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# Statistical Cross-Correlation Algorithms for Temporal Pulse Profiles

Fred A. Dilkes

**Defence R&D Canada – Ottawa**
TECHNICAL MEMORANDUM
DRDC Ottawa TM 2004-220
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## Abstract

Two statistical techniques are presented for comparing temporal pulse trains received from an Electronic Support receiver to *a priori* templates representing the expected behaviour of radar signals. The techniques implement a specialized cross-correlation procedure and incorporate the effects of non-cumulative jitter. A theoretical basis is presented in which the signal of interest is modeled to be present in a noisy channel. The channel noise may include the failure to detect some expected pulses along with the detection of unexpected pulses. The methods are compared to the more straightforward approaches based on cross-correlation histograms and a simulated example scenario is presented.



## Résumé

Le document présente deux techniques statistiques permettant de comparer des trains d'impulsions temporelles obtenus à partir d'un récepteur de soutien électronique (SE), avec des modèles a priori représentant le comportement attendu de signaux radar. Ces techniques comprennent une procédure de corrélation croisée spécialisée et intègrent les effets de gigue non cumulatifs. Le document présente une base théorique pour la modélisation du signal d'intérêt dans un canal bruyant. Le bruit du canal peut être dû entre autres à la non-détection d'impulsions attendues aussi bien qu'à la détection d'impulsions inattendues. Les méthodes sont comparées aux approches plus simples fondées sur des histogrammes de corrélations croisées, et un scénario simulé est présenté à titre d'exemple.

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# Executive summary

The field of “Electronic Support” (ES) depends heavily on the availability of a robust mechanism with which to analyze sequences of pulses. In general, the complete stream of pulses received by an ES system is composed of multiple signals exhibiting diverse features, both periodic and aperiodic, that are observed through a common, imperfect channel. The complexity of the complete ES problem has lead to the development of numerous techniques designed to separate and identify the constituent signals.

Pulses trains are typically analyzed and matched against some form of *a priori* information in order to determine the identity and/or behaviour of one or more uncooperative radar signals. The techniques for temporal pulse train analysis fall primarily into two conceptually distinct categories. The first and perhaps the more traditional technique is to base the analysis on generic periodicities that occur in the pulse train. Autocorrelation methods are commonly used to identify periodic signals and are often implemented using so-called “delta-$\tau$ histograms”. Such techniques lead ultimately to an estimation of the parameters associated with identified pulse patterns and a comparison of those estimates with some *a priori* look-up table. It has been observed that these techniques exhibit numerous shortcomings; amongst them is the inability to detect complex pulse sequences and patterns with very long periods (such as $N$-level staggers, where $N \gg 1$).

A second approach to pulse-train analysis is to search directly for specific sequences of pulses using *a priori* patterns and parameters. This approach has the advantage of being somewhat more flexible than the autocorrelation approach since the patterns of interest can be quite complicated, and need not be periodic. This approach is typically implemented using some type of cross-correlation between the observed pulse sequence and the templates in order to detect a good match.

In this memo, two alternative cross-correlation techniques are presented based on probabilistic signal models. It is argued that these approaches are theoretically well-suited to account for such effects as the uncertainties in pulse arrival times (whether induced by deliberate jitter or by receiver and channel imperfections) and other channel impairments such as the effects of missing and spurious pulses. The methods are compared to a cross-correlation histogram using a simulated pulse scenario and the results are reported.

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# Sommaire

En matière de soutien électronique (SE), il est très important de disposer d'un mécanisme robuste d'analyse de séquences d'impulsions. En général, le train d'impulsions complet reçu par un système SE se compose de multiples signaux présentant diverses caractéristiques, tant périodiques qu'apériodiques, qui sont observées dans un canal commun imparfait. La complexité de l'ensemble du problème SE a conduit au développement de nombreuses techniques en vue d'isoler et d'identifier les signaux constitutifs.

Les trains d'impulsions sont typiquement analysés et mis en correspondance avec une certaine forme d'information a priori afin de déterminer l'identité et/ou le comportement d'un ou plusieurs signaux radar non coopératifs. Les techniques d'analyse de trains d'impulsions temporelles se divisent en deux catégories distinctes du point de vue conceptuel. Les techniques de la première catégorie, qui sont peut-être les plus traditionnelles, font reposer l'analyse sur des périodicités génériques qui se produisent dans le train d'impulsions. Des méthodes d'autocorrélation sont couramment utilisées pour identifier les signaux périodiques et sont souvent mises en oeuvre à l'aide d'histogrammes dits ≪ delta-tau ≫. Ces techniques permettent finalement d'estimer les paramètres associés aux configurations d'impulsions et de comparer ces estimations avec les données d'une table de recherche a priori. On a observé qu'elles présentent de nombreuses lacunes ; entre autres, elles ne permettent pas de détecter des séquences d'impulsions complexes et des configurations à très longues périodes (p. ex. décalages à N niveaux, où $N \gg 1$).

Une deuxième façon d'analyser les trains d'impulsions consiste à rechercher directement des séquences d'impulsions spécifiques en utilisant des configurations et des paramètres a priori. Cette approche a l'avantage d'être un peu plus souple que les techniques d'autocorrélation, car les configurations d'intérêt peuvent être très compliquées, et elles ne sont pas nécessairement périodiques. Elle fait en général appel à un certain type de corrélation croisée entre la séquence d'impulsions observée et les modèles pour la détection de correspondances pertinentes.

Le document présente deux techniques de corrélation croisée fondées sur un modèle de signal probabiliste. On soutient que ces approches offrent des bases théoriques satisfaisantes pour rendre compte d'effets tels que les incertitudes dans les temps d'arrivée des impulsions (dues soit à une gigue délibérée ou à des imperfections du récepteur et du canal) et d'autres dégradations du canal, comme les effets d'impulsions manquantes ou non essentielles. Les méthodes sont comparées à un histogramme de corrélations croisées, à l'aide d'un scénario d'impulsions simulé, et les résultats obtenus sont présentés.

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# 1 Introduction

The field of "Electronic Support" (ES) depends heavily on the availability of a robust mechanism with which to analyze sequences of pulses. In general, the complete stream of pulses received by an ES system is composed of multiple signals exhibiting diverse features, both periodic and aperiodic, that are observed through a common, imperfect channel. The complexity of the complete ES problem has lead to the development of numerous techniques designed to separate and identify the constituent signals.

Pulse trains are typically analyzed and matched against some form of *a priori* information in order to determine the identity and/or behaviour of one or more uncooperative radar signals. The techniques for temporal pulse train analysis fall primarily into two conceptually distinct categories. The first and perhaps the more traditional technique is a "data-driven" technique whereby the analysis is based on generic periodicities that occur in the pulse train. Autocorrelation methods are commonly used to identify periodic signals and are often implemented using so-called "delta-$\tau$ histograms" [1, 2, 3]. Such techniques lead ultimately to an estimation of the parameters associated with the identified pulse patterns and a comparison of those estimates with some *a priori* look-up table. As discussed in [4], these techniques exhibit numerous shortcomings, amongst them is the inability to detect complex pulse sequences and patterns with very long periods (such as $N$-level staggers, where $N \gg 1$).

A second approach to pulse-train analysis is known as the "model-driven" approach whereby one searches directly for specific sequences of pulses that are known to be associated with particular systems, using *a priori* patterns and parameters. This approach has the advantage of being somewhat more flexible than the autocorrelation approach since the patterns of interest can be quite complicated, and need not be periodic. The most straightforward implementation of such an approach is to perform some form of cross-correlation between the observed pulse sequence and the templates in order to detect a good match. An example of this approach is that of Elton [5] who advocates a "cross-correlation histogram". Other related techniques have been explored as part of a collaboration under The Technical Cooperation Panel [6].

In this memo, two alternative cross-correlation techniques are presented based on probabilistic signal models. It is argued that these approaches are theoretically well-suited to account for such effects as the uncertainties in pulse arrival times (whether induced by deliberate jitter or by receiver and channel imperfections) and other channel impairments such as the effects of missing and spurious pulses. The methods are compared to the cross-correlation technique of [5] using a simulated pulse scenario and the results are reported.

# 2 Gaussian scoring profiles

In this section, a technique is presented to compute a probabilistic measure of consistency between an observed pulse sequence and an *a priori* signal template. The signal of interest is represented using a Gaussian statistical model with stationary channel impairments.

## 2.1 Stationary signal model

A Gaussian transmitted pulse sequence model is proposed. The model consists of two components: a "pulse template" $T$ and a time-of-arrival variance $\sigma^2$. The pulse template is a set of nominal times, denoted by $T = \{t_1, t_2, \cdots, t_N\}$ at which a pulse may be transmitted, modulo some absolute time offset. The time-of-arrival variance $\sigma^2$ indicates the amount of non-cumulative jitter that may be associated with each pulse in $T$. Each pulse time in $T$ is known as a "template element" and the number of such elements is denoted by $|T| = N$.

A simple statistical model of the transmitted pulse train is given as follows. If a pulse sequence "$T$ starts at time offset $\tau$" then the transmitted pulses form a set of normally distributed random variables denoted by $\tilde{X} = \{\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_N\}$. A model of "non-cumulative jitter" [7] is adopted so that the transmitted pulse times are independent random variables with Gaussian distributions

$$\tilde{x}_j \sim \mathcal{N}(t_j + \tau, \sigma^2)\,. \tag{1}$$

The mean values of these distributions are modeled to be $\mathcal{E}(x_j) = t_j + \tau$ and the variances are $\mathcal{E}(x_j - (t_j + \tau))^2 = \sigma^2$. For simplicity, we suppose that all of the transmitted pulses are contained within some observation time interval whose total time duration is denoted by $D$.

