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Understanding Bias in Proportion Production

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The Stevens exponent (β) can be obtained from proportion estimation judgments using the *power model*. In this article, the authors extend that model to proportion production, in which the relative magnitudes of 2 stimuli are adjusted to correspond to a numeric proportion (e.g., ¼ or .25). The model predicts that when $\beta < 1$, small proportions are underproduced, and large proportions are overproduced, but it predicts the reverse when $\beta > 1$, which is the opposite of the predicted patterns for estimation. Eight participants estimated and produced magnitudes and proportions with spatial volume ($\beta < 1$; Experiment 1) and color saturation ($\beta > 1$; Experiment 2). The model's predictions were generally supported. An extension of the model using reference points can account for multicycle patterns shown by some participants.

People frequently fail to make correct judgments about the magnitudes of stimuli around them. When comparing two areas or volumes, for example, people tend to overestimate the smaller quantity relative to the larger. A well-known technique for quantifying the relation between physical and perceived magnitudes is *magnitude estimation*, in which the observer is presented with a set of stimuli that vary over a range and assigns a numeric value to the perceived magnitude of each stimulus (Stevens, 1957). Stevens observed that perceived magnitude is typically related to physical magnitude by a power function, a relationship referred to as *Stevens's law*. The exponent of the function is commonly known as β or the *Stevens exponent.*

The Stevens exponent can also be estimated from the *magnitude production* technique, in which an observer adjusts the magnitude of a stimulus to correspond to a given numeric value. However, the exponents estimated from magnitude estimation and production often do not correspond, implying that one or both methods introduce a measurement bias. It is therefore desirable to develop a method of estimating the Stevens exponent that would reduce such bias.

Proportion estimation and *proportion production* offer alternate means for obtaining estimates of β (Hollands & Dyre, 2000; Spence, 1990). In proportion estimation, the observer is presented with two stimuli and assigns a numeric value to one to reflect its relative contribution to the total magnitude (e.g., 25%, ¼, .25, etc.). In *proportion production*, the observer is presented with a numeric proportion (e.g., 25%, ¼, .25) along with two stimuli and adjusts the relative magnitude of the stimuli to produce the numeric proportion. The *power model* (Spence, 1990) provides a method for estimating the Stevens exponent from proportion estimation. In this article, we extend that model to the proportion production task, test its predictions, and examine whether estimates of β from proportion estimation and production are more consistent than estimates of β from magnitude estimation and production tasks.

The organization of the article is as follows. First, we describe magnitude judgments and Stevens's power law and discuss factors that may lead to systematic biases in estimation of the Stevens exponent, β. Second, we introduce proportion judgments as an alternative method for estimating β, discuss potential advantages of proportion judgments over magnitude judgments, describe the model used to estimate β in proportion judgments, and specify the model's predictions for proportion production judgments. Finally, we present two experiments that test these predictions and we compare β values obtained from proportion and magnitude judgments for both estimation and production tasks.

Magnitude Judgments and Stevens's Law

On the basis of results from magnitude estimation experiments, Stevens (Stevens, 1975; Stevens & Galanter, 1957) showed that perceived magnitude, Ψ, is a power function of physical magnitude, Φ, for many stimulus continua. Specifically, $\Psi = \alpha\Phi^{\beta}$, where α is a scaling factor reflecting the choice of subjective units. For some continua, β is less than unity, and the psychophysical function is negatively accelerated. For example, β is typically about 0.6 for spatial volume (M. Teghtsoonian, 1965). For other continua, β is greater than unity, and the psychophysical function is positively accelerated; for example, β is about 1.7 for color saturation (Indow & Stevens, 1966). Examples of such functions are shown in Figure 1A.

Whereas magnitude estimation requires the observer to estimate the magnitude of a stimulus, magnitude production requires adjustment of the stimulus magnitude to a target magnitude, often defined as a numeric value. The power law can be applied to production tasks by taking its inverse, resulting in the function $\Phi = \alpha'\Psi^{1/\beta}$, where Φ is the magnitude produced by the observer,

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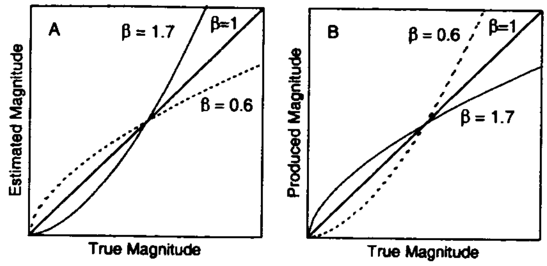

Figure 1. Psychophysical functions for β = 0.6 (geometric volume) and β = 1.7 (color saturation) based on magnitude estimation judgments (A) and magnitude production judgments (B).

α' is a scaling factor, and Ψ is the numeric value given to the observer (Stevens, 1958). Figure 1B depicts this function when β = 0.6 and 1.7. The Stevens exponent, β, can therefore be obtained by taking the reciprocal of the exponent of the function fit to magnitude production data. This implies that the Stevens exponents associated with the two tasks should be equal for a given stimulus continuum. In this article we use the notation β_{ME} to denote a Stevens exponent estimated through magnitude estimation and β_{MP} to denote a Stevens exponent estimated through magnitude production; thus, β_{ME} should equal β_{MP} for a given stimulus continuum.

Magnitude judgments of the speed of self-motion provide an illustrative example. Recarte and Nunes (1996) found that the rate of increase in observers' estimated speed became greater and greater compared with the rate of increase in actual speed. When observers adjusted the speed to correspond to given target values (speed production), they showed the opposite pattern: Produced speed increased more slowly than the target values. Recarte and Nunes fit a power function to the data from each task and obtained similar β values ($\hat{\beta}_{ME}$ = 1.5 and $\hat{\beta}_{MP}$ = 1/0.7 = 1.4).

However, the Stevens exponents obtained from magnitude estimation and magnitude production procedures do not always correspond. The range of stimuli used in magnitude estimation and production may be a contributing factor. Both the range of stimuli and the range of response values affect obtained exponents, a phenomenon known as the *range effect* (Poulton, 1975). Therefore, any difference in stimulus or response ranges between magnitude estimation and production procedures may produce a difference in exponents. For example, the commonly used *free modulus* magnitude estimation procedure allows an observer to assign any positive real number to a stimulus, thereby providing an infinite response range. However, the response range is finite in magnitude production because of limits on the range of producible magnitudes. For example, the apparatus may physically restrict the range (e.g., a computer monitor in area estimation and production). Thus, the Stevens exponents obtained with magnitude estimation and production may differ because of differences in response ranges in the two tasks.

Further, Stevens and Greenbaum (1966) argued that there is a tendency for observers' judgments to regress toward the mean value of the response range, leading to a shortened range of the response variable regardless of task. The resulting *regression effect* is that exponents tend to be larger for magnitude production than estimation. This has been shown for various continua such as loudness, duration of noise, and duration of light, and similar results have been obtained with other stimulus continua (Green, Luce, & Duncan, 1977; Hellman, 1981; Krueger, 1984; Tayama, Nakamura, & Aiba, 1987).

Stevens and Greenbaum (1966) argued that the "true" exponent is likely to lie between the estimates from magnitude estimation and production and that taking the geometric mean would be one method for obtaining the true exponent. However, the geometric mean of magnitude estimation and production exponents may not properly represent the true value either. R. Teghtsoonian and Teghtsoonian (1978) showed that the regression effect holds for large ranges, but for small ranges the opposite occurred. They also showed that the rate of increase in magnitude production exponents with increasing stimulus range was more rapid than the rate of decrease in magnitude estimation exponents. The net result is that the mean exponent becomes larger as the stimulus range increases. R. Teghtsoonian and Teghtsoonian concluded that because the exponent shifts depending on the stimulus range, no single pair of magnitude judgment experiments is sufficient to estimate a true exponent. Hence, magnitude judgments require aggregation over various stimulus–response ranges to obtain an unbiased exponent.

