by introducing variability in the system parameters (e.g., demand, travel time, travel cost). For the split delivery VRP, several vehicles can serve a customer (i.e., split deliveries are allowed).

Different methodologies and solution approaches have been developed for the VRP and its variants; these are grouped under exact and heuristic solution methods. Exact solution methods can be classified as branch-and-bound algorithms [14], branch-and-cut algorithms [15], and column generation methods [16, 17, 18]. Popular approximate methods used for the VRP are meta-heuristics such as simulated annealing [19], genetic algorithms [20], tabu search [21], and evolutionary algorithms [22].

2.2 Mathematical formulation

In broad terms, formulations for the VRP can be separated into two main categories: arc-based and path-based formulations [12]. Arc-based formulations, also referred to as vehicle flow model formulations, use integer variables associated with each arc of the problem graph to indicate whether a vehicle travels along the arc or not. They are the most frequently used models for the VRP and they are more suitable for cases in which the cost of the solution can be expressed as the sum of the costs associated with the arcs. On the other hand, path-based formulations, also referred to as set partitioning formulations, consider an integer variable for each path (or vehicle route) to indicate whether that route is part of the routing solution or not. These formulations therefore consider a number of integer variables that grow exponentially with the problem size and are typically tackled via column generation schemes.

There are two different arc-based model formulations, namely: two-index and three-index formulations. The two-index formulation uses two index-variables to indicate whether a vehicle goes from one node to another or not whereas the three-index formulation considers a third index to indicate which vehicle goes through the arc. While both formulations are used in the literature, the three-index formulation would be appropriate for the OSARP as the aircraft can visit the same hub several times to gather different items (i.e., the third index in the OSARP represents the index of the aircraft sortie).

Using the three-index formulation of the arc-based model, the classical VRP can be formulated as a mixed integer linear programming problem as follows. Let $V = \{0, 1, 2, \dots, n\}$ be a set of $n + 1$ nodes, representing n customers and one depot (node 0) and denote by d_i (a non-negative integer) the demand at each node $i \in V \setminus \{0\}$. Consider a fully connected network and denote by c_{ij} the travel cost (or time) between node i and node j ($c_{ij} \neq c_{ji}$ in general). Let K be a fleet of identical vehicles, each with capacity C to serve the customers (i.e., each customer must be served by a single vehicle). Let x_{ijk} be a binary variable indicating whether vehicle k traverses arc (i, j) in the optimal solution ($x_{ijk} = 1$) or not ($x_{ijk} = 0$), and let y_{ik} be a binary variable to indicate whether vehicle k services customer i in the optimal solution ($y_{ik} = 1$) or not ($y_{ik} = 0$). With $i \neq j$, the classical VRP model can be written as follows [12]:

$$\min \quad \sum_{i \in V} \sum_{j \in V} c_{ij} \sum_{k=1}^{K} x_{ijk} \tag{2.1}$$

subject to the following constraints, where S is a subset of customers:

$$\sum_{k=1}^{K} y_{ik} = 1 \qquad i \in V \setminus \{0\} \tag{2.2}$$

$$\sum_{k=1}^{K} y_{0k} = K \tag{2.3}$$

$$\sum_{j \in V} x_{ijk} = \sum_{j \in V} x_{jik} = y_{ik} \qquad i \in V, \quad k = 1, 2, \dots, K \tag{2.4}$$

$$\sum_{i \in V} d_i y_{ik} \leq C \qquad k = 1, 2, \dots, K \tag{2.5}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijk} \leq |S| - 1 \qquad S \subseteq V \setminus \{0\}, \ |S| \geq 2, \ k = 1, 2, \dots, K \tag{2.6}$$

$$x_{ijk}, \ y_{ik} \in \{0, 1\} \qquad i, j \in V, \quad k = 1, 2, \dots, K \tag{2.7}$$

The objective function (2.1) minimizes the total cost (or time) for all vehicles. Constraints (2.2) ensure that exactly one vehicle services every customer whereas constraint (2.3) imposes that K vehicles leave the depot. Constraints (2.4) force the same vehicle to enter and leave a given customer and also prevents a vehicle from visiting a customer if it does not serve it. Constraints (2.5) enforce the capacity restriction for every vehicle while constraints (2.6) are the subtour elimination constraints, which impose the connectivity of the route performed by each vehicle (see the next section for details).

2.3 Subtour elimination constraints

As indicated above, the aim of the subtour elimination constraints is to ensure that the solution contains no subtours disconnected from the depot. The set of constraints represented by inequalities (2.6) has a cardinality growing exponentially with the number of nodes, explaining in part the complexity of the problem. An alternative family of subtour elimination constraints with polynomial cardinality was proposed by Miller *et al.* [23] for the traveling salesman problem and was extended by Kulkarni and Bhave [24] to the capacitated VRP. The proposed formulation considers an additional continuous variable representing the load of the vehicle after visiting a given customer. For a delivery problem, the value of the load variable decreases as the vehicle moves from one customer to another along the distribution route. On the other hand, for a pickup problem, this value increases as the vehicle moves from one customer to another. The subtour elimination constraints for a delivery problem are given by ($i \neq j$):

$$u_{ik} - u_{jk} - C\,x_{ijk} \;\geq\; d_j - C \qquad i, j \in V \setminus \{0\}, \quad k = 1, 2, \ldots, K \tag{2.9}$$

$$d_i \;\leq\; u_{ik} \;\leq\; C \qquad i \in V \setminus \{0\}, \quad k = 1, 2, \ldots, K \tag{2.10}$$

where u_{ik} is the load variable of vehicle k after visiting customer i. Similarly, the subtour elimination constraints for a pickup problem can be written as ($i \neq j$):

$$u_{ik} - u_{jk} + C\,x_{ijk} \;\leq\; C - d_j \qquad i, j \in V \setminus \{0\}, \quad k = 1, 2, \ldots, K \tag{2.11}$$

$$d_i \;\leq\; u_{ik} \;\leq\; C \qquad i \in V \setminus \{0\}, \quad k = 1, 2, \ldots, K \tag{2.12}$$

Note that the above formulations of the subtour elimination constraints impose both the capacity and the connectivity requirements of the VRP. Although the previous formulations are polynomial in size, it is well known that they can lead to weak LP relaxation. To address this problem, tightening constraints for the VRP were initially proposed by Desrochers and Laporte [25] and later revised by Kara *et al*. [26]. Using the tightening constraints, inequalities (2.9) and (2.11) become, respectively:

$$u_{ik} - u_{jk} - C\,x_{ijk} - (C - d_i - d_j)\,x_{jik} \geq d_j - C \qquad i, j \in V \setminus \{0\}, \quad k = 1, 2, \ldots, K \tag{2.13}$$

$$u_{ik} - u_{jk} + C\,x_{ijk} + (C - d_i - d_j)\,x_{jik} \leq C - d_j \qquad i, j \in V \setminus \{0\}, \quad k = 1, 2, \ldots, K \tag{2.14}$$

Using the techniques described above, the operational support aircraft routing problem will be formulated as an application of the VRP. Full details follow in the next section.

