Unscented Particle Filter for Tracking a Magnetic Dipole Target

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Abstract - **In this paper we present a recursive Bayesian solution to the problem of joint tracking and classification of a target modeled at a distance by an equivalent magnetic dipole. Tracking/classification of a magnetic dipole from noisy magnetic field measurements involves the modeling of a non-linear non-Gaussian system. This system allows for complications due to multiple directions of arrival and target maneuver. The determination of target position, velocity and magnetic moment is formulated as an optimal stochastic estimation problem, which could be solved using a sequential Monte Carlo based approach known as the particle filter. In addition to the conventional particle filter, the proposed tracking and classification algorithm uses the unscented Kalman filter (UKF) to generate the transition prior as the proposal distribution.**

I. INTRODUCTION

Precise determination of target motion parameters, i.e. position, velocity, and target classification, are primary concerns in automated surveillance systems. This paper presents the use of a sequential Monte Carlo based statistical signal processing method, known as particle filter, for tracking and classifying a magnetic target.

A target containing ferromagnetic material can be adequately modeled at a distance by an equivalent magnetic dipole moment. This magnetic target can be observed by means of tri-axial magnetometers that measure the variation of the magnetic field components as a function of time as it passes. Of interest is the inverse problem of the determination of the position and magnetic parameters of the target at time step *k* from its magnetic signature collected up to and including time *k*.

One approach to solve this problem makes use of the recursive Bayesian estimation (filtering) technique. The problem is formulated in state-space form where the state variables are the position, velocity and magnetic moment of the target. Let define $\mathbf{x}_k$ as the state of the system at time step k, and $\mathbf{z}_{1:k} = \{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_k\}$ as the observation (measurements) history of a system from time 1 to *k*. Because of either noise in the state evolution process or uncertainty as to the exact nature of the process itself, the state vector $\mathbf{x}_k$ is generally regarded as a random variable. In the Bayesian filtering technique, one attempts to construct an estimate of the posterior probability density function (pdf), $p(\mathbf{x}_k \mid \mathbf{z}_{1:k})$. Since all information provided by $\mathbf{z}_{1:k}$ is conveyed by the posterior density, it may be said to be the complete solution to the estimation problem. A recursive Bayesian algorithm imposes the constrain that the estimate of $p(\mathbf{x}_k \mid \mathbf{z}_{1:k})$ should be generated solely from the previous posterior density, $p(\mathbf{x}_{k-1} \mid \mathbf{z}_{1:k-1})$, and the most recent measurement $\mathbf{z}_k$. In this way, it is not necessary to store the complete data set or to reprocess existing data when a new measurement becomes available.

The recursive propagation of the posterior density is only a conceptual solution that can be determined analytically only in a restrictive set of cases. When the analytical solution is intractable, a Monte Carlo based approach to recursive Bayesian filtering called the particle filter, is one method that approximates the optimal Bayesian solution. In the Monte Carlo method, a set of random samples (particles) are drawn from a target distribution such as $p(\mathbf{x} \mid \mathbf{z})$. In general, this distribution is not known. We will use $q(\mathbf{x}_k \mid \mathbf{z}_{1:k}) \neq p(\mathbf{x}_k \mid \mathbf{z}_{1:k})$ to denote a proposal distribution from which samples can be drawn. The main drawback of the conventional particle filter is that it uses transition prior, $p(\mathbf{x}_k \mid \mathbf{x}_{k-1})$, as the proposal distribution. The transition prior does not take into account current observation data. To overcome this difficulty, the unscented Kalman filter (UKF) was proposed to generate better proposal distributions by taking into consideration the most recent observation.

II. UNSCENTED PARTICLE FILTER

A. Recursive Bayesian Estimation

The tracking problem requires estimation of the state vector (target co-ordinates, velocity, magnetic moment) of a system that changes over time using a sequence of noisy measurements (observations) made on the system. For the specific application regarding this study, the target dynamics (the system model) is described by a linear equation, $\mathbf{f}(\bullet)$, while the system observation (the measurement model) equation, $\mathbf{h}(\bullet)$, is highly non-linear. We assume that these models are available in a probabilistic form:

$$p(\mathbf{x}_k \mid \mathbf{x}_{k-1}): \qquad \mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{v}_{k-1}) \tag{1}$$

$$p(\mathbf{z}_k \mid \mathbf{x}_k): \qquad \mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{w}_k) \tag{2}$$

where $\mathbf{x}_k$ is the n_x-dimensional state vector of the system at time step k, $\mathbf{z}_k$ is the n_z-dimensional observation vector, and $\mathbf{v}_k$ and $\mathbf{w}_k$ are vectors representing the process and measurement noise, respectively. They have the dimensions n_v and n_w. It is assumed that the noise vectors are i.i.d. and independent of current and past states.

