



Tracking a broadband acoustic source in the near field using a particle filter

Marius Birsan

Defence R&D Canada – Atlantic

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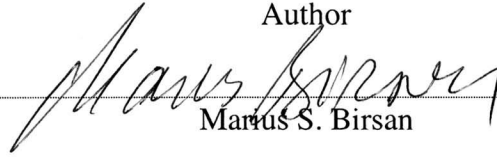
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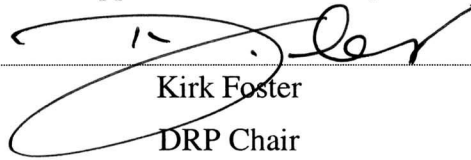
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Abstract

This report describes a numerical method that may be used to efficiently locate and track underwater sonar targets in the near field for the case of very small passive arrays. The waves emitted by the target are considered as curved rather than plane. The method uses only two pairs of acoustic sensors and is a two-step process. In the first step the time-delays are estimated for each pair of hydrophones using the generalised cross-correlation function. The second step consists of fusing this information based on the known geometry of the array to come up with the best estimate of target position. The optimal solution to the real time positioning problem is given by the recursive Bayesian filter. A transition equation describes the prior distribution of the desired parameters (target position and velocity), the so-called hidden state process, and an observation equation describes the likelihood of the observations (measurements). The determination of target position and velocity is formulated as an optimal stochastic estimation problem, which could be solved using a sequential Monte Carlo based approach known as particle filter. In addition to the conventional particle filter, the proposed tracking and classification algorithm uses the unscented Kalman filter (UKF) to generate the prior distribution of the unknown parameters. Finally, it is demonstrated the ability of the approach to track a fishing vessel over a period of time using the data collected by the Rapidly Deployment System array.

Résumé

Le présent rapport décrit une méthode numérique qui peut être utilisée pour localiser et poursuivre efficacement des cibles sonar sous-marines en champ proche dans le cas des très petits réseaux passifs. Les ondes émises par la cible sont considérées comme étant courbes plutôt que planes. La méthode ne fait appel qu'à deux paires de capteurs acoustiques et correspond à un processus à deux étapes. À la première étape, les retards sont estimés pour chaque paire d'hydrophones à l'aide de la fonction de corrélation croisée généralisée. La seconde étape consiste à fusionner cette information en fonction de la géométrie connue du réseau pour en arriver à la meilleure estimation de la position de la cible. La solution optimale du problème de positionnement en temps réel est fournie par l'algorithme récursif du filtre bayésien. Une équation de transition décrit la distribution a priori des paramètres voulus (position et vitesse de la cible), le processus dit d'état caché, et une équation d'observation décrit la probabilité des observations (mesures). La détermination de la position et de la vitesse de la cible est formulée comme un problème d'estimation stochastique optimale, qui pourrait être résolu au moyen d'une méthode Monte Carlo séquentielle appelée filtre particulière. En plus du filtre particulière classique, l'algorithme de poursuite et de classification proposé fait appel à l'estimateur de Julier et Uhlmann pour générer la distribution a priori des paramètres inconnus. Finalement, on démontre la capacité de la méthode à poursuivre un navire de pêche pendant un certain temps au moyen des données collectées par le réseau à déploiement rapide.

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Executive summary

Introduction

A requirement exists to improve the Navy's capability to conduct anti-submarine warfare (ASW) and intelligence, surveillance and reconnaissance (ISR) operations in littoral waters. To address this problem, a technology demonstration project (TDP) was set up to develop a Rapidly Deployable System (RDS). The RDS concept includes an array of acoustic, electric and magnetic sensors, communication links using radio frequency buoys and underwater acoustic modems, the necessary hardware, and automated (in-node) signal processing of the acoustic, electric and magnetic data. Once the data was collected and processed, the transmitted information would allow the detection, localization and classification of any vessel navigating in the area. Therefore, one of the objectives of the automated signal-processing task in the RDS project is to develop, test and integrate algorithms and software that improve the detection, localization and classification of the target making use of the magnetic data. The work described in this report is part of the development of the RDS automated signal processing capability.

Work description and results

This report presents a numerical method that may be used to efficiently locate and track underwater sonar targets in the near field for the case of very small passive arrays. The method is applied to track a fishing boat using real acoustic data obtained from the RDS array. The RDS array can be divided into sub-arrays with the minimum requirement of having two hydrophones per sub-array. The sensor system for the present method consists of two array sensors (from the RDS array), each one with two hydrophones. The tracking calculation has two steps: in the first step the time-delays are estimated for each pair of hydrophones; the second step consists of combining or fusing this information based on the known geometry of the array to come up with the best estimate of target position.

The common technique to calculate the time delay between broadband signals is the generalized cross-correlation (GCC) method. The estimated time delay for a pair of sensors is assumed to be the delay that maximizes the GCC function for that pair. Fusing the pair-wise time delay estimates to obtain the best estimate of the target position is done using the Unscented Particle Filter (UPF) algorithm. The particle filter is an alternative for real-time applications classically approached by model-based Kalman filter techniques. For this application, a one second update is considered real time. The tracking problem is formulated in state-space form where the state variables are the position and the velocity of the target. The results demonstrate the ability of the approach to track a real target that generates a broadband acoustic signal. By determining the elevation of the target relative to the array, which is placed on the ocean floor, the classification of the vessel as a surface or submerged ship is also possible supposing the water depth in the area is known.

Significance and future work

The need for tracking under realistic conditions has motivated a series of trials where acoustic and magnetic signals from various ships were recorded. The application of the presented algorithms to the experimental data indicated the possibility to incorporate them into the Rapidly Deployable System TD project.

Birsan M. 2005. Broadband acoustic source tracking in near field using particle filter.
DRDC Atlantic TM 2005-079. Defense R&D Canada – Atlantic.

Sommaire

Introduction

Il existe un besoin d'améliorer la capacité des forces navales à mener la guerre anti-sous-marine (GASM) et les opérations de renseignement, de surveillance et de reconnaissance (RSR) en eaux littorales. À cette fin, un projet de démonstration de technologies (PDT) a été mis sur pied pour développer un système à déploiement rapide (SDR). Le concept du SDR comprend un réseau de capteurs acoustiques, électriques et magnétiques, des liaisons de communication faisant appel à des radiophares et à des modems acoustiques sous-marins, le matériel nécessaire et le traitement automatisé (par les nœuds) des signaux (données acoustiques, électriques et magnétiques). Une fois que les données collectées seront traitées, l'information transmise permettra la détection, la localisation et la classification de tout navire évoluant dans les parages. Par conséquent, un des objectifs de la tâche de traitement automatisé des signaux du projet du SDR consiste à élaborer, à essayer et à intégrer des algorithmes et du logiciel qui améliorent la détection, la localisation et la classification de la cible au moyen des données magnétiques. Les travaux décrits dans le présent rapport font partie du développement de la capacité de traitement automatisé des signaux du SDR.

Description du travail et résultats

Le présent rapport décrit une méthode numérique qui peut être utilisée pour localiser et poursuivre efficacement des cibles sonar sous-marines en champ proche dans le cas des très petits réseaux passifs. Cette méthode est appliquée à la poursuite d'un navire de pêche au moyen de données acoustiques réelles obtenues du réseau SDR. Le réseau SDR peut se diviser en sous-réseaux avec un minimum de deux hydrophones par sous-réseau. Le système de capteurs de la méthode en cause comprend deux capteurs du réseau SDR, reliés chacun à deux hydrophones. Le calcul de poursuite comprend deux étapes : À la première étape, les retards sont estimés pour chaque paire d'hydrophones à l'aide de la fonction de corrélation croisée généralisée; la seconde étape consiste à combiner ou fusionner cette information en fonction de la géométrie connue du réseau pour en arriver à la meilleure estimation de la position de la cible.

