Refining physical optics for near-field computations

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It is shown how standard physical optics formulations can easily be adapted to perform near-field computations by introducing locally expanded phase approximations used in conjunction with surface partitioning. This approach leads to a more robust radiation procedure while preserving the simplicity of standard physical optics expressions.

Introduction: High frequency techniques such as physical optics (PO) remain the methods of choice to estimate the radar cross-section of electrically large scatterers [1]. Despite their inherently approximate nature, they can yield accurate results for a variety of configurations provided underlying assumptions are borne in mind. One such assumption for PO is that the observation point be located at infinity. This greatly simplifies the radiation integral for surface currents and makes possible practical closed-form solutions, conferring on PO an attractive combination of simplicity and computational efficiency. Predictably, the far-field assumption severely undermines the ability to estimate scattered fields closer in to the scatterer and there is surprisingly little material in the literature devoted to this problem. The most obvious way of overcoming the shortcomings of PO in the near field is to rely on a more accurate representation of the Green function, an approach taken by Pouliguen and Desclos [2] as well as by Neto [3] who use an exact Green function. This provides excellent accuracy but at the steep price of requiring the numerical integration of the surface currents, which is a highly undesirable prospect for large scatterers. Instead, we choose to preserve the simplicity of standard PO formulations, an approach also considered by Gordon [4], and resolve this difficulty by means of a phase approximation with an expansion centre that lies in the neighbourhood of sources of interest. The derivation provided herein illustrates clearly the ease with which existing formulations can be modified and also serves as an example of how the expressions in [4], which Gordon stated without supporting derivations, can be obtained.

Fundamentals: Consider an electrically large finite scatterer of maximum extent D, perfectly electrically conducting, with surface S and surface normal $\hat{\boldsymbol{n}}$. Given an incident field $\boldsymbol{H}_i(\boldsymbol{r})$, the tangent plane approximation [1] leads to the surface current estimate $\boldsymbol{J}(\boldsymbol{r}) = 2\hat{\boldsymbol{n}} \times \boldsymbol{H}_i(\boldsymbol{r})$ over the lit portion S_{lit} of S. Provided the observation range $r \gg \max(1, k, r')$, where $\boldsymbol{r}' = \hat{\boldsymbol{r}}' r'$ denotes source points, the scattered electric field may be approximated by

$$\boldsymbol{E}(\boldsymbol{r}) \simeq -j\frac{Z}{\lambda}\int_{S_{lit}} (\hat{\boldsymbol{n}} \times \boldsymbol{H}_i(\boldsymbol{r}')) \cdot (\bar{\bar{\boldsymbol{I}}} - \hat{\boldsymbol{R}}\hat{\boldsymbol{R}})\frac{e^{-jk\hat{\boldsymbol{R}}\cdot(\boldsymbol{r}-\boldsymbol{r}')}}{R} dS' \qquad (1)$$

where k denotes the wavenumber, Z the intrinsic impedance and $\boldsymbol{R} = R\hat{\boldsymbol{R}} = \boldsymbol{r} - \boldsymbol{r}'$ so that $\hat{\boldsymbol{R}} \cdot (\boldsymbol{r} - \boldsymbol{r}') = R$. The above is an intermediate result; the integral in (1) can be further simplified since, consistent with the above assumption on r, $\hat{\boldsymbol{R}}\hat{\boldsymbol{R}} \simeq \hat{\boldsymbol{r}}\hat{\boldsymbol{r}}$ and $R \simeq r$. The phase term, requiring higher accuracy, is approximated using a Taylor series. Discarding terms $\mathcal{O}(1/r)$ in the series, the Green function becomes

$$\frac{e^{-jk\hat{\boldsymbol{R}}\cdot(\boldsymbol{r}-\boldsymbol{r}')}}{R} \simeq \frac{e^{-jk\hat{\boldsymbol{r}}\cdot(\boldsymbol{r}-\boldsymbol{r}')}}{r} \qquad (2)$$

and the standard PO expression

$$\boldsymbol{E}(\boldsymbol{r}) \simeq -j\frac{Z}{\lambda}\frac{e^{-jk\hat{\boldsymbol{r}}\cdot\boldsymbol{r}}}{r}\int_{S_{lit}} (\hat{\boldsymbol{n}} \times \boldsymbol{H}_i(\boldsymbol{r}')) \cdot (\bar{\bar{\boldsymbol{I}}} - \hat{\boldsymbol{r}}\hat{\boldsymbol{r}})e^{jk\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'} dS' \qquad (3)$$

is recovered. It could be obtained directly from (1) by letting $R \to r$ and $\hat{\boldsymbol{R}} \to \hat{\boldsymbol{r}}$. The linear behaviour of the exponent with respect to r' permits closed-form solutions for the integral in (3) and Gordon [5] provides a good example with his derivation of an elegant expression for an arbitrary flat polygonal surface.