The transmitted pulse set $\tilde{X}$ is randomly mapped into a set of received pulse times $X$ by a stationary channel impairment process. Each pulse in $\tilde{X}$, independently of the other pulses, has a probability $P_\mathrm{d}$ of being detected and appearing in the set $X$; conversely, the probability of the pulse being missed, and not appearing in $X$, is $1 - P_\mathrm{d}$. In addition, the detected pulses are interleaved with spurious pulses. For simplicity, suppose that the spurious pulses are distributed according to a Poisson process with pulse density $\rho_\mathrm{s}$ over a the observation interval. The resulting set of received pulse times is denoted by $X = \{x_1, x_2, \cdots, x_M\}$ and includes all detected and spurious pulses. The number of pulses in this set is denoted by $|X| = M$.

## 2.2 Gaussian scores

The objective of this section is to develop a method for determining a measure of consistency between a particular template model $(T, \sigma^2)$ and some observed pulse sequence $X$. Here, we begin by expressing the likelihood of making the observation using the particular Gaussian model, in the presence of the stationary channel noise. An exact expression for such a likelihood function can be expressed by enumerating every possible joint classification of the pulses in $X$ in terms of spurious and detected pulses. In order to facilitate this, it is convenient to define the following notion:

An "association" $(t, x) \in T \times X$ is a 2-tuple that identifies a possible correspondence between a template element $t \in T$ and an observed pulse $x \in X$. An "association relation" $K \subset T \times X$ is a set of associations that identifies a one-to-one correspondence between a subset of elements of template $T$ and a subset of observed pulses in $X$. Each observed pulse should be associated with no more than one template element, and *vice versa*. Some template elements and/or pulses may not be associated at all. The number of associations in the relation $K$ is denoted by $|K| \leq \min\{|X|, |T|\}$.

The likelihood density for observing $X$ given that "$T$ starts at time offset $\tau$" using the above Gaussian model with stationary channel impairments is given by

$$\begin{aligned} \tilde{p}_{\mathcal{N}}(X|T,\sigma^2,P_{\rm d},\rho_{\rm s};\tau) &= \sum_K \Big[ \prod_{(t,x)\in K} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t+\tau-x)^2}{2\sigma^2}} \Big] \\ &\quad \times (1-P_{\rm d})^{|T|-|K|} P_{\rm d}^{|K|} \rho_{\rm s}^{|X|-|K|} e^{-\rho_{\rm s} D} . \end{aligned} \tag{2}$$

Here, the sum $\sum_K$ extends over every possible association relation $K$ between $X$ and $T$; the product $\prod_{(t,k)\in K}$ extends over every association pair within the relation $K$. The likelihood function given in (2) may be interpreted as an exact "score" with which to measure the consistency of the observation $X$ with the hypothesis that the template $T$ starts at time offset $\tau$.

Several of the factors in (2) are independent of association relation $K$ and can be dismissed as overall scaling constants. After some elementary manipulations, it is found that the likelihood density can be decomposed as follows:

$$\tilde{p}_{\mathcal{N}}(X|T,\sigma^2,P_{\rm d},\rho_{\rm s};\tau) = p_{\mathcal{N}}(X|T,\sigma^2,c_0;\tau)(1-P_{\rm d})^{|T|}\rho_{\rm s}^{|X|}e^{-\rho_{\rm s}D}$$

where

$$p_{\mathcal{N}}(X|T,\sigma^2,c_0;\tau) \equiv \sum_K \exp\Big( - \sum_{(t,x)\in K} c(t,x,\tau) \Big) , \tag{3}$$

$$c(t,x,\tau) \equiv \frac{(t+\tau-x)^2}{2\sigma^2} + c_0 ,$$

and

$$c_0 \equiv \ln\left( \frac{(1-P_{\rm d})\sigma\rho_{\rm s}}{P_{\rm d}} \right) + \frac{1}{2}\ln(2\pi) .$$

Hereafter, it is assumed that the parameters have been chosen in such a way so that $c_0 < 0$; this is generally true unless the pulse environment is extremely dense or the signal is highly jittered. The expression $c(t,x,\tau)$ is known below as the "cost" of the association $(t,x)$ for a particular value of the time offset $\tau$. All costs and probabilities appearing in (3) are dimensionless.

It is not practical to enumerate every association relation in order to evaluate the above likelihood functions since (i) the number of association relations is exponential in both $|X|$ and $|T|$ and (ii) for a given value of the time offset $\tau$ most of the association relations have very large values of the total cost $\sum c(t,x,\tau)$ and do not contribute significantly to (3). As a result, a sub-optimal strategy is needed to appropriately estimate the score as a function of $\tau$.

An alternative scoring function that could be used to approximate (3) is given by

$$q_{\mathcal{N}}(X|T,\sigma^2,c_0;\tau) \equiv \max_K \exp\Big(-\sum_{(t,x)\in K} c(t,x,\tau)\Big)\,. \tag{4}$$

The score given in (4) represents a lower bound for (3),

$$q_{\mathcal{N}}(X|T,\sigma^2,c_0;\tau) \leq p_{\mathcal{N}}(X|T,\sigma^2,c_0;\tau)\,.$$

The score in (4) is known hereafter as the "$q_{\mathcal{N}}$-score" and is more often represented by its logarithm. It may be of interest to note that the $q_{\mathcal{N}}$-score is restricted to the interval

$$0 \leq \ln q_{\mathcal{N}}(X|T,\sigma^2,c_0;\tau) \leq |T||c_0|\,.$$

## 2.3 Scoring intervals

The $q_{\mathcal{N}}$-score (4) has the advantage of having a finite representation that can be computed without further approximation. To see this, consider some particular scenario described by $(X,T,\sigma^2,c_0)$, and define

$$\hat{K}(\tau) \equiv \arg\min_K \sum_{(t,x)\in K} c(t,x,\tau)$$

to be the optimal association relation between $X$ and $T$ at time offset $\tau$. This relation-valued function is necessarily piecewise constant in the sense that there must exist a partition of the real line $\mathbb{R}$ into intervals characterized by a finite set of partition points $\mathcal{T}_{\mathcal{N}} = \{\tau_s | s = 0\ldots S\}$ labeled monotonically

$$\tau_0 < \tau_1 < \tau_2 < \cdots < \tau_S\,,$$

so that $\hat{K}(\tau)$ is unchanged over each interval of the form $\tau \in (\tau_{s-1},\tau_s)$. This relation is denoted by $\hat{K}_s$ so that

$$\hat{K}(\tau) = \hat{K}_s \text{ for } \tau \in (\tau_{s-1},\tau_s)\,.$$

Since $T$ and $X$ are finite sets, it can be shown that the partition can be chosen so that $S < 2|X||T|$ and

$$\hat{K}(\tau) = \emptyset \text{ for } \tau < \tau_0 \text{ or } \tau > \tau_S\,.$$

Correspondingly the logarithm of the $q_{\mathcal{N}}$-score

$$\ln q_{\mathcal{N}}(X|T,\sigma^2,c_0;\tau) = \begin{cases} -\sum_{(t,x)\in\hat{K}_s} c(t,x,\tau) & \text{if } \tau \in (\tau_{s-1},\tau_s),\, s = 1\ldots S\,, \\ 0 & \text{if } \tau < \tau_0 \text{ or } \tau > \tau_S\,, \end{cases}$$

is a quadratic function of $\tau$ over each interval.

In order to illustrate these concepts consider case in which the observations and template are respectively given by

$$\begin{aligned} X &= \{-2, 2.1, 10, 26, 31.4, 41, 69.2, 70, 73.3\}\,, \\ T &= \{0, 30, 70\}\,. \end{aligned} \tag{5}$$

The remaining parameters are taken to be $\sigma^2 = 25\mu s^2$, $\rho_s = 0.02/\mu s$ and $P_d = 0.9$, leading to $c_0 = -3.581$. The $q_{\mathcal{N}}$-score is illustrated in Figure 1(a) and the partition points are illustrated by vertical dashed lines. The horizontal dotted grid lines indicate multiples of $|c_0|$. (Figures 1(b), 1(c) and 1(d) will be discussed in subsequent sections.)

The segmentation of the real line into intervals $(\tau_{s-1}, \tau_s)$ can be done using a straightforward inductive algorithm with linear complexity. The algorithm is discussed in in Annex A.

# 3 Uniform scoring profiles

The reasoning described in the preceding section may also be applied to an probabilistic template based on uniform distributions. In this case, we suppose the template consists of a pulse template $T$ and some time interval parameter $w$. As in the previous section, $T$ is the set of nominal times, but now $w$ is a positive parameter describing the width of a uniform distribution interval. In this case, instead of (1), the transmitted time variables are distributed by

$$\tilde{x}_j \sim \mathcal{U}(t_j - w/2 + \tau, t_j + w/2 + \tau)\,. \tag{6}$$

In other words, $\tilde{x}_j$ has an equal probability of taking on any value that satisfies $|t_j + \tau - \tilde{x}_j| < w/2$.