La technique courante de calculer le retard entre les signaux à large bande est la méthode de corrélation croisée généralisée (CCG). C'est le retard qui maximise la fonction CCG pour une paire de capteurs qui est pris comme le retard estimé pour cette paire. Le fusionnement des estimations du retard par paire de capteurs pour obtenir la meilleure estimation de la position de la cible est réalisé au moyen de l'algorithme de l'estimateur de Julier et Uhlmann. Cet estimateur est une solution de rechange pour les applications en temps réel anciennement réalisées au moyen des techniques faisant appel au filtre de Kalman basé sur un modèle. Pour cette application, une mise à jour de une seconde est considérée comme du temps réel. Le problème de poursuite est formulé sous la forme d'une équation d'espace d'états où les variables d'état sont la position et la vitesse de la cible. Les résultats démontrent la capacité de la méthode à poursuivre une cible réelle qui produit un signal acoustique à large bande. De plus, la détermination du site de la cible par rapport au réseau, qui est placé sur le fond marin, permet de classer le navire comme un navire de surface ou sous-marin, en supposant que la profondeur de l'eau de la zone soit connue.

Portée et recherches futures

Le besoin de la poursuite en conditions réalistes a donné lieu à une série d'essais pendant lesquels les signaux acoustiques et magnétiques de divers navires ont été enregistrés. L'application des algorithmes présentés aux données expérimentales a indiqué la possibilité de les incorporer dans le projet DT du système à déploiement rapide.

Birsan, M. 2005. Broadband acoustic source tracking in near field using particle filter (poursuite de sources acoustiques en champ proche au moyen d'un filtre particulaire). RDDC Atlantique TM 2005-079. R & D pour la défense Canada – Atlantique.

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1. Introduction

Recent defence strategy requires the monitoring of the activity of surface and underwater vessels in a certain area. For this reason, a surveillance Rapidly Deployable System (RDS) was conceived that includes an array of acoustic and magnetic sensors, communication links using radio frequency buoys and underwater acoustic modems, the necessary hardware, and automated (in-node) signal processing of the acoustic and magnetic data. The sensor system for this design consists of 34 hydrophones and 3 tri-axial magnetometers. One of the objectives of the Rapidly Deployable System (RDS) Project is to determine the location of an acoustic source relative to the sensor system.

This report describes a numerical method that may be used to efficiently locate and track underwater sonar targets in the near field for the case of passive arrays with a very small number of sensors. The sensors system used to exercise this method consists of two hydrophone array sensors. The waves emitted by the target should be considered as curved rather than plane, meaning that the present method is more general than the classical beamforming technique. From the acoustic RDS array one can construct sub-arrays with the minimum requirement of having two hydrophones per sub-array. The method presented here uses only two pairs of sensors in the RDS array and is a two-step process. In the first step the time-delays are estimated for each pair of hydrophones. The second step consists of combining or fusing this information based on the known geometry of the array to come up with the best estimate of target position.

The common technique to calculate the time delay between broadband signals is the generalized cross-correlation (GCC) method. The estimated time delay for a pair of sensors is assumed to be the delay that maximizes the GCC function for that pair. In one possible modification of the GCC method, called the phase transform (PHAT), the amplitude information of the signals is discarded by cross-correlating only the phases. The GCC-PHAT function is better dealing with reverberation.

Fusing the pair-wise time delay estimates to obtain the best estimate of the target position is done using the Unscented Particle Filter (UPF) algorithm. The particle filter is an alternative for real-time applications classically approached by model-based Kalman filter techniques. For this application, a one second update is considered real time.

The target tracking is regarded as an optimal stochastic estimation problem where one seeks the probability distribution of the unknown parameters (target position and velocity) conditioned to the measured data. The measurements are the range differences from the target position to the sensor arrays. The problem is formulated in state-space form where the state variables are the position and velocity of the target. Define a vector \mathbf{x}_k as the state of the system at time step k , a sequence $\mathbf{z}_{1:k} = \{\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_k\}$ as the observation (measurements) history of a system from time 1 to k , the estimate of this state (conditional mean) $\hat{\mathbf{x}}(i|j) = E\{\mathbf{x}(i) | \mathbf{z}_{1:j}\}$ and the estimate covariance $\mathbf{P}(i|j) = E\{\hat{\mathbf{x}}(i|j)\hat{\mathbf{x}}^T(i|j) | \mathbf{z}_{1:j}\}$. Because of either noise in the state evolution process or uncertainty as to the exact nature of the process itself, the state vector \mathbf{x}_k is generally regarded as a random variable. The unknown states follow a first order Markov process of initial distribution $p(\mathbf{x}_0)$ and transition equation $p(\mathbf{x}_k | \mathbf{x}_{k-1})$ that describes the prior distribution of the signal of interest $\{\mathbf{x}_k, k \in N\}$.

In the Bayesian filtering technique, one attempts to construct an estimate of the posterior probability density function (PDF), $p(\mathbf{x}_k | \mathbf{z}_{1:k})$. Since all information provided by $\mathbf{z}_{1:k}$ is conveyed by the posterior density, it may be said to be the complete solution to the

estimation problem. A recursive Bayesian algorithm imposes the constraint that the estimate of $p(\mathbf{x}_k | \mathbf{z}_{1:k})$ should be generated solely from the previous posterior density, $p(\mathbf{x}_{k-1} | \mathbf{z}_{1:k-1})$, and the most recent measurement \mathbf{z}_k . In this way, it is not necessary to store the complete data set or to reprocess existing data when a new measurement becomes available.

The recursive propagation of the posterior density is only a conceptual solution that can be determined analytically only in a restrictive set of cases. When the analytical solution is intractable, a Monte Carlo based approach to recursive Bayesian filtering called the particle filter, is one method that approximates the optimal Bayesian solution. In the Monte Carlo method, a set of random samples (particles) are drawn from a target distribution such as $p(\mathbf{x} | \mathbf{z})$. In general, this distribution is not known. We will use $q(\mathbf{x}_k | \mathbf{z}_{1:k}) \neq p(\mathbf{x}_k | \mathbf{z}_{1:k})$ to denote a proposal distribution from which samples can be drawn. The main drawback of the conventional particle filter is that it uses transition prior, $p(\mathbf{x}_k | \mathbf{x}_{k-1})$, as the proposal distribution. The transition prior does not take into account current observation data. To overcome this difficulty, the unscented Kalman filter (UKF) was proposed to generate better proposal distributions by taking into consideration the most recent observation.

Finally, using real data collected by an RDS array, the ability to track a fishing vessel over a period of time is demonstrated.

2. Principle of position estimation

Three different localization methods are mainly used today: angle-of-arrival (AOA), received-signal-strength (RSS), and propagation-time based measurements, which include time-of-arrival (TOA) and time-difference-of-arrival (TDOA). Usually time-based positioning method outperforms the AOA method [1], and most of the available solutions today are time-based positioning system. In general, direct TOA results in two problems. First, all transmitters and receivers in the system have to be precisely synchronized. Second, the transmitting signal must be labeled with a timestamp in order for the receiver to discern the distance the signal has traveled. The method works well for the Global Positioning System (GPS).

RSS is based on propagation-loss equations. It uses a known mathematical model describing the path loss attenuation with distance. Since a measurement of signal strength provides a distance estimate between the mobile target and the base stations, the target must lie on a circle centered at the base station. By using multiple base stations, the location of the target can be determined.

