Development of a Reliability Constraint for Medium Support Vehicle System (MSVS) Fleet Contenders

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Abstract

When evaluating possible contenders to replace a fleet of Canadian Forces (CF) vehicles, it is not enough to look solely at the contenders' performance, in this case Mean Kilometres Between Mission Failures (MKBMF). In support of a tasking from the project management office for the Medium Support Vehicle System (PMO MSVS) to the Directorate of Materiel Group Operational Research (DMGOR), a procedure was developed to factor in the contenders' experience (fleet usage) in order to determine the reliability of the observed MKBMF. We wish to determine the minimum MKBMF that a contending fleet can observe in order to satisfy the requirement that there be no less than *K* kilometres between mission failures, 95% of the time. It is shown that for a contending fleet with a total of *T* vehicle kilometres of in-service usage, the observed MKBMF must exceed the value $MKBMF_{min}$, where $MKBMF_{min} = T/((T/K) - 1.645(T/K)^{1/2})$.

Résumé

Lors de l'évaluation de candidats potentiels pour le remplacement d'une flotte de véhicules des Forces Canadiennes (FC), il n'est pas suffisant de regarder uniquement la performance de chaque candidat, ici le Kilométrage Moyen de Bon Fonctionnement (KMBF). En soutien à une tâche attribuée par le bureau de gestion de projet pour le Système Mobile de Véhicule de Soutien (BGP SMVS) au Directeur de la Recherche Opérationnelle Groupe du Matériel (DRO GM), une procédure a été développée pour permettre de tenir compte de l'expérience de chaque candidat pour déterminer la fiabilité de la valeur observée de KMBF. L'objectif est de déterminer le KMBF minimum qu'une flotte de véhicules peut démontrer pour pouvoir satisfaire à la contrainte qu'il n'y ait pas plus de *K* kilomètres entre deux pannes, dans 95% des cas, une fois les véhicules achetés. Il est démontré que pour une flotte avec un total de *T* véhicules-kilomètres de service, le KMBF observé doit dépasser la valeur $KMBF_{min}$, où $KMBF_{min} = T/((T/K) - 1.645(T/K)^{1/2})$.

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Executive Summary

BACKGROUND

In April 2004 the Central OR Team (CORT) received a request from the Director Material Group OR (DMGOR) to comment on a proposed methodology to determine qualified bidders for the Medium Support Vehicle System (MSVS) project. The proposal recommended that only contenders with an in-service fleet of at least 355 vehicles be considered, based on fleet reliability targets stated in the MSVS Statement of Operational Requirement (SOR).

Specifically, the SOR [1] states that "Mean Kilometres Between Mission Failures[1] (MKBMF) shall not be less than 6000 km, desirable 10,000 km."

The CORT response indicated serious reservations with the proposed recommendation and it was agreed that CORT would undertake a project to develop a practical and mathematically sound procedure.

PROBLEM STATEMENT

The aim of the project is to develop a procedure to determine a contending fleet's minimal demonstrated operational reliability, based on the fleet's level of experience (i.e. kilometres of fleet usage). The resulting reliability constraint will constitute a basis for accepting or rejecting a contending fleet.

METHODOLOGY

In order to develop a criterion for fleet reliability, the situation can be considered as a problem of decision under uncertainty. In this case, given a contender with fleet usage T_i km and N_i mission failures, the hypothesis to consider is whether or not the contending fleet has a true reliability less than the stated requirement of 6000 km between mission failures. The "true reliability" of a fleet is the number of kilometres between mission failures that can be expect if the fleet is acquired.

The formal test of hypothesis is written as follows:

> H_0 (null hypothesis): The true fleet MKBMF is less than 6000 km.
>
> H_a (alternate hypothesis): The true fleet MKBMF is greater than 6000 km.

The decision criterion is to disqualify the contending bidder if the observed evidence is insufficient for us to reject the null hypothesis.

---

1 A mission failure is defined in the draft Statement of Interest and Qualification [2] as an instance when system components are not operating within their specified set of parameters resulting in the inability of the system to accomplish the mission.

To develop the statistical procedure for the test of hypothesis, the assumption was made that the failures accumulate according to a Poisson process, characterized by the parameter λ, representing the mission failure rate per km of fleet usage. Using the properties of the Poisson process, an expression can be derived for N_0 , the maximum acceptable number of observed mission failures, as a function of the fleet's experience:

$$N_0 = \frac{T}{6000} - 1.645\sqrt{\frac{T}{6000}} \quad (5)$$

Alternately, this expression can be re-written in terms of the minimum required Mean Kilometres Between Mission Failures (MKBMF):

$$MKBMF_{\min} = \frac{T}{\frac{T}{6000} - 1.645\sqrt{\frac{T}{6000}}} \quad (6)$$

This decision criterion, to determine the minimal in-service demonstrated reliability for a contending vehicle fleet, is mathematically sound and can be seen to be simple and clear, fair to all contenders, and can cater for all possible circumstances.

More generally, the methodology can be applied with almost no modification to the evaluation of the reliability of any system or sub-system with randomly occurring failures.

Sommaire

CONTEXTE

En avril 2004, l'Équipe Centrale de Recherche Opérationnelle (ÉROC) a été sollicitée par le Directeur de la Recherche Opérationnelle - Groupe du Matériel (DRO GM) pour donner son avis concernant une méthodologie proposée pour déterminer les soumissionnaires qualifiés pour le projet de Système Mobile de Véhicule de Soutien (SMVS).

Plus spécifiquement, l'énoncé des exigences opérationnelles [1] mentionne que le "kilométrage moyen de bon fonctionnement (KMBF) ne devra pas être inférieur à 6000 km, idéalement 10 000 km".[2]

La réponse de ÉROC indiquant de sérieuses réservations quant aux recommandations proposées, il a été convenu que ÉROC entreprendrait le projet de développer une procédure pratique et mathématiquement fondée.