In this case, we can replace (2) by a likelihood density based on the uniform jitter model,

$$\begin{aligned} \tilde{p}_{\mathcal{U}}(X|T,w,P_d,\rho_s;\tau) &= \sum_K \Big[ \prod_{(t,x)\in K} w^{-1} I\big(|t+\tau-x| < w/2\big) \Big] \\ &\quad \times (1-P_d)^{|T|-|K|} P_d^{|K|} \rho_s^{|X|-|K|} e^{-\rho_s D} \end{aligned} \tag{7}$$

where $I\big(|t+\tau-x| < w/2\big)$ represents the indicator function

$$I\big(|t+\tau-x| < w/2\big) = \begin{cases} 1 & \text{if } |t+\tau-x| < w/2\,, \\ 0 & \text{if } |t+\tau-x| \geq w/2\,. \end{cases}$$

For simplicity, it is assumed that the elements of the template $T$ are well separated compared with the width $w$ so that, for each value of $\tau - x$ there is no more than one template element $t \in T$ that satisfies $|t+\tau-x| < w/2$.

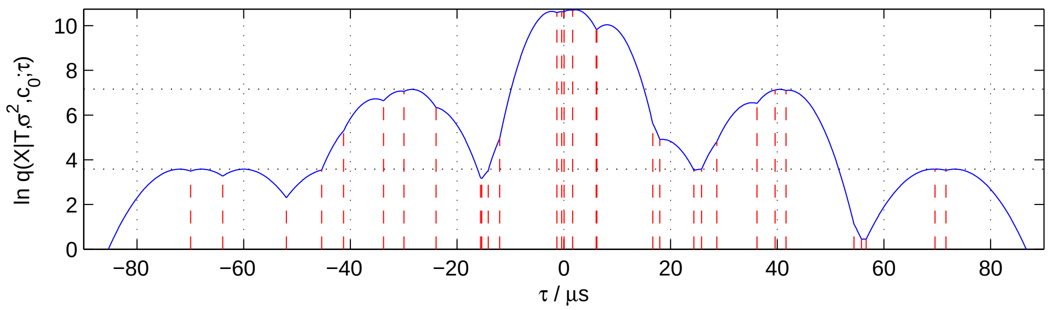
Now, for $\tau \in \mathbb{R}$ and $t \in T$, let $l_t(\tau)$ denote the number of received pulses $x \in X$ that satisfy $|t+\tau-x| < w/2$. It is not difficult to demonstrate that (7) can be rewritten as

$$\tilde{p}_{\mathcal{U}}(X|T,w,P_d,\rho_s;\tau) = p_{\mathcal{U}}(X|T,w,d_0;\tau)(1-P_d)^{|T|}\rho_s^{|X|} e^{-\rho_s D}$$

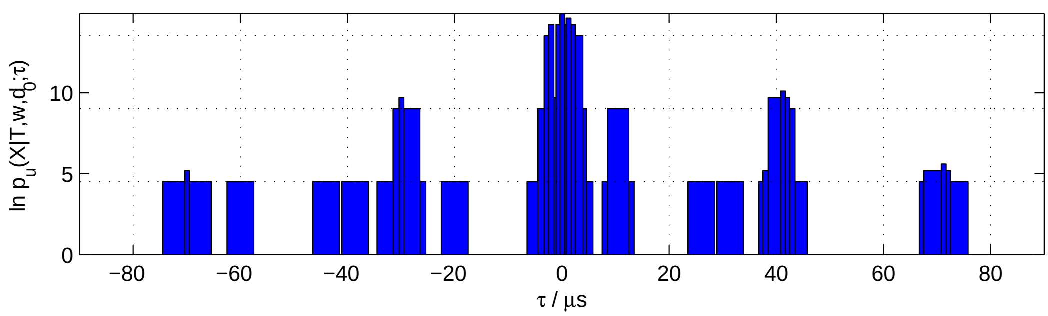
where

$$\ln p_{\mathcal{U}}(X|T,w,d_0;\tau) = \sum_{t\in T} \ln \left[ \frac{P_d l_t(\tau)}{(1-P_d)w\rho_s} + 1 \right]. \tag{8}$$

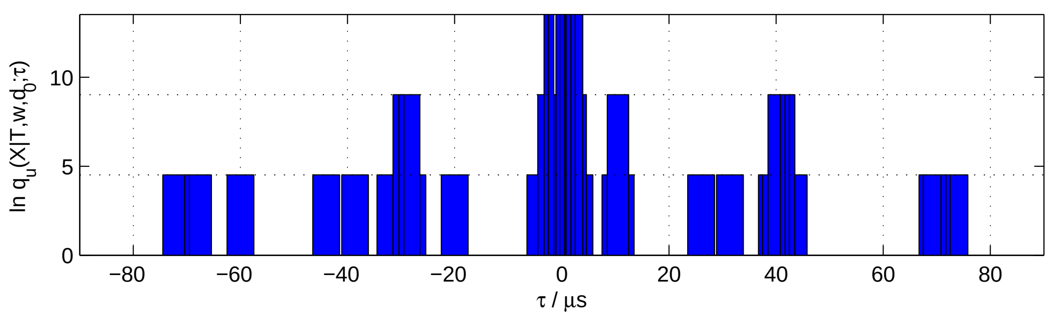
Here it has been recognized that the dependence of the right hand side of the this equation on the parameters $P_d$, $\rho_s$ and $w$ can be summarized by a single composite parameter, denoted



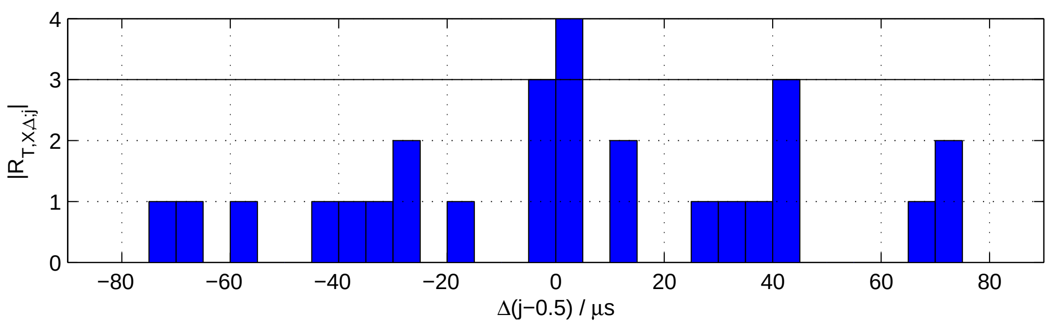

(a) $q_{\mathcal{N}}$-score $\ln q_{\mathcal{N}}(X|T,\sigma^2,c_0;\tau)$ using $\sigma^2 = 25$, $c_0 = -3.581$




(b) $p_{\mathcal{U}}$-score $\ln p_{\mathcal{U}}(X|T,w,d_0;\tau)$ using $w = 5$ , $d_0 = -4.511$.




(c) $q_{\mathcal{U}}$-score $\ln q_{\mathcal{U}}(X|T,w,d_0;\tau)$ using $w = 5$, $d_0 = -4.511$.




(d) Cross-correlation histogram $|R_{T,X,\Delta;j}|$ using $\Delta = 5$

***Figure 1:*** *Scores for the example configuration* (5).

by $d_0$. For later convenience, this parameter is taken to be

$$d_0 \equiv -\ln\left[\frac{P_\mathrm{d}}{(1-P_\mathrm{d})w\rho_\mathrm{s}}+1\right] < 0\,.$$

Since $l_t(\tau)$ is a discrete-valued, and therefore piecewise constant, function of $\tau$, so must be $\ln p_{\mathcal{U}}$. That is, it must be possible to partition the real line at a finite number of points so that $\ln p_{\mathcal{U}}(X|T,w,d_0;\tau)$ is constant over each interval. In fact, the set of discontinuities in $\ln p_{\mathcal{U}}$ is almost surely equal to the finite set

$$\mathcal{T}_{\mathcal{U}} = \{\tau \in \mathbb{R} | \exists x \in X, t \in T, \tau = x - t \pm w\}\,.$$

Consequently, a brute-force approach to computing the complete partition is to first calculate each of the $2|X||T|$ differences $x-t\pm w$, and then sort the resulting set.

Figure 1(b) illustrates the the $p_{\mathcal{U}}$-score for the example in (5) using parameter values $w=5$, $\rho_\mathrm{s}=0.02$, $P_\mathrm{d}=0.9$. Note that the dotted horizontal grid lines represent multiples of $|d_0| = 4.511$.

If, instead of the summation $\sum_K$ in (7), a $\max_K$ operation is used, then the result would become

$$\begin{aligned}\tilde{q}_{\mathcal{U}}(X|T,w,P_\mathrm{d},\rho_\mathrm{s};\tau) &\equiv \max_K \prod_{(t,x)\in K} \left[w^{-1} I\big(|t+\tau-x| < w/2\big)\right] \\ &\quad \times (1-P_\mathrm{d})^{|T|-|K|} P_\mathrm{d}^{|K|} \rho_\mathrm{s}^{|X|-|K|} e^{-\rho_\mathrm{s} D} \\ &= q_{\mathcal{U}}(X|T,w,d_0;\tau)(1-P_\mathrm{d})^{|T|}\rho_\mathrm{s}^{|X|} e^{-\rho_\mathrm{s} D}\,.\end{aligned}$$

where

$$\ln q_{\mathcal{U}}(X|T,w,d_0;\tau) = N(\tau)|d_0|\,. \tag{9}$$

Here $N(\tau)$ indicates the number of template elements $t \in T$ for which there is at least one pulse $x \in X$ satisfying $|t+\tau-x| < w/2$. Like $\ln p_{\mathcal{U}}$, this function is piecewise constant; its discontinuities form a subset of $\mathcal{T}_{\mathcal{U}}$.