From the Bayesian perspective, it is required to estimate $p(\mathbf{x}_k \mid \mathbf{z}_{1:k})$ assuming that the pdf at time (*k*-1), $p(\mathbf{x}_{k-1} \mid \mathbf{z}_{1:k-1})$, is available. The first step in this process is called prediction and makes use of equation (1), which is assumed to describe a Markov process of order one:

$$p(\mathbf{x}_k \mid \mathbf{z}_{1:k-1}) = \int p(\mathbf{x}_k \mid \mathbf{x}_{k-1})\, p(\mathbf{x}_{k-1} \mid \mathbf{z}_{1:k-1})\, d\mathbf{x}_{k-1} \qquad (3)$$

The second step, the measurement update, uses the most recent observation to produce the desired pdf via Bayes' rule:

$$p(\mathbf{x}_k \mid \mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k \mid \mathbf{x}_k)\, p(\mathbf{x}_k \mid \mathbf{z}_{1:k-1})}{p(\mathbf{z}_k \mid \mathbf{z}_{1:k-1})}$$

$$p(\mathbf{z}_k \mid \mathbf{z}_{1:k-1}) = \int p(\mathbf{z}_k \mid \mathbf{x}_k)\, p(\mathbf{x}_k \mid \mathbf{z}_{1:k-1})\, d\mathbf{x}_k \qquad (4)$$

where the second equation is the normalization constant. Once the posterior pdf is determined, it is straightforward conceptually to produce any desired statistic of $\mathbf{x}_k$. For instance, the minimum mean-square error (MMSE) estimate of the current state could be found by computing the conditional mean:

$$\hat{\mathbf{x}}_k = \int \mathbf{x}_k\ p(\mathbf{x}_k \mid \mathbf{z}_{1:k})\, d\mathbf{x}_k \qquad (5)$$

The conditional covariance matrix is obtained in a similar way:

$$\mathbf{P}_k = \int (\mathbf{x}_k - \hat{\mathbf{x}}_k)(\mathbf{x}_k - \hat{\mathbf{x}}_k)^T\ p(\mathbf{x}_k \mid \mathbf{z}_{1:k})\, d\mathbf{x}_k \qquad (6)$$

In general, the recursive propagation of the posterior density cannot be determined analytically because the integrals in (3) and (4) do not have closed-form solutions. Solutions do exist in a restrictive set of cases. For example, if $\mathbf{f}(\bullet)$ and $\mathbf{h}(\bullet)$ are linear functions and if Gaussian distributions are assumed for **x**, **v**, and **w,** the estimation of states is reduced to the well-known Kalman filter.

The problem of tracking a magnetic dipole does not satisfy the original Kalman filter requirements because the system observation is non-linear. Moreover, because the target can approach the sensors from any direction and can maneuver at any time, the true posterior density is multi-modal and a Gaussian description will be inaccurate.

In order to deal with non-linear systems and/or non-Gaussian reality, two categories of techniques have been developed: parametric and non-parametric. The parametric techniques are based on improvements of the Kalman filter. These filters (for example, extended and unscented Kalman filters) can handle non-linear equations, but they implicitly approximate the posterior density as Gaussian. The non-parametric techniques are based on Monte Carlo simulations and are the subject of the present study. These filters assume no functional form, but instead use a set of random samples (particles) to estimate the posteriors. The advantage is that the particle filters can accommodate simultaneous alternative hypotheses that can describe a multi-modal distribution well.

B. Particle Filter Implementation

The basic idea of the Monte Carlo based approach to an intractable Bayesian filtering case is to approximate an unknown distribution, *p*, by a set of properly weighted particles drawn from a known distribution, *q*. In this way, the difficult problem of distribution estimation is converted to an easy problem of weight estimation. The exact form of the proposal distribution *q* is a critical issue in designing the particle filter and is usually approximated to facilitate easy sampling.