ÉNONCÉ DU PROBLÈME

L'objectif du projet est de développer une procédure mathématiquement bien fondée pour déterminer la fiabilité opérationnelle minimale d'un soumissionnaire, à partir du niveau d'expérience de la flotte (i.e. kilomètres accumulés par la flotte de véhicules). La contrainte de fiabilité découlant de cette procédure constituera un fondement pour accepter ou rejeter une flotte de véhicules.

MÉTHODOLOGIE

Pour développer un critère de fiabilité opérationnelle, on considère la situation comme un problème de décision en présence d'incertitude. Dans ce cas-ci, étant donné un concurrent dont la flotte a une utilisation de T_i km et N_i pannes[3], l'hypothèse à considérer est la suivante : la fiabilité réelle de cette flotte de véhicule est-elle ou non inférieure à l'exigence opérationnelle énoncée de 6000 km entre les pannes. La fiabilité "réelle" d'une flotte est le nombre de kilomètres moyen de bon fonctionnement qu'on peut espérer observer une fois la flotte acquise.

Le test d'hypothèse est énoncé de la façon suivante :

> H_0 (hypothèse nulle): Le KMBF réel de la flotte est inférieur à 6000 km.
>
> H_a (hypothèse alternative): Le KMBF réel de la flotte est supérieur à 6000 km.

Le critère de décision est de disqualifier le soumissionnaire concurrent si les faits présentés sont insuffisants pour nous permettre de rejeter l'hypothèse nulle.

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[2] Traduction libre
[3] Une panne est définie dans l'Énoncé des Intérêts et Conditions d'Attribution préliminaire [2] comme l'occurrence d'une situation où les composantes du système n'opèrent pas selon leur mode normal, entraînant l'incapacité du système à accomplir sa mission.

Pour développer la procédure statistique pour le test d'hypothèses, on suppose que les pannes s'accumulent suivant un processus de Poisson, caractérisé par le paramètre λ, qui représente le taux de panne par km d'utilisation de la flotte. En utilisant les propriétés des processus de Poisson, on trouve une expression pour N_0 , le nombre maximal acceptable de pannes observées, en fonction de l'expérience de la flotte :

$$N_0 = \frac{T}{6000} - 1.645\sqrt{\frac{T}{6000}} \quad (5)$$

Cette expression peut également être écrite en terme de la valeur minimale requise pour le kilométrage moyen de bon fonctionnement (KMBF):

$$KMBF_{\min} = \frac{T}{\frac{T}{6000} - 1.645\sqrt{\frac{T}{6000}}} \quad (6)$$

Ce critère de décision, pour déterminer la fiabilité opérationnelle minimale à démontrer par une flotte concurrente, est bien fondé mathématiquement et peut être perçu comme simple, clair et équitable pour tous les concurrents.

De façon plus générale, la méthodologie peut être appliquée pratiquement sans modification à l'évaluation de la fiabilité de tout système ou sous-système pouvant présenter des arrêts de fonctionnement aléatoires.

Table of contents

List of figures

List of tables

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1. Introduction

1.1 Context

In April 2004 the Central OR Team (CORT) received a request from the Director Material Group OR (DMGOR) to comment on a proposed methodology to determine qualified bidders for the Medium Support Vehicle System (MSVS) project. The proposal, given in Annex A to this report, recommended that only contenders with an in-service fleet of at least 355 vehicles be considered, based on fleet reliability targets stated in the MSVS Statement of Operational Requirement (SOR) [1].

Specifically, requirement 4.11.3 states that "Mean Kilometres Between Mission Failures[1] (MKBMF) shall not be less than 6000 km, desirable 10,000 km."

The CORT response indicated serious reservations with the proposed recommendation. Accordingly, DMGOR convened a meeting with CORT and staff from the MSVS Project Management Office to explore possible remedies. It was agreed, based on CORT's mandate to provide OR advice and assistance on particularly complex problems and with the approval of the Director of Operational Research (Corporate), that CORT undertake a project to develop a practical and mathematically sound procedure.

1.2 Aim of the Project

The aim of the project as seen by the MSVS Project Director is stated in Annex B as the need to "determine the minimal in-service fleet size required so that operational data may be used to predict the Army's stated reliability requirement".

Although the Project Director states the aim as determining the minimal in-service fleet size, in fact the aim is better stated in terms of demonstrated operational reliability. In other words, the aim is to develop a mathematically sound procedure to determine a contending fleet's minimal demonstrated operational reliability, based on the fleet's level of experience (i.e. kilometres of fleet usage). The resulting reliability constraint will constitute a basis for accepting or rejecting a contending fleet.

Stating the aim this way, a contender with a highly reliable but small in-service fleet would be acceptable as a bidder as long as the in-service fleet had accumulated sufficient operational data – expressed as the total number of kilometres of fleet usage – to show that its fleet reliability met the targets stated in the SOR.

[1] A mission failure is defined in the draft Statement of Interest and Qualification [2] as an instance when system components are not operating within their specified set of parameters resulting in the inability of the system to accomplish the mission.

1.3 Requirements for the Applicability of the Procedure

Any procedure to determine the minimal in-service demonstrated reliability must not only be mathematically sound, but must also be seen to be simple and clear, and fair to all contenders. Further, since the bidding is open to vehicle manufacturers world-wide, the procedure must cater for all possible circumstances. We do not want to be too severe in eliminating legitimate bidders, nor do we want to accept bidders whose fleets are marginal in terms of the reliability requirements.

Throughout the text, a number of assumptions will be made concerning the occurrence of mission failures. Although these assumptions are standard in the modeling of system failures [4], it should be verified that they are appropriate to the system being studied to ensure that the results are scientifically valid.

The mathematical procedure developed also assumes that all operational data are comparable and that the definition of mission failures is consistent for all contenders (and all failures). If it is not the case, for example if one vehicle was used mostly in desert conditions while others operated in urban settings, the data will need to be adjusted to reflect the different operational environments. This pre-processing step is required before applying the statistical methodology.

Although the procedure is presented here in the context of contenders for the MSVS, it can be applied more generally to determine the ability of any system to perform according to a certain reliability standard for a given amount of time, based on the system's level of operational usage and its observed rate of failure.

2. Mathematical Derivation of the Reliability Criterion

2.1 Examples

To understand the problem in more concrete terms, consider the 4 hypothetical cases below.