The score in (9) is a lower bound for (8)

$$q_{\mathcal{U}}(X|T,w,d_0;\tau) \le p_{\mathcal{U}}(X|T,w,d_0;\tau)\,,$$

and $\ln q_{\mathcal{U}}$ is restricted to the interval

$$0 \le \ln q_{\mathcal{U}}(X|T,\sigma^2,c_0;\tau) \le |T||d_0|\,.$$

The $q_{\mathcal{U}}$-score is illustrated in Figure 1(c) for the example (5) using parameter values $w=5$, $\rho_\mathrm{s}=0.02$, $P_\mathrm{d}=0.9$ and $d_0=-4.511$.

# 4 Histogram scoring profiles

In Section 5, the $q_{\mathcal{N}}$- and $p_{\mathcal{U}}$-scoring techniques are contrasted against the cross-correlation histogram presented in [5]. The latter technique is briefly paraphrased here. In that approach, the received pulse time set $X$ is compared to the template set $T$ using the cross-correlation function

$$S_{T,X}(\tau) \equiv \sum_{x \in X} \sum_{t \in T} \delta(t + \tau - x),$$

where $\delta(\cdot)$ refers to the delta function of Dirac. This function is then smoothed in some fashion. In [5], the smoothing is accomplished by integrating over sequential intervals of the form $((j-1)\Delta, j\Delta]$ where $\Delta$ is some specified bin width and $j$ is a bin counter. The result is

$$\int_{(j-1)\Delta}^{j\Delta} S_{T,X}(\tau)\, \mathrm{d}\tau = \left| R_{T,X,\Delta;j} \right|,$$

where $\left| R_{T,X,\Delta;j} \right|$ counts the number of elements in the set

$$R_{T,X,\Delta;j} \equiv \{(t,x) | t \in T, x \in X, (j-1)\Delta < x - t \leq j\Delta\}.$$

Like the association relations introduced in Subsection 2.2, each set $R_{T,X,\Delta;j} \subset T \times X$ is a collection of associations between elements of $T$ and $X$. However, unlike the previous relations, there is no one-to-one requirement on $R_{T,X,\Delta;j}$ so that two or more different pulses in $X$ may be associated with the same template element in $T$. Although the perfect association would have exactly $|T|$ elements, there is no fundamental upper bound on $|R_{T,X,\Delta;j}|$.

By plotting $|R_{T,X,\Delta;j}|$ against the bin centers $(j-0.5)\Delta$, for sequential values of $j$, one obtains the cross-correlation histogram of described in [5].

For illustration, the cross-correlation histogram for the configuration in (5) is shown in Figure 1(d) using a bin-width of $\Delta = 5$. The optimal number of associations is $|T| = 3$ and is indicated by a solid horizontal line. Note that the bin corresponding to $\tau \in (0,5]$ contains $|R_{T,X,\Delta;1}| = 4$ associations, despite $T$ containing only three elements. This function can be compared with the other scoring functions shown in Figure 1 for the same example configuration.

# 5 Simulation

This section demonstrates the effectiveness of the scoring approaches introduced here and compares them with the cross-correlation histogram of [5] by using a synthetic data set.

## 5.1 Configuration

Following the example of [5], a synthetic experiment is conducted in which three signals are interleaved and observed through a noisy channel.

The characteristics of these signals are as follows:

1. The first signal is a set of transmitted pulses $\tilde{X}_1$ described by a constant pulse-to-pulse interval of 875$\mu$s, without jitter.
2. The second signal $\tilde{X}_2$ is a 5 element/5 position staggered sequence whose pulse-to-pulse intervals are $\{620\mu s, 325\mu s, 220\mu s, 475\mu s, 490\mu s\}$. These intervals are repeated in cyclic order, without jitter
3. The third signal $\tilde{X}_3$ exhibits non-cumulative jitter with a nominal pulse-to-pulse interval of 875$\mu$s. The actual transmission time of each pulse is a random variable normally distributed about a nominal time with a standard deviation of 12$\mu$s.

In the simulation, each of these signals is persistent over a long period of observation and each is characterized by a starting phase that has been selected from a uniform distribution.

The complete set of transmitted pulses is the union $\tilde{X} = \tilde{X}_1 \cup \tilde{X}_2 \cup \tilde{X}_3$. This sequence is then mapped into a received pulse sequence $\tilde{X} \mapsto X$ by a stationary noise model in which (i) each pulse in $\tilde{X}$ has a probability of 0.9 of being detected and appearing in $X$, and (ii) $X$ includes additional spurious pulses that are distributed according to a Poisson process over the observation width with average density of 1 pulse every 150$\mu$s. The received set $X$ is the union of all detected and spurious pulses.

## 5.2 Results

Several scores for this scenario are computed for values of the offset in the interval $0 \leq \tau \leq 6000\mu$s. These are illustrated in Figures 2-7. In each figure, the three panels show $q_{\mathcal{N}}$-, $p_{\mathcal{U}}$- and histogram-scores with the horizontal dotted grid lines indicating multiples of $|c_0|$, $|d_0|$ or 1 respectively. The $q_{\mathcal{U}}$-scores have been omitted since they are almost indistinguishable from the $p_{\mathcal{U}}$-scores. Where relevant, the scoring parameters $\rho_s = 0.02\mu s^{-1}$, and $P_d = 0.9$ have been used to compute $c_0$ and $d_0$.

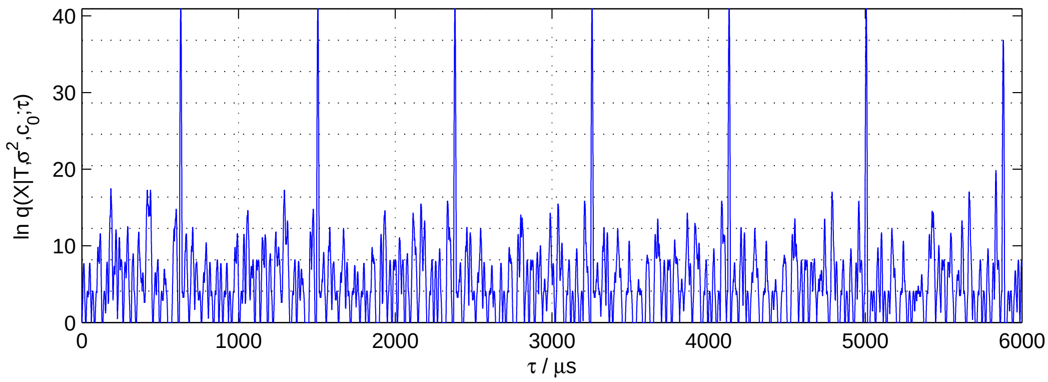
Figures 2 and 3 show the results of comparing $X$ with a pulse template containing eleven equally spaced elements,

$$\begin{aligned} T_1 &= \{0.0, 875.0, 1750.0, 2625.0, 3500.0, 4375.0, \\ &\quad 5250.0, 6125.0, 7000.0, 7875.0, 8750.0\}, \end{aligned}$$

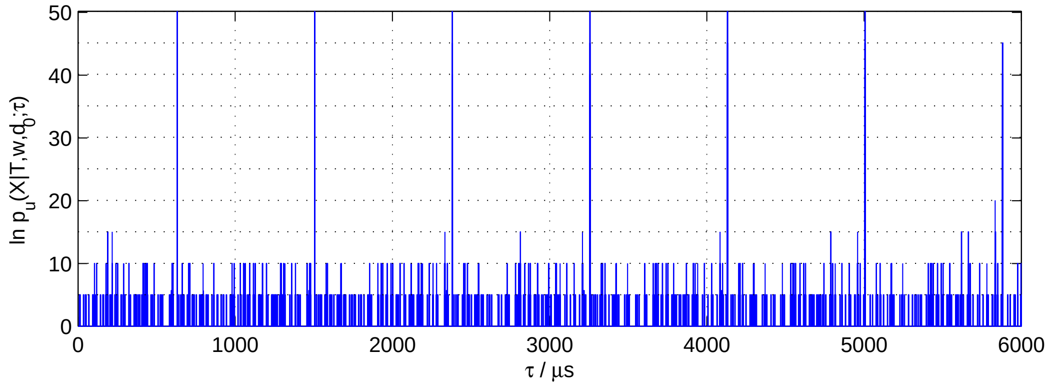
corresponding to the expected behaviour of signal $\tilde{X}_1$. In Figure 2, the $q_{\mathcal{N}}$- and $p_{\mathcal{U}}$-scores are calculated, respectively, using $\sigma = 3\mu$s and $w = 3\mu$s. The histogram is computed using $\Delta = 3\mu$s. Figure 3 illustrates the $q_{\mathcal{N}}$-, $p_{\mathcal{U}}$- and histogram-scores using, respectively, $\sigma = 12\mu$s, $w = 12\mu$s and $\Delta = 12\mu$s. It should be noted that, since there is no jitter on the signal $\tilde{X}_1$, the scoring approaches with the smaller values of $\sigma$, $w$ or $\Delta$ are expected to be more suitable than those with the larger values.