A numerical approximation to the recursive Bayesian filtering method given by the equation (3) and (4) is the following algorithm:

1. **Initialization:** sample N particles $\mathbf{x}_k^{(i)}$, i = 1, 2, …, N, from the proposal distribution. The proposal distribution can be the transition prior as used in the conventional particle filters, or more advanced distributions like the one used in this study.
2. **Measurement update:** update the importance weights. The Bayesian sequential importance sampling (SIS) procedure gives a recursive calculation of the normalized weight [1]:

$$w_k^{(i)} = w_{k-1}^{(i)}\ \frac{p(\mathbf{z}_k \mid \mathbf{x}_k^{(i)})\ p(\mathbf{x}_k^{(i)} \mid \mathbf{x}_{k-1}^{(i)})}{q(\mathbf{x}_k^{(i)} \mid \mathbf{z}_{1:k})} \qquad (7)$$

$$w_k^{(i)} = \frac{w_k^{(i)}}{\sum_{i=1,N} w_k^{(i)}}$$

As an approximation to (5) take:

$$\hat{\mathbf{x}}_k \approx \sum_{i=1,N} w_k^{(i)}\ \mathbf{x}_k^{(i)}$$

3. **Re-sampling** is a necessary step introduced in particle filtering algorithms to reduce the degeneration of samples. In practice it was noticed that, after a few iterations, one of the importance weights tends to one, while the others become zero. To avoid the degeneracy, the sampling importance re-sampling (SIR) method selects N samples with replacement from the set $\mathbf{x}_k^{(i)}$, where the probability to take sample 'i' is $w_k^{(i)}$. Then set $w_k^{(i)} = 1/N$, i = 1, 2, …, N.
4. **Prediction:** assuming that the probability of the process noise is known, use equation (1) to simulate $\mathbf{x}_{k+1}^{(i)}$, i = 1, 2, …, N.
5. Set k = k +1, and iterate to item 2.

C. The Unscented Kalman Filter

As mentioned, the deficiency of the sequential importance sampling (SIS) approximation is that the proposal distribution may be very different from the posterior distribution, especially if using the transition prior as the proposal distribution. An improved proposal distribution must incorporate the current observation data with the optimal Gaussian approximation of the state.

In a previous study [2] on the magnetic dipole tracking application, it was shown that the unscented Kalman filter

(UKF) is the best Kalman filter for the non-linear systems. The UKF is so named because it implements the Kalman recursion using the sample points provided by the unscented transform. The unscented transform deterministically generates a set of points that have a certain mean and sample covariance. The non-linear function is then applied to each of the sample points, yielding a transformed sample from which the predicted mean and covariance are calculated. The estimate of the conditional mean provided by the UKF is shown to be correct up to the second order of its Taylor series expansion. Reference [3] gives the implementation of UKF algorithm.

Because the UKF is the best in accurately propagating the mean and covariance of the Gaussian approximation to the state distribution, it can be used to generate the proposal distribution for the particle filter. In this way, one obtains a parametric/non-parametric hybrid filter called the unscented particle filter (UPF).

III. MAGNETIC TARGET TRACKING

The mathematical model used for the target is a moving magnetic dipole. The target is fully characterized by its motion parameters (position and velocity) and the value of the magnetic dipole moment. Its maneuvers and/or a non-linear trajectory are modeled in the state update through the process noise on velocity. The simplifying assumption made is that the target is moving horizontally. Also the magnetic mass of the target remains constant during the measurement and can be estimated from the values of the equivalent magnetic dipole moment.