Table 1: *Examples*

	FLEET SIZE	FLEET TOTAL KM	MISSION FAILURES	FLEET MKBMF
Case 1	1200	14,000,000	1650	8485
Case 2	800	4,000,000	660	6061
Case 3	340	3,500,000	320	10938
Case 4	400	1,000,000	160	6250

Case 1 is an example of a mature vehicle fleet with reliability data indicating that its MKBMF is well within the 6000 to 10000 km range specified by the SOR. Intuitively we would not wish to reject this contender on the criterion of fleet reliability. Case 2, on the other hand, is an example of a less mature and evidently less reliable fleet whose MKBMF is barely within the required range. This contender appears to be marginally acceptable but risky in terms of its fleet reliability history. Case 3 is an example of a small fleet with an excellent reliability record, exceeding the desirable MKBMF target of 10000 km. Although this fleet is small, we may wish to consider it as an acceptable contender. Finally, Case 4 is an example of a small fleet with a demonstrated MKBMF within the acceptable range. However, the small in-service fleet size and low fleet usage makes this contender more risky and we may wish to disqualify it on this basis.

2.2 Formulation as a Decision Problem

For purposes of developing a mathematically sound reliability criterion, we can characterize each contending fleet by its total fleet usage measured in km and its demonstrated reliability measured in km between mission failures, i.e. MKBMF. For a contender i, let the total fleet usage be T_i km, and let the number of mission failures be N_i . The fleet reliability, denoted by K_i , is calculated by dividing T_i by N_i .

In order to develop a criterion for fleet reliability, we consider the situation as a problem of decision under uncertainty. To ease the discussion, we will use the language of statistical hypothesis testing. As such, given a contender with fleet usage T_i km and N_i mission failures, the hypothesis we need to consider is whether or not the contending fleet has a true reliability less than the stated requirement of 6000 km between mission failures. The “true reliability” of a fleet is the number of kilometres between mission failures that we can expect to observe if the fleet is acquired.

We will write the formal test of hypothesis as follows:

H_0 (null hypothesis): The true fleet MKBMF is less than 6000 km.

H_a (alternate hypothesis): The true fleet MKBMF is greater than 6000 km.

The decision criterion is to disqualify the contending bidder if the observed evidence is insufficient for us to reject the null hypothesis. Although it sounds like a negative approach to the problem to assume a priori that a contending vehicle fleet does not meet the reliability target, this scientifically[2] crafted problem formulation makes it clear that the onus is on the bidder to provide sufficient evidence in terms of fleet reliability history in order to demonstrate compliance with the statement of requirement.

As in any statistical decision problem we have the usual two types of error to consider.

Type I Error

If we reject H_0 when it is in fact true, we will make a Type I error. In other words, a Type I error consists of admitting a contending fleet when in fact we should have disqualified it because its true MKBMF is less than 6000 km. This could happen for example if a fleet of vehicles with true MKBMF less than 6000 km but with little operational usage experienced by chance a particularly low number of mission failures. Since we strongly wish to avoid this situation, we will set the probability of Type I error (denoted by the Greek letter α) to the conventionally low value of 0.05 or 1 in 20. This means that with high probability vehicle fleets with true MKBMF less than 6000 km will be disqualified. Note that the 'high probability' is at least 19 times out of 20 or 0.95, the worst case being when the fleet MKBMF is exactly 6000 km. When the fleet MKBMF is lower than 6000 km, the probability of disqualification will be greater than 0.95.

Type II Error

If we accept H_0 when it is false, we will make a Type II error. In other words, a Type II error consists of disqualifying a contending vehicle fleet when its true MKBMF is greater than 6000 km (and therefore acceptable). This situation is likely to happen when a vehicle fleet with true MKBMF greater than 6000 km has insufficient evidence in terms of operational usage to demonstrate compliance with the target reliability standard or has experienced by chance a particularly high number of mission failures. We cannot completely control Type II error because it depends on the particular MKBMF value for each vehicle fleet and on how much operational usage the fleet has accumulated. However, we can minimize Type II error overall by selecting the optimal statistical test of hypothesis and we can graph the resulting probability of Type II error for a selection of parameter values to demonstrate the performance of the statistical procedure.

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[2] In the sense of Popper's philosophy of science where falsifiability is the fundamental criterion. [7]

2.3 Statistical Procedure for the Test of Hypotheses

To develop the statistical procedure for the test of hypothesis we first assume that the mission failures occur randomly with time. We can make several observations concerning the stochastic process which counts the number of mission failures over time:

a) The number of failures at time 0 is 0;

b) The time increments are independent and stationary. In other words the number of failures at time t_1 is independent of the number of failures at time t_2 and the expected number of failures in two time intervals of equal size follows the same distribution;

c) In any time interval, no matter how small, there is a non-zero probability that a failure will occur but it is not certain that a failure will occur, and,

d) Failures cannot be simultaneous. In other words, two failures occurring at the same time will be counted as one single mission failure.

By a standard theorem of stochastic processes (see p.119 of [4]), these observations allow us to make the assumption that mission failures accumulate according to a Poisson process. In the case of a vehicle fleet (where mission failures over distance (rather than time) are important) this means that mission failures happen randomly and independently as the vehicle fleet accumulates kilometres of operational usage.

Poisson processes are typically used to model the occurrence of events in time or space and are frequently used in reliability theory to model the accumulation of system failures.