3. Operational support aircraft routing problem

In the previous section, an overview of the classical VRP was presented. In this section, the operational support routing problem is formulated using the VRP model. The mathematical formulation and the underlying assumptions of the problem are discussed and presented.

3.1 Problem description

The OSARP considers the routing of a fleet of transport aircrafts for the movement of cargo and supplies during sustainment operations. Supplies are located in Canada (i.e., national hubs) and at various international operational support hubs. As discussed in the introduction, the OSARP is a multi-commodity, multi-depot, and multi-trip problem that involves pickup and delivery operations using capacitated aircrafts. Some classes of supply (ammunition and subsistence) cannot be mixed together on the same flight (this restriction is not considered in the model). Table 1 depicts the Canadian standard classes of supply that are used for planning sustainment packages.

Table 1: *Canadian Standard Classes of Supply.*

Class	Title	Description
I	Subsistence	Food, rations, and water
II	Clothing & Individual equipment	Clothing, individual equipment, tentage, organizational tool sets and tool kits, hand tools, maps, and administrative and housekeeping supplies and equipment
III	POL	Petroleum fuels; lubricants; hydraulic and insulating oils; preservatives; liquid and compressed gasses; bulk chemical products; coolants; deicing and antifreeze compounds, together with components and additives of such products; and coal
IV	Fortification and barrier materials	Construction materials, including installed equipment and all fortification/barrier materials
V	Ammunition	Ammunition of all types, including chemical and special weapons, bombs, explosives, mines, fuses, detonators, missiles, rockets, propellants, and other associated items.
VI	Personal Items	Personal demand items (nonmilitary sales items).
VII	Major End Items	Major end items: a final combination of end products that are ready for their intended use, for example, tanks, launchers, mobile machine shops, and vehicles.
VIII	Medical supplies	Medical materiel, including medical-peculiar repair parts.
IX	Repair Parts	Repair parts (less medical-peculiar repair parts): all repair parts and components, to include kits, assemblies, and subassemblies required for maintenance support of all equipment.
X	Miscellaneous	Materiel to support nonmilitary programs, such as agricultural economic development, not included in Classes I through IX.

Supply classes I, II (but only for individual equipment), VI, VIII, and IX could potentially be pre-positioned or locally procured at the hub locations. Supply classes III and IV are in general provided locally through Third Party Logistics Supply Services (TPLSS). Supply classes II (but only for clothing) and V are Canadian unique items and should be shipped directly from Canada. The remaining supply classes (VII and X) are not routine sustainment items and are usually delivered on request.

3.2 OSARP model

An OSARP model is developed to address aircraft routing problems associated with the hub-based operational support concept. The model is formulated as a variant of the classical VRP.

3.2.1 Assumptions

There are several important assumptions that underlie the development of the aircraft routing model for the hub-based operational support:

1. The study considers a single line of support scenario (i.e., one mission) and assumes that the deployment duration is divided into identical sustainment planning periods—a time interval (e.g., month) at which the operational demand and the stock level are evaluated. The replenishment of the support hubs is also assumed to be at the beginning of each sustainment planning period.
2. The study is restricted to the strategic lift of cargo and supplies during the sustainment phase. The costs associated with personnel rotations, hub operating and maintenance costs and local transportation costs are excluded.
3. The sustainment operations are conducted by airlift and great circle distances are used to estimate the airlift time, neglecting issues such as diplomatic over-flight clearances or weather conditions. In addition, the cost associated with the aircraft refuelling stops is neglected (i.e., the additional fuel consumption required for landing and taking off would be included in the flying rate).
4. The stock level at each support hub and the operational demand are known within each sustainment period (i.e., deterministic problem). These quantities are expressed in number of aircraft pallets for each class of supply.
5. The study considers the problem of routing a single aircraft; the CF assumes that one aircraft is available for sustainment at any given time (i.e., the model needs to be reformulated to address multiple aircraft routing problems with identical or different aircraft types). It is assumed that the aircraft is positioned in Canada at the beginning of each sustainment period. The aircraft maintenance scheduling problem is not considered in this study.
6. The study does not consider backhauls of materiel to hubs or Canada. This would be addressed in a follow-up study.

3.2.2 Mathematical formulation

The OSARP model is formulated as a pickup VRP. Let $V = \{0, 1, 2, \dots, n, n+1\}$ be a set of $n + 2$ distribution nodes, where node 0 represents the hub in Canada, node $n + 1$ indicates the destination node in theatre and nodes 1, 2, … n denote the operational support hubs. Consider a fully connected network and denote by d_{ij} the great circle distance (km) between nodes i and $j \in V$. The distance d_{ij} is in principle equal to the distance d_{ji} and the lift time between nodes i and j

is considered symmetrical. However, due to the aircraft load and jet streams, the speed of the aircraft may vary and the lift time (and consequently the lift cost) between nodes *i* and *j* could be different. A non symmetrical lift time model variant will be discussed.

Consider a single aircraft routing problem and let K be the number of aircraft sorties planned during a given sustainment period (i.e., in the model, K is not a variable and is calculated using the aircraft payload characteristics and the operational demand. Future implementations of the model should consider a variable number of aircraft sorties), L the aircraft maximum payload (kg), C its pallet capacity (i.e., maximum number of pallet positions), v its average cruise speed (km/h), and r its hourly flying cost ($/h). Let x_{ijk} be a binary variable to indicate whether sortie k traverses arc (i, j) in the optimal solution ($x_{ijk} = 1$ if so, $x_{ijk} = 0$ otherwise), and let y_{ik} be a binary variable to indicate whether sortie k goes through hub i in the optimal solution ($y_{ik} = 1$ if so, $y_{ik} = 0$ otherwise).

Let M be the number of classes of supply, w_p the average weight (kg) of a pallet of supply class p and D_p the operational demand (in number of pallets) for supply class p. Let the variable q_{ipk} be the number of pallets of supply class p to pick up from node i at sortie k ($q_{n+1,pk} = 0$), and the variable Q_{ipk} be the number of pallets of supply class p at node i before sortie k (it is assumed that each supply node i has a limited number of pallets of each class (a_{ip}) that is replenished at every sustainment period).

The minimum number of aircraft sorties can be determined using the operational demand and the aircraft maximum payload and capacity as follows:

$$K = \left\lceil \max\left\{ \frac{\sum_{p=1}^{M} D_p}{C} \;,\; \frac{\sum_{p=1}^{M} w_p D_p}{L} \right\} \right\rceil \tag{3.1}$$

where the symbol $\lceil . \rceil$ represents the ceiling operator.

Equation (3.1) provides a lower bound of the number of aircraft sorties. In the model, an iterative approach would be required to solve the routing problem. The problem should first be solved with the lower bound (K). If the optimal solution cannot be found then the value of (K) should be increased by one until the convergence.