For more practical means of position location of an acoustic source in the ocean, the TDOA method is proposed. Unlike TOA, TDOA only require that the fixed location receivers have precisely synchronized clocks, making TDOA more realistic than requiring each mobile unit to have an accurate clock.

The idea behind TDOA is to determine the relative position of the mobile transmitter by examining the difference in time at which the signal arrives at multiple base station receivers. Therefore, each TDOA measurement determines that the transmitter must lie on a hyperboloid with a constant range difference between the two receivers. The equation of the hyperboloid is given by:

$$R_{i,j} = \sqrt{(r_x - r_i^x)^2 + (r_y - r_i^y)^2 + (r_z - r_i^z)^2} - \sqrt{(r_x - r_j^x)^2 + (r_y - r_j^y)^2 + (r_z - r_j^z)^2} \quad (1)$$

where (r_i^x, r_i^y, r_i^z) and (r_j^x, r_j^y, r_j^z) represent the fixed receiver i and j , and (r_x, r_y, r_z) represents the coordinate of the target.

A two-dimensional target location can be estimated from the intersection of two or more TDOA measurements, as shown in Figure 1. Two hyperbolas are formed from TDOA measurements at three fixed base stations to provide an intersection point that locates the target. Three dimensional location estimates require at least four independent TDOA measurements.

2.1 Computing TDOA

Sources of error in acoustic location systems include multi-path propagation, and correlated noise. Multi-path propagation is the primary reason for inaccuracies observed in the AOA and RSS systems. Multi-path also affects the time-based location systems, causing errors in the timing estimates. Therefore, some efforts must be taken to mitigate these impairments to improve the location accuracy. Note that a small error in making bearing

estimates or a small error in knowing the location of the sensors can make a significant difference in the estimated range of the acoustic source.

The conventional methods for computing these time estimates are to use correlation techniques. TDOA estimation can be from the cross-correlation between signals received at a pair of base stations. Suppose the transmitted signal $s(t)$, and the signal $x_i(t)$ is received at base station i , corrupted by the noise $n_i(t)$ and delayed by d_i such that $x_i(t) = s(t - d_i) + n_i(t)$. Similarly, the signal $x_j(t) = s(t - d_j) + n_j(t)$ that arrives at base station j is delayed by d_j and corrupted by the noise $n_j(t)$.

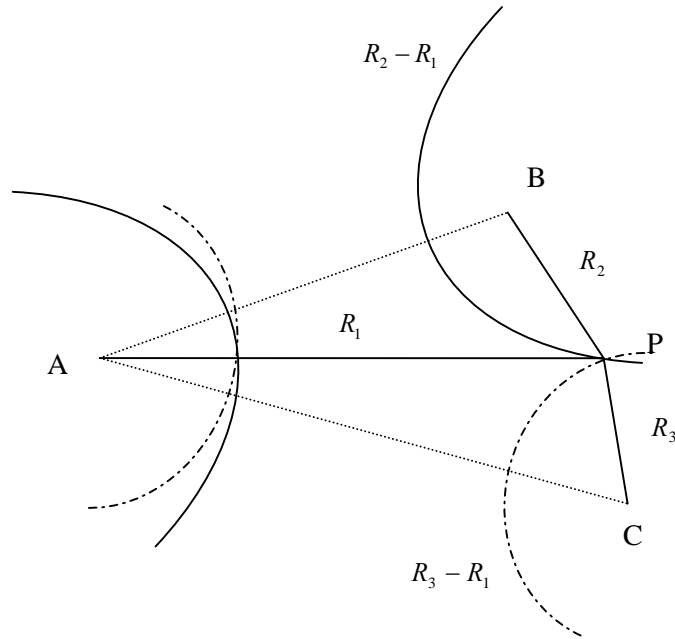


Figure 1. 2-D positioning of P based on TDOA measurements from stations A , B and C .

The cross-correlation function of these signals is given by integrating the lag product of two received signal over a sufficiently long time period T .

$$C_{x_i, x_j}(\tau) = \frac{1}{T} \int_0^T x_i(t) x_j(t - \tau) dt \quad (2)$$

The TDOA estimate is the value, τ , that maximizes $C_{x_i, x_j}(\tau)$. This approach requires that base stations share a precise time reference and reference signals, but does not impose any requirement on the signal transmitted.

The cross-correlation (2) can be written in the frequency domain:

$$C_{x_i, x_j}(\tau) = \frac{1}{2\pi} \int_{-\pi}^{\pi} G_{x_i, x_j}(\omega) e^{j\omega\tau} d\omega \quad (3)$$

where $G(\omega)$ is the Fourier transform of $C(\tau)$. In the most simplified case [2], the following assumptions are made:

1. signal and noise are un-correlated
2. noises at two sensors are un-correlated
3. there is no reverberation.

Because in the real world the assumptions 2 and 3 are not valid most of the time, the estimation of TDOA based on (3) can easily break down. To deal with this problem, various frequency weighting functions have been proposed, and the resulting framework is called generalized cross-correlation (GCC):

$$C_{x_i, x_j}(\tau) = \frac{1}{2\pi} \int_{-\pi}^{\pi} W(\omega) G_{x_i, x_j}(\omega) e^{j\omega\tau} d\omega \quad (4)$$

where $W(\omega)$ is the frequency weighting function. Previous research [3] suggests that the maximum likelihood (ML) weighting function is robust to ambient noise and that the phase transform (PHAT) weighting function is better dealing with reverberation. All these and more GCCs are described in [2].

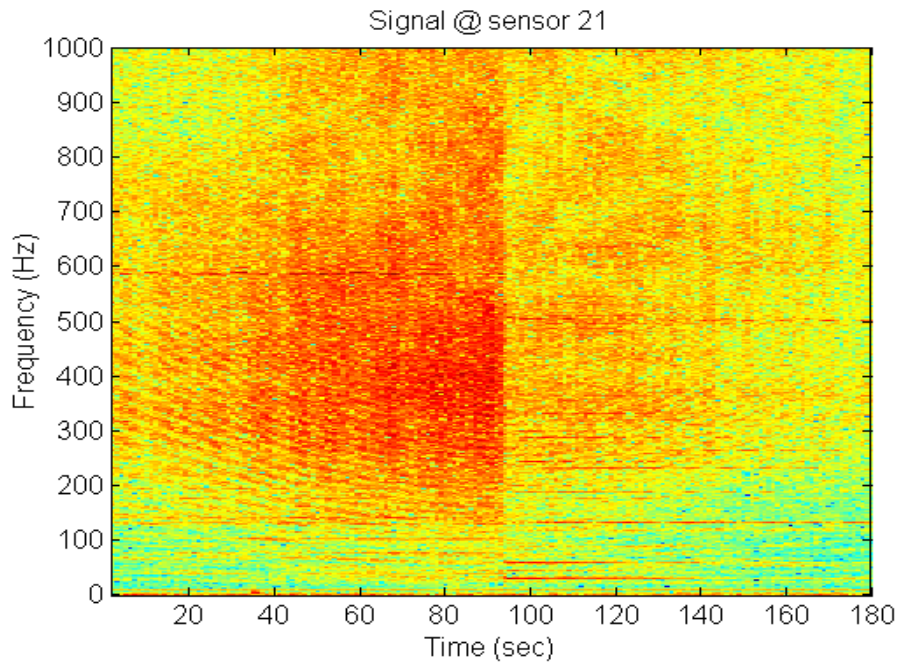


Figure 2. Time-frequency representation of the acoustic signal at sensor 21.