Let $N(T)$ be the stochastic process which counts the number of mission failures in T km of vehicle fleet usage. Since we are assuming a Poisson Process, we can state the probability law of $N(T)$ as follows.

$$\text{Prob}\{N(T) = j\} \quad = \quad \frac{e^{-\lambda T}(\lambda T)^j}{j!} \qquad j = 1, 2, \ldots \qquad (1)$$

The Poisson Process $N(T)$ is characterized by the parameter λ representing the mission failure rate per kilometre of fleet usage. In terms of the more convenient fleet MKBMF value used in the discussion above we have the following relationship, with K representing the MKBMF of a fleet.

$$\lambda \quad = \quad \frac{1}{K}.$$

We proceed by assuming as a worst case that the true MKBMF for a contending vehicle fleet with T km of usage is 6000 km. To control the Type II error, we wish to accept H_0 in this case 19 times out of 20 [3], thereby disqualifying this fleet as a contender. It can be shown that a decision criterion based on the observed number of mission failures in T km of usage is optimal[4] for our test of hypothesis (see p292 of [3]). The idea is to reject H_0 when the number of mission failures in T km of usage is particularly low, such that the desired probability of Type I error is achieved. Specifically, we reject H_0 when the number of mission failures in T km is less than or equal to N_0 such that the following equation is satisfied.

$$\text{Prob}\{N(T) \le N_0 \mid K = 6000\} \quad \le \quad 0.05$$

$$\sum_{i=0}^{N_0} \frac{e^{-\frac{T}{6000}}\left(\frac{T}{6000}\right)^i}{i!} \quad \le \quad 0.05 \qquad (2)$$

For a given value of T we can solve inequality (2) for the largest value of N_0 such that the condition is satisfied. Our criterion is to reject H_0 when a fleet with T km of usage has N_0 or fewer mission failures. In other words, a contending vehicle fleet will be considered to have met the reliability target if there have been N_0 or fewer mission failures in the T km of operational usage to date.

2.4 Examples of the Application of the Decision Criterion

Consider Case 1 discussed above (mature fleet with good reliability history) in which the contending fleet had accumulated 14 million km of operational usage. Substituting T = 14 million in inequality (2) we get the following.

$$\sum_{i=0}^{N_0} \frac{e^{-2333.33}(2333.33)^i}{i!} \quad \le \quad 0.05$$

Solving numerically for N_0 gives a decision criterion of 2253 mission failures, i.e. the maximum acceptable number of mission failures is 2253. Since the observed number of mission failures for this fleet was 1650, we reject H_0 and accordingly declare that this vehicle fleet easily meets the reliability target as expected.

In Case 2 discussed above (a less mature and evidently less reliable fleet whose MKBMF is barely within the required range) the fleet in question had accumulated a total of 4 million km. Substituting T = 4 million in inequality (2) we get the following.

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[3] The use of a 0.05 decision rule (or 19 times out of 20) is arbitrary, but does define the point where, in many situations, researchers define reasonable doubt as the value of the test statistic that is equaled or exceeded only 5% of the time (when H_0 is true). (p290 of [8])

[4] A decision criterion based on the observed number of failures is optimal in the sense that the Type II error is minimized for a given Type I error, according to the Neyman-Pearson lemma.

$$\sum_{i=0}^{N_0} \frac{e^{-666.67}(666.67)^i}{i!} \quad \leq \quad 0.05$$

Solving numerically for N_0 gives a decision criterion of 623 mission failures. Since the observed number of mission failures for this fleet was 660, we accept H_0 and accordingly disqualify this vehicle fleet based on insufficient operational usage data to guarantee with high probability that the fleet reliability target has been achieved.

In Case 3 discussed above (small fleet with an excellent reliability record, exceeding the desirable MKBMF target of 10000 km) the fleet in question had accumulated a total of 3.5 million km. Substituting T = 3.5 million in inequality (2) we get the following.

$$\sum_{i=0}^{N_0} \frac{e^{-583.33}(583.33)^i}{i!} \quad \leq \quad 0.05$$

Solving numerically for N_0 gives a decision criterion of 543 mission failures. Since the observed number of mission failures for this fleet was 320, we reject H_0 and accordingly declare that this vehicle fleet easily meets the reliability target as expected.

In Case 4 discussed above (small fleet with low usage history but with a demonstrated MKBMF within the acceptable range) the fleet in question had accumulated a total of only 1million km. Substituting T = 1 million in inequality (2) we get the following.

$$\sum_{i=0}^{N_0} \frac{e^{-166.67}(166.67)^i}{i!} \quad \leq \quad 0.05$$

Solving numerically for N_0 gives a decision criterion of 145 mission failures. Since the observed number of mission failures for this fleet was 160, we accept H_0 and accordingly disqualify this vehicle fleet based on insufficient operational usage data to guarantee with high probability that the fleet reliability target has been achieved.

As shown in the four examples above, the decision criterion behaves in a manner consistent with the expectations in each case. The procedure is mathematically sound, fair to all contenders and caters for all fleet history circumstances.

2.5 Simplifying the Expression of the Decision Criterion

Because the criterion is based on an inequality and must be solved numerically for each separate value of fleet usage, it is open to criticism that it is perhaps too complex for a statement of requirement which is to be distributed to possible bidders worldwide.

Fortunately this difficulty can be easily overcome as follows. First, we note that as long as the total fleet operational usage for a contender is reasonably large we can approximate the cumulative Poisson in equality (2) by a Normal distribution (Annex C provides a justification of the use of a Normal approximation to the Poisson distribution).

As shown in Annex C, the sum of Poisson variables of parameter T/K can be approximated by a Normal of mean T/6000 and variance T/6000, provided that T is sufficiently large. The approximation is increasingly good as T increases and can be considered reliable for T/6000 greater than 10, or in other words for T greater than 60 000 km.

We therefore re-write the left-hand side of inequality (2) as follows.

$$\sum_{i=0}^{N_0} \frac{e^{-\frac{T}{6000}}(\frac{T}{6000})^i}{i!} \approx \int_{-\infty}^{N_0} Normal(\mu = \frac{T}{6000}; \sigma^2 = \frac{T}{6000})$$

Inequality (2) is then approximated by the following.

$$\int_{-\infty}^{N_0} Normal(\mu = \frac{T}{6000}; \sigma^2 = \frac{T}{6000}) \leq 0.05 \quad (3)$$

After using the fact that the area under the standardized Normal distribution to the left of –1.645 is equal to 0.05, we can re-write (3) as follows.

$$\frac{N_0 - \frac{T}{6000}}{\sqrt{\frac{T}{6000}}} = -1.645 \qquad (4)$$

Rearranging (4) to get (5) gives us a simple form of the reliability criterion N_0. A contending vehicle fleet with *T* km of operational usage must have experienced N_0 or fewer mission failures in order to meet the fleet reliability criterion.