The objective of the OSARP model is to determine the optimal aircraft route and the number of pallets of each supply class to pick up from the distribution nodes in order to minimize the total transportation cost under various operational constraints (e.g., stock availability, operational demand, etc). The model can be expressed as a Mixed Integer Nonlinear Program (MINLP). A MINLP is a mathematical program with a nonlinear objective function and/or constraints in which a specified subset of the variables is required to take on integer values. Using the VRP arc-based formulation, the OSARP model can be formulated as follows:

$$\text{Minimize} \quad \sum_{i \in V} \sum_{j \in V} r \frac{d_{ij}}{v} \sum_{k=1}^{K} x_{ijk} \tag{3.2}$$

subject to:

$$\sum_{j\in V} x_{ij1} = \sum_{j\in V} x_{ji1} = y_{i1} \qquad i \in V \setminus \{0, n+1\} \qquad (3.3)$$

$$\sum_{j\in V} x_{0j1} = y_{01} = 1 \qquad (3.4)$$

$$\sum_{j\in V} x_{ijk} = \sum_{j\in V} x_{jik} = y_{ik} \qquad i \in V \setminus \{n+1\},\ k = 2,\dots,K \qquad (3.5)$$

$$\sum_{i\in V} x_{i,n+1,k} = 1 \qquad k = 1,2,\dots,K \qquad (3.6)$$

$$\sum_{j\in V} x_{n+1,jk} = 1 \qquad k = 2,\dots,K \qquad (3.7)$$

$$\sum_{i\in V\setminus\{n+1\}} \sum_{p=1}^{M} w_p\, q_{ipk} \leq L \qquad k = 1,2,\dots,K \qquad (3.8)$$

$$\sum_{i\in V\setminus\{n+1\}} \sum_{p=1}^{M} q_{ipk} \leq C \qquad k = 1,2,\dots,K \qquad (3.9)$$

$$\sum_{i\in V\setminus\{n+1\}} \sum_{k=1}^{K} q_{ipk} = D_p \qquad p = 1,2,\dots,M \qquad (3.10)$$

$$q_{ipk} \leq y_{ik}\, Q_{ipk} \qquad i \in V\setminus\{n+1\};\ k = 1,2,..,K;\ p = 1,2,..,M \qquad (3.11)$$

$$\sum_{i\in S} \sum_{j\in S} x_{ijk} \leq |S| - 1 \qquad S \subseteq V\setminus\{n+1\};\ |S| \geq 2;\ k = 1,2,..,K \qquad (3.12)$$

$$q_{n+1,pk} = 0 \qquad k = 1,2,..,K;\ p = 1,2,..,M \qquad (3.13)$$

$$x_{iik} = 0 \qquad i \in V;\ k = 1,2,\dots,K \qquad (3.14)$$

$$x_{n+1,0K} = 1 \qquad (3.15)$$

$$x_{ijk},\ y_{ik} \in \{0,1\} \qquad i,\ j \in V;\ k = 1,2,\dots,K \qquad (3.16)$$

$$q_{ipk} \text{ integer} \qquad i \in V;\ k = 1,2,\dots,K;\ p = 1,2,..,M \qquad (3.17)$$

The objective function (3.2) minimizes the total travel cost for all aircraft sorties. Constraints (3.3) ensure that if the aircraft enters a hub node during the first sortie then it should leave it. Constraints (3.4) enforce that the first aircraft sortie starts in Canada. Constraints (3.5) are degree balance constraints for the hub nodes after the first sortie to ensure that if a hub is selected at sortie k, then two arcs should be created in the solution (one for entering and one for leaving the hub). Constraints (3.6) guarantee that the aircraft arrives at the theatre node for each sortie and constraints (3.7) ensure that the aircraft departs from the theatre node after the first sortie. Constraints (3.8) and (3.9) enforce the aircraft payload and the pallet capacity restrictions while constraints (3.10) enforce the demand requirements. Constraints (3.11) guarantee the availability of supplies at the distribution nodes. Constraints (3.12) are the subtour elimination constraints, which impose the connectivity of the route at each sortie and eliminate the routes that do not contain the theatre node. Constraints (3.13) indicate that no item is picked up from the theatre node and constraints (3.14) ensure that for a given leg the node of origin and the node of destination are different. Constraint (3.15) ensures that the aircraft returns to its main base after the last sortie. Finally, constraints (3.16) and (3.17) are variable domain constraints.

The residual number of pallets at each node (Q_{ipk}) can be formulated as follows:

$$\begin{cases} Q_{ipk} = Q_{ipk-1} - q_{ipk} \quad ; \quad k \geq 2 \\ Q_{ip1} = a_{ip} \end{cases} \tag{3.18}$$

where a_{ip} is the number of pallets of supply class p that is stored at node i at the beginning of the sustainment period. Note that the operational support aircraft routing model is nonlinear and could be difficult to solve, particularly for larger distribution systems (e.g., number of nodes greater than 10).

3.2.3 Subtour elimination constraints

As for the VRP, one can introduce a variable u_{ik} to represent the aircraft load (in volume) at sortie k after visiting hub i. Using equation (2.14) for a pickup problem, the lifted subtour elimination constraints for the OSARP problem can be written as ($i \neq j$):

$$u_{ik} - u_{jk} + C\, x_{ijk} + x_{jik}\left(C - f_{ik} - f_{jk}\right) \leq C - f_{jk} \qquad i, j \in V \setminus \{n+1\},\ k = 1,2,\ldots,K \tag{3.19}$$

By analogy with the classical VRP, f_{ik} represents the demand (d_i) in equation (2.14) and is defined as the quantity of supplies that should be picked up from hub i at sortie k. As indicated in Section 2, the lifted subtour elimination constraints impose both the vehicle capacity and the route connectivity requirements. For the OSARP, the aircraft payload capacity is enforced by constraint (3.8) and the aircraft volume capacity is enforced in the subtour elimination constraints. As such, f_{ik} indicates the total number of pallets of all supply classes that should be picked up from hub i at sortie k and can be written as*:*

$$f_{ik} = \sum_{p=1}^{M} q_{ipk} \qquad i \in V \setminus \{n+1\},\ k = 1,2,\ldots,K \tag{3.20}$$

and the aircraft payload bounds are (similar to equation 2.12):

$$f_{ik} \leq u_{ik} \leq C \qquad i \in V \setminus \{n+1\},\ k = 1,2,\ldots,K \tag{3.21}$$

3.2.4 Model implementation

The OSARP model has been implemented using LINGO[1]. LINGO is a modeling tool for solving linear, integer, and nonlinear optimization problems. Annex *A* presents the program source code

[1] LINGO. Lindo Systems Inc (www.lindo.com).

for future reference. The interested reader may reproduce the results presented in this paper by copying these codes into a LINGO file and executing them within the LINGO environment.

3.3 Variable aircraft speed

In the basic OSARP model formulation, it is assumed that the aircraft speed is constant. This sub-section discusses an OSARP model variant where this assumption is modified, called the *Variable Aircraft Speed* sub-model.