In summary, results from previous studies (Poulton, 1975; Stevens & Greenbaum, 1966; R. Teghtsoonian & Teghtsoonian, 1978; Zwislocki & Goodman, 1980) suggest that it is difficult to obtain equal exponents from magnitude estimation and magnitude production, and taking the average of exponents is also problematic.

Proportion Judgments

Proportion estimation and production are potentially more reliable alternatives to magnitude estimation and production. In proportion estimation, the observer is shown two stimulus magnitudes and makes judgments about one stimulus magnitude relative to the total magnitude. For example, an observer estimates a proportion when presented with two piles of coins and is asked to estimate the magnitude of one pile relative to the total. The observer can respond using a percentage (25%), a proper fraction (¼), or a decimal fraction (.25), or by dividing a line in two parts (Spence, 1990). We would also classify the *constant sum method* (Comrey, 1950; Metfessel, 1947), in which an observer divides 100 points between two stimuli, as a particular type of proportion judgment. For proportion production, numeric values between 0 and 1 (or between 0% and 100%) are presented to the observer, who adjusts the relative magnitudes of two stimuli to correspond. For example, dividing a group of coins into two piles so that one is 60% of the total is proportion production.

We note two potential advantages for proportion judgments over magnitude judgments (Hollands & Dyre, 2000; Spence, 1990). First, proportion judgments appear less subject to the measurement biases seen with estimation and production of single magnitudes described earlier. Both stimulus and response ranges for proportion judgments are equalized between 0 and 1 (or between 0% and 100%). Thus, the range and regression effects seen in magnitude judgments should not affect proportion judgments when the stimulus–response relationship is reversed. Whether proportion judgments produce unbiased estimates of the Stevens exponent can be tested by comparing estimates from both estimation and production tasks. If proportion judgments are free from the range effect present in magnitude judgments, estimated β values from proportion estimation and production should be in closer correspondence than β_{ME} and β_{MP}.

A second advantage of proportion judgments is a reduction in error due to memory. In magnitude judgments, the observer must set a standard or a modulus, or make judgments on the basis of the preceding stimulus. However, the mental representation of the modulus or the preceding stimulus may drift within the experiment (Spence, 1990). On the other hand, in proportion judgments, the observer makes judgments about one magnitude relative to the total magnitude, both of which are perceptually available to the observer (Krantz, 1972). Therefore, such judgments should not be subject to memory drift over time.

A Model for Proportion Judgments

Spence (1990) proposed the power model as a method for estimating Stevens exponents from proportion estimation judgments. Subjective proportion is expressed as the ratio of subjective estimates of part and whole, specifically,

$$P = \frac{\Pi^{\beta}}{\Pi^{\beta} + (1 - \Pi)^{\beta}}, \tag{1}$$

where P is the subjective proportion, Π is the physical magnitude of one part, $(1 - \Pi)$ is the magnitude of its complement, and β is the Stevens exponent. Stimulus and response are expressed in the same units, so one can compute a bias score by taking the difference between an estimated and true proportion. A positive difference indicates overestimation, a negative one indicates underestimation. When $\beta < 1$, Equation 1 predicts an over-then-under pattern; in contrast, if $\beta > 1$, the opposite occurs (see Figure 2A).

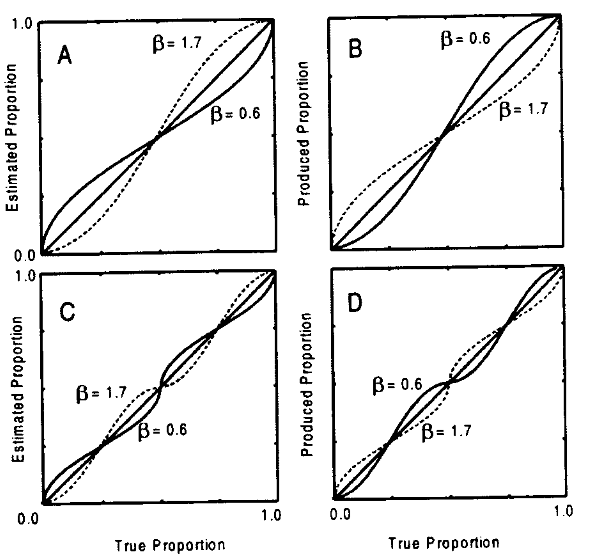

Figure 2. A and B: Predictions of the power model when $\beta = 0.6$ and $\beta = 1.7$. A: Estimated proportion as a function of true proportion. B: Produced proportion as a function of true (numeric) proportion. C and D: Predictions of the two-cycle version of the cyclical power model when $\beta = 0.6$ and 1.7. C: Estimated proportion as a function of true proportion. D: Produced proportion as a function of true (numeric) proportion.

The power model may be extended to proportion production by solving for Π (see Appendix A). When the observer performs a production task, Π denotes the physical proportion produced by the observer, and P denotes the numeric proportion given by the experimenter such that

$$\Pi = \frac{P^{1/\beta}}{P^{1/\beta} + (1 - P)^{1/\beta}}. \tag{2}$$

For proportion production, the exponent is $1/\beta$, the reciprocal of the exponent for proportion estimation. Hence, the value of β in proportion estimation (β_{PE}) should be equal to the value of β in proportion production (β_{PP}); that is, $\beta_{PE} = \beta_{PP}$. If $\beta_{PP} < 1$, Equation 2 predicts that an under-then-over (S-shape) pattern will occur, whereas if $\beta_{PP} > 1$, an over-then-under (reversed S-shape) pattern is predicted. Thus, a pattern opposite to that seen in the proportion estimation task should be observed (Figure 2B).

To test these predictions we investigated two types of judgments (magnitude and proportion) with two types of tasks (estimation and production) for two stimulus continua: volume ($\beta < 1$) and color saturation ($\beta > 1$). The power model predicts that when $\beta < 1$, an over-then-under pattern should occur for proportion estimation but an under-then-over pattern should occur for proportion production. We tested this in Experiment 1 using geometric volume. The model makes the opposite set of predictions when $\beta > 1$; we tested this in Experiment 2 using color saturation.

We noted earlier that the exponents for magnitude estimation and production do not always correspond, but there should be greater correspondence between proportion estimation and production exponents. If proportion judgments produce more consistent β values than magnitude judgments, the difference in β values between proportion estimation and production should be smaller than the difference in β values between magnitude estimation and production. This should be true regardless of stimulus continuum. We tested this in both experiments.

Experiment 1

In Experiment 1, participants were shown simulated spheres that varied in size. Four task–judgment combinations were performed: magnitude estimation, magnitude production, proportion estimation, and proportion production. Spatial volume typically produces Stevens exponents less than unity (about 0.6; M. Teghtsoonian, 1965). The power model predicts that when $\beta < 1$, an over-then-under pattern should occur for proportion estimation but an under-then-over pattern should occur for proportion production. We also expected greater discrepancy between the Stevens exponents obtained through magnitude estimation and production than between exponents obtained through proportion estimation and production.

Method

Participants. Eight university students participated in this study. The participants were screened for 20/20 Snellen acuity (normal or corrected) before the experiment. Participants were naïve as to the experimental hypotheses, and they received course credits for participation.