TDOA based methods are directly applicable to broadband signals. A drawback of these methods is that they are useful only for the case of a single acoustic signal impinging on the array. An example of TDOA calculation is presented using data collected from the RDS array during a trial in St. Margaret's Bay, Nova Scotia, where the water depth is about 65m. The acoustic source is a fishing vessel generating a broadband signal whose time-frequency representation is shown in Figure 2. The signal was recorded at 2000Hz sampling rate. For the tracking application, the signal was windowed to obtain a one second update. The GCC-PHAT of channels 24 and 28 (called "array 1") of the RDS array is presented in Fig. 3 using 2000 samples (one second signal recording). To obtain a better (between samples) localization of the peak in the GCC function, a 3-point interpolation curve was used.

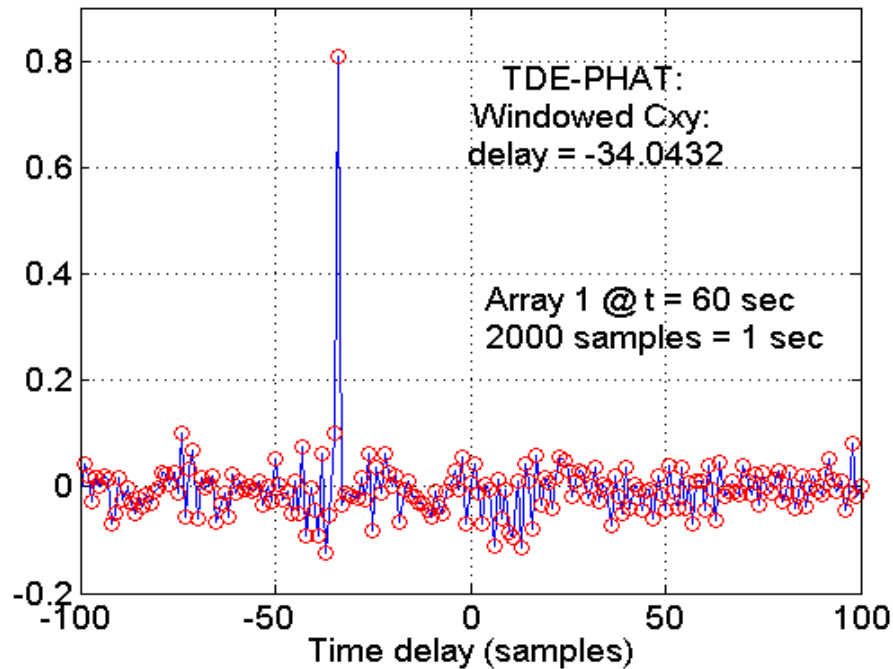


Figure 3. TDOA as calculated by the GCC-PHAT.

One can see in Fig. 3 that the reverberation does not introduce errors in the TDOA calculation in the sense that the peak of the GCC function is well localized. The small peaks on the left that might be due to later arrivals are not a factor in the TDOA calculation.

2.2 Determining position of the target

When TDOA method is used, a straightforward approach to locate the target uses a geometric method to compute the intersection points of hyperbolas of TDOA. The theory for location estimation has been thoroughly developed in the literature. The exact solutions to the hyperbolic TDOA equations can be found in [4]. An easier method of solution is to linearize the equations through use of a Taylor-series expansion and create an iterative algorithm [5].

As mentioned, a three-dimensional location estimate requires at least four fixed base stations. The stations could be on two different planes, but not on the same line, to obtain the position at the intersection of three hyperboloids. Obviously, this is not the situation in the case of the RDS system, which is a one-dimensional array. Let us assume that the X-axis is along the RDS array baseline with the origin at one of the hydrophones. If two or more colinear sub-arrays are used, the range difference measurements will localize the target on a circle situated in the Y-Z plane at the intersection of two or more cones (degenerated hyperboloids). In this case, there is a concern if the location of the source can be determined from the array data uniquely, i.e. about the system observability. A system is observable when its state vector can be reconstructed from the measurements of its output. In the practical problem discussed here, it can be seen that the system is not observable and additional information may be required.

Observability of a time-varying system depends on both the measurements and the state equations (motion model). When the measurements can resolve the target parameters (e.g. range differences) uniquely, we only need to examine the observability of the target motion dynamics. The observability test consists in a rank test of the Jacobian of the state vector with respect to the measurement vector [6]. For a system to be observable, the Jacobian needs to be full rank (has non-zero determinant). In the following, to achieve the system observability condition, the measurements will include the time gradient of the range differences. With this information added to the system and by imposing the motion in the horizontal plane, the state vector is uniquely determined. Now the algebraic method to compute the tracking parameters becomes difficult, if not impossible.

To solve the problem, we propose a recursive Bayesian filter algorithm for tracking a moving acoustic source by passive arrays. The target motion is assumed to be locally linear, that is the target is moving uniformly in the horizontal plane with unknown speed and direction of heading.

3. Tracking algorithm

3.1 Non-linear non-Gaussian Bayesian filter

The tracking problem requires estimation of the state vector (target co-ordinates and velocity) of a system that changes over time using a sequence of noisy measurements (observations) made on the system. The state vector is an unobservable (hidden) Markov process of initial distribution $p(\mathbf{x}_0)$ and transition equation $p(\mathbf{x}_k | \mathbf{x}_{k-1})$. For the specific application of this study, the target dynamics (the system model) is described by a linear equation, $\mathbf{f}(\bullet)$, while the system observation (the measurement model) equation, $\mathbf{h}(\bullet)$, is highly non-linear. We assume that these models are available in a probabilistic form:

$$p(\mathbf{x}_k | \mathbf{x}_{k-1}): \quad \mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{v}_{k-1}) \quad (5)$$

$$p(\mathbf{z}_k | \mathbf{x}_k): \quad \mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k) \quad (6)$$

where \mathbf{x}_k is the n_x -dimensional state vector of the system at time step k , \mathbf{u}_k is the input vector, \mathbf{z}_k is the n_z -dimensional observation vector, and \mathbf{v}_k and \mathbf{w}_k are vectors representing the process and measurement noise, respectively. They have the dimensions n_v and n_w . It is assumed that the noise vectors are i.i.d. (independent identically distributed) and that they are independent of current and past states.

From the Bayesian perspective, it is required to estimate $p(\mathbf{x}_k | \mathbf{z}_{1:k})$ assuming that the PDF at time $(k-1)$, $p(\mathbf{x}_{k-1} | \mathbf{z}_{1:k-1})$, is available. The first step in this process is called prediction and makes use of equation (5), which is assumed to describe a Markov process of order one:

$$p(\mathbf{x}_k | \mathbf{z}_{1:k-1}) = \int p(\mathbf{x}_k | \mathbf{x}_{k-1}) p(\mathbf{x}_{k-1} | \mathbf{z}_{1:k-1}) d\mathbf{x}_{k-1} \quad (7)$$

The second step, the measurement update, uses the most recent observation to produce the desired PDF via Bayes' rule:

$$p(\mathbf{x}_k | \mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{z}_{1:k-1})}{p(\mathbf{z}_k | \mathbf{z}_{1:k-1})} \quad (8)$$

$$p(\mathbf{z}_k | \mathbf{z}_{1:k-1}) = \int p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{z}_{1:k-1}) d\mathbf{x}_k$$

where the second equation is the normalization constant. Once the posterior PDF is determined, it is straightforward conceptually to produce any desired statistic of \mathbf{x}_k . For instance, the minimum mean-square error (MMSE) estimate of the current state could be found by computing the conditional mean:

$$\hat{\mathbf{x}}_k = \int \mathbf{x}_k p(\mathbf{x}_k | \mathbf{z}_{1:k}) d\mathbf{x}_k \quad (9)$$

The conditional covariance matrix is obtained in a similar way:

$$\mathbf{P}_k = \int (\mathbf{x}_k - \hat{\mathbf{x}}_k)(\mathbf{x}_k - \hat{\mathbf{x}}_k)^T p(\mathbf{x}_k | \mathbf{z}_{1:k}) d\mathbf{x}_k \quad (10)$$

In general, the recursive propagation of the posterior density cannot be determined analytically because the integrals in (7) and (8) do not have closed-form solutions. Solutions do exist in a restrictive set of cases. For example, if $\mathbf{f}(\bullet)$ and $\mathbf{h}(\bullet)$ are linear functions and if Gaussian distributions are assumed for \mathbf{x} , \mathbf{v} , and \mathbf{w} , the estimation of states is reduced to the well-known Kalman filter.

