Bayesian Theory Used in Designing the Ocean Floor Electromagnetic Sounding Experiment

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Abstract - In the Bayesian formulation of inversion the information content for each unknown parameter is quantified in terms of its marginal posterior probability distribution, which defines the accuracy expected in inversion. The problem of seafloor electromagnetic sounding is defined in terms of recovering the electrical conductivity profile beneath the seafloor from measurements of the electromagnetic field. Electromagnetic inversion represents a strongly non-linear problem for which a direct solution is not available. A matched-field approach to this problem can be formulated based on Bayesian inversion theory, which provides environmental parameter estimates and their uncertainties. This paper investigates the contribution of various experimental factors to the information content of the inversion parameters aiming to recover the conductivity profile from measurements of the electromagnetic field.

I. INTRODUCTION

The controlled source electromagnetic (CSEM) sounding is a technique developed for determining the electrical conductivity structure of the seabed. The method uses a transmitted EM signal between two points located near the seafloor and was developed for geophysical exploration of the Earth's crust [1, 2]. As interest is focused on deep sounding of the ocean basement, the air-seawater interface is ignored assuming that the seawater thickness is much greater that the ocean skin depth. Also, information about the thin sediment layers (1-20m) between the seawater and rock is ignored. Because electrical conductivity is related to porosity, EM sounding may offer an alternative to acoustic methods for seafloor characterization especially in environments where the acoustic methods often are inadequate such as shallow waters, surf zones, and coastal regions. In these cases, the effect of the upper surface of the seawater layer cannot be ignored and, as a result, the inversion algorithm must use the full wave propagation model.

One approach for determining seabed parameters from measured electromagnetic (EM) fields is matched-field inversion (MFI). In the simplest terms, MFI searches a multi-dimensional model parameter space to obtain the best fit between the modeled (replica) and measured data. The correlation between the predicted field values, $\mathbf{w}_f(\mathbf{m})$, obtained from the forward propagation model, and the array data, $\mathbf{d}^{obs}$, is realized by the objective function. Conventional MFI is achieved by minimizing the objective function with a reasonable number of calls to the propagation model. The model parameter vector $\mathbf{m}$ that produces the best match between the calculated and measured data is taken as an estimate of the true parameters.

Searching over a parameter space for the unknown parameters can be considered to be an inverse problem that may be solved using non-linear optimization, which calculates only a single point estimate of the parameters. To be able to investigate the influence of different experiment factors on the state of knowledge of the unknown parameters, one needs to estimate the accuracy of the inversion. Solving the inverse problem in terms of the Bayes' conditional probability represents a possibility to quantify the information content available in the inversion results.

An approach for determining seabed parameters from measured EM fields using the Bayesian theory of inversion was proposed in [3]. The result of inversion is the *a posteriori* probability density for the estimated parameters from which information such as moments and marginal distributions can be extracted. Marginal distributions quantify the accuracy expected in inversion function on the available information provided by a set of measured data. Relatively little work has been carried out to investigate the influence of experimental factors in estimating seabed parameters related to underwater EM propagation and the present study addresses this problem. Only a brief description of the Bayesian theory applied to matched field inversion is presented here and it follows the rigorous likelihood-based approach developed by Gerstoft and Mecklenbrauker [4]. The method has been recently used to estimate the seabed geoacoustic properties from measured ocean acoustic fields [5, 6].

This paper is directed toward determining to what level the experiment design influences the estimates of the parameters for remotely sensing the electrical conductivity structure. We assume that, at least within a limited range, the environment can be modeled by a set of horizontal layers. Also, the conductivity and the thickness of the seawater layer are usually known with a high degree of accuracy. In this study, Bayesian inversion is applied to synthetic test cases. Complex valued signal measurements from a horizontal electric dipole (HED) to an array of receivers are generated for a specific geometry. The propagation of the electromagnetic field in a multi-layered marine environment is modeled by solving the Maxwell's equations and the boundary conditions for a horizontal electric dipole placed in seawater. The effect of the incomplete forward model errors (environmental mismatch) is not considered and the same propagation model is used for generating both the synthetic data and the replica vectors.