Stimuli and apparatus. Stimuli were simulated three-dimensional spheres displayed on a computer monitor. The depth dimension was defined by shading, texture (checkerboard surface pattern), and motion (sinusoidal rotation of ±60° at 1 Hz about the vertical axis). All stimuli were presented on a 43.2-cm (17-in.) cathode ray tube (CRT) monitor at 1,280 × 1,024 resolution controlled by a graphics workstation.

We used 21 size levels (i.e., 21 spheres) with the following radii: 4.68, 8.01, 10.09, 11.55, 12.71, 13.69, 14.55, 15.32, 16.02, 16.66, 17.25, 17.81, 18.33, 18.83, 19.30, 19.75, 20.18, 20.59, 20.99, 21.37, and 21.67 mm. Each level can be expressed as a percentage of the total volume shown on a given trial in the proportion task (43,030 mm^3). These were 1, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, and 99%. For magnitude estimation, one sphere was shown on each trial. For proportion estimation, two spheres whose sum equaled 100% were shown on each trial. For magnitude production, a sphere with a radius of 17.25 mm (50%) was presented at trial commencement. For proportion production, two spheres each having a radius of 17.25 mm (50% each) were shown at trial commencement. For both types of proportion judgments, the stimuli had a constant total volume across trials; thus, when one increased, the other decreased. A numeric keypad was used to enter values in the estimation tasks, and a joystick was used to adjust the magnitude of stimuli in the production tasks.

Design and procedure. A 2 (judgment: magnitude vs. proportion) × 2 (task: estimation vs. production) within-subjects factorial design was used. The order of the four factorial combinations of judgment and task was counterbalanced across blocks of trials, and the magnitudes and proportions to be judged were presented in random order within each block. For each of the four task–judgment combinations, each of the 21 stimuli was shown five times.

At the beginning of the experiment, the visual acuity of each participant was tested using the Snellen eye chart. They were told that they were to perform four tasks (magnitude estimation, magnitude production, proportion estimation, and proportion production), and that they would be shown an instruction screen at the beginning of each task that described the stimuli and how to respond to them. For magnitude estimation, participants were told to assign a number to the particular volume shown on each trial, that the number should correspond to the magnitude of the stimulus, and that any numbers could be used. For proportion estimation, participants were told to enter a percentage value representing the volume of the left sphere relative to the combined volume of the two spheres. For magnitude production, participants were told to adjust the sphere's volume to correspond to a given number. The numbers used were 1, 5, 10, 15, . . . , 50, . . . , 90, 95, and 99. Participants adjusted the sphere's volume with a joystick (pushing to the left on the joystick decreased the sphere's size; pushing to the right increased it). For proportion production, participants were told to adjust the left sphere's volume to correspond to a given percentage relative to the total volume of both spheres. Participants were allowed to take as much time as necessary to make their judgments. When they finished all sessions, they were debriefed. The experiment took approximately one hour to complete.

Results

Fits to averaged data. A mean response value was computed for each physical magnitude or proportion for each of the four conditions, averaging across participants. These data are shown in Figure 3. Using nonlinear regression, Stevens's power law was fit

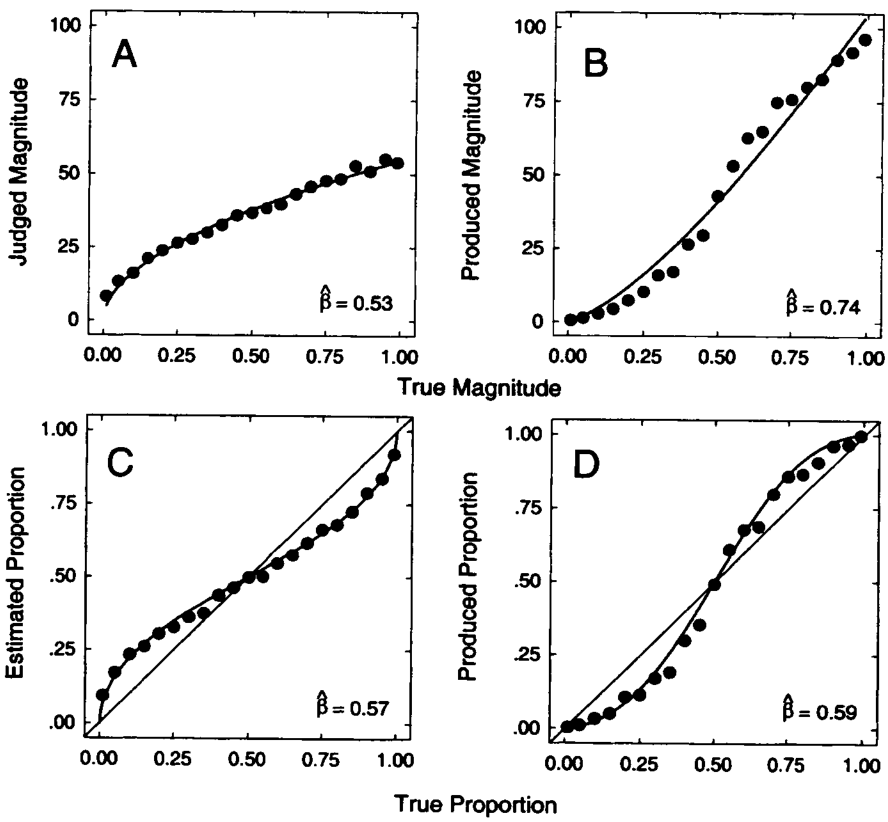

Figure 3. Experiment 1 (volume) results. A: Judged magnitude as a function of physical magnitude. B: Produced magnitude as a function of numeric magnitude. C: Judged proportion as a function of true proportion. D: Produced proportion as a function of numeric proportion. Dots represent the mean values averaged across all participants and blocks. Continuous functions represent the best fitting versions of Stevens's law (A and B) and the cyclical power model (C and D).

to the mean magnitude judgment data and the power model was fit to the mean proportion judgment data. For magnitude estimation, $\hat{\beta}_{ME} = 0.53$, $R^2 = .99$; for magnitude production, $\hat{\beta}_{MP} = 0.74$, $R^2 = .97$. For proportion estimation, $\hat{\beta}_{PE} = 0.57$, $R^2 = .99$; for proportion production, $\hat{\beta}_{PP} = 0.59$, $R^2 = .99$.

Fits to individual participants and analysis of variance (ANOVA). Stevens exponents were obtained from each participant in each condition, using Stevens's law for magnitudes and the power model for proportions. Figure 4 shows the mean exponents and standard errors. A 2 × 2 within-subjects factorial ANOVA was performed on the exponents. As Figure 4 shows, the exponent was greater for magnitude production than for any other condition, $F(1, 7) = 29.43$, $MSE = 0.003$, $p < .001$ (Tukey's honestly significant difference [HSD], $p < .05$). The other three conditions did not differ from each other (Tukey's HSD, $p > .05$).

Discussion

In general, the Stevens exponents obtained from all four tasks were less than unity. As predicted by the extension of the power model to proportion production, the over-then-under pattern seen with proportion estimation was accompanied by an under-then-over pattern for proportion production. Also as predicted, there was greater discrepancy between the Stevens exponents obtained through magnitude estimation and production than between exponents obtained through proportion estimation and production.