The problem of tracking an underwater acoustic source does not satisfy the original Kalman filter requirements because the system observation is non-linear. Moreover, because the target can approach the sensors from any direction and can maneuver at any time, the true posterior density is multi-modal and a Gaussian description will be inaccurate.

In order to deal with non-linear systems and/or non-Gaussian reality, two categories of techniques have been developed: parametric and non-parametric. The parametric techniques are based on improvements of the Kalman filter. These filters (for example, extended and unscented Kalman filters) can handle non-linear equations, but they implicitly approximate the posterior density as Gaussian. The non-parametric techniques are based on Monte Carlo simulations [7] and are the subject of the present study. These filters assume no functional form, but instead use a set of random samples (particles) to estimate the posterior PDF's. The advantage is that the particle filters can accommodate simultaneous alternative hypotheses that can describe well a multi-modal distribution.

3.2 Particle filter implementation

The basic idea of the Monte Carlo based approach to an intractable Bayesian filtering case is to approximate an unknown distribution, p , by a set of properly weighted particles drawn from a known distribution q . In this way, the difficult problem of distribution estimation is converted to an easy problem of weight estimation. The exact form of the proposal distribution q is a critical issue in designing the particle filter and is usually approximated to facilitate easy sampling.

A numerical approximation to the recursive Bayesian filtering method given by the equations (5) and (6) is the following algorithm [7]:

1. **Initialization:** sample N particles $\mathbf{x}_k^{(i)}$, $i = 1, 2, \dots, N$, from the proposal distribution. The proposal distribution can be the transition prior as used in the conventional particle filters, or more advanced distributions like the one used in this study.
2. **Measurement update:** update the importance weights. The Bayesian sequential importance sampling (SIS) procedure gives a recursive calculation of the normalized weight:

$$w_k^{(i)} = w_{k-1}^{(i)} \frac{p(\mathbf{z}_k | \mathbf{x}_k^{(i)}) p(\mathbf{x}_k^{(i)} | \mathbf{x}_{k-1}^{(i)})}{q(\mathbf{x}_k^{(i)} | \mathbf{z}_{1:k})}; \quad w_k^{(i)} = \frac{w_k^{(i)}}{\sum_{i=1, N} w_k^{(i)}} \quad (11)$$

As an approximation to (5) take:

$$\hat{\mathbf{x}}_k \approx \sum_{i=1, N} w_k^{(i)} \mathbf{x}_k^{(i)}$$

3. **Re-sampling** is a necessary step introduced in particle filtering algorithms to reduce the degeneration of samples. In practice it was noticed that, after a few iterations, one of the importance weights tends to one, while the others become zero. To avoid the degeneracy, the sampling importance re-sampling (SIR) method selects N samples with replacement from the set $\mathbf{x}_k^{(i)}$, where the probability to take sample 'i' is $w_k^{(i)}$. Then set $w_k^{(i)} = 1/N$, $i = 1, 2, \dots, N$.
4. **Prediction:** assuming that the probability of the process noise is known, use equation (5) to simulate $\mathbf{x}_{k+1}^{(i)}$, $i = 1, 2, \dots, N$.
5. Set $k = k + 1$, and iterate to step 2.

3.2.1 The unscented particle filter

As mentioned, the deficiency of the sequential importance sampling (SIS) approximation is that the proposal distribution may be very different from the posterior distribution, especially if using the transition prior as the proposal distribution. An improved proposal distribution must incorporate the current observation data with the optimal Gaussian approximation of the state.

In a previous study [8] on the magnetic dipole tracking application, it was shown that the unscented Kalman filter (UKF) is the best Kalman filter for the non-linear systems. The UKF is so named because it implements the Kalman recursion using the sample points provided by the unscented transform. The unscented transform deterministically generates a set of points that have a certain mean and sample covariance. The non-linear function is then applied to each of the sample points, yielding a transformed sample from which the predicted mean and covariance are calculated. The estimate of the conditional mean provided by the UKF is shown to be correct up to the second order of its Taylor series expansion. Reference [9] gives the implementation of UKF algorithm.

Because the UKF is the best to accurately propagate the mean and covariance of the Gaussian approximation to the state distribution, it can be used to generate the proposal distribution for the particle filter. In this way, one obtains a parametric/non-parametric hybrid filter called the unscented particle filter (UPF).

3.3 Target dynamics and measurements

For the tracking problem under investigation, the mathematical model used for the target is a moving acoustic source in the near field. The model is quite general being applicable even if the distance between the real target (vessel) and sensor is increased. For a far field target, the interval between successive delays is constant for a linear array with uniform spacing between elements. This situation is a particular case in the proposed TDOA method. The target is fully characterized by its motion parameters (position, velocity, and acceleration). When all these parameters are known the target may be classified as a surface

or submerged vessel, so that the determination of the target elevation relative to the array is important.

To allow for maneuvers and/or a non-linear trajectory, ship motion is described using a variable velocity model. The only simplifying assumption made is that the target is moving horizontally. This assumption is valid especially in shallow waters where it is less probable that the target undergoes abrupt vertical maneuvers.

Let the time increment between the data samples be Δt seconds, \mathbf{v} is the velocity vector in m/sec, and \mathbf{r} is the position vector from the point of origin to the source in meters. For a full characterization of the target, the entire system at time step k can be represented by the state vector:

$$\mathbf{x}_k = (r_x \ r_y \ r_z \ V_x \ V_y)^T \quad (12)$$

The discrete equations of target motion are obtained using the piece-wise approximation:

$$\begin{aligned} r_x(k) &= r_x(k-1) + \Delta t V_x \\ r_y(k) &= r_y(k-1) + \Delta t V_y \\ r_z(k) &= r_z(k-1) \\ V_x(k) &= V_x(k-1) \\ V_y(k) &= V_y(k-1) \end{aligned} \quad (13)$$

and $V_z = 0$. The target maneuvers (variable velocity) can be included in the state update through the state noise \mathbf{v} . Two pairs of acoustic sensors produce the observation data. If we assume that the RDS array is oriented along the X axis, the range difference and its time derivative between sensor i and sensor j is given by the formula:

$$\begin{aligned} R_{i,j} &= \sqrt{(r_x - r_i^x)^2 + r_y^2 + r_z^2} - \sqrt{(r_x - r_j^x)^2 + r_y^2 + r_z^2} \\ \mathcal{R}_{i,j} &= \frac{(r_x - r_i^x)V_x + r_y V_y}{\sqrt{(r_x - r_i^x)^2 + r_y^2 + r_z^2}} - \frac{(r_x - r_j^x)V_x + r_y V_y}{\sqrt{(r_x - r_j^x)^2 + r_y^2 + r_z^2}} \end{aligned}$$

For two pairs of sensors, the measurement vector at time k has the form:

$$\mathbf{z}_k = (R_{1,2} \ R_{3,4} \ \mathcal{R}_{1,2} \ \mathcal{R}_{3,4})^T \quad (14)$$

where \mathcal{R} represents the time derivative (gradient) of R .

