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Novel Weight Stability Analysis for Net-Flow Based Multiple-Criteria Method Applied to Courses of Action Analysis

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Abstract

This report presents a novel type of approach for weight stability analysis developed for multiple criteria methods, which is implemented in the Commander's Advisory System for Airspace Protection (CASAP) prototype. Criteria weighting is a complex preference elicitation process. In these circumstances, it is helpful to determine to what extent the solution (ranking) obtained is sensitive to variations in the relative importance coefficients. Stability analysis provides the decision maker (DM) with a precise idea about the sensitivity of a decision to any change in the weighting parameters. In this report we developed, implemented and tested a novel type of stability analysis approach for Multiple Criteria Decision Analysis (MCDA) net flow based methods. This approach is based on the concept of searching for a maximal geometrical box around a centric point (*centroid*). Then, a multi-objective problem is formulated to find the maximal stability zones around the weight vector fixed by the DM. The problem is then transformed into a lexicographical mathematical program. The implementation of the approach and some experimental results are presented. The empirical results are very significant: It is possible to identify the most sensitive criteria in the decision outcome. It is also possible to identify the most conflicting ones. This report also discusses how we think such an approach could be extended to other MCDA methods as well as how it could be used for machine learning. For instance, we show its applicability in the case of PROMETHEE, MAUT/MAVT and weighted sum type MCDA methods. Another extension of this work will consider other types of mathematical norms to define other *centroid*.

Résumé

Ce rapport présente un nouveau type d'approche d'analyse de stabilité de poids développée pour les méthodes multicritères mises en oeuvre dans le prototype du système-conseil du Commandant pour la Protection de l'espace aérien (CASAP). La détermination de l'importance relative des critères est un processus complexe d'articulation des préférences. Dans ces circonstances, il est utile de déterminer dans quelle mesure la solution obtenue est sensible aux variations des coefficients d'importance relative. L'analyse de stabilité fournit au décideur une idée précise de la sensibilité de sa décision aux imperfections des données.

Dans ce rapport nous avons développé un nouveau type d'approche d'analyse de stabilité des poids pour les méthodes multicritères basées sur la notion de bilan de flux. Cette approche est basée sur la recherche de la boîte géométrique maximale autour d'un point central (*centroïde*). Puis, un problème multiobjectif est formulé pour trouver les zones de stabilité maximales autour du vecteur de poids fixé par le décideur. Le problème est alors transformé en un programme mathématique lexicographique. La mise en oeuvre de l'approche et des résultats expérimentaux sont présentés. Les résultats empiriques sont très significatifs : il est possible de déterminer les critères qui influencent beaucoup la décision. Il est aussi possible d'identifier les critères les plus contradictoires. Ce rapport aborde ainsi comment nous pensons étendre une telle approche à d'autres méthodes multicritères et aussi comment employer l'apprentissage. Par exemple, nous avons montré que l'approche est applicable dans le cas de PROMETHEE, MAUT/MAVT et de la somme pondérée. Une autre extension de ce travail tiendra compte d'autres types de normes pour définir d'autres types de centroïde.

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Executive summary

This report presents a novel approach for weight stability analysis developed for multiple criteria methods, which is implemented in the Commander's Advisory System for Airspace Protection (CASAP) prototype. This approach is providing the decision-maker (Commander) with valuable information to balance a decision when considering many criteria. It is the case when one has to select the most appropriate course of action to a situation. Based on the formulation of the problem, a novel model is developed to represent the order-preserving weight intervals. The order-preserving concept refers to generating neighbourhood orders

Those weights play a major role in major multiple criteria methods. Weighting the criteria comes to determining the relative importance coefficients of the criteria. These coefficients are in reality an estimate of the relative importance that the decision maker (Commander) gives to each criterion in order to balance a decision. In a risky situation, the Commander needs to analyse the stability of a decision given all the information considered. Even with graphical and comprehensive tools to set up these relative importance coefficients, it is very hard to estimate these parameters with precision. It is impossible to eliminate completely the imprecision and vagueness of human judgment. Criteria weighting is a complex preference elicitation process.