Experiment 2

In Experiment 2, participants were shown color patches whose excitation purity (subjectively, color saturation) varied from gray to red. Approximate changes in excitation purity were produced by manipulating the relative brightness of red, green, and blue pixels on a CRT display. Participants performed the same four task–judgment combinations as in Experiment 1 (magnitude estimation and production, proportion estimation and production) except that they judged or adjusted color saturation levels rather than volumes. Color saturation typically produces Stevens exponents greater than unity (about 1.7; Indow & Stevens, 1966). As we mentioned earlier, the power model predicts that when $\beta > 1$, an under-then-over pattern occurs for proportion estimation, but an over-then-under pattern occurs for proportion production. As in Experiment 1, we expected greater discrepancy between the Stevens exponents obtained through magnitude estimation and production than between exponents obtained through proportion estimation and production.

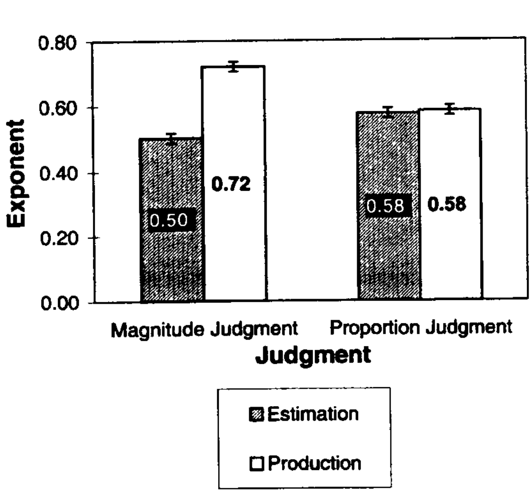

Figure 4. Experiment 1 (volume) results. Mean estimated Stevens exponents obtained from magnitude estimation, magnitude production, proportion estimation, and proportion production data. The power model was used to fit proportion estimation and production data. Error bars indicate standard errors of the means.

Method

In general, the methods used in Experiment 2 were identical to those used in Experiment 1 with the exception of the stimulus continuum. We note differences below.

Color patches in the shape of equilateral triangles (base and height = 73 mm) were shown on the same computer monitor used in Experiment 1. To manipulate the excitation purity of the triangular patches, we varied the magnitudes of the three color parameters (λ_R, λ_G, or λ_B) passed to the OpenGL function `glColor3f` (OpenGL Architecture Review Board, 1992). To scale the manipulation of red excitation purity (γ_R) from 1% to 99% (.01 to .99), the photopic luminance of each pixel type (red, green, or blue) was independently measured as a function of the magnitude of its corresponding OpenGL color parameter (λ_R, λ_G, or λ_B). An International Light photometer (Model IL1400A with SEL033 detector, R diffuser, and Y [photopic] filter; Newburyport, MA) was used for luminance measurements. For the particular monitor (Iayama Model S700) and video card (Hewlett Packard Visualize FX4) used, the luminance of the pixels was approximately linearly related to the magnitude of the OpenGL color parameters. On the basis of these luminance measurements we derived the following function for scaling the OpenGL red color parameter as a function of red excitation purity: $\lambda_R(\gamma_R) = 0.35 + 0.44 \times \gamma_R$. To maintain approximately constant overall luminance as γ_R changed from 1% to 99%, we derived two additional functions of γ_R that varied the luminance contribution of the green and blue pixels: $\lambda_G(\gamma_R) = \lambda_B(\gamma_R) = 0.361 \times \{1 - \exp[-2.81(1 - \gamma_R)]\}$. These three functions $[\lambda_R(\gamma_R), \lambda_G(\gamma_R)$, and $\lambda_B(\gamma_R)]$ were then used to determine the red, green, and blue parameter values used for the `glColor3f` function. Measurements of color patch luminance varied by less than 3% as γ_R changed from 1% to 99% (for comparison, this is less than the change in luminance when the same stimulus was presented at different locations on the computer monitor).[1]

For magnitude estimation, participants were told to assign a number to the particular saturation level shown on each trial, and that any number could be used. For proportion estimation, participants were told to enter a

[1] This method of manipulating excitation purity is not necessarily linear. Hence, obtained exponents may not be precise for saturation. However, our interest was to compare different types of judgments (proportion vs. magnitude) using a stimulus continuum for which $\beta > 1$. Thus, the validity of our manipulation depended on whether we reliably obtained estimates of $\beta > 1$ and whether the estimates were reasonably close to those obtained in previous experiments (e.g., $\beta = 1.7$ obtained by Indow & Stevens, 1966), rather than whether we varied excitation purity linearly. A more important concern was to ensure that luminance remained approximately constant as excitation purity changed, because brightness has $\beta \approx 0.3$–0.5 (Stevens, 1975). If judgments were affected by changes in brightness then β would be systematically underestimated and perhaps even be less than 1.0. The exponents obtained from magnitude estimation and production data obtained in Experiment 2 were similar to those obtained in previous studies (e.g., Indow & Stevens, 1966), which suggests that our manipulation of excitation purity was approximately linear.

percentage value representing the saturation of the left patch relative to the combined saturation of both patches. For magnitude production, participants were told to adjust the saturation of the patch to correspond to a given number. They adjusted the saturation with a joystick (pushing to the left on the joystick decreased saturation; pushing to the right increased it). For proportion production, participants were told to adjust the saturation of the left patch to correspond to a given percentage relative to the total volume of both patches. Pushing to the left on the joystick increased the saturation of the left patch (and simultaneously decreased the saturation of the right patch); pushing to the right had the opposite effect.

Results

Fits to averaged data. As in Experiment 1, a mean subjective estimate value was computed for each physical magnitude or proportion, for each condition, averaged across participants. These data are shown in Figure 5. Using nonlinear estimation, Stevens's power law was fit to the mean magnitude judgment data and the power model was fit to the mean proportion judgment data. For magnitude estimation, $\hat{\beta}_{ME} = 1.60$, $R^2 = .99$; for magnitude production, $\hat{\beta}_{MP} = 1.84$, $R^2 = .97$. For proportion estimation, $\hat{\beta}_{PE} = 1.18$, $R^2 = .99$; for proportion production, $\hat{\beta}_{PP} = 1.62$, $R^2 = .96$. However, Figure 5 shows a distinct two-cycle bias pattern for proportion production (over-under, over-under). This is discussed further below.

Fits to individual participants and ANOVA. As in Experiment 1, Stevens exponents were obtained from each participant in each condition. Figure 6 shows the mean exponents as a function of the four task–judgment combinations. The ANOVA showed only that exponents were smaller for estimation tasks than production tasks ($Ms = 1.42$ and 1.77, respectively), $F(1, 7) = 6.84$, $MSE = 0.144$, $p < .05$. However, examination of results for individual participants in both proportion production and estimation showed two-cycle patterns. Cyclical bias patterns were also evident in the magnitude production data for some participants. These results are discussed further below.

Discussion

In general, the Stevens exponents obtained from all four tasks were greater than unity. As predicted by the power model, the under-then-over pattern seen with proportion estimation was accompanied by an over-then-under pattern for proportion production. However, contrary to predictions, Stevens exponents obtained from proportion estimation and production were no closer than those obtained from magnitude estimation and production. Proportion judgments tended to produce β values closer to unity than did magnitude judgments.