II. BASIC CONCEPTS OF BAYESIAN THEORY

In the Bayesian formulation, the result of inversion is characterized by its *a posteriori* probability density (PPD) of the estimated parameters, $P(\mathbf{m}|\mathbf{d}^{obs})$, which is the conditional probability density function of $\mathbf{m}$ given $\mathbf{d}^{obs}$. From the *a posteriori* probability density function, one can calculate the moments such as the mean, covariance, and marginal distributions, which provide parameter estimates and uncertainties. One important result of applying the Bayesian formalism to inversion problems is that it includes the data errors, both measurement and theoretical, into the solution. In general, information about the statistical distribution of the measured EM field is not available *a priori*, but reasonable approximations may be used. Under the assumption that the errors on data from N-sensors are uncorrelated across frequency and time, zero-mean Gaussian distributed random variables, the data variance estimate is [4]:

$$\hat{\upsilon}_f = B_f\left(\mathbf{m}^{ML}\right)\left|\mathbf{d}^{obs}\right|^2 / N \qquad (1)$$

Here the objective function is the so-called normalized Bartlett mismatch function at a single frequency, f:

$$B_f(\mathbf{m}) = 1 - \frac{\left|\mathbf{w}_f^{\mathrm{T}}(\mathbf{m})\,\mathbf{d}_f^{obs}\right|^2}{\left|\mathbf{w}_f(\mathbf{m})\right|^2 \left|\mathbf{d}_f^{obs}\right|^2} \qquad (2)$$

where T indicates the conjugate transpose. The maximum likelihood (ML) model, $\mathbf{m}^{ML}$, for multi-frequency data is found by minimizing the objective function:

$$\Phi(\mathbf{m}) = \prod_{f=1}^{F} B_f(\mathbf{m}) \qquad (3)$$

using a global optimization procedure.

One method, used in this study, which allows us to obtain an estimate of the PPD is by calculating the Gibbs' probability distribution function (PDF) using the Gibbs' sampling technique. An implementation of Gibbs' sampling is the simulated annealing (SA) algorithm at constant temperature, T = 1. The Gibbs' probability distribution function (PDF) to be calculated is:

$$P_G\left(\mathbf{m}\,|\,\mathbf{d}^{obs}\right) \propto \exp\left(-E(\mathbf{m})/T\right) \qquad (4)$$

where the energy or error function is [5]:

$$E(\mathbf{m}) = \sum_{f=1}^{F} B_f(\mathbf{m})\left|\mathbf{d}^{obs}\right|^2 / \hat{\upsilon}_f \qquad (5)$$

It can be shown that after a suitable large number of accepted perturbations in the SA algorithm, the sampling distribution of the unknown parameters is given by the Gibbs' PDF [7]. The two functions, the PPD of an inverse problem and the Gibbs' PDF at constant temperature, T = 1, are identical in the case of uniform prior distribution [5], which is the standard case in MFI. The advantage of using Gibbs' sampling (SA at T = 1) to estimate the PPD is that the integral properties (moments) of the distribution, such as the posterior mean, covariance, and marginal PPD for

parameter m^j are computed directly from the set of models [5].

For meaningful results it is necessary to verify that the estimates have converged. Convergence was established by collecting two independent samples in parallel and periodically comparing the PPD moments estimated from each sample. The procedure is terminated when the difference between two cumulative marginal distributions for all parameters is less than 0.1.