In those circumstances, it is helpful to determine to what extent the solution (ranking) obtained is sensitive to the relative importance coefficient variations. Stability analysis provides the decision maker with a precise idea about the sensitivity of a decision to any change in the weighting parameters.

In this report we developed, implemented and tested a novel type of stability analysis approach for Multiple Criteria Decision Analysis (MCDA) net flow based methods. It is developed for those methods generating a total preorder of the alternatives. This approach is based on the concept of searching for geometrical box around a centroid. This box is determining through the analysis of weight sensitivity for each criterion separately. Then, once the form and the dimension of that box are fixed, a multi-objective problem is formulated to find the maximal stability zones around the weight vector fixed by the DM. The problem is then transformed into a lexicographical mathematical program. The implementation of the approach and the experimental results are very significant. It is possible to identify the most sensitive criteria in the decision outcome. It is also possible to identify the most conflicting ones.

This report discusses how we think to extend such approach to other MCDA methods and also how to use it for machine learning. For instance, we showed its applicability in the case of PROMETHEE, MAUT/MAVT and weighted sum type MCDA methods. Another extension of this work will consider other types of mathematical norms to define other types of *centroid*.

Guitouni, Adel and Lang, Pascal. 2008. Novel Weight Stability Analysis for Net-Flow Based Multiple-Criteria Method Applied to Courses of Action Analysis. DRDC Valcartier TR 2001-216.

Sommaire

Ce rapport présente une nouvelle approche d'analyse de stabilité de poids développée pour les méthodes multicritères qui sont mises en oeuvre dans le prototype du système-conseil du Commandant pour la Protection de l'espace aérien (CASAP). Cette approche devrait fournir au décideur (le Commandant) de l'information utile pour équilibrer sa décision. Basé sur la formulation du problème, un nouveau modèle est proposé pour représenter les intervalles de poids qui préservent le classement des options (Course of Action: COA). Le concept de préservation du classement réfère à l'interdiction de renverser l'ordre des préférences.

Les poids jouent un rôle important dans les méthodes multicritères. Ces coefficients sont en réalité une évaluation de l'importance relative que le décideur (le Commandant) donne à chaque critère pour équilibrer sa décision. Dans une situation risquée, le Commandant a besoin d'analyser la stabilité de sa décision étant donné l'information considérée. Même avec des outils graphiques pour expliciter ces coefficients d'importance relatifs, il est très difficile d'estimer ces paramètres avec précision. De toute façon, il est impossible d'éliminer complètement l'imprécision et le manque de précision du jugement humain. La détermination de l'importance relative de chaque critère est un processus d'articulation de préférences très complexe.

Dans les circonstances, il est utile de déterminer dans quelle mesure la solution obtenue est sensible aux variations des coefficients d'importance relative. L'analyse de stabilité fournit au décideur une idée précise de la sensibilité de sa décision face à n'importe quel changement des coefficients d'importance relative des critères.

Dans ce rapport, nous avons développé une nouvelle approche permettant de déterminer les intervalles de stabilité des coefficients d'importance relative utilisés par les méthodes multicritères basées sur le bilan de flux. Ces méthodes génèrent des solutions de type préordre total. Cette approche est basée sur le concept de recherche d'une forme géométrique autour d'un *centroïde*; de type pavé. Cette boîte est construite au moyen de l'analyse de sensibilité de poids pour chaque critère pris séparément. Alors, une fois la forme et la dimension de cette boîte fixée, un problème multiobjectif est formulé pour trouver les zones de stabilité maximales autour du vecteur de poids fixé par le décideur. Le problème est alors transformé en un programme mathématique lexicographique. La mise en oeuvre de l'approche et les résultats expérimentaux sont très significatifs. Cette approche permet de déterminer les critères les plus critiques pour la décision. Il est aussi possible de déterminer les critères les plus contradictoires.

Ce rapport aborde comment nous pensons étendre une telle approche à d'autres méthodes multicritères et aussi comment l'employer pour l'apprentissage. Par exemple, nous avons montré que cette approche s'applique au cas PROMETHEE, MAUT/MAVT et la somme pondérée. Une autre extension de ce travail tiendra compte d'autres types de normes définir d'autres types de *centroïde*.