While several factors (e.g., weather conditions, aircraft payload) can affect the aircraft speed, only the aircraft load is examined in this model variant. In order to do that, the aircraft average speed is represented as a function of the aircraft load (in weight) using a linear relationship as follows.

$$v_{ik} = \left(\frac{v_m - v_e}{L}\right) u'_{ik} + v_e \qquad i \in V,\ k = 1,2,...,K \qquad (3.22)$$

where v_{ik} is the aircraft speed at sortie k after node i, u'_{ik} is the aircraft load weight at sortie k after node i, v_m is the aircraft speed at maximum load, and v_e is the empty aircraft speed. Equation (3.22) indicates that the aircraft speed is proportional to the aircraft load. If the aircraft is empty then it has a maximum speed and vice versa. As for the aircraft load volume variable (u_{ik}), the aircraft load weight variable (u'_{ik}) can be bounded as follows:

$$f'_{ik} \leq u'_{ik} \leq L \qquad i \in V \setminus \{n+1\},\ k = 1,2,...,K \qquad (3.23)$$

where f'_{ik} represents the total weight of pallets of all supply classes that should be picked up from hub i at sortie k and can be written as:

$$f'_{ik} = \sum_{p=1}^{M} w_p q_{ipk} \qquad i \in V \setminus \{n+1\},\ k = 1,2,...,K \qquad (3.23)$$

Introducing the variable speed representation in the aircraft routing model would further increase the nonlinearity and the difficulty of the problem particularly for larger supply distribution systems (e.g., more than 10 distribution nodes).

While the speed of an aircraft does not change with its load in practice, an aircraft with a small load will have the capacity to carry more fuel in order to avoid (or reduce) refuelling stops. For example, return flights to Canada with an empty load would not require refuelling stops as the aircraft can carry more fuel to travel long distances. The variable aircraft speed model would be used to address this issue.

To illustrate the application of the OSARP model, an example is detailed in the next section.

4. Example application

The section presents an example application of the OSARP model to illustrate the methodology for planning airlift routes during sustainment operations. The example considers a typical CF deployment scenario and randomly generated sustainment data.

4.1 Data

The data required for the analysis include a deployment scenario for which the sustainment airlift routing will be examined, characteristics of the supply distribution network (i.e., hub locations, stock level, classes of supply, etc), and characteristics of the logistics transportation assets.

4.1.1 Deployment scenario

The example considers a deployment scenario based upon the historical Operation ATHENA deployment – Canada's contribution to the International Security Assistance Force (ISAF) in Afghanistan. In support of ISAF, Canada deployed a task force of about 1200 troops comprised of a light mechanized battle group and 350 vehicles. Force sustainment was conducted using a mix of chartered and CF flights flown directly from Trenton to the destination in theatre. The total weight of cargo for re-supplying Operation ATHENA can be determined using the historical sustainment flight loads. A previous study [2] indicated that the operational demand of supplies could be estimated at 200 pallets per month. As an example, it is assumed that 60% of the supply requirements (i.e., 120 pallets) is Canadian unique items (e.g. ammunition, clothing, etc) and needs to be shipped directly from Canada. The remaining 80 pallets of the demand could be satisfied from Canada and/or the operational support hubs; they are the subject of this study.

4.1.2 Supply distribution network

Figure 2 highlights seven regions identified by the CF where future potential operational support hubs could be located. These regions are selected based on their proximity to failed and failing states. Failed and failing states are determined using a combination of twelve social, economic and political/military indicators developed by the Fund for Peace[2]. The geographical distribution of the top 60 failed and failing states is presented in Figure 2, where the different colours represent varying levels of country stability, with red indicating "Critical", orange indicating "In Danger" and yellow indicating "Borderline".

Representative locations (Dakar, Senegal; Spangdahlem, Germany; Mombasa, Kenya; Dubai, United Arab Emirates; Singapore, Singapore; Seoul, South Korea; Panama, Panama) have been identified in the regions for the support hubs [27]; they do not necessarily reflect the likelihood of their future use by the CF. These locations are selected to meet several hub operational requirements including commercial distribution (i.e., connecting major off-continent air and seaports to the regional sea, air and land routes), geography, climate, infrastructure, and political considerations. Detailed descriptions of these requirements can be found in [27].

In this study, the supply distribution network was composed of the various operational support hub locations, along with Trenton in Canada. For this scenario (i.e., deployment to Afghanistan), the Panama hub is dominated by Trenton (i.e., the sustainment lift from Trenton would be more cost effective than from Panama, assuming that Trenton always has enough supply of all classes

[2] http://www.fundforpeace.org

to satisfy the demand), therefore it was removed from the analysis. It was assumed that four classes of supply (i.e., class I, individual equipment of class II, class VI, and class IX) were pre-positioned or can be procured at the hub locations. The stock levels in the different hubs at the beginning of each sustainment period are presented in Table 2. These quantities are randomly selected and are used in the analysis to demonstrate the methodology. The operational demand and the average pallet weight of each supply are also randomly selected and are presented in Table 3.

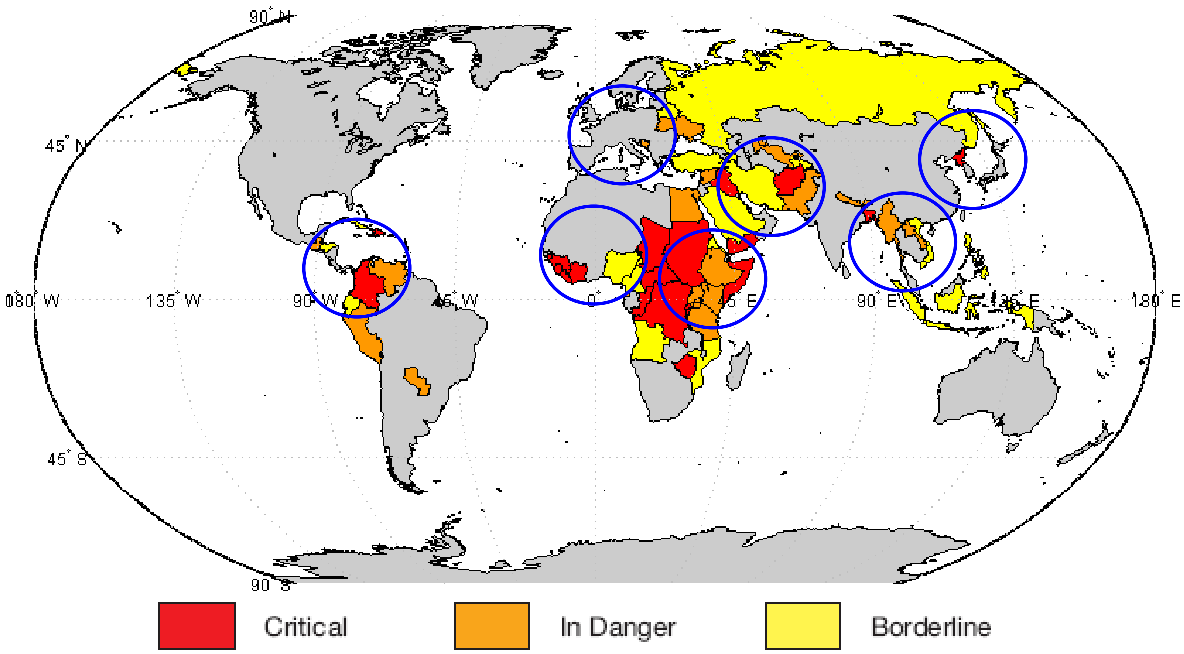

Figure 2: *Potential Support Hubs with Respect to Failed and Failing States Distribution*