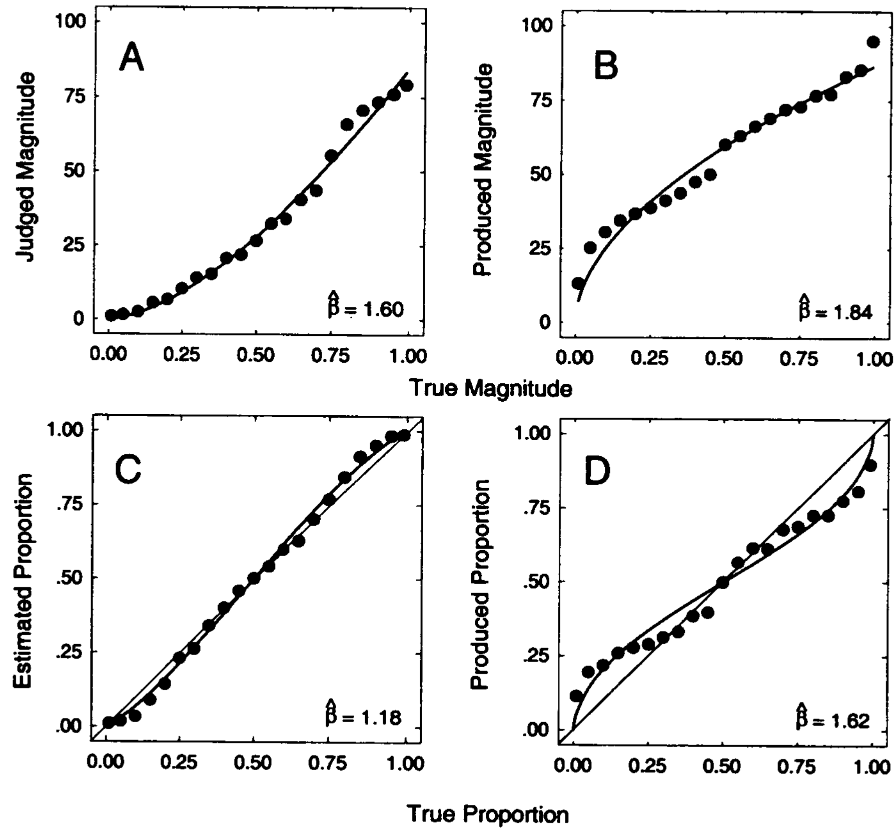

Figure 5. Experiment 2 (color saturation) results. A: Judged magnitude as a function of physical magnitude. B: Produced magnitude as a function of numeric magnitude. C: Judged proportion as a function of true proportion. D: Produced proportion as a function of numeric proportion. Dots represent the mean values averaged across all participants and blocks. Continuous functions represent the best fitting versions of Stevens's law (A and B) and the cyclical power model (C and D).

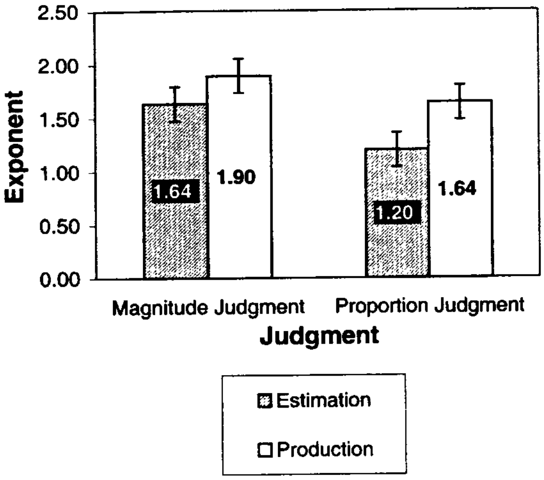

Figure 6. Experiment 2 (color saturation) results. Mean estimated Stevens exponents obtained from magnitude estimation, magnitude production, proportion estimation, and proportion production data. The power model was used to fit proportion estimation and production data. Error bars indicate standard errors of the means.

Two-cycle patterns were observed in the proportion estimation and production data. Such patterns are commonly observed in proportion judgments (Hollands & Dyre, 2000) and may be related to observers making judgments relative to a reference value, such as 50-50 (equal saturation of the two triangles). In the next section, we describe a general version of the power model that can account for multiple-cycle patterns in proportion estimation and we extend that model to proportion production. Then we describe fits of the general model to data from Experiment 2.

Accounting for Multicycle Patterns in Proportion Estimation and Production

Recently, Hollands and Dyre (2000) extended the power model to a more general form, the *cyclical power model* (CPM), to account for the multiple cycles of S-shaped bias sometimes found in proportion estimation. The power model implies that observers estimate proportions on the basis of the relative magnitudes of part and whole; however, in some cases observers implicitly or explicitly use intermediate points of reference, such as .5 or 50%. According to the CPM the frequency of the bias pattern is related to the number of reference points used by the observer.

The CPM is formally defined as

$$P = \frac{(\Pi - R_{i-1})^{\beta}}{(\Pi - R_{i-1})^{\beta} + (R_i - \Pi)^{\beta}} \times \frac{R_i - R_{i-1}}{R_n} + \frac{R_{i-1}}{R_n}, \quad R_{i-1} < \Pi < R_i, \tag{3}$$

where P is the subjective proportion, Π is the physical magnitude of the part, and R_{i-1}, R_i, and R_n, are reference values such that

$$R_i = \frac{R_n}{n} \times i,\ i = 0, 1, 2, \ldots, n. \tag{4}$$

When there are only two reference points, the two reference values are 0 and 1, R_n is equal to 1, Equation 3 simplifies to Equation 1, and the cyclical power model is equivalent to the power model. Thus, the pattern seen in Figure 2A results. However, when intermediate reference points are used, the model predicts higher frequency bias patterns. Such patterns are commonly observed (e.g., Huttenlocher, Hedges, & Duncan, 1991; Spence & Krizel, 1994). For example, people typically show two-cycle bias patterns when judging proportions shown in pie charts (Hollands & Dyre, 2000; Spence & Krizel, 1994). According to the cyclical power model, when three reference values (0, ½, and 1) are used, a two-cycle bias pattern is predicted. Thus, for the pie chart a reference value of ½ implies the use of a reference point at the six o'clock position of the pie (Hollands & Dyre, 2000). In general, intermediate reference points are defined such that $R_{i-1} < \Pi < R_i$, and the cycle of S or reversed-S patterns will repeat n times. Figure 2C shows a two-cycle pattern for proportion estimation. When $\beta < 1$, an over-under-over-under pattern occurs; when $\beta > 1$, the pattern is under-over-under-over.

The choice of reference points is strategic, and we propose that it is affected by stimulus characteristics (e.g., points of symmetry in a display), the nature of the response method, instructions, or training. In these experiments we constructed the stimuli so that possible intermediate reference points were not obvious to us; however, the strategic selection of reference points is subjective and may therefore show variability among participants.

It is possible to extend the CPM to proportion production. If the produced magnitude of one part is expressed as Π, then

$$\Pi = \frac{R_{i-1}(R_i - P)^{1/\beta} + R_i(P - R_{i-1})^{1/\beta}}{(R_i - P)^{1/\beta} + (P - R_{i-1})^{1/\beta}}, \tag{5}$$

where P is the numeric value given to the observer, $i = 0, 1, \ldots, n$, and $R_{i-1} < P < R_i$ (Appendix B shows the derivation for this equation). Figure 2D shows a two-cycle bias pattern for proportion production. The version of CPM shown in Equation 5 can therefore be used to account for multiple-cycle patterns observed in proportion estimation and production.

The simple power model (and its extension to proportion production described in Appendix A) cannot properly account for multiple-cycle patterns and will produce a β value that does not properly represent the bias pattern if fit to multiple-cycle data. It may be more informative in these cases to use the more general CPM to account for the bias. Therefore, we fit multicycle versions of the CPM to estimation and production data from individual participants in both experiments.

Results

For each experiment, one-, two-, four-, and eight-cycle versions of the CPM were fit to the mean data for each participant using nonlinear estimation (regression). The estimation and production versions of the model were fit to proportion estimation and proportion production data, respectively.