Let consider that the time increment between the data samples is Δt seconds, $\mathbf{m}^S$ is the magnetic moment vector of the dipole expressed in kA-m^2, **v** is the velocity vector in m/sec, and **r** is the position vector from the observation point to the dipole center in meters. For a full characterization of the target, the entire system at time step *k* can be represented by the state vector:

$$\mathbf{x}_k = \left(r_X \ r_Y \ r_Z \ v_X \ v_Y \ m_X^S \ m_Y^S \ m_Z^S\right)^T \tag{8}$$

The discrete equations of target motion are obtained using the piece-wise approximation:

$$\begin{aligned} r_X(k) &= r_X(k-1) + \Delta t\, v_X \\ v_X(k) &= v_X(k-1) \\ m_X^S(k) &= m_X^S(k-1) \end{aligned} \tag{9}$$

and similar relations exist for the Y and Z components ($v_Z = 0$). Tri-axial magnetic sensors produce the observation data. The magnetic flux density vector **B** at a given point due to a magnetic dipole is given by the formula:

$$\mathbf{B} = \frac{\mu}{4\pi}\left[3\langle \mathbf{r}, \mathbf{m}\rangle \mathbf{r} - |\mathbf{r}|^2 \mathbf{m}\right] / |\mathbf{r}|^5 \tag{10}$$

where μ is the permeability of the medium (= $4\pi 10^{-7}$), and <•,•> is the inner product operator. In the above formula, all vectors are defined in the same coordinate system, which normally are the sensor coordinates. Because the desired magnetic moment, $\mathbf{m}^S$, is related to the ship reference frame, it is necessary to apply a rotation:

$$\begin{aligned} m_X &= m_X^S \frac{v_X}{\sqrt{v_X^2 + v_Y^2}} + m_Y^S \frac{v_Y}{\sqrt{v_X^2 + v_Y^2}} \\ m_Y &= m_X^S \frac{-v_Y}{\sqrt{v_X^2 + v_Y^2}} + m_Y^S \frac{v_X}{\sqrt{v_X^2 + v_Y^2}} \\ m_Z &= m_Z^S \end{aligned} \tag{11}$$

For a single sensor, the measurement vector at time k has the form:

$$\mathbf{z}_k = \left(B_x \quad B_y \quad B_z\right)^T \tag{12}$$

These are the process and measurement equations used by the Bayesian filter for tracking and classification of a magnetic dipole. As one can see, the process function **f**(•) in equation (1) is linear, and the measurement function **h**(•) in equation (2) is highly non-linear.

IV. SIMULATED EXPERIMENT

One of the problems encountered in designing an experiment is the system observability. A system is observable when its state vector can be reconstructed from the measurements of its output. Because the magnetic flux density depends on both states **m** and **r**, it is not possible to reconstruct these (six) states from a single sensor, which produces at most 3 data points. The 2-observers situation allows the complete problem to be solved [4], i.e. to determine unambiguously the horizontal position, speed and depth, and the equivalent dipole moment of the moving target.

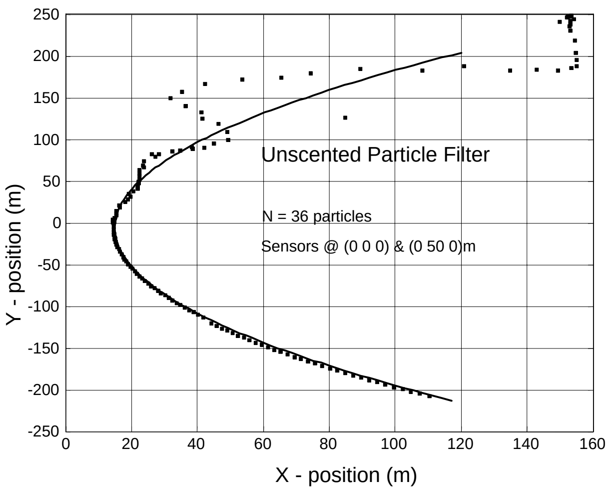

Fig. 1. Estimated (▪) and true (-) X and Y values.

Simulated observations are used in this study: a true non-linear (U-turn) trajectory is assumed for a magnetic dipole having a known moment, observations are computed using equation (11) and (10), and contaminated with Gaussian noise with the SNR of 10 dB. Two tri-axial magnetometers (observers) placed on the seafloor have the positions given by $\mathbf{s_1}$ = (0, 0, 0) and $\mathbf{s_2}$ = (0, 50, 0) meters, respectively. The target is represented by a magnetic dipole with a constant moment over time, $\mathbf{m}^S$ = (50, -5, 125) kA•m^2. The target approaches the observers with a time variable velocity, and at a distance from the bottom of 20m. Discrete observations (12) are taken at a sampling period of one second.