$$N_0 = \frac{T}{6000} - 1.645\sqrt{\frac{T}{6000}} \quad (5)$$

We can also re-write the criterion (5) in terms of the minimum allowable fleet MKBMF, denoted $MKBMF_{min}$. This form of the fleet reliability criterion is given in (6) below. A contending vehicle fleet with T km of operational usage must have a value of MKBMF which exceeds $MKBMF_{min}$ in order to meet the fleet reliability criterion.

$$MKBMF_{\min} = \frac{T}{\frac{T}{6000} - 1.645\sqrt{\frac{T}{6000}}} \quad (6)$$

2.6 Analysis of Type II Error for the Decision Criterion

By nature of its construction, the decision criterion (2) has probability of Type I error at most 0.05 in all circumstances. However, the probability of Type II error depends on the true value of the fleet MKBMF and the total usage T for the fleet in question.

As a preliminary, we first consider the behaviour of $MKBMF_{min}$ as a function of the total fleet operational usage using equation (6) above. Figure 1 shows the graph. First note that the value of $MKBMF_{min}$ is always greater than 6000 km. Also note that as the total usage T grows large, the value of $MKBMF_{min}$ asymptotically approaches 6000 km. This behaviour is typical for statistical decision problems. It shows very clearly the increased reliability requirement placed on fleets with less operational data.

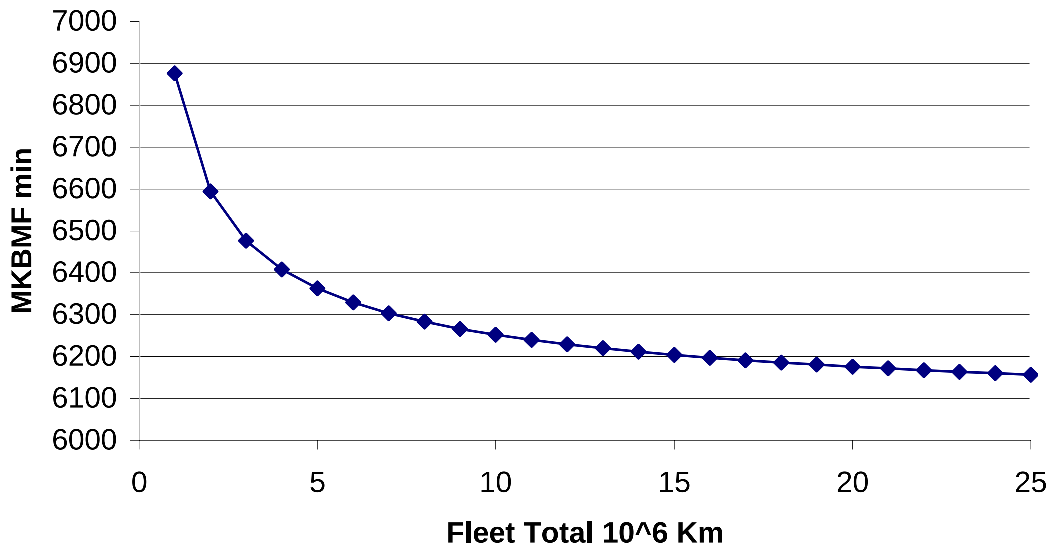

Figure 1. *Total fleet usage required to satisfy the reliability criterion as a function of the observed MKBMF*

Next we consider the probability of Type II error as a function of the true fleet MKBMF for several representative values of fleet operational usage *T*. These graphs are given in Figure 2. In all cases the probability of disqualifying a fleet whose true MKBMF exceeds the minimum target value of 6000 km approaches zero as the true MKBMF increases. This behaviour is again typical of statistical decision problems, reflecting the increased power of the test procedure when the true MKBMF value greatly exceeds the minimum acceptable value. Note in particular that the Type II error probability graphs get steeper around the critical value as operational experience *T* increases. This phenomenon reflects the benefit of increased operational experience (sample size) and is very typical of statistical decision problems.