Figures 4 and 5 show similar results using the template

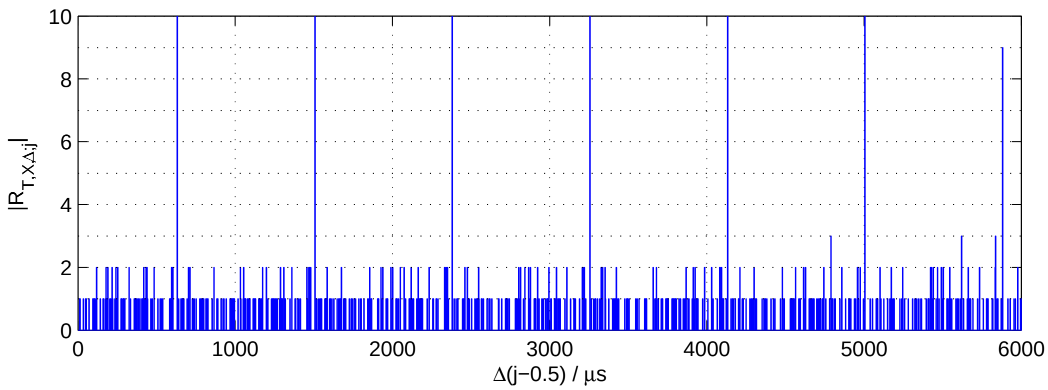
$$\begin{aligned} T_2 &= \{0.0, 620.0, 945.0, 1165.0, 1640.0, 2130.0, \\ &\quad 2750.0, 3075.0, 3295.0, 3770.0, 4260.0\}, \end{aligned}$$




(a) $q_{\mathcal{N}}$-score with $\sigma = 3\mu$s , $c_0 = -4.092$.

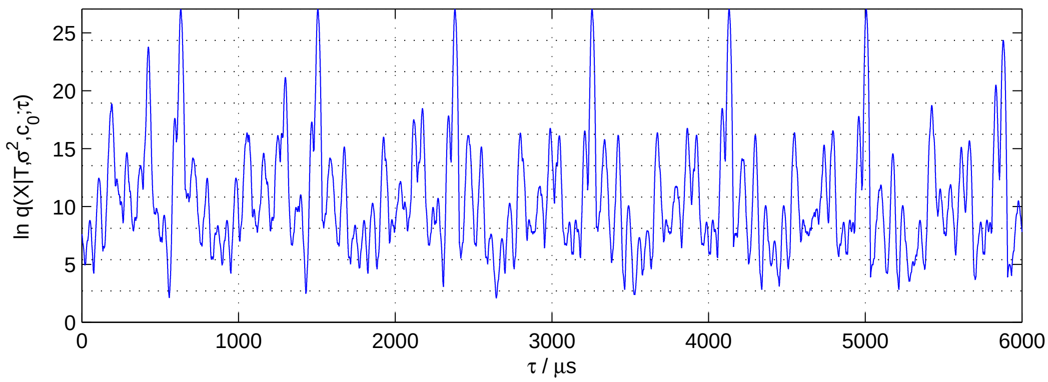



(b) $p_{\mathcal{U}}$-score with $w = 3\mu$s , $d_0 = -5.017$.

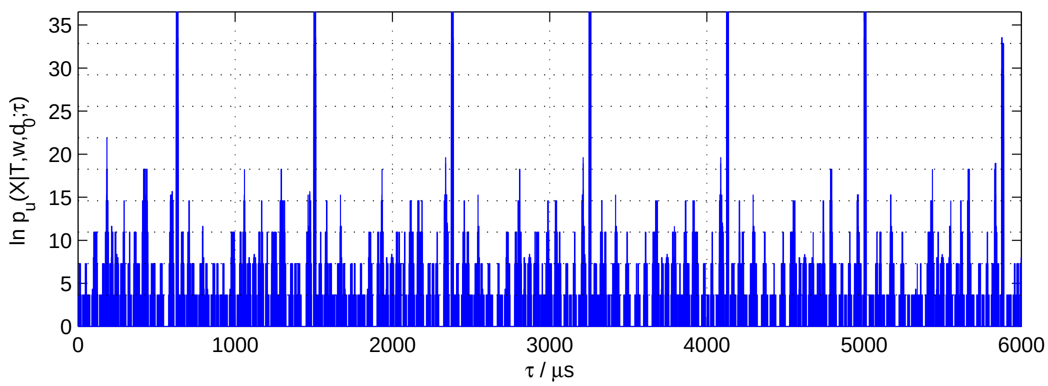



(c) Histogram with $\Delta = 3$.

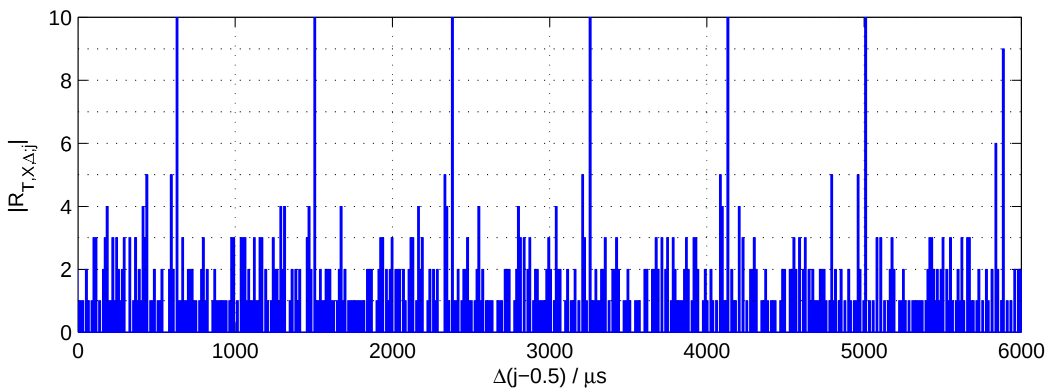
***Figure 2:*** *Cross-correlation scores for $X$ computed with template $T_1$ and narrow tolerance.*




(a) $q_{\mathcal{N}}$-score with $\sigma = 12\mu s$ , $c_0 = -2.705$.




(b) $p_{\mathcal{U}}$-score with $w = 12\mu s$ , $d_0 = -3.651$.




(c) Histogram with $\Delta = 12$.

***Figure 3:*** *Cross-correlation scores for $X$ computed with template $T_1$ and wide tolerance.*

corresponding to the expected behaviour of signal $\tilde{X}_2$. Like the previous signal, $\tilde{X}_2$ has no jitter on the arrival times, and the smaller values of $\sigma$, $w$ or $\Delta$ shown in Figure 4 should be considered to more suitable than the larger values shown in Figure 5.

Finally, Figures 6 and 7 show the scores when the template

$$\begin{aligned} T_3 &= \{0.0, 600.0, 1200.0, 1800.0, 2400.0, 3000.0, \\ &\quad 3600.0, 4200.0, 4800.0, 5400.0, 6000.0\}, \end{aligned}$$

is applied. This template represents the expected behaviour of $\tilde{X}_3$. Since the signal is associated with a non-cumulative $1\sigma$ jitter of $12\mu$s, the smaller values $\sigma = 3\mu$s, $w = 3\mu$s or $\Delta = 3\mu$s shown in Figure 6 are not expected to perform as well as the larger values $\sigma = 12\mu$s, $w = 12\mu$s or $\Delta = 12\mu$s shown in Figure 7.

Some observations about these results are in order. Both $\tilde{X}_1$ and $\tilde{X}_2$ represent idealized signals that contain no jitter, or time-of-arrival uncertainty. Although this situation is somewhat idealized, it illustrates the limiting behaviour of the various scoring techniques. When the templates $T_1$ and $T_2$ are applied to the interleaved signal, we see that, in all cases, it is preferable to use smaller values of the parameters $\sigma$, $w$ and $\Delta$. Moreover, the histogram performs as well or better than the $q$- and $q_u$ scores.

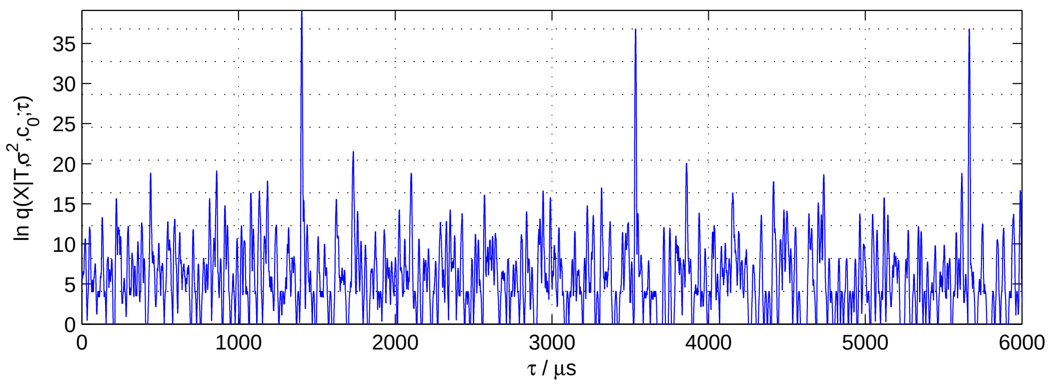
The signal $\tilde{X}_3$ is more realistic. It is fairly clear that the larger parameter shown in Figure 7 result in a cleaner signal detection than do the smaller values shown in Figures 6. In fact, the $\Delta = 3\mu$s histogram results in the most ambiguous results. By contrast, the cleanest and most consistent results are generated by the $q_{\mathcal{N}}$-score with $\sigma = 12\mu$s, possibly owing to the fact that this score is a most closely matches the signal generation model.