The results from Experiment 1 showed that the one-cycle version of the CPM fit best in every case (i.e., for each participant the R^2 value was largest for the one-cycle version of the CPM). The one-cycle version of the CPM is equivalent to Spence's (1990) power model, so the Experiment 1 results and interpretation were unchanged.

In Experiment 2, however, the results were less consistent, with two-cycle patterns occurring in both proportion estimation and production. Two participants showed the best fit with a two-cycle model for proportion estimation; 4 participants showed the best fit with a two-cycle model for proportion production. The remaining participants showed the best fit with a one-cycle model for both tasks. The fits are summarized in Table 1. Some examples of two-cycle data and fits are shown in Figure 7.

The Stevens exponent from the best fitting model version was substituted for the one-cycle version for proportion estimation and production, and an ANOVA was conducted on this set of exponents along with the original exponents estimated from Stevens's power law for the magnitude estimation and production tasks. The analysis showed that exponents were larger for proportion production than proportion estimation, $F(1, 7) = 6.52$, $MSE = 0.123$, $p < .05$ (Tukey's HSD, $p < .05$), but that proportion production exponents were not greater than either magnitude estimation or production exponents (Tukey's HSD, $p < .05$). Figure 8 shows the mean exponents as a function of the four task–judgment combinations. A comparison of Figures 6 and 8 shows that the use of best-fit functions greatly increased the exponents for proportion production.

Discussion

In summary, the multicycle patterns produced by some participants were better fit by the CPM than by the power model. The implication is that these participants used 50% as a reference value. This result occurred for both estimation and production judgments. Indeed, half the participants showed a two-cycle pattern in proportion production. This implies that observers are not restricted to using intermediate reference points just for proportion estimation but can also use them in proportion production. It also implies that one should use the best fitting version of the CPM to estimate Stevens exponents, rather than using the power model alone.

Nonetheless, this does not explain why estimated exponents for proportion estimation were so different from proportion production. This result contradicted our prediction that proportion estimation and production should provide more consistent estimates than magnitude estimation and production. The Experiment 2 results may reflect observers' uncertainty about how to decompose color saturation across two stimuli, an unfamiliar task relative to decomposing volume, as in Experiment 1 (Schneider & Bissett, 1988). For proportion production, when the observer adjusts the saturation of one patch, the other is adjusted in complementary fashion. The observer can therefore examine how a total saturation is decomposable into two color patches. In contrast, for proportion estimation, the display is static. For less decomposable continua such as color saturation, proportion production may be a more appropriate method for estimating Stevens exponents than proportion estimation.

Cyclical bias patterns were also evident in the magnitude production data for some participants in both experiments. In the following section, application of the power model to magnitude estimation and production, along with a competing model, are discussed.

Table 1
Experiment 2 (Color Saturation) Results: Fits of One-, Two-, Four-, and Eight-Cycle Versions of the Cyclical Power Model (CPM) to Proportion Estimation and Production Data

	Number of cycles							
	One		Two		Four		Eight	
Participant	β	R^2	β	R^2	β	R^2	β	R^2
Proportion estimation								
1	1.04	.02	1.44[a]	.71[a]	1.07	.00	1.06	.00
2	1.25[a]	.30[a]	1.60	.27	1.03	.00	1.06	.00
3	1.58[a]	.93[a]	1.15	.02	1.22	.00	1.39	.00
4	1.15[a]	.43[a]	1.14	.07	0.83	.03	1.41	.03
5	1.12[a]	.19[a]	1.10	.00	1.09	.00	0.89	.00
6	0.95	.02	1.42[a]	.44[a]	0.90	.00	0.62	.02
7	1.25[a]	.54[a]	1.33	.20	1.08	.00	1.21	.00
8	1.25[a]	.53[a]	1.63	.50	0.99	.00	1.29	.01
Proportion production								
1	1.78[a]	.80[a]	2.29	.35	7.87	.23	>100	.09
2	1.68	.61	2.96[a]	.65[a]	4.38	.16	6.75	.04
3	1.71[a]	.83[a]	2.07	.33	2.84	.08	5.48	.03
4	1.82[a]	.77[a]	2.60	.44	5.71	.18	3.63	.02
5	1.55	.56	2.35[a]	.59[a]	2.04	.09	1.61	.01
6	1.45	.25	3.11[a]	.70[a]	6.26	.14	>100	.01
7	1.18	.09	1.90[a]	.65[a]	1.26	.00	1.03	.00
8	1.97[a]	.86[a]	2.59	.39	3.26	.08	8.95	.03

Note. R^2 values for bias were used to better distinguish among the different model versions, because when judged proportion or produced proportion was predicted, R^2 values were generally at ceiling near unity (see Hollands & Dyre, 2000, on this point).
[a] Best fitting version of the CPM for each participant.

Cyclical Bias in Magnitude Production

The cyclical pattern was not limited to proportion judgments. About half of the participants in each experiment showed cyclical patterns in magnitude production; examples are shown in Figure 9. Most of these participants showed one-cycle patterns; only 1 participant (from Experiment 2) showed a two-cycle pattern. There was little evidence for cyclical patterns in the magnitude estimation data. In this section, we discuss two possible interpretations for cyclical patterns in magnitude production.

One possibility is that observers treated magnitude production judgments like proportion production judgments. That is, the observer treated the single magnitude presented (which was an integer between 1 and 100) as a percentage value and attempted to produce a magnitude that corresponded to a percentage of some remembered standard magnitude. If the chosen standard was always the largest stimulus presented, a one-cycle pattern should result, and the power model (as extended to production) would be a more valid model of the process than Stevens's power law. Multicycle patterns could arise because other values (e.g., a stimulus in the middle of the range) were used similarly, and the CPM (as extended to production) would be an appropriate model in this case.

A second interpretation is based on an observer's choice of numbers. R. Teghtsoonian, Teghtsoonian, and Baird (1995) noted that sinusoidal functions often occur in magnitude estimation and production experiments when there are many stimulus levels and each is tested multiple times (e.g., Green & Luce, 1974). To account for this result, they proposed a "preferred numbers" model

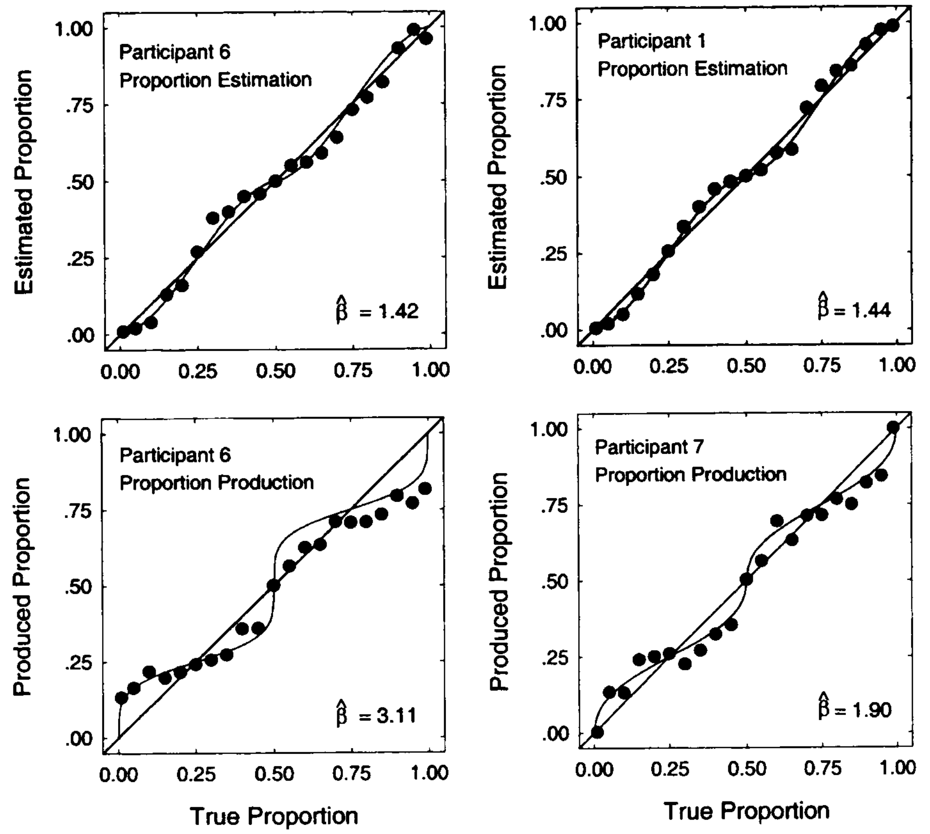

Figure 7. Experiment 2 (color saturation) results for some individual participants. Examples of two-cycle bias patterns in proportion estimation (top panels) and production (bottom panels).