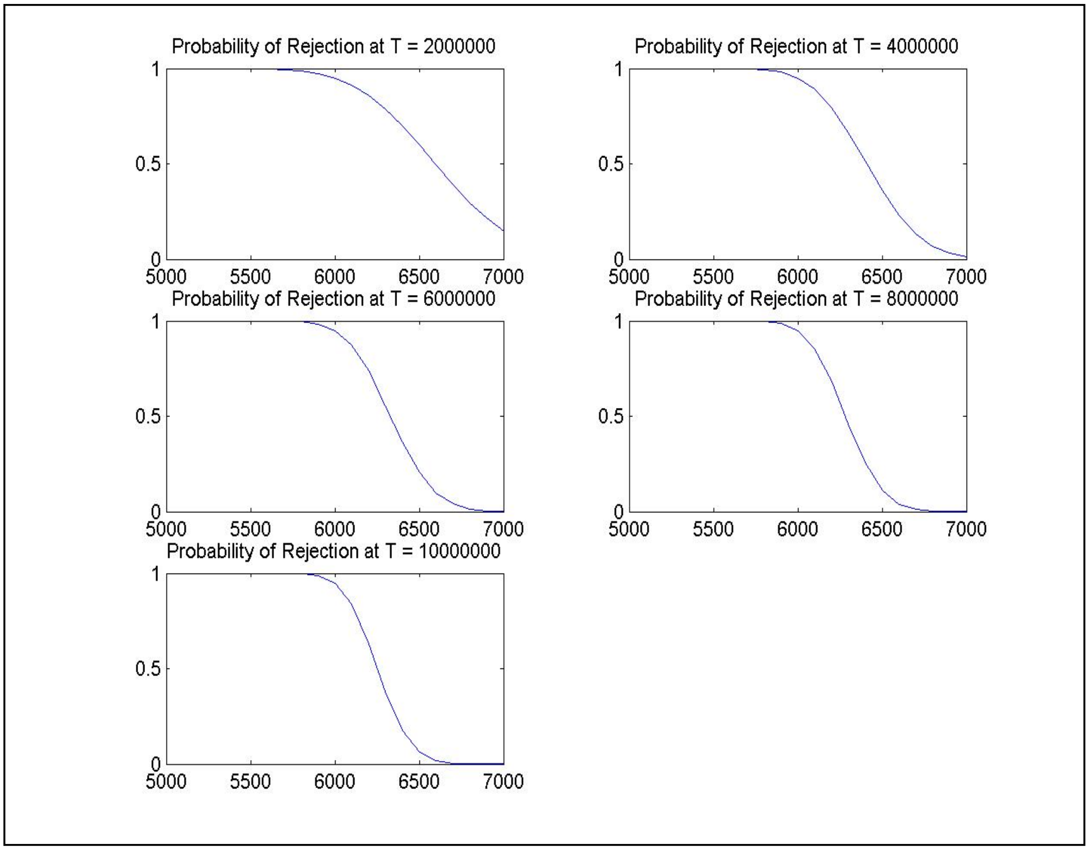

Figure 2. *Probability of rejection versus MKBMF for various levels of fleet usage*

3. Conclusion

A decision criterion to determine the minimal in-service demonstrated reliability for a contending vehicle fleet has been developed in this report. Two equivalent forms of the criterion are given by equations (5) and (6). Equation (5) expresses the critical number of failures as a function of the kilometres of fleet usage :

$$N_0 = \frac{T}{6000} - 1.645\sqrt{\frac{T}{6000}}$$

Equivalently, equation 6 is a closed-form formula for the minimum acceptable MKBMF :

$$MKBMF_{\min} = \frac{T}{\frac{T}{6000} - 1.645\sqrt{\frac{T}{6000}}}$$

The procedure is mathematically sound and can be seen to be simple and clear and fair to all contenders. The procedure is statistically optimal in the sense that it holds Type I error at a constant 5 percent and provides the best possible Type II error levels. Contending vehicle fleets with marginally acceptable MKBMF values or low operational usage will have a higher chance of being disqualified while mature fleets with MKBMF values well above the minimum acceptable level will have a high chance of being admitted to the competition.

The procedure has the advantage of providing to all contenders a reliability target that is dependent on their amount of operational experience (total kilometres of fleet usage), making for a competition that is (and appears) fair to all bidders.

In more general terms, the procedure that was presented here is useful in all situations where systems with different amounts of operational data must be compared. The case of contenders for the replacement of a fleet of vehicles was presented here but the methodology could be applied with almost no modification to the evaluation of the reliability of any system with random occurring failures, like a communication network for example. The methodology is also equally applicable to parts of a system or to the system as a whole (system of systems), as long as mission failure is defined appropriately in relation to the system or sub-system under study.

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Annex A : Original Proposal for a Methodology to Determine Required Fleet Size

DRAFT BRIEFING NOTE – DETERMINATION OF FLEET SIZE

13 April 2004

References: A. Blanchard, B.S., and Fabrycky, W.J., *Systems Engineering and Analysis*, 3 ed, Prentice Hall, Upper Saddle River, NJ, 1998
B. Website *How to determine Sample Size, Determining Sample Size* http://www.isixsigma.com/library/content/c000709ex.asp
C. British Columbia Institute of Technology Website *The Chi Squared Distribution* http://www.math.bcit.ca/faculty/david_sabo/apples/math2441/section8/onevariance/chisqtable/chisqtable.htm.
D. Draft Briefing Note on Comparative Maintenance workload between MLVW and FMTV, prepared by Maj Barrett

Background

1. The Medium Support Vehicle System project is in the process of developing a Solicitation of Interest and Qualification (SOIQ) to determine qualified bidders for it's vehicle contract. In order to minimize risk, the project's acquisition strategy relies on the procurement of an in-service fleet of vehicles to meet the Army's requirement. The Project Director has requested that a statistically sound methodology be developed to determine the minimal in-service fleet size required so that operational data may be used to predict the Army's stated reliability requirements.

Aim

2. The aim of this briefing note is to determine an empirical methodology to determine the fleet size needed such that operational data can be used to evaluate contender bids.

Discussion

3. The boundaries of this problem are provided by the essential requirements articulated in the Statement of Requirement (SOR). Specifically, requirements 3.1.1.1 and 3.1.1.2 which state:

a. It is essential that the design has been introduced or has been significantly updated in the last 8 years; and

b. It is essential that the vehicle be in service with a military service of a NATO country.

From these essentials, it is clear that the Army wants to procure a proven product and it can be assumed that compliant vehicle bids will have had sufficient time to correct initial "teething" problems with their design. This is important, since from a reliability perspective we can assume that the design has passed the "infant mortality" stage and will have a constant failure rate (see figure 1) and ref A. This further implies that the exponential failure law can be used to predict the reliability of the vehicle fleet and probability functions such as the Chi-Squared distribution can be used to determine the likely distribution of sample data about the mean. In the case of the MSVS it could be used to predict the required distribution of operational data about the reliability target stated in the Statement of Requirements (SOR).

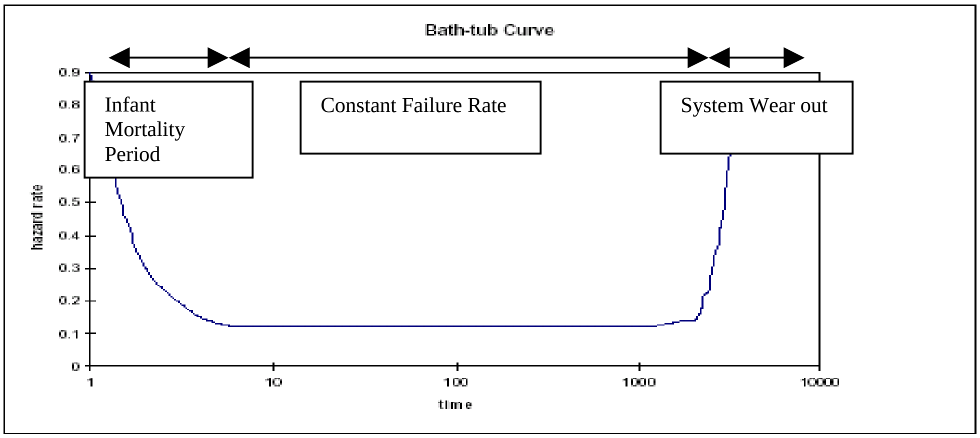

Figure 1 – Bath Tub Curve indicating failure rates with respect to product life cycle.