# 6 Conclusions

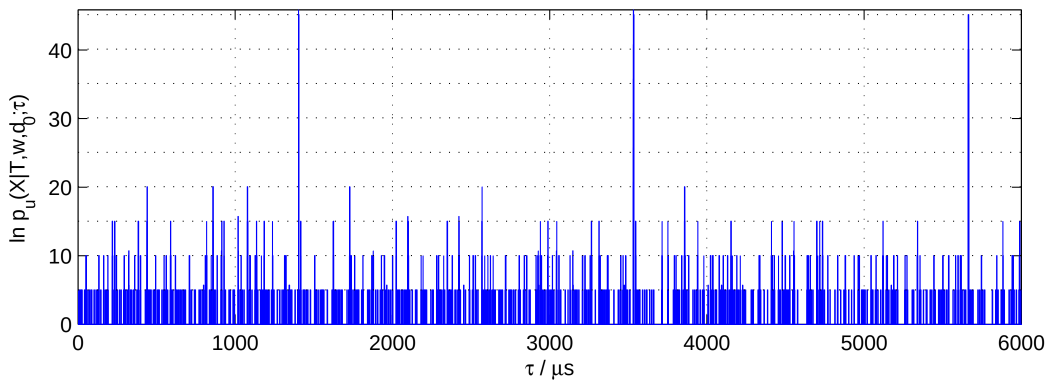
This paper has described candidate techniques that may be applied in Electronic Support for identifying fixed patterns of pulses in an observed sequence specified using an *a priori* template. The techniques described apply to temporal pulse sequences in which the pulse descriptor words consist of pulse arrival times. However, it is not difficult to modify these techniques to include more general pulse parameters, such as carrier frequency, and pulse modulation parameters whose measured values may follow some known probability distribution.

The first method, known herein as the $q_{\mathcal{N}}$-score, attempts to formulate a lower bound for the likelihood of observing a measured sequence given some *a priori* model. The complete model consists of a characteristic template of relative pulse arrival times, a Gaussian non-cumulative jitter variance and two stationary parameters describing the spurious pulse density and pulse detection probability. An algorithm is presented whereby an exact representation for the $q_{\mathcal{N}}$-score can be evaluated with linear time complexity.

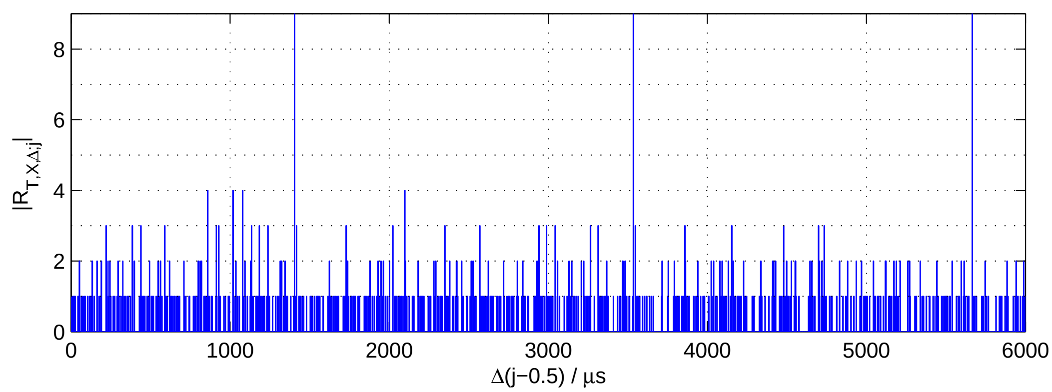
The second method, known as $p_{\mathcal{U}}$-score, is similar to the $q_{\mathcal{N}}$-score, but attempts to compute the likelihood of observing a sequence using a non-cumulative jitter model that follows a




(a) $q_{\mathcal{N}}$-score with $\sigma = 3\mu s$ , $c_0 = -4.092$.

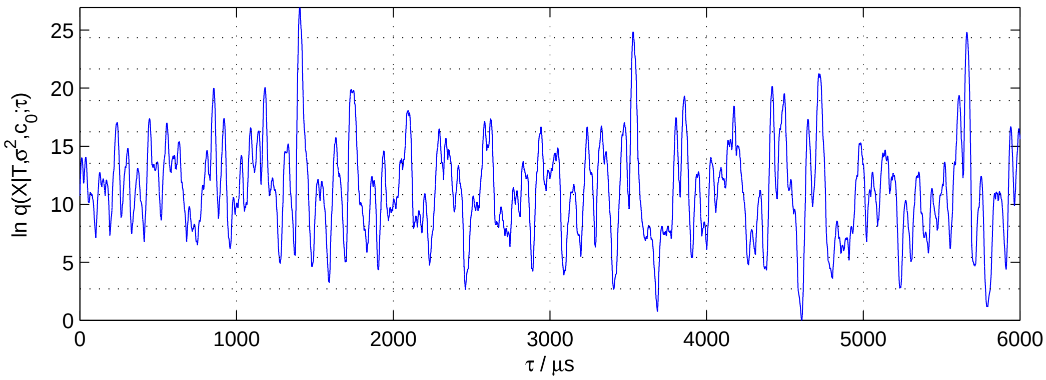



(b) $p_{\mathcal{U}}$-score with $w = 3\mu s$ , $d_0 = -5.017$.

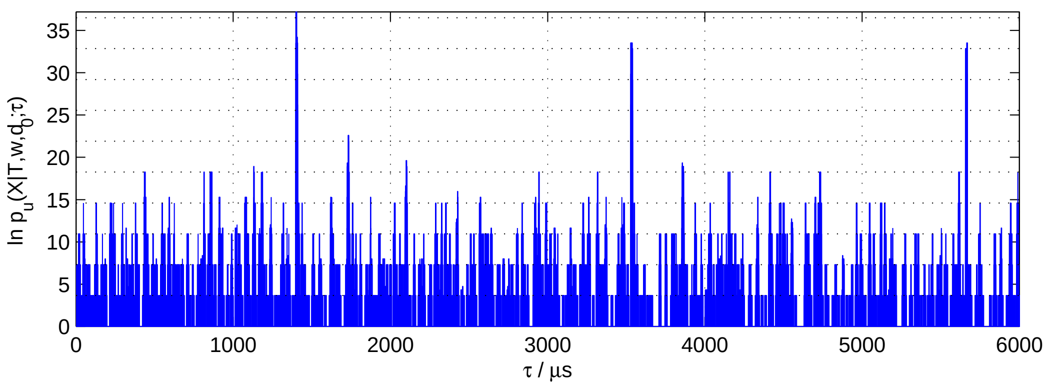



(c) Histogram with $\Delta = 3$.

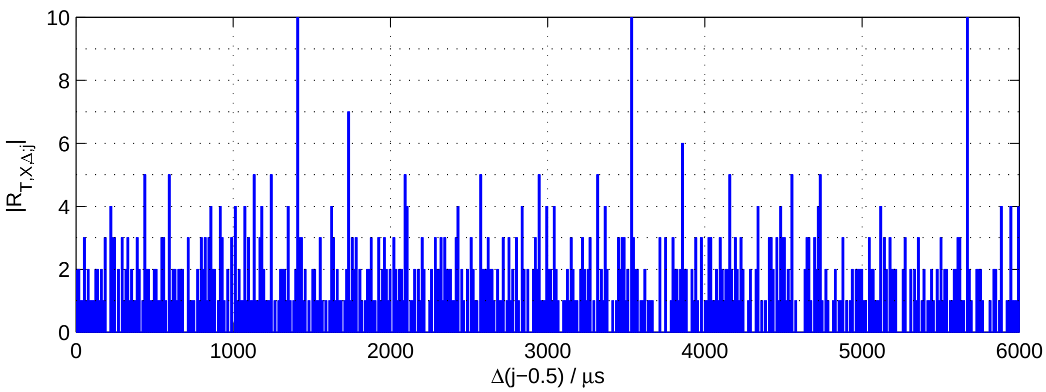
***Figure 4:*** *Cross-correlation scores for $X$ computed with template $T_2$ and narrow tolerance.*




(a) $q_{\mathcal{N}}$-score with $\sigma = 12\mu s$ , $c_0 = -2.705$.

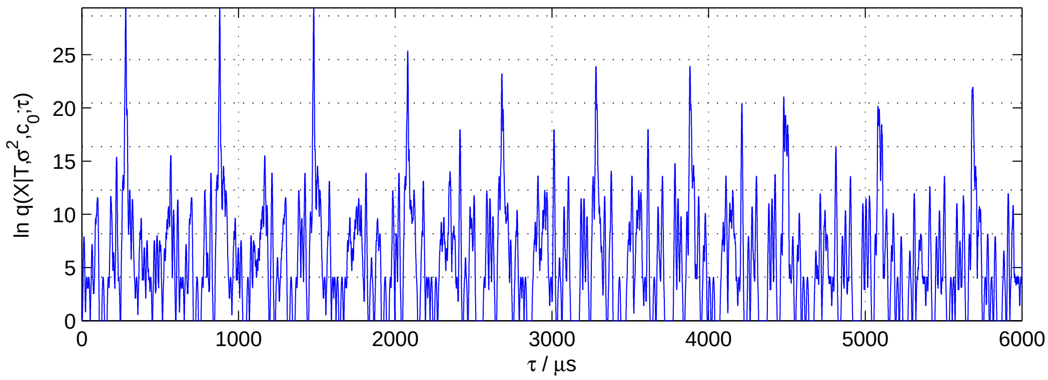



(b) $p_{\mathcal{U}}$-score with $w = 12\mu s$ , $d_0 = -3.651$.