The phase accuracy of the simplified Green function (2) can be qualified in terms of r and r' by examining the expansion centre of the series which satisfies

$$\frac{r'^2 - 2\boldsymbol{r}\cdot\boldsymbol{r}'}{r^2} = 0 \qquad (4)$$

Two trivial cases of concern where (4) holds are (i) when $r \to \infty$, guaranteeing recovery of the exact behaviour in the far field, and (ii) when $r' \to 0$ for finite r. Given a finite observation range r, the latter suggests (2) inevitably looses accuracy with increasing r'. This is anticipated by the far-field criterion [6] which estimates, given a maximum phase error $\delta\phi = \pi/8$ rad, the minimum observation range at $r_{ff} = 2D^2/\lambda$: r_{ff} is thus proportional to the square of $\max(r')$.

Refined formulation: A good way of overcoming the difficulties of PO in the near field is to resort to phase approximations of the same nature as (2) but with an arbitrary expansion centre $\boldsymbol{r}_n$ as opposed to one essentially fixed at the origin. To see how to proceed, we write

$$R = |\boldsymbol{r} - \boldsymbol{r}'| = |\boldsymbol{r} - \boldsymbol{r}_n - \boldsymbol{r}' + \boldsymbol{r}_n| = |\boldsymbol{q} - \boldsymbol{q}'| \qquad (5)$$

where $\boldsymbol{q} = \boldsymbol{r} - \boldsymbol{r}_n$ and $\boldsymbol{q}' = \boldsymbol{r}' - \boldsymbol{r}_n$. It is immediately apparent from (5) that we can modify the standard expressions by simply letting $\boldsymbol{r} \to \boldsymbol{q}$ and $\boldsymbol{r}' \to \boldsymbol{q}'$. For instance, the expansion centre of the series now satisfies (see (4))

$$\frac{|\boldsymbol{r}' - \boldsymbol{r}_n|^2 - 2(\boldsymbol{r} - \boldsymbol{r}_n)\cdot(\boldsymbol{r}' - \boldsymbol{r}_n)}{|\boldsymbol{r} - \boldsymbol{r}_n|^2} = 0 \qquad (6)$$

which holds when $\boldsymbol{r}$ recedes to infinity. More interestingly (with $\boldsymbol{r}$ finite) the equality is satisfied if $\boldsymbol{r}' = \boldsymbol{r}_n$ and the expansion is therefore highly accurate when $\boldsymbol{r}'$ lies in the neighbourhood of $\boldsymbol{r}_n$. The substitution process described above for $\boldsymbol{q}$ and $\boldsymbol{q}'$ produces an approximate Green function centred at $\boldsymbol{r}_n$, *viz.*

$$\frac{e^{-jk\hat{\boldsymbol{R}}\cdot(\boldsymbol{r}-\boldsymbol{r}')}}{R} \simeq \frac{e^{-jk\hat{\boldsymbol{\rho}}_n\cdot(\boldsymbol{r}-\boldsymbol{r}')}}{r} \qquad (7)$$

where the unit vector

$$\hat{\boldsymbol{\rho}}_n = \frac{\boldsymbol{r} - \boldsymbol{r}_n}{|\boldsymbol{r} - \boldsymbol{r}_n|} \qquad (8)$$

Approximation (7), which recovers (2) if $\boldsymbol{r}_n = 0$, is accurate provided $\boldsymbol{r}'$ lies in the vicinity of the expansion point $\boldsymbol{r}_n$ and $r \gg r'$.

The associated expression for the electric field is obtained by substituting (7) in (1) and, assuming $\boldsymbol{r}'$ is in the neighbourhood of $\boldsymbol{r}_n$, replacing $\bar{\bar{\boldsymbol{I}}} - \hat{\boldsymbol{R}}\hat{\boldsymbol{R}}$ with $\bar{\bar{\boldsymbol{I}}} - \hat{\boldsymbol{\rho}}_n\hat{\boldsymbol{\rho}}_n$. Finally, subdividing S_{lit} into surface elements S_n (each associated with an expansion point $\boldsymbol{r}_n$) we obtain

$$\boldsymbol{E}(\boldsymbol{r}) \simeq -j\frac{Z}{\lambda}\sum_n \frac{e^{-jk\hat{\boldsymbol{\rho}}_n\cdot\boldsymbol{r}}}{r}\int_{S_n} (\hat{\boldsymbol{n}} \times \boldsymbol{H}_i(\boldsymbol{r}')) \times (\bar{\bar{\boldsymbol{I}}} - \hat{\boldsymbol{\rho}}_n\hat{\boldsymbol{\rho}}_n)e^{jk\hat{\boldsymbol{\rho}}_n\cdot\boldsymbol{r}'} dS' \qquad (9)$$