III. CONTROLLED SOURCE EM SOUNDING

In controlled source EM experiments, an EM signal is injected into the ocean seafloor using a deep-towed transmitter and a series of measurements is made with a set of electric sensors placed on the seafloor at different ranges (geometrical sounding). The electrical conductivity structure of the rocks below the sea may be determined by measuring the spatial character of the signal attenuation. It is also possible to achieve similar results by keeping the transmitter and receiver a fixed distance apart and varying the frequency of the current (frequency sounding).
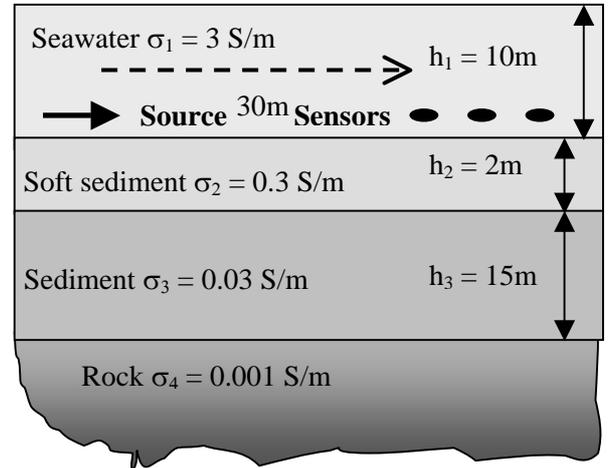

Fig. 1. Geometry of the marine environment.

For the problem considered in this work we used a synthetic marine environment consisting of five horizontal layers. The electromagnetic properties of each layer are assumed to be homogeneous, linear and isotropic. It is understood that the magnetic permeability μ_i ($i = 0, 1, 2, 3, \ldots$) for each layer can be replaced by the constant $\mu = \mu_0$, the permeability of the free space. The x and y directions denote the horizontal plan in which the structure is uniform in its electromagnetic parameters. The z direction is the vertical direction pointing downwards in which the structure varies in its properties. The origin of this Cartesian reference frame is located on the interface of air and seawater, yielding positive z values in the layers of interest. The very first and last layers of the propagation model are air ($\sigma_0 = 0$ S/m) and the lithosphere ($\sigma_4 = 0.001$ S/m), respectively. These layers are semi-infinite. The next layer from the top is seawater with the conductivity σ_1 for $0 < z < h_1$, where h_1 is the sea depth. Two layers of sediments, whose spatial conductivity distributions (σ_2, h_2, σ_3, h_3) are to be determined by inversion, are interposed between the seawater and lithosphere. The geometry considered for the

forward EM propagation model is illustrated in Fig. 1.

The EM propagation modeling is done in the frequency domain for low frequencies where the quasi-static approximation (displacement current neglected) is valid. In seafloor controlled-source EM sounding applications, the source and the field observation points are situated in the seawater. An EM field solution for the multi-layered conducting half-space medium was derived in [3, 8].

In the simulated experiment devised for this study, signals of different frequencies are injected into the water by a horizontal electric dipole (HED) with a moment of 10 A-m that can be dragged along the seafloor by a surface ship, for example. The receivers are placed at various ranges along the x-axis of the HED and detect only the x-component of the EM field. Practically, the electric sensors may be attached on the same cable in extension to the current source. The water depth and conductivity are 10m and 3.0S/m, respectively. The objective of the inversion problem is to recover the spatial distribution of conductivity, i.e. h_2, σ_2, h_3, and σ_3 in this case.
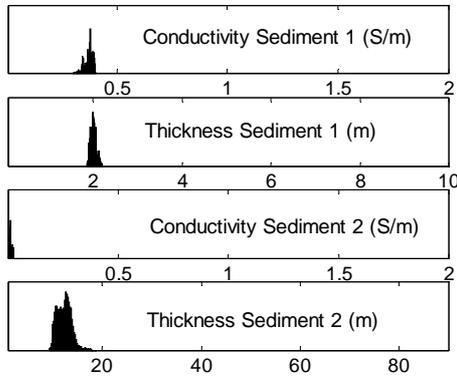

Fig. 2. Marginal PPDs estimated from single frequency (30kHz) data with a SNR of 30dB.

Before designing any experiment of this kind, it is useful to have a qualitative evaluation of the geometry based on a simplified model that considers only two half-spaces: (1) seawater with the wave-number $k_1 \cong (i\omega\mu\sigma_1)^{1/2}$ and (2) rock or deep sediment with the wave-number $k_2 \cong (i\omega\mu\sigma_2)^{1/2}$. The EM field equations for this model can be found in references [1, 9]. Let us consider that the separation distance between the horizontal electric dipole (HED) and receiver is r_0. In accord with the EM theory, the range (of frequencies) for which the parameter $|k_1 \, r_0|$ is small (< 1) is of no practical interest because over this range the signal is almost entirely controlled by eddy currents in the seawater. On the other hand, it is clearly demonstrated that for large parametric values when $|k_1 \, r_0| \gg 1$ the effect of the basement (rock) becomes important and the intermediate (soft) sediment layers between seawater and basement are ignored.