Table 2: *Stock Level in Pallets at Hubs.*

Hub	**Supply Class**			
	I	**II**	**VI**	**IX**
Trenton	1000	1000	1000	1000
Spangdahlem	20	10	10	15
Dakar	10	0	5	0
Mombasa	10	0	5	20
Dubai	20	10	10	10
Singapore	20	0	10	0
Seoul	10	5	0	20

Table 3: *Operational Demand and Pallet Weight per Supply Class*

	Supply Class			
	I	**II**	**VI**	**IX**
Demand (# pallets)	40	5	10	25
Pallet Weight (tonne)	3	2	2.5	2.8

4.1.3 Transport assets

In the study, the CC-177 aircraft was considered for the transport of supplies from Canada and the hubs into the theatre of operation. Other potential airlift assets such as the Antonov 124 (AN-124) aircraft could also be used for the sustainment airlift. Table 4 presents the performance characteristics and the payloads of the selected strategic lift aircraft. The charter costs and the average payloads of the assets are estimated based on historical data[3].

Table 4: *Selected Characteristics of Several Strategic Lift Aircraft.*

Aircraft	Payload (tonne)	Capacity (pallets)	Flying cost (CA $/h)	Cruise speed at max payload (km/h)	Cruise speed at empty payload (km/h)
CC-177	50	18	13,000	600	800
AN-124	80	28	28,000	600	800

4.2 Aircraft routing analysis

The first step of the aircraft routing analysis process is to determine the number of aircraft sorties required to satisfy the operational demand. Using equation (3.1), the number of aircraft sorties using the CC-177 aircraft payload and capacity characteristics is given by:

$$K = \left\lceil \max\left\{ \frac{\sum_{p=1}^{M} D_p}{C} \,,\, \frac{\sum_{p=1}^{M} w_p D_p}{L} \right\} \right\rceil$$

$$= \left\lceil \max\left\{ \frac{80}{18} \,,\, \frac{40\times 3 + 5\times 2 + 10\times 2.5 + 25\times 2.8}{50} \right\} \right\rceil$$

$$= 5$$

[3] The C-177 payload is 76 tonnes but its operational load during JTF Afghanistan sustainment flights is about 50 tonnes due to various operational constraints in Kandahar airfield (e.g., infrastructure).

To analyse the aircraft routing problem, a baseline scenario using one CC-177 aircraft positioned in Trenton was considered (a constant aircraft speed was used in this analysis). The first aircraft sortie starts at Trenton and the remaining four sorties depart from Kandahar. The sequence of support hubs to visit during a given sustainment period defines an aircraft route. Different route options could be obtained depending on the hub location, aircraft capacity, stock level at hubs, etc. The optimal aircraft route is defined as the most cost effective sequence of hubs to visit in order to satisfy the operational demand while respecting the aircraft load requirements and the stock availability constraints. The optimal route for this example is determined as follows:

Trenton → Kandahar → Dubai → Kandahar → Dubai → Kandahar →
Mombasa → Kandahar → Mombasa → Kandahar → Trenton.

The total airlift cost (i.e., objective function) using the optimal route is $994,717. Table 5 presents the pickup nodes and an aircraft load configuration (i.e., the number and types of pallets to load in the aircraft) for each sortie. It is important to note that the aircraft load configuration is not unique and different loading solutions could be obtained for the same route.

Table 5: *Pickup Nodes and Aircraft Loads for the Baseline Scenario*

Sortie	Pickup Node	# of Pallets of Supply Class				Aircraft Load	
		I	II	VI	IX	Total Pallets	Weight (tonne)
1	Trenton	16				16	48
2	Dubai	4		4	10	18	50
3	Dubai	10	5	1		16	42.5
4	Mombasa	10			5	15	44
5	Mombasa			5	10	15	40.5
Total		**40**	**5**	**10**	**25**	**80**	**225**

The analysis indicates that only two support hubs (i.e., Dubai and Mombasa) would be required along with Trenton to satisfy the operational demand in this example. In fact, these support hubs represent the closest nodes to the theatre of operation. In addition, the optimal route could be different in each sustainment period. Therefore, the stock level at each hub and the operational demand should be evaluated accurately every sustainment period. Finally, the order of the pickup nodes in the optimal route is not important in this particular case (i.e., the optimal solution is made of single segment routes and is not unique). This property of the solution may not always be true. For example, the following route is also a valid optimal route:

Trenton → Kandahar → Mombasa → Kandahar → Dubai → Kandahar →
Mombasa → Kandahar → Dubai → Kandahar → Trenton.

4.3 Analysis of model parameters

An analysis was conducted to address the impact of key operational parameters and underlying assumptions on the aircraft routing solution. Three typical operational parameters (aircraft speed, aircraft type, stock level at the support hubs, and operational demand) are examined.

Aircraft speed

In the baseline scenario, a constant aircraft speed of 600 km/h for the CC-177 is used to determine the airlift time and cost. In this analysis, a more realistic representation of the aircraft speed, dependent on the aircraft load (i.e., equation from Section 3.3), is considered. Table 6 presents the analysis results and indicates that the optimal route remained the same as the baseline scenario but the total airlift cost decreased from $994,717 to $841,147. The load configuration in this scenario is also different from the baseline scenario. Indeed, the routing model tends to maximize the aircraft load weights for the short legs (e.g., Mombasa–Kandahar) and to minimize the aircraft load weights for the long legs (e.g., Trenton–Kandahar).

Table 6: *Pickup Nodes and Aircraft Loads for the Aircraft Speed Sensitivity Analysis*

Sortie	Pickup Node	# of Pallets of Supply Class				Aircraft Load	
		I	II	VI	IX	Total Pallets	Weight (tonne)
1	Trenton	10				10	30
2	Dubai	12			5	17	50
3	Dubai	8	5	5		18	46.5
4	Mombasa	10			7	17	49.6
5	Mombasa			5	13	18	48.9
Total		**40**	**5**	**10**	**25**	**80**	**225**

Aircraft type

In this analysis, the impact of changing the aircraft type on the routing solution is examined using the AN-124 performance characteristics and flying rate. Using equation (3.1), the number of aircraft sorties using the AN-124 aircraft payload and capacity characteristics is given by:

$$K = \left\lceil \max\left\{ \frac{\sum_{p=1}^{M} D_p}{C} \; , \; \frac{\sum_{p=1}^{M} w_p D_p}{L} \right\} \right\rceil$$

$$= \left\lceil \max\left\{\frac{80}{28}, \frac{40\times 3+5\times 2+10\times 2.5+25\times 2.8}{80}\right\}\right\rceil$$

$$= 3$$

The optimal route using the AN-124 aircraft is determined as follows:

Trenton → Kandahar → Mombasa → Kandahar → Dubai → Kandahar → Trenton.