These are the process and measurement equations used by the Bayesian filtering technique for tracking and classification of an acoustic source. As one can see, the process function $\mathbf{f}(\bullet)$ in equation (13) is linear, and the measurement function $\mathbf{h}(\bullet)$ in (14) is highly non-linear. Discrete observations (14) are taken at a sampling period of one second. It can be also verified that, in this case, the Jacobian of \mathbf{z}_k at \mathbf{x}_k is full rank, so that the system is observable.

In applying the filtering technique to the system, the initial conditions and the noise covariance matrices need to be specified. In the initialization step, the particles should be drawn from an unknown proposal distribution. The basic assumption is that the target can approach the sensors from any horizontal direction. Therefore, the filter must accommodate simultaneous alternative hypotheses until they can be rejected by future measurements. A reasonable initial estimate of the horizontal position is an approximate circle around the sensors with a radius of about 200m. In the present example, 36 particles were used with the horizontal positions spread over a circle every 10°, from 0° to 350°. For the vertical position, an initial estimate between zero and the approximate water depth can be given. Because we have no information about the magnitude of velocity, a good initial estimate of this vector is merely the null vector.

The initial covariance matrix, $\mathbf{P}(0|0)$, gives a measure of belief in the initial state estimate. It is assumed that initially all the states are un-correlated, so that the matrix is diagonal. This matrix is not known and has to be sufficiently large, but the initial $\mathbf{P}(0|0)$ is forgotten as more data is processed.

The measurement noise covariance matrix can be estimated directly from the actual data and, once calculated, it does not change during the filter run.

The process noise covariance is zero for a deterministic process. However, it was practically proved to be a good idea to introduce random perturbations in the target position and velocity. These small perturbations account for the mis-modeling such as target maneuvers and prevent divergence, so that the process noise covariance may be regarded as a tuning parameter of the filter.

3.4 Tracking results

Having outlined the background to this problem and suggested how a broadband underwater acoustic target positioning in real time might be carried out, these ideas are next applied to a set of real data collected with the RDS array. The array location was not known precisely, so that the following geometry was used: the one-dimensional array sits on the bottom of the ocean along the X-axis having zero value at the center of the array. The Y-axis is perpendicular to the array also on the ocean floor and Z-axis is oriented upwards. Consequently, the target coordinates will be given in this reference frame.

First, the GCC-PHAT method was used to calculate the TDOA between sensors 24 and 28, and 34 and 35, each pair being separated by a distance of about 40.5m along the same line. Also, 4.8m separates sensors #28 and #34. Actually, the present method requires only a three sensors array, so that the selection of sensors in the RDS array was arbitrary. Using 2000 readings per second, the TDOA for each pair of sensors can be calculated in real time. Knowing the sound speed in the ocean ($\approx 1470\text{m/sec}$) and the windowing interval (1 sec), from the TDOA calculations one can determine the range differences and their time derivatives. The results of these calculations, together with the estimated filter output, are presented in Fig. 4. For the unscented particle filter (UPF), they represent the measurements (observations).

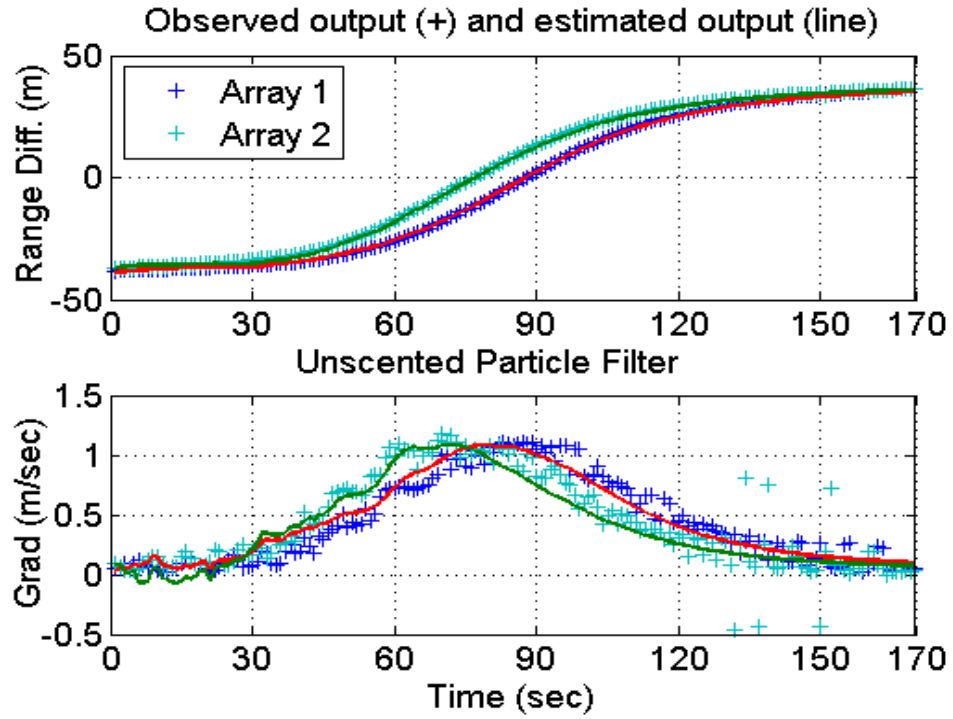


Figure 4. Measured data and estimated filter output.

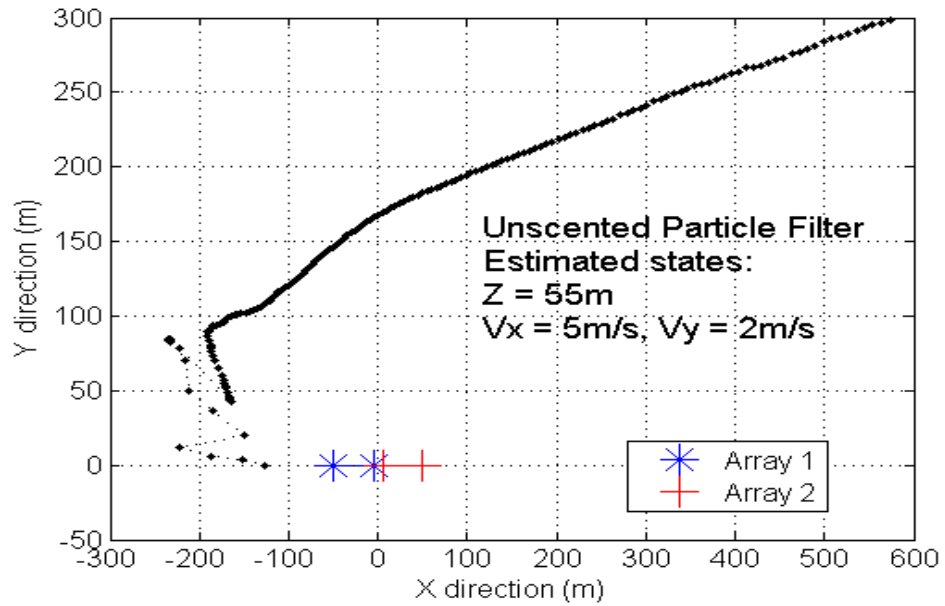


Figure 5. The X-Y co-ordinates of the vessel.

Figure 5 shows the track of the ship in the X-Y plane relative to the two arrays. The ship travels the distance in about 170 seconds having the estimated velocities $V_x = 5\text{m/sec}$ and $V_y = 2\text{m/sec}$ (Fig. 6b). The plot in Fig. 5 makes use of the state vector calculation results presented in Fig. 6a, where an estimate of the Z coordinate of about 55m is also shown. Even if the actual position and orientation of the array is not known exactly, these results are realistic when compared with the GPS tracking of the ship (Fig. 7). Only a portion (red colored, shown enlarged and rotated 90° below) of recorded data was used. However, the estimate of the ship elevation is less than the water depth (65m). As mentioned, the errors in ship positioning may be caused by the errors in the array location.