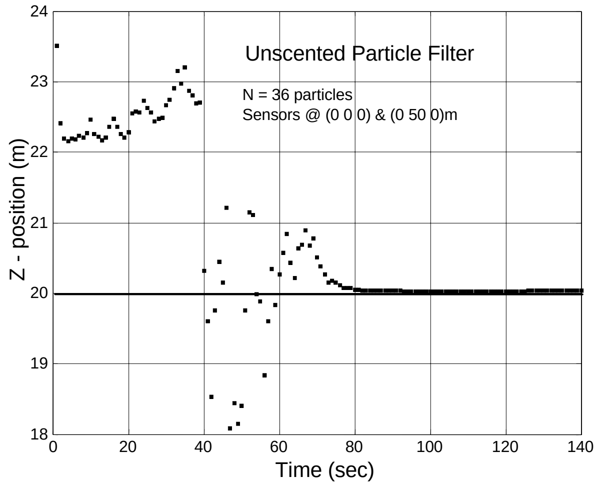

Fig. 2. Estimated (■) and true (-) Z values.

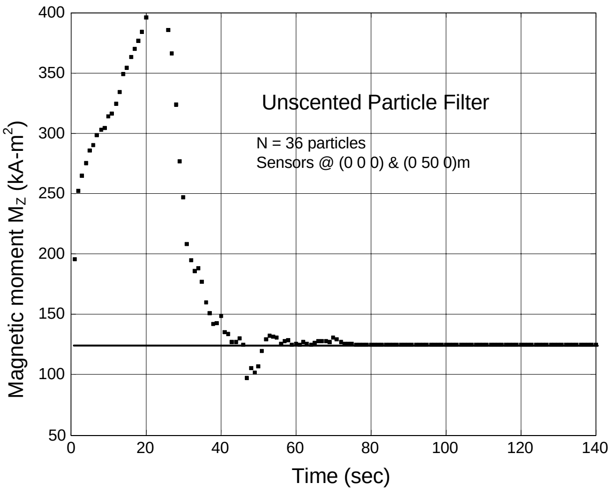

Fig. 3. Estimated (■) and true (-) m_Z values.

In applying the filtering technique to the system, the initial conditions and the noise covariance matrixes need to be specified. In the initialization step, the particles should be drawn from an unknown proposal distribution. The basic assumption is that the target can approach the sensors from any horizontal direction. Therefore, the filter must accommodate simultaneous alternative hypotheses until they can be disambiguated by future measurements. A reasonable initial estimate of the horizontal position is an approximate circle around the sensors with a radius of about 200m from where the magnetic signal becomes sizable. In the present example, 36 particles were used with the horizontal positions spread over a circle every 10° from 0° to 350°. For the vertical position, an initial estimate between zero and the approximate water depth can be given. Because we have no information about the magnitude of velocity, acceleration and magnetic dipole moments, a good initial estimate of these vectors are merely the null vectors.

The initial covariance matrix, $\mathbf{P}(0|0)$, gives a measure of belief in the initial state estimate. It is assumed that initially all the states are un-correlated, so that the matrix is diagonal. This matrix is not known and has to be sufficiently large, but the initial $\mathbf{P}(0|0)$ is forgotten as more data is processed. The measurement noise covariance matrix can be estimated directly from the actual data and, once calculated, it does not change during the filter run.

The process noise covariance is zero for a deterministic process. However, it was practically proved to be a good idea to introduce random perturbations in the target position and velocity. These small perturbations account for the target maneuvers and prevent divergence, so that the process noise covariance may be regarded as a tuning parameter of the filter.

The results are presented in figures 1 to 3 where the true values of state variables are plotted together with the state estimates obtained from this filter. The performance of the UPF is good.

V. CONCLUSIONS

This study presents the application of the unscented particle filter (UPF) in solving the joint problem of tracking and classification of a target modeled as an equivalent magnetic dipole of arbitrary orientation. The problem is formulated in state-space form where the state variables are the position, velocity, and magnetic moment of the target. The advantage of using the particle filter for this application is the flexibility in selecting the initial state vector to cover the possible directions of arrival. Imposing the initial state vector arbitrarily represents a major limitation when using the non-linear Kalman filters. On the other hand, the non-linear Kalman filter used in this study, the unscented Kalman filter, offers a better proposal distribution than the one used in conventional particle filters by taking into account the most recent measurement.

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