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1.0 Introduction

The Canadian Operational Planning process comprises six main steps: initiation, orientation, course of action development, decision, plan development and plan review. The *Course of Action (COA) Development* step involves the entire staff. The Commander's guidance and intent helps the staff to focus on the development of comprehensive and flexible plans within the available time frame. These COAs "should answer the fundamental questions of when, who, what, where, why and how" [CFC Toronto, 2000b; US Army, 1995]. Each COA should be suitable, feasible, acceptable, exclusive and complete. A good COA positions the force for future operations and provides flexibility to meet unforeseen events during its execution [US Army, 1995]. The "who" in a COA does not specify individual units, but rather uses generic assets and capabilities. During the COA development step, staffs analyse the relative combat power of friendly and enemy forces, and generate comprehensive COAs.

The *Decision* step is based on the analysis and comparison of the proposed COAs, and the primary approach used in this analysis is war gaming. The central framework used by the staff in the war-gaming is a discussion of the actions, reactions and counter-reactions [US Army, 1995; US ACGSC, 1995]. It relies heavily on a doctrinal foundation, critical judgement, and experience. During a war-gaming session, the staff takes a COA and determines its strengths and weaknesses by pitting it against potential enemy COAs. As a result of this analysis, the Commander and staff may make changes to an existing COA or develop an entirely new one. Prior to the war-gaming session, the Commander will identify a list of evaluation criteria. These criteria represent the factors to measure the relative effectiveness and efficiency of each COA.

The COA comparison highlights the respective advantages and disadvantages of each COA. The most commonly used technique is the decision matrix, which used pre-defined evaluation criteria to assess the evaluation of each COA. Each staff officer is free to use his/her own matrix – with the Commander's criteria – for comparison in his/her own field of expertise. Typically, these matrices did not provide a decision solution and, in practice, it is the Chief of Staff (COS) who determines each criterion's relative importance. An ad hoc aggregation process lead to one or several recommendations and the COS then decided which one would be recommended to the Commander during the Decision Brief.

COA approval consists of a choice of the best COA according to the Commander's beliefs and estimates. If the Commander is to reject all of the proposed COAs, then the staff will be required to start the process all over again. Once a COA is chosen, the Commander still has the opportunity to refine his/her intent, guidance and priorities for execution planning. By deciding on a COA, the Commander assesses what residual risk is acceptable, and based on his/her decision, and final guidance, the staff then refines the COA, completes the planning process and issues orders. The aim of the plan development step is to provide a set of orders based on the Commander's decision. Orders provide all of the necessary information to subordinate and supporting units to initiate planning or execution of operations. In the final step, the Commander conducts a final review of the plan, and grants approval for orders to be disseminated.

The Commander's Advisory System for Airspace Protection (CASAP) prototype has been designed to help the Air Operation Center (AOC) staff managing counter-drug events and their related COAs. This command and control (C&C) tool also helps the Commander or the Chief of Staff of the AOC to screen and prioritise the proposed COAs to overcome emergency situation. During different knowledge acquisition sessions, it was established that, in such situations, the Commander needs to balance several conflicting and incommensurable criteria to make *wise* decisions. Therefore, Multiple Criteria Decision Analysis is deemed to be appropriate to deal with Canadian airspace protection decision-making situations. PAMSSEM[1], a Multiple Criteria aggregation procedure, was then identified and adapted to the C&C requirements in a context of counter-drug operations [Guitouni *et al.*, 1999]. PAMSSEM is implemented in CASAP, with different add-ins to help the Decision-Maker analysing the COAs, and minimising the risk component introduced by the subjectivity and the uncertainty of the evaluation process [Guitouni *et al.*, 1999].