The total airlift cost using the optimal route is $1,576,120. While the number of sorties of the AN-124 is smaller than the number of sorties of the CC-177, the airlift cost using the AN-124 is higher than the airlift cost using the CC-177. This can be explained by comparing the airlift ratio of each aircraft type. The airlift ratio (λ_L) is defined as the ratio of the flying rate (r) to the aircraft payload (L) times the aircraft cruise speed (v) and can be written as:

$$\lambda_L = \frac{r}{v\times L} \quad (4.1)$$

The airlift ratio can also be calculated using the aircraft capacity (C) instead of the aircraft payload (L) as follows.

$$\lambda_C = \frac{r}{v\times C} \quad (4.2)$$

Using equations (4.1) and (4.2), the airlift ratio of the CC-177 is $0.43/tonne×km or $1.20/pallet×km and the airlift ratio of the AN-124 is $0.58/tonne×km or $1.67/pallet×km, which explains the higher airlift cost with the AN-124. Table 7 presents the pickup nodes and a potential load configuration for the three AN-124 sorties.

Table 7: *Pickup Nodes and Aircraft Loads for the Aircraft Type Sensitivity Analysis*

Sortie	Pickup Node	# of Pallets of Supply Class				Aircraft Load	
		I	II	VI	IX	Total Pallets	Weight (tonne)
1	Trenton	12		10	4	26	72.2
2	Mombasa	8			20	28	80.0
3	Dubai	20	5		1	26	72.8
Total		**40**	**5**	**10**	**25**	**80**	**225**

Stock level

In this analysis, the impact of the stock level on the routing solution is examined by changing the stock availability at a given support hub in the baseline model. The analysis is conducted using the baseline scenario but assuming that the stock level in Dubai is composed of 20 pallets of supply class I only. The optimal route for this scenario is determined as:

Trenton → Kandahar → Dubai → Kandahar → Mombasa → Kandahar →
Mombasa → Kandahar → Spangdahlem → Kandahar → Trenton.

Compared with the baseline scenario, the hub in Dubai is used once and the hub in Spangdahlem would be used to complete the operational demand. The total airlift cost using the optimal route is $1,167,617 which corresponds to an increase of $172,900 with respect to the baseline scenario. The analysis indicates that the aircraft routing solution is sensitive to the stock level. Table 8 presents the pickup nodes and a potential aircraft load configuration for each sortie.

Table 8: *Pickup Nodes and Aircraft Loads for the Stock Level Sensitivity Analysis*

Sortie	**Pickup Node**	**# of Pallets of Supply Class**				**Aircraft Load**	
		I	**II**	**VI**	**IX**	**Total Pallets**	**Weight (tonne)**
1	Trenton	4		5	6	15	41.3
2	Dubai	16				16	48
3	Mombasa	10		5	2	17	48.1
4	Mombasa				17	17	47.6
5	Spangdahlem	10	5			15	40
Total		**40**	**5**	**10**	**25**	**80**	**225**

Operational demand

In this analysis, the impact of the operational demand on the routing solution is examined by changing the total number of pallets required for the mission. In addition to the baseline scenario data, we supposed some pallets of national items are also required. The analysis is conducted using the baseline scenario but assuming an additional number of pallets (e.g., 20, 40, and 60) of national items (e.g., Class V with an average pallet weight of 2.5 tonnes) to move from Trenton. Table 9 presents the optimal route and the lift cost for different demand requirements. For example, a demand of 100 pallets corresponds to the demand of the baseline scenario (80 pallets) plus an additional 20 pallets of national items.

The number of sorties for each scenario is calculated using equation (3.1). The analysis indicates that the optimal routes for the different demand scenarios are similar to the baseline scenario optimal route with one additional sortie from Trenton for each additional 20 pallets. While optimal solutions could be obtained for these scenarios, the time required to run an additional sortie increased significantly. For example, it took about 45 minutes to find an optimal solution for the baseline scenario but it took more than 6 hours for the scenario with 140 pallets. The solution runtime is sensitive to the size of the problem (number of the sorties, number of hubs, number of supply classes) and most probably to the weight/volume combination of classes of supply.

Table 9: *Optimal Route and Lift Cost for Different Operational Demands*

Demand (Class V)	Total Demand (pallets)	Number of Sorties	Optimal Route	Lift Cost ($)
0 (baseline)	80	5	Trenton → Kandahar → Dubai → Kandahar → Dubai → Kandahar → Mombasa → Kandahar → Mombasa → Kandahar → Trenton	994,717
20	100	6	Trenton → Kandahar → Trenton → Kandahar → Dubai → Kandahar → Dubai → Kandahar → Mombasa → Kandahar → Mombasa → Kandahar → Trenton	1,463,540
40	120	7	Trenton → Kandahar → Trenton → Kandahar → Trenton → Kandahar → Dubai → Kandahar → Dubai → Kandahar → Mombasa → Kandahar → Mombasa → Kandahar → Trenton	1,932,364
60	140	8	Trenton → Kandahar → Trenton → Kandahar → Trenton → Kandahar → Trenton → Kandahar → Dubai → Kandahar → Dubai → Kandahar → Mombasa → Kandahar → Mombasa → Kandahar → Trenton	2,401,187

4.4 Discussion

Following this study, several important points need to be discussed to clarify the scope and the limitations of the developed model. While there are many technical and operational factors that should be considered in the operational support aircraft routing problem, this study focused only on the cost of the materiel distribution lift. Delivery date, aircraft maintenance constraints (i.e., aircraft maintenance should be performed after a certain number of flying hours), crew change requirements (i.e., crew has a maximum number of flying hours per day), and multiple aircrafts are also important factors for consideration in planning sustainment operations. To address these issues, required delivery times for the different classes of supply should be estimated and included in the operational demand. In addition, aircraft maintenance and crew change schedules should be introduced in the problem formulation. By adding these parameters and constraints in the aircraft routing model, the problem would become more complicated (i.e., integrating routing and scheduling) and difficult to solve with Branch-and Bound algorithms. Other combinatorial optimization techniques such as metaheuristic algorithms could be required to address the complexity of the problem. For example, an iterative procedure using evolutionary algorithms could be appropriate for this problem.

While the aircraft routing model is formulated as a pickup problem, it could easily be adapted to address delivery problems such as the movement of materiel during force repatriation. In this case, the demand represents the materiel to be moved from the theatre and the hubs represent the different repatriation destinations. The aircraft routing model could also be used to address the impact of hub disruption (e.g., following a terrorist attack) or hub closure on the sustainment cost.