that assumes that an observer's judgments are not distributed continuously around a true value but are instead taken from a discrete set of preferred values falling within a band around the true value. Because stimulus values vary in their distance from these preferred values, estimated values tend to vary correspondingly, leading to cyclical patterns when averages are taken. The model is flexible as to the particular set of values used, but in simulations, R. Teghtsoonian et al. had success using a power function that generated certain multiples of 5 and 10 (see R. Teghtsoonian et al., 1995, and also Baird & Noma, 1975, for details). Although R. Teghtsoonian et al. did not extend the model to magnitude production, it seems reasonable that an observer could be biased by the particular set of numbers chosen by the experimenter (in our case, predominantly multiples of 5), and this could affect the pattern of produced magnitudes obtained. Thus, this model also serves as a viable explanation for the cyclical bias in magnitude production observed in our experiments.

Experiments in which instructions encourage the adjustment of a single stimulus magnitude with respect to a remembered standard, or the use of certain numbers for the response, are necessary before any conclusions can be drawn about the best explanation for cyclical patterns in magnitude production. Nonetheless, the CPM shows potential utility as an explanatory framework for cyclical bias in magnitude production data.

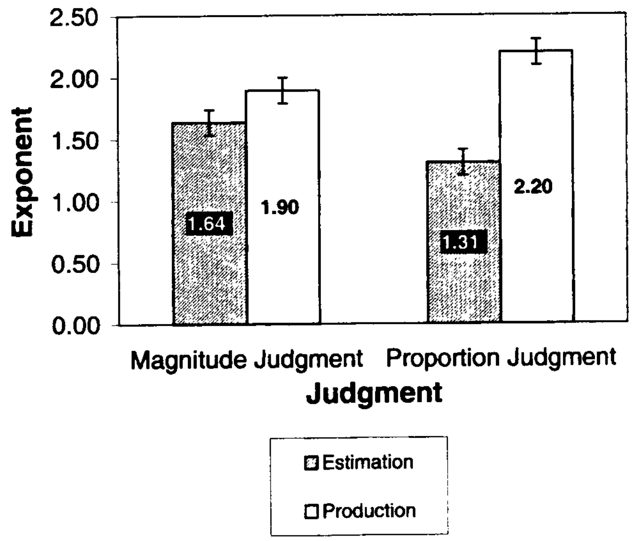

Figure 8. Experiment 2 (color saturation) results. Mean estimated Stevens exponents obtained from magnitude estimation, magnitude production, proportion estimation, and proportion production data. The best fitting version of the cyclical power model was used for proportion estimation and production data. Error bars indicate standard errors of the means.

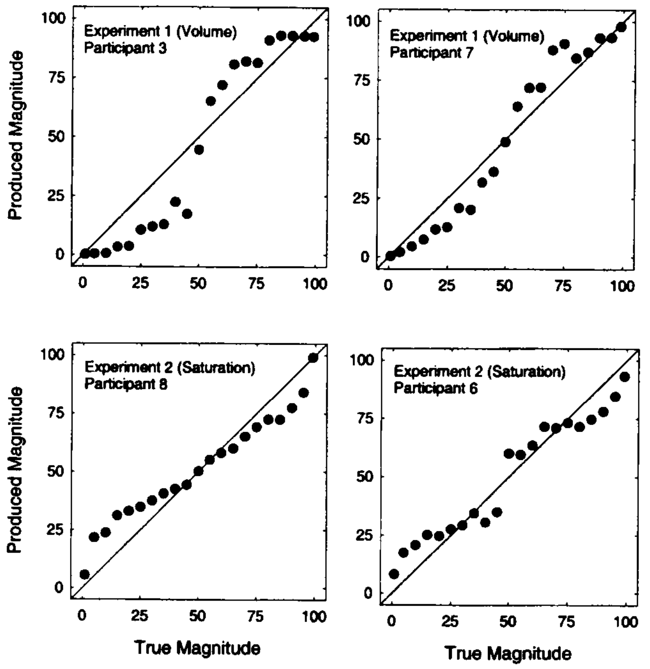

Figure 9. Experiment 2 (color saturation) results for 4 participants. Examples of cyclical bias in proportion estimation (top panels) and production (bottom panels).

Cross-Experiment Correlational Analysis

One method for examining the effect of stimulus continuum on the Stevens exponent is to correlate the four estimates ($\hat{\beta}_{ME}$, $\hat{\beta}_{MP}$, $\hat{\beta}_{PE}$, $\hat{\beta}_{PP}$) across the two experiments—that is, across the two stimulus continua. When the stimulus continuum is changed from geometric volume to color saturation, the increase in β values obtained from magnitude judgments should be associated with an increase in β values obtained from proportion judgments, and the increase in β values obtained from estimation tasks should be associated with an increase in β values obtained from production tasks. For estimates of the Stevens exponent to be reliable, they must be positively correlated.

Table 2 shows the correlation matrix for all possible pairs of estimates. For proportion estimation and production, the Stevens exponent from the best fitting model version of CPM was used. All measures correlated well with all other measures. In particular, we note that Stevens exponents obtained from magnitude estimation and proportion estimation were correlated, $r(14) = .89, p < .001$, consistent with the finding of Hollands and Dyre (2000). A slightly lower correlation was found between β values obtained from magnitude production and proportion production, $r(14) = .81, p < .001$. The correlation between exponents obtained from magnitude estimation and production, $r(14) = .76, p < .005$, was lower than the correlation between exponents obtained from proportion estimation and production, $r(14) = .85, p < .001$. This result indicates a greater correspondence between estimation and production for proportions than for single magnitudes. In general, Stevens exponents estimated using the same task but different judgment types were related, justifying the use of proportion judgments (particularly proportion production) for the estimation of β values.

General Discussion

In this article, we showed how the power model could be extended to proportion production and tested the model's predictions empirically. When $\beta < 1$, the power model predicted an over-then-under pattern for proportion estimation but an under-then-over pattern for proportion production. This was verified in Experiment 1 using volume as the stimulus continuum. The model makes the opposite set of predictions when $\beta > 1$, and the results of Experiment 2 confirmed these predictions. In addition, an extension of the multiple-cycle version of the power model accounted for multiple-cycle bias patterns observed in proportion production. In summary, it appears that the extension of the power model to the production of proportions is warranted and can account for the bias observed in such judgments.