4. According to ref B, the Chi- Squared distribution is used for calculating confidence interval estimates for population variances and standard deviations, and in testing hypotheses involving the same parameters. The graph below provides an indication of the shape of the distribution. As the number of degrees of freedom (independent variables) "v" increase in value, the distribution appears to become more symmetric (bell-like). The peak, which would roughly locate the mean value, is shifting to the right as the number of degrees of freedom "**v"** increase. In fact, for values of ν larger than about 30, the χ^2-distribution can be well approximated as a normal distribution with the indicated mean and standard deviation

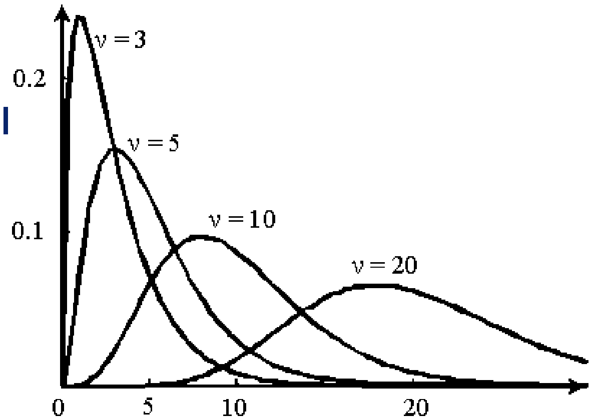

Figure 2 – Chi-Squared Distribution

5. Degrees of Freedom. Degrees of Freedom are defined as independent variables that can have an impact upon the overall reliability of the system. In the case of the SMP MSVS, the vehicle is a complex system of systems with potentially many independent sources of failure (engine, transmission, electrical, hydraulics, CTI, ABS, etc). If we assume that as v approaches a large number greater than 30 the sample mean distribution can be solved using a Normal distribution then the calculations become fairly simple.

Analysis

6. In order to determine the minimum fleet size a number of assumptions need to be made, specifically:

a. Operational data from the HLVW will provide a suitable proxy to determine operational reliability data standard deviation for the new MSVS, see ref D;

b. a 95% confidence interval; and

c. a sample error of 5%.

We are solving for the sample size n.
A 95% degree confidence corresponds to α = 0.05. Each of the shaded tails in the following figure has an area of $\alpha/2$ = 0.025. The region to the left of $z_{\alpha/2}$ and to the right of z = 0 is 0.5 - 0.025, or 0.475. In the Table of Standard Normal Distribution, an area of 0.475 corresponds to a z value of 1.96. The critical value is therefore $z_{\alpha/2}$ = 1.96.

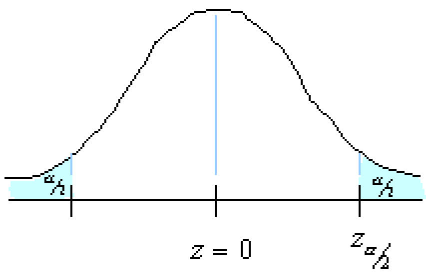

Figure 3 – Normal Distribution with a Z value = 1.96

Based upon the operational data at Annex A, the margin of error is E = 0.35 and the standard deviation is σ = 3.363. Using the formula below, we can calculate n:

$$n = \left[\frac{z_{\alpha/2}\sigma}{E} \right]^2$$

((1.96 x 3.363)/0.35)^2 = 354.6

So we will need to sample at least 355 (rounded up) randomly selected vehicles. With this sample we will be 95% confident that the sample mean $\bar{x}$ will be within 5% of the true mean of the population of vehicles.

Conclusion

7. Based upon the above calculations it would be prudent to consider only contenders that have an in-service fleet of at least 355 vehicles. It should be noted that the larger the fleet size the greater confidence that operational data will be representative of the entire fleet (in the case of the MSVS 1500 vehicles).

Annex A – HLVW Standard deviation calculations based on historical data 1995-98.

Prepared for Maj JM Morin, PD MSVS

By Maj AC Barrett, ILSM MSVS

Annex A
Draft Briefing Note on Fleet Size
15-Apr-04

CFR	1995	1996	1997	1998	AVG CM	Deviation fm mean
92104	11	7	6	5	7.25	0.097656
92122	4	9	7	7	6.75	0.035156
92131	8	9	7	4	7	0.003906
92133	9	11	6	5	7.75	0.660156
92145	4	23	9	7	10.75	14.53516
92184	8	10	6	9	8.25	1.722656
92202	1	0	6	6	3.25	13.59766
92205	0	3	5	2	2.5	19.69141
92218	15	12	6	8	10.25	10.97266
92220	4	3	1	13	5.25	2.847656
92446	8	12	9	5	8.5	2.441406
92551	4	10	4	3	5.25	2.847656
92555	4	6	3	2	3.75	10.16016
92629	10	5	3	3	5.25	2.847656
92724	8	19	9	9	11.25	18.59766
92797	17	21	12	16	16.5	91.44141
92805	4	3	4	2	3.25	13.59766
92844	2	11	7	3	5.75	1.410156
92878	2	7	7	7	5.75	1.410156
93195	4	5	4	5	4.5	5.941406
				Sample Avg	**6.9375**	
				Std Dev		**3.362795**

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Annex B : Aim of the Project as Seen by MSVS Project Director

-----Original Message-----
From: Morin Maj JJMR@CLS DLR@Ottawa-Hull
Sent: Wednesday, 28 April, 2004 12:06
To: Desmier PE@ADM(Mat) DMGOR@Ottawa-Hull
Subject: MSVS fleet size

The aim of the MSVS fleet size determination is to devise a statistically sound methodology be developed to determine the minimal in-service fleet size required so that operational data may be used to predict the Army's stated reliability requirements. Also this ensures that the vehicles proposed are not developmental in nature and that any kinks have been worked out. Modern vehicles manufacturers will claim that the engine they intend to use is used commercially with a half million Km between failures (probable) and the transmissions, axles, transfer case, etc components all meet similar longevity. The key point that is never mentioned is that these components were never integrated into a package (or truck) with the mission profile requested. Therefore, the overall performance is unknown, components compatibility is unknown, weak points are unknown. At the end, rubber must meet the road (or dirt) to confirm the engineering thinking. This is an integration problem that has to be validated and corrected before the vehicle can reach the predicted reliability. Simple he says......