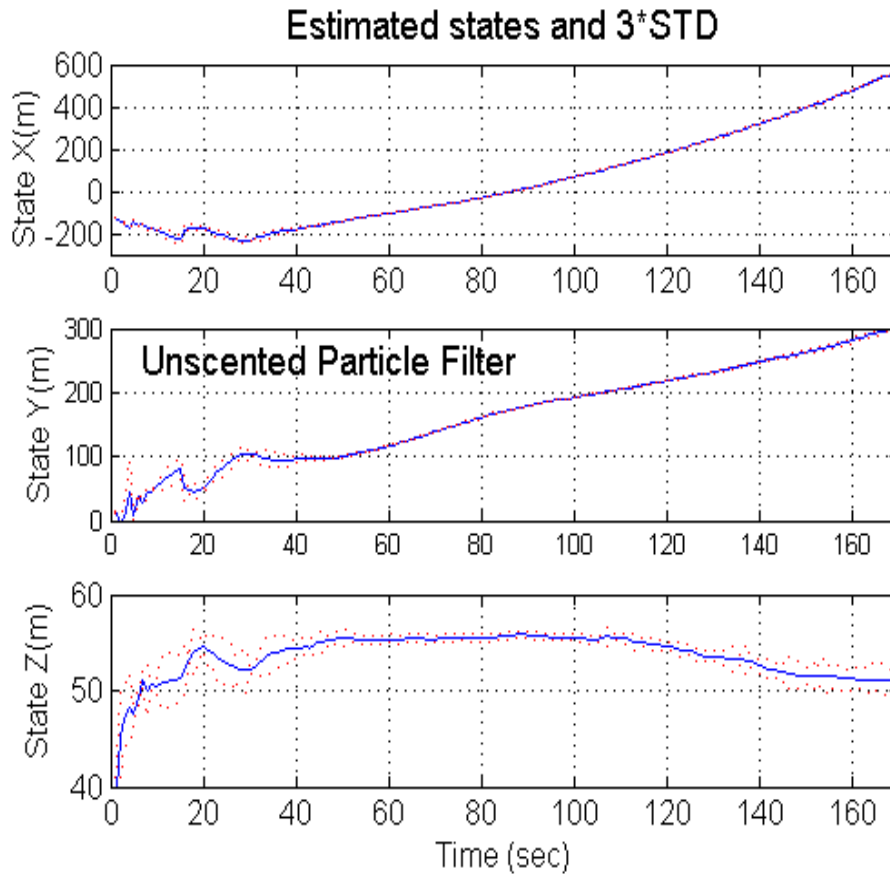


Figure 6a. State (X, Y, Z) estimates from the UPF with $\pm 3\sigma$

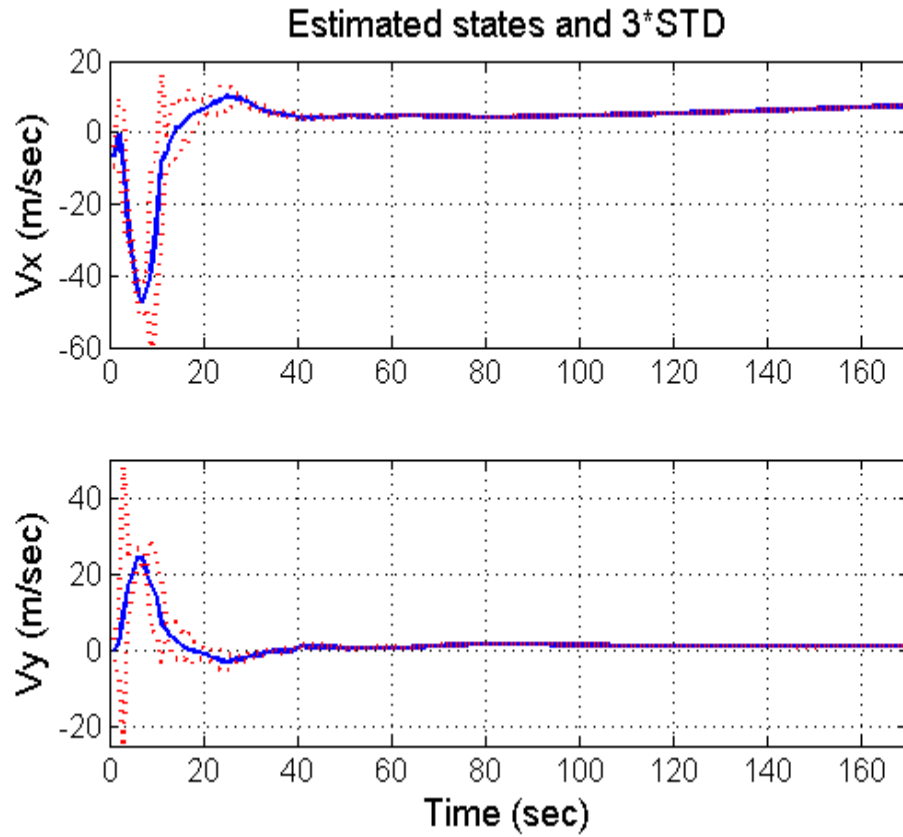


Figure 6b. State (V_x and V_y) estimates from the UPF with $\pm 3\sigma$

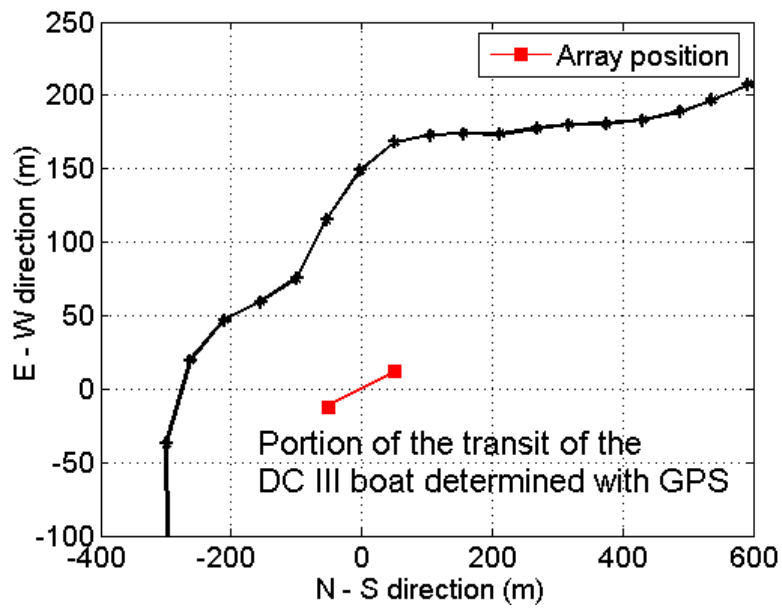
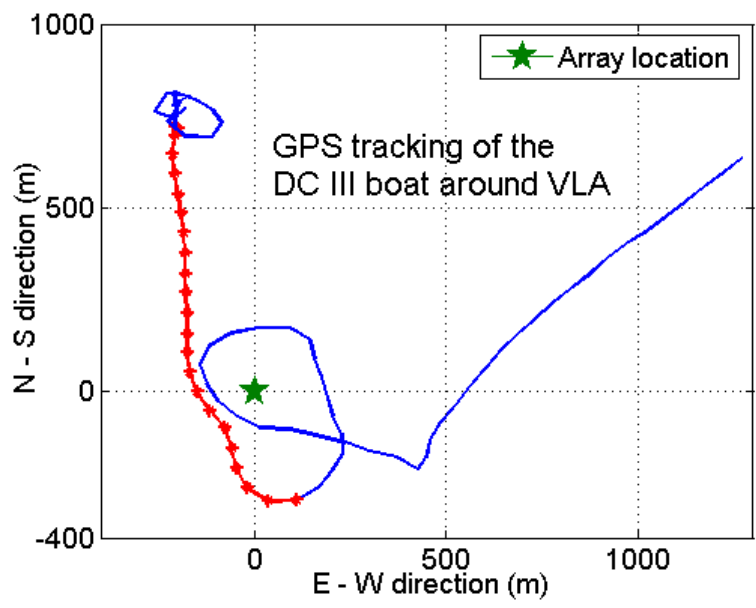


Figure 7. Vessel tracking using the GPS together with the estimated array position. Only the data from the red portion (shown below enlarged and rotated 90°) was used.