In the two half-space models, the EM field consists of three parts: (1) the direct field, (2) the reflected field, and (3) lateral-waves that travel outward a distance r_0 in the more resistive sediment layer. Information about the conductivity of the seafloor is only included in the lateral-wave field. In a multi-layered seafloor, additional to the lateral wave, the field generated by a HED at a given distance inside the seawater includes reflections of the EM wave that travels

down into horizontal layers of different conductivities. These contributions to the total field were considered in the present propagation model.
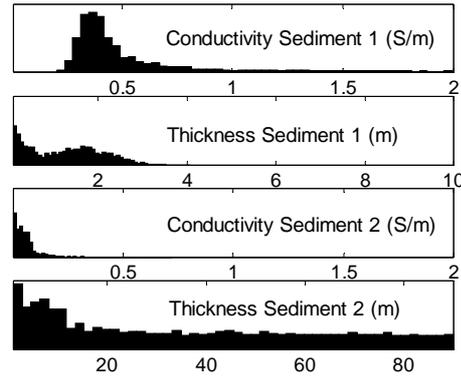

Fig. 3. Estimated marginal PPDs from 4 sensors and 9 frequencies data with a SNR of 15dB and 30m separation distance.

The field associated with the lateral wave is quite complicated, but it can be expressed [9] as a function of the exponential factor $\exp[ik_2r_0]$. The dependence of this field component of k_2 makes it valuable for seafloor characterization. In this case, the parameter of interest is $|k_2 \, r_0|$ which governs the amplitude variation of the received signal. The range where $|k_2 \, r_0| < 1$ must be avoided because in this case $\exp(ik_2 \, r_0) \approx 1$ and the lateral wave field (thus the total field) is independent of σ_2. From the practical viewpoint, the observation of the field in the range of r_0 and frequencies where the rapid exponential decrease of the lateral-wave field is dependent on σ_2 (when this is not too small) offers a promising means for seafloor exploration.

The lowest conductivity used in the environmental model ($\sigma_4 = 0.001$ S/m) imposes the limit for the maximum frequency at about 30 kHz to maintain the validity of the quasi-static approximation. As a result, a minimum separation distance of about 30m between the electric source and the sensor array is obtained from the condition of exponential decrease in the second sediment layer, for example taking $|k_3 \, r_0| = 2.5$. This distance is considered small enough for a local characterization of the seafloor, which is also one of the objectives of this study.
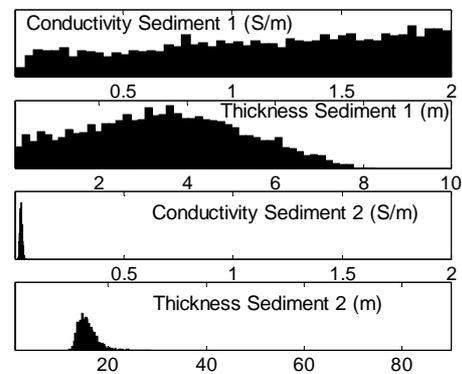

Fig. 4. Estimated marginal PPDs from 10 sensors single frequency data with a SNR of 15dB and 30m separation distance.

VI. EXAMPLES

This section illustrates the use of marginal PPDs to evaluate the information obtained from the seafloor EM sounding inversion by considering the effects of a number of experimental factors including : number of sensors in the receiving array, source frequency, number of frequencies, source-array range, and SNR.