J-M Morin

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Annex C : Normal Approximation to the Poisson Distribution

The Central Limit Theorem states that if X_1, X_2..., X_n are independent random variables with constant mean $E(X_i) = \mu$ and constant variance $Var(X_i) = \sigma^2$, then for n sufficiently large, the sum $X_1 + \ldots + X_n$ is approximately Normal with mean $n \cdot \mu$ and variance $n \cdot \sigma^2$ and the average $\overline{X} = (\sum_{i=1,\ldots,n} X_i)/n$ is approximately Normal with mean μ and variance σ^2 / n.

It can easily be verified that a Poisson variable of parameter λ can be considered as the sum of λ Poisson variables of parameter 1. Assuming that a Poisson variable X describes the number of occurrences of a phenomenon over a period of time λ, we can divide the time interval in n subintervals of time λ / n and call Y_i the number of occurrences of the phenomenon in the i^{th} subinterval. By definition of the Poisson process, the Y_i's are independent Poisson variables and their mean and variance are given by λ / n.

Hence the total number of occurrences X is the sum of the individual Y_i's and the Central Limit Theorem can be applied to conclude that the distribution of X can be approximated by a Normal distribution with mean λ and variance λ.

Example

Assume a Poisson variable X with parameter λ. We will calculate the probability that X is more than 10% greater than λ using the Normal approximation for different values of λ. The values in Table 2 are calculated using the two following formulas for the exact and approximate evaluation of the probability.

Exact Evaluation (Poisson): $\text{Prob}\{X > N\} \;=\; 1 - \sum_{i=0}^{N} \frac{e^{-\lambda} \cdot \lambda^i}{i!}$.

Normal Approximation: $\text{Prob}\{X > N\} \;=\; 1 - \Phi(\frac{N + 0.5 - \lambda}{\sqrt{\lambda}})$. [5]

[5] Note that a continuity correction of 0.5 is required because we are approximating a discrete random variable with a continuous random variable.

__Table 2:__ Comparison of the exact evaluation and the Normal approximation of different Poisson probabilities for increasing values of the parameter λ.

λ	Probability to Evaluate	Exact Evaluation (Poisson)	Normal Approximation	Absolute Error	Relative Error
5	Prob{X > 6}	0.2378	0.2512	0.0134	5.64%
10	Prob{X > 11}	0.3032	0.3179	0.0144	4.75%
15	Prob{X > 17}	0.2511	0.2593	0.0082	3.27%
20	Prob{X > 22}	0.2794	0.2881	0.0087	3.11%
25	Prob{X > 28}	0.2366	0.2420	0.0054	2.28%
50	Prob{X > 55}	0.2155	0.2183	0.0028	1.30%
100	Prob{X >110}	0.1471	0.1469	0.0002	0.14%

We can see that the Normal approximation gets more accurate as λ gets large. Starting at λ=10, the relative error is less than 5%.

We have established in the main document that the parameter of the Poisson distribution of interest for this problem is given by the number of kilometres of fleet usage divided by the fleet's MKBMF. With the requirement that the MKBMF be no less than 6000 km, we can expect the Normal to be a good approximation for values of fleet usage that exceed 60 000 km (λ=10).

List of symbols/abbreviations/acronyms/initialisms

ABS	Anti Lock Break System
BGP	Bureau de Gestion de Projet
CF	Canadian Forces
CFR	Contender Failure Rate
CORT	Central Operational Research Team
CTI	Computer Telephony Integration
DGOR	Director General Operational Research
DMGOR	Director Materiel Group Operational Research
DND	Department of National Defence
DRO GM	Directeur de la Recherche Opérationnelle Groupe du Matériel
ÉROC	Équipe de Recherche Opérationnelle Centrale
FC	Forces Canadiennes
FMTV	Family of Medium Tactical Vehicles
HLVW	Heavy Logistics Vehicle Wheeled
ILS	Integrated Logistics Support
ILSM	Integrated Logistics Support Manager
KMBF	Kilométrage Moyen de Bon Fonctionnement
MKBMF	Mean Kilometres Between Mission Failures
MLVW	Medium Logistics Vehicle Wheeled
MSVS	Medium Support Vehicle System
PD	Project Director
PMO	Project Management Office
SMP	Standard Military Pattern

SMVS	Système Mobile de Véhicule de Soutien
SOIQ	Statement of Interest and Qualification
SOR	Statement of Operational Requirement

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4. AUTHORS (last name, first name, middle initial)
Emond, Edward, J. and Yazbeck, T.

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When evaluating possible contenders to replace a fleet of Canadian Forces (CF) vehicles, it is not enough to look solely at the contenders' performance, in this case Mean Kilometres Between Mission Failures (MKBMF). In support of a tasking from the project management office for the Medium Support Vehicle System (PMO MSVS) to the Directorate of Materiel Group Operational Research (DMGOR), a procedure was developed to factor in the contenders' experience (fleet usage) in order to determine the reliability of the observed MKBMF. We wish to determine the minimum MKBMF that a contending fleet can observe in order to satisfy the requirement that there be no less than K kilometres between mission failures, 95% of the time. It is shown that for a contending fleet with a total of T vehicle kilometres of in-service usage, the observed MKBMF must exceed the value $MKBMF_{min}$, where $MKBMF_{min} = T/((T/K) - 1.645(T/K)^{1/2})$.

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MTBF
Mean Kilometres Between Mission Failures
MKBMF
Operational Reliability

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