4. Conclusions

This report investigates the possibility of using the Unscented Particle Filter (UPF) for tracking and classification of an acoustic target. By determining the elevation of the target relative to the array, which is placed on the ocean floor, the classification of the vessel as a surface or submerged ship is possible supposing the water depth in the area is known.

The acoustic waves emitted by the target are considered as curved rather than plane, so that the method is more general than the classical beamforming technique. The method uses two pairs of acoustic sensors (arrays) and is a two-step process. In the first step the time-delays are estimated for each pair of hydrophones using the generalised cross-correlation (GCC) function. The second step consists of fusing this information based on the known geometry of the array to come up with the best estimate of target position. The problem is formulated in state-space form where the state variables are the position and velocity of the target.

A common concern in source location is the system observability, i.e. the unique determination of the states based on the observations. To achieve the system observability condition, the measurements include the time gradient of the range differences. Even so, the results preserve the inherent right-left ambiguity of the one-dimensional array.

The filter is applied to a 2-observers situation (two arrays) that allows the complete tracking and classification problem to be solved. All the vessel parameters like the heading, speed and depth, are estimated with a good level of accuracy by the UPF.

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Annexes

Implementing the Unscented Kalman Filter

The unscented transformation uses a set of weighted samples or ‘sigma points’, $S_i = \{W_i, \chi_i\}$, to completely capture the true mean and covariance of the random variable \mathbf{x} and then propagates the sigma points through the non-linear function. We use the following notation: \mathbf{K} is the Kalman gain, W_i are the weights, λ is the scaling parameter, $n_a = n_x + n_v + n_w$, and

$$\mathbf{x}^a = [\mathbf{x}^T \mathbf{v}^T \mathbf{w}^T]^T, \quad \mathcal{S}^a = \left[(\mathcal{S}^x)^T (\mathcal{S}^v)^T (\mathcal{S}^w)^T \right]^T$$

1. Initialize with:

$$\bar{\mathbf{x}}_0 = E[\mathbf{x}_0], \quad \mathbf{P}_0 = E[(\mathbf{x}_0 - \bar{\mathbf{x}}_0)(\mathbf{x}_0 - \bar{\mathbf{x}}_0)^T], \quad \bar{\mathbf{x}}_0^a = E[\mathbf{x}^a] = [\bar{\mathbf{x}}_0^T \mathbf{0} \mathbf{0}]^T$$

$$\mathbf{P}_0^a = E[(\mathbf{x}_0^a - \bar{\mathbf{x}}_0^a)(\mathbf{x}_0^a - \bar{\mathbf{x}}_0^a)^T] = \begin{bmatrix} \mathbf{P}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix}$$

2. For discrete time samples, $k = 1, 2, \dots, K$,

(a) Calculate sigma points:

$$\mathcal{S}_{k-1}^a = \left[\bar{\mathbf{x}}_{k-1}^a \quad \bar{\mathbf{x}}_{k-1}^a \pm \sqrt{(n_a + \lambda) \mathbf{P}_{k-1}^a} \right]$$

(b) Time update:

$$\mathcal{S}_{k|k-1}^a = \mathbf{f}(\mathcal{S}_{k-1}^x, \mathcal{S}_{k-1}^v)$$

$$\bar{\mathbf{x}}_{k|k-1} = \sum_{i=0}^{2n_a} W_i^{(m)} \mathcal{S}_{i,k|k-1}^x$$

$$\mathbf{P}_{k|k-1} = \sum_{i=0}^{2n_a} W_i^{(c)} \left[\mathcal{S}_{i,k|k-1}^x - \bar{\mathbf{x}}_{k|k-1} \right] \left[\mathcal{S}_{i,k|k-1}^x - \bar{\mathbf{x}}_{k|k-1} \right]^T$$

$$\mathbf{z}_{k|k-1} = \mathbf{h}(\mathbf{s}_{k|k-1}^x, \mathbf{s}_{k-1}^w)$$

$$\bar{\mathbf{z}}_{k|k-1} = \sum_{i=0}^{2n_a} W_i^{(m)} Z_{i,k|k-1}$$

(c) Measurement update equations:

$$\mathbf{P}_{k|k}^{ZZ} = \sum_{i=0}^{2n_a} W_i^{(c)} [Z_{i,k|k-1} - \bar{\mathbf{z}}_{k|k-1}] [Z_{i,k|k-1} - \bar{\mathbf{z}}_{k|k-1}]^T$$

$$\mathbf{P}_{k|k}^{XZ} = \sum_{i=0}^{2n_a} W_i^{(c)} [\mathbf{s}_{i,k|k-1} - \bar{\mathbf{x}}_{k|k-1}] [Z_{i,k|k-1} - \bar{\mathbf{z}}_{k|k-1}]^T$$

Where \mathbf{P}^{ZZ} , \mathbf{P}^{XZ} are respectively the covariance matrix of the measurement and the cross-covariance of the measurement and the state variable. Next:

$$\mathbf{K}_k = \mathbf{P}_{k|k}^{XZ} [\mathbf{P}_{k|k}^{ZZ}]^{-1}$$

$$\bar{\mathbf{x}}_k = \bar{\mathbf{x}}_{k|k-1} + \mathbf{K}_k (\mathbf{z}_k - \bar{\mathbf{z}}_{k|k-1})$$

$$\mathbf{P}_k = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{P}_{k|k}^{ZZ} \mathbf{K}_k^T$$

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13. **ABSTRACT** (a brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), (R), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual).

This report describes a numerical method that may be used to efficiently locate and track underwater sonar targets in the near field for the case of very small passive arrays. The waves emitted by the target are considered as curved rather than plane. The method uses only two pairs of acoustic sensors and is a two-step process. In the first step the time-delays are estimated for each pair of hydrophones using the generalised cross-correlation function. The second step consists of fusing this information based on the known geometry of the array to come up with the best estimate of target position. The optimal solution to the real time positioning problem is given by the recursive Bayesian filter. A transition equation describes the prior distribution of the desired parameters (target position and velocity), the so-called hidden state process, and an observation equation describes the likelihood of the observations (measurements). The determination of target position and velocity is formulated as an optimal stochastic estimation problem, which could be solved using a sequential Monte Carlo based approach known as particle filter. In addition to the conventional particle filter, the proposed tracking and classification algorithm uses the unscented Kalman filter (UKF) to generate the prior distribution of the unknown parameters. Finally, it is demonstrated the ability of the approach to track a fishing vessel over a period of time using the data collected by the Rapidly Deployment System array.

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unscented Kalman filter, tracking algorithm, acoustic source, generalized cross-correlation

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