Summation aside this has the same form as (3), showing that existing closed-form expressions for PO integrals can simply be modified by (mainly) letting $\hat{\boldsymbol{r}} \to \hat{\boldsymbol{\rho}}_n$. While (3) is identically recovered if $\boldsymbol{r}_n = 0$, the phase error is greatly reduced by locating the expansion points $\boldsymbol{r}_n$ at (say) the centre of their respective surface element S_n. The far-field criterion then becomes element specific: if the surface element S_n has maximum dimension d_n, the estimated minimum observation radius, centred at $\boldsymbol{r}_n$, is $r_{ff,n} = 2d_n^2/\lambda$. In principle at least, the radiation of the PO currents can now be made arbitrarily more accurate, irrespective of D, by increasing surface element density. Also note that apart from the required summation, the computational complexity is essentially the same as the standard representation and certainly much more attractive than a numerical surface integration. As previously mentioned, the above derivation also serves to illustrate how the expressions stated by Gordon [4], of the same nature as (9), can be obtained.

Fig. 1 provides a simple comparison between the phase error $\delta\phi$ of the approximate Green functions (2) and (7) as the source point $\boldsymbol{r}'$ moves away from the origin. The error is shown as a function of source location $\boldsymbol{r}' = y'\hat{\boldsymbol{y}}$, $0 \le y' \le 50$, and observation point $\boldsymbol{r} = x\hat{\boldsymbol{x}}$, $0 \le x \le 10^4$, when the wavelength $\lambda = 0.03$ m. For the standard formulation (2), the phase error $|\delta\phi| = |\angle\exp[-jk(R - \hat{\boldsymbol{r}}\cdot\boldsymbol{R})]|$ is depicted in the top portion of the graph. In this instance $D/2 = 50$ m and $r_{ff} = 2D^2/\lambda = 667$ km. By this measure, the observation points are mostly situated well within the near field and, as expected, the phase error is considerable for all sources save those in the neighbourhood of the origin. Suppose the y

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axis is now subdivided into 2.5 m segments and (7) is used with r_n located at the segment centre for all sources in a given segment. The far field is now estimated at $r_{ff,n} = 2(2.5)^2/\lambda = 417$ m, a staggering difference of three orders of magnitude. As shown on the bottom half of the Figure, regions of large phase error $|\delta\phi| = |\angle \exp[-jk(R - \hat{\rho}_n \cdot R)]|$ are now located within hundreds of metres of the sources and the accuracy is dramatically improved over the standard formulation.

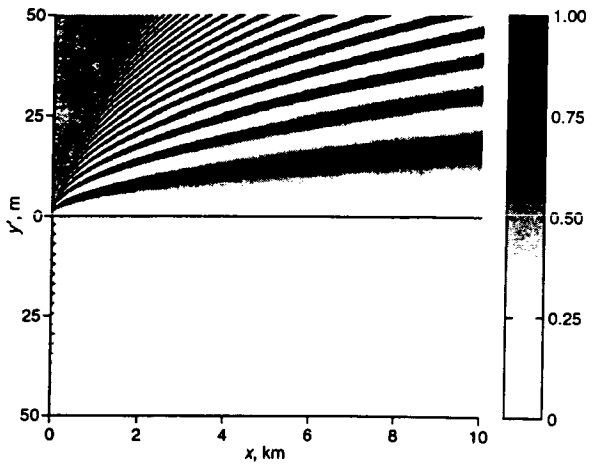

Fig. 1 *Comparison of phase errors $|\delta\phi|$ (grey scale, fraction of π rad) for standard approximation [(2), top half] and proposed approximation with local expansion points [(7), bottom half]*

Conclusion: The standard PO formulation was adapted to near-field computations using a locally expanded approximation together with surface partitioning. It was shown that it provides improved accuracy in the near field with the dimension of concern for determining the far-field boundary now being that of the surface elements as opposed to the global dimensions of the scatterer. Both the form and computational efficiency of standard formulations are preserved, making modifications to existing implementations relatively straightforward. More generally, the proposed technique could be applied to any computational method requiring accurate radiation of surface currents in the near field.

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