Finally, in the aircraft routing model, the cost associated with refuelling services is neglected to simplify the problem solving. To address this issue, an additional cost for refuelling should be included in the model. The objective function (equation 3.2) would be replaced by the following equation:

$$\sum_{i \in V} \sum_{j \in V} \left(r \frac{d_{ij}}{v} + \alpha \left(\left\lceil \frac{d_{ij}}{g} \right\rceil - 1 \right) \right) \sum_{k=1}^{K} x_{ijk} \tag{4.3}$$

where α is the refuelling service cost ($) and g is the aircraft range (km). This formulation further increases the degree of nonlinearity and the difficulty of the problem.

In equation (4.3), the refuelling cost factor is assumed to be constant. In practice, the cost for refuelling services depends on refuelling locations. The additional cost associated with the travel time to reach refuelling stops should also be included. To take into consideration these factors, the model should be modified to include the aircraft range limit and some potential refuelling locations. Another alternative would be to modify the travel distance to include an equivalent to the refuelling cost (i.e., including additional distance flown to the refuelling locations). This would imply using an asymmetrical model.

5. Conclusions and recommendations

5.1 Conclusions

This paper addressed a complex military aircraft routing problem related to the CANOSCOM hub-based operational support concept. A mathematical model was developed to determine optimal aircraft routes for the movement of cargo and supplies from various support hubs to a theatre of operation. The model was formulated as a vehicle routing problem and was implemented using mixed integer nonlinear programming. A variable aircraft speed variant of the model was also formulated and examined. An example using historical deployment scenarios and randomly generated sustainment data was presented to illustrate the methodology.

The model developed in this study could be used by CANOSCOM to plan sustainment flights for deployed operations, to determine optimal airlift routes for supporting domestic operations or exercises, to conduct movement options and 'What-If" scenarios (e.g., hub disruption) analysis, and to further study the impact of various logistics parameters and constraints (e.g., hub capacity, hub location, operational demand, etc) on the operational support performance. The model is flexible and can easily be adapted and applied to address other military airlift problems (e.g., personnel deployment lift, troop rotations) or commercial transportation problems.

5.2 Recommendations

A number of recommendations for the decision makers appear to logically follow from this study:

1. To effectively use the hub-based support concept, CANOSCOM should consider the operational support aircraft routing model for planning future sustainment flights. Movement planners and supply Subject Matter Experts (SMEs) should provide data required to implement the methodology.

2. When using the aircraft routing model, the stock levels at hubs should be evaluated accurately during each sustainment period, as they could affect the optimal aircraft routing solution.

3. While the model is formulated as a pickup problem, it could easily be adapted to address delivery problems such as the movement of materiel during force repatriation. It is recommended that CANOSCOM consider the model to plan movement options for the drawdown of Joint Task Force Afghanistan. As such, SMEs should identify the materiel that should be shipped back to Canada and the materiel that should be post-positioned at the support hubs (e.g., Camp Mirage, Spangdahlem) in order to facilitate the route planning and optimization.

4. Finally, further analysis should be conducted to address other aspects of the operational support aircraft routing problem, including movement of personnel; priority of the items; aircraft maintenance; crew change requirements; stochastic demand of the items; stock availability at the hubs, and backhaul problems.

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Annex A

Operational support aircraft routing model

The operational support aircraft routing model was implemented in LINGO. LINGO is an optimization tool for solving linear, integer, and nonlinear programming problems. For nonlinear or mixed integer nonlinear problems, LINGO uses a *Global Server* solver. The solver combines a series of range bounding algorithms (e.g., interval analysis and convex analysis) and range reduction techniques (e.g., linear programming and constraint propagation) within a branch-and-bound framework to find global solutions to non-convex nonlinear problems. This Annex presents the LINGO code for the aircraft routing model. The code is for the basic model and does not include the variable aircraft speed model variant. The code also includes the data for the example of Section 4.

```
Model:
! Operational Support Aircraft Routing Model. This model was formulated
to address aircraft routing problems associated with the CANOSCOM Hub-
based support concept. The problem considers multi-depots (operational
hubs), a single customer (one line of operation), multiple commodities
(classes of supply), one vehicle (aircraft) with limited capacity
(payload, volume) and multiple trips (aircraft sorties).


**********************************************************************

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                        DRDC CORA, Ottawa, ON, Canada
                        2009

**********************************************************************;

! Distribution Nodes:

      1. Trenton  2. Spangdahlem    3. Dakar    4. Mombasa
      5. Dubai    6. Singapore      7. Seoul    8. Kandahar (Theatre);


SETS:
   Hub / 1..8/;
   Supply / 1..4/: Demand, Pallet_Weight;
   Sortie / 1..5/;
   Hub_Hub(Hub, Hub): Distance;
   Hub_Sortie(Hub, Sortie): Y, U, W;
   Hub_Supply(Hub, Supply): Stock_0;
   Hub_Hub_Sortie(Hub, Hub, Sortie): X;
   Hub_Supply_Sortie(Hub, Supply, Sortie): Q, Stock;
ENDSETS

! Objective Function;
   [OBJ]  Min  =  (Rate/Speed)  *  @SUM(Sortie(K):  @SUM(Hub_Hub(I,  J):
   Distance(I, J) * X(I, J, K)));

! N is the number of nodes and also the index of the theatre node
   N = @SIZE(Hub);
```

```
   M = @SIZE(sortie);

! Hub Nodes for sortie 1;
   ! Constraint (3.3);
   @FOR(Hub(I)  |  I  #NE#  1  #and#  I  #NE#  N:  @SUM(Hub(J):  X(I,J,1))  =
   @SUM(Hub (J): X(J, I, 1)));
   @FOR(Hub(I) | I #NE# 1 #and# I #NE# N: @SUM(Hub (J): X(I, J, 1)) =
   Y(I, 1));

   ! Constraint (3.4);
   @SUM(Hub (J): X(1,J,1)) = 1;
   Y(1, 1) = 1;

! Hub Nodes;
   ! Constraint (3.5);
   @FOR(Sortie(K) | K #NE# 1: @FOR(Hub(I) | I #NE# N: @SUM(Hub(J): X(I,
   J, K)) =  @SUM(Hub(J) : X(J, I, K))));
   @FOR(Sortie(K) | K #NE# 1: @FOR(Hub(I) | I #NE# N: @SUM(Hub(J): X(I,
   J, K)) =  Y(I, K)));

! Theatre Node;
   ! Constraint (3.6);
   @FOR(Sortie(K) : @SUM( Hub (I): X(I, N, K)) = 1);
   ! Constraint (3.7);
   @FOR(Sortie(K) | K #NE# 1 : @SUM( Hub (J): X(N, J, K)) = 1);

! Case for i=j (Constraint 3.14);
   @FOR(Sortie(K) : @FOR(Hub(I) : X(I, I, K) = 0));

! Aircraft to return to main base (Constraint 3.15);
   X(N, 1, M) = 1;

! Aircraft Payload Constraints (3.8);
   @FOR(Sortie(K): @SUM(Hub(I) | I #NE# N: @SUM(Supply(P):
   Pallet_Weight(P) * Q(I, P, K))) <= Payload);

! Demand Constraint (3.10);
   @FOR(Supply(P)  :  @SUM(Sortie(K)  :  @SUM(Hub(I)  |  I  #NE#  N  :  Q(I,  P,
   K))) = Demand(P));


! Stock Availability Constraint (3.11);
   @FOR(Hub_Sortie(I,  K)  |  I  #NE#  N  :  @FOR(Supply(P):  Q(I,  P,  K)  <=
   Y(I, K) * Stock(I, P, K)));

! Stock Availability at theatre node (Constraint 3.13);
   @FOR(Sortie(K) : @FOR(Supply(P): Q(N, P, K) =  0));

! Stock Level Constraint (3.18);
   @FOR(Hub_Supply_Sortie  (I,  P,  K)  |  I  #NE#  N  #AND#  K  #GT#  1  :
   Stock(I, P, K) = Stock (I, P, K-1) - Q(I, P, K-1));
   @FOR(Hub_Supply(I, P) | I #NE# N : Stock(I, P, 1) = Stock_0 (I, P));

! Sub-tour Elimination Constraints (Constraint 3.19 including 3.20);
   @FOR(Sortie(K)  :  @FOR(Hub(I)  |  I  #NE#  N  :  @FOR(Hub(J)  |  J  #NE#  I
   #AND# J #NE# N: U(I, K) - U(J, K) + X(I, J, K) * Capacity  + X(J, I,
```

```
   K) * (Capacity - @SUM(Supply(P) :Q(I, P, K)) - @SUM(Supply(P) : Q(J,
   P, K))) <= Capacity - @SUM(Supply(P) : Q(J, P, K)))));

   ! Constraint 3.21 including 3.20;
   @FOR(Sortie(K) : @FOR(Hub(I) | I #NE# N : U(I, K) <=  Capacity));
   @FOR(Sortie(K) : @FOR(Hub(I) | I #NE# N : U(I, K) >= @SUM(Supply(P)
   : Q(I, P, K))));

! Variable Domains;
   @FOR(Hub_Hub_Sortie : @BIN(X));
   @FOR(Hub_Supply : @GIN(Stock_0));
   @FOR(Hub_Supply_Sortie : @GIN(Q));
   @FOR(Hub_Sortie : @BIN(Y));
   @FOR(Hub_Sortie : @GIN(U));
   @FOR(Hub_Supply_Sortie : @GIN(Stock));

! Model parameters;
  DATA:

  ! Operational Demand;
  Demand          =     40,     5,    10,     25;

  ! Pallet Weight;
  Pallet_Weight   =      3,     2,   2.5,    2.8;

  ! Aircraft;
  Capacity  =     18;
  Payload   =     50;
  Rate      =     13;               (in $1000)
  Speed     =     600;              (in km/h)

  ! Travel Distance;
  Distance =

!         Tren     Spang    Dakar    Momb     Dub     Sing    Seoul    Kand;
!Tren;       0,     6069,    6494,  12455,  10930,  14949,  10621,  10819,
!Spang;  6069,        0,    4483,   6779,   4966,  10407,   8637,   5230,
!Dakar;  6494,     4483,       0,   6619,   7609,  13327,  13085,   8533,
!Momb;  12455,     6779,    6619,      0,   3671,   7177,   9998,   4828,
!Dub;   10930,     4966,    7609,   3671,      0,   5846,   6727,   1240,
!Sing;  14949,    10407,   13327,   7177,   5846,      0,   4624,   5217,
!Seoul; 10621,     8637,   13085,   9998,   6727,   4624,      0,   5495,
!Kand;  10819,     5230,    8533,   4828,   1240,   5217,   5495,      0;

  !Initial Stock Level;
   Stock_0     =   1000,    1000,    1000,    1000,
                     20,      10,      10,      15,
                     10,       0,       5,       0,
                     10,       0,       5,      20,
                     20,      10,      10,      10,
                     20,       0,      10,       0,
                     10,       5,       0,      20,
                      0,       0,       0,       0;

ENDDATA
end
```