We also predicted greater correspondence between proportion estimation and production exponents than between magnitude estimation and production exponents. In Experiment 1 ($\beta < 1$) the difference in exponents obtained from proportion estimation and proportion production was smaller than the difference in β values obtained from magnitude estimation and production, consistent with the prediction. The Experiment 1 data were consistent with the argument that magnitude estimation underestimates the true exponent and magnitude production overestimates it (Stevens & Greenbaum, 1966). As shown in Figure 4, the β value estimated from magnitude estimation was smaller than the β value estimated from magnitude production. More importantly, Stevens exponents obtained from proportion estimation and production were close to the intermediate value of magnitude judgment exponents in the volume condition. The results from Experiment 1 indicate that a single experiment involving proportion estimation or production is sufficient to produce an unbiased estimate of the exponent, whereas multiple experiments may be required with magnitude judgments to account for range and regression effects.

Table 2
Correlations Among Stevens Exponents Obtained Using Each of the Four Methods

Method	Magnitude judgment: Estimation	Magnitude judgment: Production	Proportion judgment: Estimation	Proportion judgment: Production
Magnitude estimation	—	.76**	.89***	.80***
Magnitude production		—	.70*	.81***
Proportion estimation			—	.85***
Proportion production				—

* $p = .002$. ** $p = .001$. *** $p < .001$.

In contrast, the results of Experiment 2 were not consistent with predictions. With color saturation, proportion estimation produced a smaller Stevens exponent than did proportion production, whose exponent did not differ from the exponents obtained with either kind of magnitude judgment. Proportion production may offer a better method than proportion estimation when a stimulus continuum is not easily decomposable. Although it is difficult to perform the proportion estimation judgment on saturation, proportion production allows the observer to adjust color patches dynamically and observe how the patches are colored in complementary fashion, making total saturation more decomposable.

The correlation between magnitude and proportion judgment exponents provides further support for the assumption that the source of bias in proportion judgments is related to positive or negative acceleration of the psychophysical function obtained from magnitude judgments (Hollands & Dyre, 2000). In addition, the correlation between estimation and production for proportions was greater than that for single magnitudes, showing greater stability of measurement for proportion judgments.

Some participants in Experiment 2 showed cyclical patterns for magnitude production. According to the CPM, the cyclical bias may imply that observers were using minimum and maximum volumes or saturations as reference values and compared the stimulus with this reference. Alternatively, the cyclical bias may be due to a preferred number strategy (R. Teghtsoonian et al., 1995) in which the varying distances of true magnitudes from preferred numbers introduces the cyclical pattern. Additional experimental work is necessary to determine whether cyclical patterns observed in magnitude estimation and production data are due to the CPM or to the preferred number strategy.

In summary, the results show that the power model can be extended to proportion production judgments with success. The predicted reversals from over-then-under to under-then-over were seen for both volume and color saturation. Proportion judgments produced more stable exponents than magnitude judgments for volume. Perhaps this occurs when $\beta < 1$ more generally, although this remains untested. Whereas proportion estimation appeared problematic with color saturation as the stimulus continuum, proportion production appeared a more fruitful technique for estimating Stevens exponents. Finally, we noted that the CPM provides a possible explanation for observed cyclical bias in magnitude production data.

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(Appendixes follow)

Appendix A

Extending the Power Model to Proportion Production

Appendix A provides a mathematical proof that the power model (Spence, 1990) can be extended to predict bias observed in production judgments. To do this, the equation for the power model will be re-expressed so as to predict a produced proportion based on a given numeric proportion.

In the power model, the subjective proportion, P, is expressed as

$$P = \frac{\Pi^{\beta}}{\Pi^{\beta} + (1 - \Pi)^{\beta}},$$

where Π is the physical proportion to be judged and β is the Stevens exponent. This equation can be inverted to predict the physical proportion, Π, produced given a target proportion, P.

First, by rearranging terms one obtains

$$\Pi^{\beta}(1 - P) = P(1 - \Pi)^{\beta}.$$

Next, by taking logarithms, one can obtain

$$\log[\Pi^{\beta}(1 - P)] = \log[P(1 - \Pi)^{\beta}].$$

According to the logarithmic law, the function above is equivalent to

$$\log \Pi^{\beta} + \log(1 - P) = \log P + \log(1 - \Pi)^{\beta},$$

and hence,

$$\beta \log \Pi + \log(1 - P) = \log P + \beta \log(1 - \Pi).$$

By rearranging terms, one can obtain

$$\log \Pi - \log(1 - \Pi) = \frac{\log P - \log(1 - P)}{\beta},$$

and by logarithmic law,

$$\log\left(\frac{\Pi}{1 - \Pi}\right) = \frac{1}{\beta}\log P - \frac{1}{\beta}\log(1 - P),$$

hence,

$$\log\left(\frac{\Pi}{1 - \Pi}\right) = \log\left[\frac{P^{1/\beta}}{(1 - P)^{1/\beta}}\right].$$

Removing logarithms yields

$$\frac{\Pi}{1 - \Pi} = \frac{P^{1/\beta}}{(1 - P)^{1/\beta}},$$

and rearranging terms produces

$$\Pi = \frac{P^{1/\beta}}{(1 - P)^{1/\beta} + P^{1/\beta}},$$

where Π reflects the subjective magnitude adjusted by the observer and P, the numeric values presented to the observer. The bias in production, expressed as $1/\beta$, is therefore the reciprocal of the proportion estimation exponent, β.

Appendix B

Extending the Cyclical Power Model to Proportion Production

Appendix B provides a mathematical proof for inverting the cyclical power model (CPM; Hollands & Dyre, 2000) to predict proportion production β values allowing for the use of intermediate reference points.

According to the CPM, subjective proportion is expressed as

$$P = \frac{(\Pi - R_{i-1})^{\beta}}{(\Pi - R_{i-1})^{\beta} + (R_i - \Pi)^{\beta}} \times \frac{R_i - R_{i-1}}{R_n} + \frac{R_{i-1}}{R_n},$$

where Π is the physical proportion to be judged, β is the Stevens exponent, R_i, R_{i-1}, and R_n are reference values such that $R_{i-1} < \Pi < R_i$, and $i = 0, 1, \ldots, n$. This equation can be inverted to predict the physical proportion, Π, produced given a numeric proportion, P. Because the task is proportion production, let $R_n = 1$, and therefore

$$P = \frac{(\Pi - R_{i-1})^{\beta}}{(\Pi - R_{i-1})^{\beta} + (R_i - \Pi)^{\beta}} \times (R_i - R_{i-1}) + R_{i-1}.$$

By rearranging terms one obtains

$$(P - R_{i-1})(R_i - \Pi)^{\beta} = (\Pi - R_{i-1})^{\beta}(R_i - P),$$

and by taking logarithms,

$$\log(P - R_{i-1}) + \beta \log(R_i - \Pi) = \beta \log(\Pi - R_{i-1}) + \log(R_i - P).$$

By rearranging terms under logarithmic law, one obtains

$$\log\left(\frac{R_i - \Pi}{\Pi - R_{i-1}}\right) = \frac{1}{\beta}\left(\frac{R_i - P}{P - R_{i-1}}\right).$$

Removing logarithms yields

$$\frac{R_i - \Pi}{\Pi - R_{i-1}} = \frac{(R_i - P)^{1/\beta}}{(P - R_{i-1})^{1/\beta}}.$$

Rearranging terms leads to

$$\Pi = \frac{R_{i-1}(R_i - P)^{1/\beta} + R_i(P - R_{i-1})^{1/\beta}}{(R_i - P)^{1/\beta} + (P - R_{i-1})^{1/\beta}},$$

where Π reflects subjective magnitude adjusted by the observer, P, the numeric value given to the observer, $i = 0, 1, \ldots, n$, $R_{i-1} < P < R_i$.

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