The first experimental design is shown in Fig. 1and is based on the theoretical considerations presented in the previous section. Complex valued synthetic data was generated using the environmental parameters for a single frequency of 30 kHz, and 30m separation between the HED and the sensor array. The EM fields are received at four electric sensors separated by a 3m distance. White Gaussian noise was added to the data vectors to obtain a SNR of either 30 or 15 dB. Reasonably wide parameter search bounds (abscissa values for each parameter plot) were adopted to assess the resolving power of the EM data with limited prior information. The marginal distributions throughout this paper will be plotted as histograms of the sampled models discretized into 50 bins and scaled so that the total area is one when the search interval is scaled from 0 to 1.
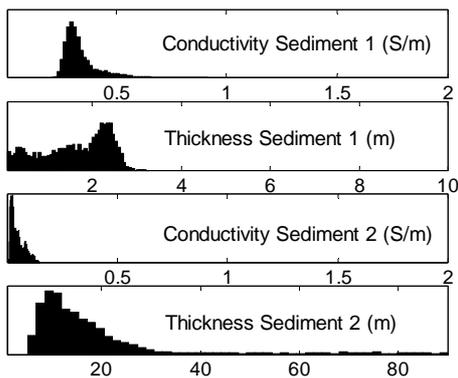

Fig. 5. Estimated marginal PPDs from multi-frequency $(10 - 30 \, kHz)$ data with 15dB SNR and 50m separation distance.

The estimated marginal PPDs for a SNR of 30 dB are presented in Fig. 2. In this case, the conductivity profile is estimated quite accurately. However, for more noise the ability of the inversion algorithm to recover the parameters is lost when the SNR is decreased to 15 dB. The marginal PPDs indicate large uncertainties for all parameters.

One approach to reduce the uncertainties is to include more information by using EM fields recorded at multiple frequencies preserving the number of sensors and/or to increase the length of the array. Several synthetic data with a SNR of 15 dB were generated for a frequency range between 10 and 30 kHz. Fig. 3 shows the marginal PPDs computed for data at 9 frequencies and still the only parameter reasonably resolved is σ_2. This means that, for the values of $|k_3 \, r_0|$ considered here (between 1.4 and 2.5), the field dependence on σ_3 is not strong enough to overcome the perturbation produced by the noise. According to the previous qualitative evaluation of the field, the dependency of the field on σ_3 might be improved by increasing the value of the exponential factor $|k_3 \, r_0|$, for example by increasing the radial distance between source

and receivers. One possibility is to increase the length of the array by increasing the number of sensors. The effect of increasing the number of sensors from four to ten is illustrated in Fig 4. Because of the increased length of the array, the second layer parameters are inverted well, but the first layer is still neglected. Another solution is to increase the distance between the source and the receivers preserving the same number of sensors. Fig. 5 shows the marginal PPDs for the parameters computed by applying MFI to EM synthetic data with SNR of 15 dB calculated at 5 frequencies evenly spaced between 10 and 30 kHz for a source-array distance of 50m. All parameters are reasonably inverted in this case.

V. SUMMARY AND CONCLUSIONS

This study considered the problem of evaluating the effects of various experimental factors on the inversion of the spatial conductivity in the seafloor EM sounding experiment. The Bayesian approach to inversion provides a measure for the information content of the resulting data sets in terms of marginal probability distributions of the unknown parameters. Various experiment factors, such as the number of sensors, source-receiver distance, source frequency, number of frequencies, and SNR, affect the inversion of conductivity profile. At lower SNR, the increase of the quantity of information in the measured data increased the accuracy of the estimates. For each specific experimental setup, the marginal distributions were computed using the Gibbs sampling method to evaluate the accuracy expected in inversion. The simulation results show that both conductivity and thickness of the first sediment layers can be determined simultaneously even for low values of SNR.

The problem considered here was a difficult one, with two intermediate sediment layers between seawater and rock and the air-seawater interface, where the full wave solution for the field was required. However, in designing the experiment we used qualitative evaluation of the electric field in the two conducting half-space model. In agreement with the EM theory applied in a layered structure, it was noted that an important factor determining the quality of inversion is the exponential factor $|k_i \, r_0|$, $i = 2, 3$, being the sediment layer of interest. On the other hand, in the seawater layer the parameter $|k_1 \, r_0|$ must be kept within reasonable limits to preserve the effectiveness of the method. This qualitative reasoning shows the possible existence of an optimal source and receiver configuration for a certain frequency range that should be used to determine the environment.

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