Note that some distances exceed the aircraft range. If the travel distance is greater than the aircraft range then refuelling stops are required. The cost associated with refuelling services is neglected in the model. Future implementation of the model should include the aircraft range parameter as discussed in Section 4.4.

List of acronyms

AN-124	Antonov An-124 Ruslan
CANOSCOM	Canadian Operational Support Command
CC-177	Canadian version of Boeing C-17 Globemaster III
CDS	Chief of Defence Staff
CF	Canadian Forces
ISAF	International Security Assistance Force
JTF	Joint Task Force
LP	Linear programming
MINLP	Mixed Integer Nonlinear Programming
MN ITD	Multinational Intra-Theatre Distribution
NP-hard	Nondeterministic Polynomial-time hard
OSARP	Operational Support Aircraft Routing Problem
SME	Subject Matter Expert
STRATL	Strategic Mobility Optimization Model for Lift Capability Analysis
TPLSS	Third Party Logistics Supply Services
VRP	Vehicle Routing Problem

Distribution letter

September 2009

Distribution List

DISTRIBUTION OF DRDC CORA TM 2009-044, AN OPERATIONAL SUPPORT AIRCRAFT ROUTING MODEL

1. Enclosed for your information is DRDC CORA TM 2009-044 entitled “An Operational Support Aircraft Routing Model” prepared by Dr Ahmed Ghanmi of the CANOSCOM OR&A team of DRDC CORA.

2. The Canadian Forces is seeking to establish permanent and temporary operational support hubs at strategic locations around the globe to improve its logistics support effectiveness and responsiveness for deployed operations. These hubs will be used for cross-loading between modes of transportation as well as for the pre-positioning of non-perishable supplies (i.e., various non-perishable supplies could be procured at local area and positioned at hubs for future delivery) to reduce transportation costs and to improve the speed of delivery. This paper presents a mathematical model developed to address aircraft routing problems associated with the Canadian Operational Support Command hub-based support concept. The model could be used for planning sustainment flights to determine cost (or time) effective aircraft routes for the movement of cargo and supplies from various support hubs to a theatre of operation. The model is formulated as a vehicle routing problem and is implemented using mixed integer nonlinear programming. An example using historical deployment and sustainment data is presented to illustrate the methodology.

3. Comments or suggestions on this report are welcome and should be addressed to Dr. Ahmed Ghanmi at 613-945-2166 or Ahmed.Ghanmi@forces.gc.ca.

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The Canadian Forces is seeking to establish permanent and temporary operational support hubs at strategic locations around the globe to improve its logistics support effectiveness and responsiveness for deployed operations. These hubs will be used for cross-loading between modes of transportation as well as for the pre-positioning of non-perishable supplies to reduce transportation costs. This paper presents a mathematical model developed to address aircraft routing problems associated with the Canadian Operational Support Command hub-based support concept. The model could be used for planning sustainment flights to determine cost (or time) effective aircraft routes for the movement of cargo and supplies from various support hubs to a theatre of operation. The model was formulated as a vehicle routing problem and was implemented using mixed integer nonlinear programming. An example using historical deployment and sustainment data was presented to illustrate the methodology.

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