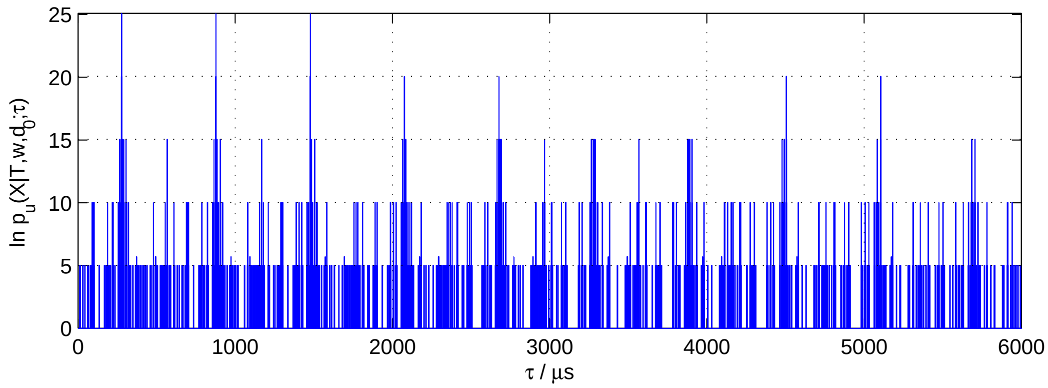



(c) Histogram with $\Delta = 12$.

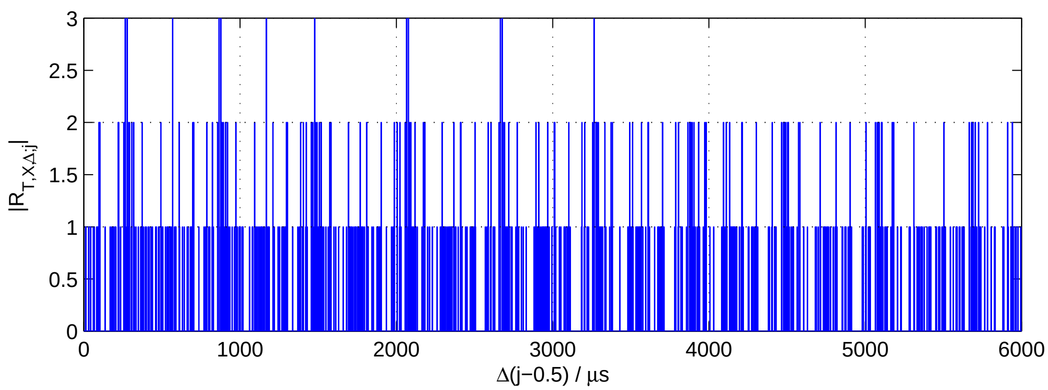
***Figure 5:*** *Cross-correlation scores for $X$ computed with template $T_2$ and wide tolerance.*




(a) $q_{\mathcal{N}}$-score with $\sigma = 3\mu s$ , $c_0 = -4.092$.

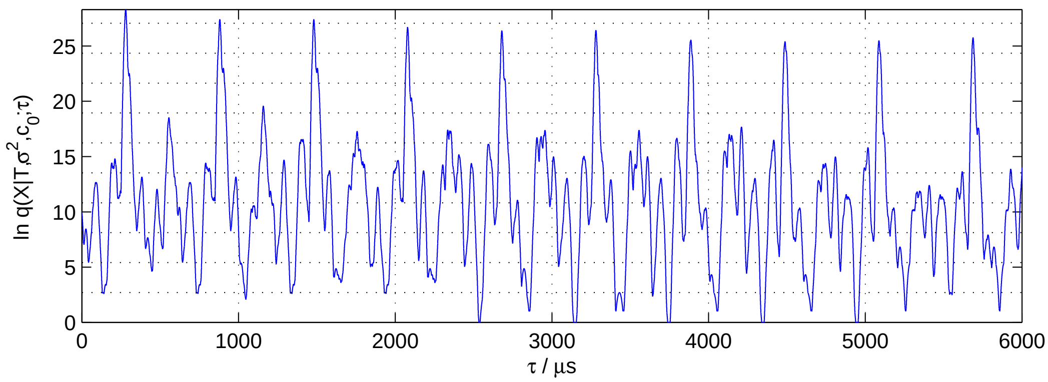



(b) $p_{\mathcal{U}}$-score with $w = 3\mu s$ , $d_0 = -5.017$.

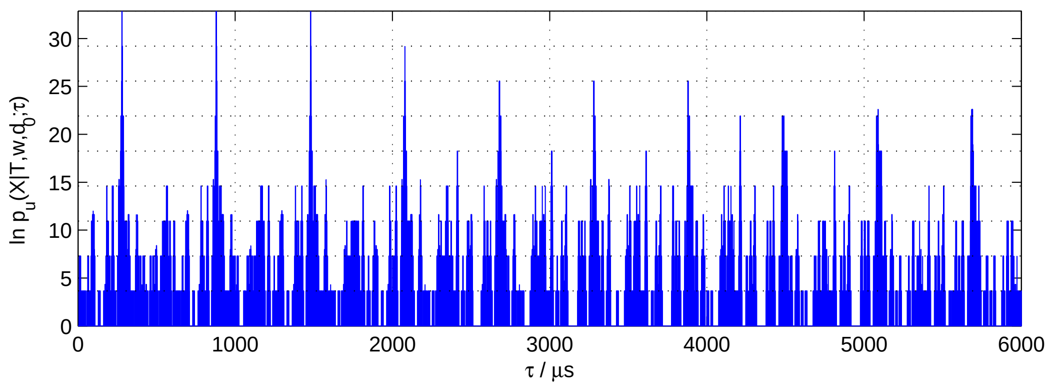



(c) Histogram with $\Delta = 3$.

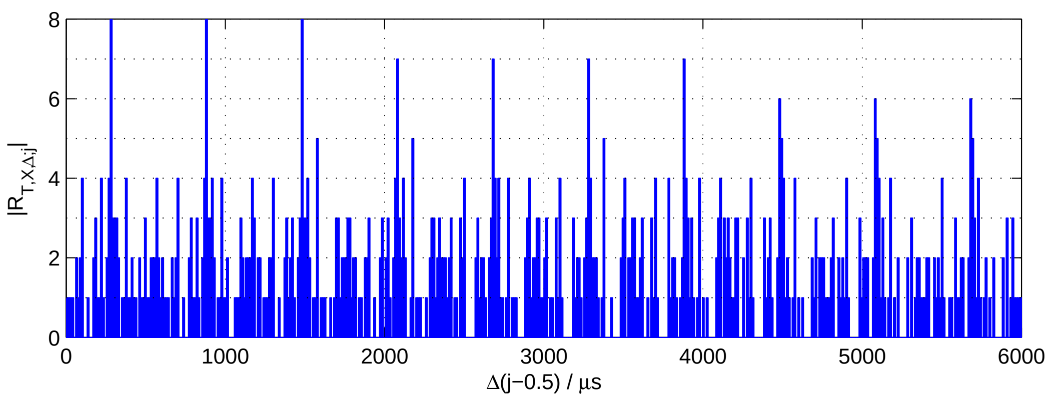
***Figure 6:*** *Cross-correlation scores for $X$ computed with template $T_3$ and narrow tolerance.*




(a) $q_{\mathcal{N}}$-score with $\sigma = 12\mu s$ , $c_0 = -2.705$.




(b) $p_{\mathcal{U}}$-score with $w = 12\mu s$ , $d_0 = -3.651$.




(c) Histogram with $\Delta = 12$.

***Figure 7:*** *Cross-correlation scores for $X$ computed with template $T_3$ and wide tolerance.*

uniform distribution. A lower bound for the $p_{\mathcal{U}}$-score, known as the $q_{\mathcal{U}}$-score, is also presented and is found to be nearly identical to the $p_{\mathcal{U}}$-score in practice.

Each of the $q_{\mathcal{N}}$- and $p_{\mathcal{U}}$-scores depend on two parameters. One parameter (either $\sigma$ or $w$ respectively) measures the tolerance of the arrival time around some expected value. The second parameter (either $c_0$ or $d_0$) measures the cost of associating elements of the template with pulses in the observation and depends on the estimated pulse density and detection probability.

The two methods are compared against a cross-correlation histogram introduced in [5]. It is found, using a synthetic experiment, that for signals without jitter or time-of-arrival uncertainties, the $q_{\mathcal{N}}$-and $p_{\mathcal{U}}$-scores have similar or slightly worse discriminating power than the cross-correlation histogram. However, for a signal with a significant degree of jitter, the $q_{\mathcal{N}}$-score whose parameters are chosen to match the expected jitter variance provides a cleaner template detection measure than either the $p_{\mathcal{U}}$-score or the cross-correlation histogram.

# Annex A
# Segmentation Algorithm

In Subsection 2.3, it was claimed that the $q_{\mathcal{N}}$-score defined in (4) is piecewise smooth and quadratic over intervals of the form $(\tau_{s-1}, \tau_s)$ where $\mathcal{T}_{\mathcal{N}} = \{\tau_s | s = 0 \ldots S\}$ represents some collection of partition points. This appendix describes an efficient recursive algorithm with which one can determine the elements of $\mathcal{T}_{\mathcal{N}}$, in increasing order.

Suppose that some partition point $\tau_{s-1}$ is known and it is required to find the next subsequent partition point $\tau_s$. Within the interval $\tau \in (\tau_{s-1}, \tau_s)$, the optimal association relation $\hat{K}_s$ is assumed to have been determined. For each $t \in T$, there are two possibilities:

1. It may be that for all $x \in X$, the cost of associating $t$ with $x$ is non-negative $c(t,x,\tau) \geq 0$ for $\forall\tau \in (\tau_{s-1}, \tau_s)$. If this is the case, then the optimal association relation $\hat{K}_s$ does not include any association for $t$ and we may say that $t$ is "unassociated" on this interval.

2. Alternatively, there may exist $x \in X$ such that $c(t,x,\tau) < 0$ for $\tau \in (\tau_{s-1}, \tau_s)$. In this case, we can define $\hat{x} = \arg\min_{x \in X} c(t,x,\tau)$. Then, $(t, \hat{x}) \in \hat{K}_s$ and $t$ is said to be "associated with $\hat{x}$" on $(\tau_{s-1}, \tau_s)$.

Now, for a particular value of $t \in T$, it is of interest to find the smallest value of $\tau$ exceeding $\tau_{s-1}$ at which the association for $t$ within $\hat{K}(\tau)$ exhibits a change. This is denoted by $\tau_{s,t}$ and is guaranteed to satisfy $\tau_{s-1} < \tau_s \leq \tau_{s,t}$. The criterion for determining $\tau_{s,t}$ depends on whether or not $t$ is associated in $\hat{K}_s$.

1. If $t$ is unassociated on $(\tau_{s-1}, \tau_s)$, then $\tau_{s,t}$ is the first value beyond $\tau_{s-1}$ at which it becomes beneficial to add an association for $t$. More precisely, we have

$$\tau_{s,t} = \inf_{\tau} \left\{ \tau > \tau_{s-1} \middle| \exists x, c(t,x,\tau) < 0 \right\}.$$

2. However, if $t$ is associated with some $\hat{x}$ on $(\tau_{s-1}, \tau_s)$, then $\tau_{s,t}$ is the first value after $\tau_{s-1}$ at which it becomes beneficial either to remove the association $(t, \hat{x})$ or to replace it with a different association for $t$,

$$\tau_{s,t} = \inf_{\tau} \left\{ \tau > \tau_{s-1} \middle| \forall x, c(t,x,\tau) \geq 0 \right\} \cup \left\{ \tau > \tau_{s-1} \middle| \exists x, c(t,x,\tau) < c(t,\hat{x},\tau) \right\}.$$

For sufficiently large values of $\tau_{s-1}$, some values of $\tau_{s,t}$ may not exist since the relevant infimum may extend over an empty set. If none of these values exists, then we may conclude that there are no partition points exceeding $\tau_{s-1}$ so that $S = s - 1$. However, if some of the infima exist then

$$\begin{aligned} \tau_s &= \min_t \{\tau_{s,t}\} \\ \hat{t}_s &= \arg\min_t \{\tau_{s,t}\} \end{aligned}$$

It is then straightforward to determine $K_{s+1}$ from $K_s$, depending on which of the above cases gave rise to the relevant infimum, by either adding, removing or replacing the association involving the $\hat{t}_s$.

For the implementation of the algorithm discussed below, it is required to assume that the elements of $T = \{t_n | n = 1 \ldots N\}$ and $X = \{x_m | m = 1 \ldots M\}$ are indexed in increasing order so that order so that $t_1 < t_2 < \cdots < t_N$ and $x_1 < x_2 < \cdots < x_M$. As discussed above, the algorithm operates on the premise that a particular $t_n \in T$ need be associated only if there exists some $x \in X$ so that $c(t, x, \tau) < 0$. Referring to (3), it is found that this condition is equivalent to

$$t_n^- < x - \tau < t_n^+$$

where $t_n^\pm = t_n \pm \sqrt{-2\sigma^2 c_0}$. It is furthermore assumed that these thresholds do not overlap, ensuring that

$$t_1^- < t_1 < t_1^+ < t_2^- < t_2 < t_2^+ < t_3^- < \ldots < t_{n-1}^+ < t_n^- < t_n < t_n^+ \, .$$

When applied methodically, these variables can be used to evaluate the infima described above.

For convenience, the algorithm has been divided into several procedures shown in Figures A.1, A.2, and A.3. The entry point is the method “IteratePartition” that takes, as parameters, all values of $t_n$, $t_n^\pm$ and $x_m$. Although the only explicit output of this algorithm is the set of partition points $\mathcal{T}_\mathcal{N}$, additional processing can be applied on an interval-by-interval basis by inserting an optional method called “ProcessAssociationRelation”.

**procedure** $\mathcal{T}_{\mathcal{N}}$ = IteratePartition$(\{(t_n^-, t_n, t_n^+)|n = 1 \ldots N\}, \{x_m|m = 1 \ldots M\})$
    **for** $\forall n = 1 \ldots N$ **do**
        $\hat{m}_n \leftarrow 1$
        $q_n \leftarrow$ Unassociated
        $\tau_n \leftarrow x_{\hat{m}_n} - t_n^+$
    **end**
    $\tilde{N} \leftarrow N$
    $\hat{n} \leftarrow N$
    $\mathcal{T} \leftarrow \{\tau_{\hat{n}}\}$
    **repeat**
        **if** $q_{\hat{n}} =$ AssociatedImprove
            $\hat{m}_{\hat{n}} \leftarrow \hat{m}_{\hat{n}} + 1$
            MakeAssociation
        **elseif** $q_{\hat{n}} =$ Unassociated
            MakeAssociation
        **elseif** $q_{\hat{n}} \leftarrow$ AssociatedLose
            RemoveAssociation
            **if** $\tilde{N} = 0$ **return end**
        **end**
        $\hat{n} \leftarrow \arg\min_n \{\hat{\tau}_n | n = 1 \ldots \tilde{N}\}$
        $\mathcal{T}_{\mathcal{N}} \leftarrow \mathcal{T}_{\mathcal{N}} \cup \{\hat{\tau}_{\hat{n}}\}$
        ProcessAssociationRelation
    **end**
**end**

***Figure A.1:*** *"IteratePartition" iterates through the interval partitions $\mathcal{T}_{\mathcal{N}} = \{\tau_s | s = 0 \ldots S\}$.*

**procedure** MakeAssociation
    $\tau_- \leftarrow x_{\hat{m}_{\hat{n}}} - t_{\hat{n}}^-$
    **if** $\hat{m}_{\hat{n}} < M$ **do**
        $\tau \leftarrow (x_{\hat{m}_{\hat{n}}} + x_{\hat{m}_{\hat{n}}+1})/2 - t_{\hat{n}}$
        **if** $\tau < \tau_-$ **do**
            $q_{\hat{n}} =$ AssociatedImprove
            $\tau_{\hat{n}} = \tau$
            **return**
        **end**
    **end**
    $q_{\hat{n}} =$ AssociatedLose
    $\tau_{\hat{n}} = \tau_-$
    **return**
**end**

***Figure A.2:*** *"MakeAssociation" determines the state parameters for the current interval after an association has been made.*

**procedure** RemoveAssociation

$q_{\hat{n}} = \text{Unassociated}$

$\hat{m}_{\hat{n}} \leftarrow \hat{m}_{\hat{n}} + 1$

**if** $\hat{m}_{\hat{n}} \leq M$ **do**

$\hat{\tau}_{\hat{n}} \leftarrow x_{\hat{m}_{\hat{n}}} - t_{\hat{n}}^{+}$

**else**

$\tilde{N} \leftarrow \hat{n} - 1$

**end**

**return**

**end**

***Figure A.3:*** *"RemoveAssociation" determines the state parameters for the current interval after an association has been eliminated.*

# References


1. Wiley, R. G. (1993). Electronic intelligence: the analysis of radar signals, Second ed. Norwood, MA: Artech House.

2. Mardia, H. K. (1989). New techniques for the deinterleaving of repetitive sequences. *IEE Proc. F, Commun., Radar & Signal Process.*, **136**(4), 149–154.

3. Milojević, D. J. and Popović, B. M. (1992). Improved algorithm for the deinterleaving of radar pulses. *IEE Proc. F, Commun., Radar & Signal Process.*, **139**(1), 98–104.

4. Driscoll, D. E. and Howard, S. D. (1999). The Detection of Radar Pulse Sequences by Means of a Continuous Wavelet Transform. In *Proceedings of the 1999 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP'99)*, Vol. 3, pp. 1389–1392. Pheonix, Arizona, USA.

5. Elton, S. D. (2001). A cross-correlation technique for radar emitter recognition using pulse time of arrival information. In *Proceedings of 2001 Workshop on Defence Applications of Signal Processing*, pp. 19–23. Borossa Valley, South Australia.

6. Roe, J., Elton, S., Huntington-Thresher, I., Noone, G., Lavoie, P., Dworkin, K., and Reid, C. (2002). Emitter deinterleaving and identification using *a priori* temporal signal profiles - Interim Report. TTCP Technical Report, TR-EWS-TP4-1-2002. UNCLASSIFIED.

7. Elton, S. D. (1999). Jitter identification techniques for a regular event-based process. In *Proceedings of the 1999 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP'99)*, Vol. 5, pp. 2801–2804. Pheonix, Arizona, USA.

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Two statistical techniques are presented for comparing temporal pulse trains received from an Electronic Support receiver to *a priori* templates representing the expected behaviour of radar signals. The techniques implement a specialized cross-correlation procedure and incorporate the effects of non-cumulative jitter. A theoretical basis is presented in which the signal of interest is modeled to be present in a noisy channel. The channel noise may include the failure to detect some expected pulses along with the detection of unexpected pulses. The methods are compared to the more straightforward approaches based on cross-correlation histograms and a simulated example scenario is presented.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus-identified. If it not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title).

Electronic Support
Electronic Warfare
Pattern Recognition
Pulse Processing
Radar
Cross-correlation

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