In this report, we present a novel type of weight stability analysis for net-flow based multiple-criteria method implemented in CASAP. These weights play a major role in multiple criteria methods. Weighting the criteria comes to determine the relative importance coefficients of the criteria. These coefficients are in reality an estimate of the relative importance (π_j) that the decision maker (DM) gives to each criterion in order to balance a decision. In a risky situation, the Commander needs to analyse the stability of his/her decision given all the information considered. For example, the Commander may need to see if in any case the COA selected is well balanced. One has to perform what-if analysis if weights are reconsidered. Or, he may be interested to analyse the decision frontier, or identify more sensitive criteria that are influencing the decision. In order to address these concerns and more, we developed a novel type of weight stability analysis, which has been implemented and tested within CASAP.

Chapter 2 presents a short introduction to multiple criteria decision analysis framework. In chapter 3, we discuss the role of the weights or the relative important coefficients in major multiple criteria analysis (MCDA) methods, and we formulate the problem. In chapter 4, we formulate the stability analysis approach developed. We show the originality of such approach. Then, in chapter 5, we present the implementation of this approach in CASAP and discuss some computational results. In chapter 6, we discuss the extension of the proposed approach to other MCDA methods and also to other domains. Finally, we present a short conclusion in chapter 7.

[1] PAMSSEM stands for "Procédure d'Agrégation Multicrittère de type Surclassement de Synthèse pour Évaluations Mixtes"

2.0 Multiple Criteria Decision Analysis

Solving a decision problem within the traditional framework of decision theory and operational research consists in finding the feasible solution that maximises a single objective function (e.g., utility function, cost, benefits). In this framework, it is acknowledged that in order to help a decision maker to make a "better" decision, there is generally one criterion (economic function, utility function...) recognised by all stakeholders as having the ability to establish the "right course" for changing the system in question. The premise prevailing is the "homo-economicus", which means that the rational decision maker (DM) always prefers the solution that maximises his/her welfare. Tackling a decision making situation within this paradigm supposes that the situation is isolable, with shape boundaries, and stable with a 'good' structure that can be handled by the mathematical models. This perspective also supposes that the DM is able to articulate his/her preferences according to the strict preference ($\succ$) and the indifference (~) relations. Hence, the maximisation of the DM satisfaction is correlated to the optimisation of an objective function over a set of feasible solutions. The preference structure $\{\succ, \sim\}$ leads to a total preorder of alternatives. As discussed by many authors [Ackoff, 1979; Bell and Olick, 1989; Landry et al., 1993; Ngwenyama et al., 1989; Roy, 1985, 1987, 1990, 1993; van Gigch, 1989; Whitley, 1984; Zeleny, 1992], the classical perspective stands on a non-realistic hypothesis (e.g. the transitivity of $\succ$ and ~). Adopting this perspective implies that the problem exists by itself and should be considered without taking into account the DM subjectivity [Landry, 1995]. The knowledge is mainly originating from the problem that is external and independent of the knowing subject.

In reality, a DM faced with a decision problem is often called upon to reconcile several aspects or points of view, which are often conflicting and incommensurable. It is then needed to develop novel tools to help the DM to differentiate between various options available to him/her while considering several decision criteria. Advances in modeling, mathematics, and new computer technologies together have stimulated research in the development of additional tools able to respond to this concern. It is as a result of these activities that procedures such as ELECTRE, PROMETHEE, TACTIC, MAUT, AHP, PAMSSEM, PRIAM and NAIADE were developed [Guitouni, 1998].

MCDA, often recognised as being synonymous with decision aid science, is a discipline that has undergone significant development during the last three decades. Several manuals and textbooks have been published [Keeney and Raifa, 1976; Saaty, 1980, 1987; Hwang and Yoon, 1981; Goicoechea *et al.* (eds.), 1982; Roy, 1985; Schärlig, 1985, 1996; Vincke, 1992; Bana and Costa (ed.), 1990; Bogetoft and Pruzan, 1991; Chen and Hwang, 1992; Keeney, 1992; Pomerol and Barba-Romero, 1993; Roy and Bouyssou, 1993; Paruccini, 1994; Pardalos *et al.* (eds.), 1995; Olson, 1996; Climaco (*ed.*), 1997]. Several Ph.D. theses have even been published in book form; Andenmatten (1995), Munda (1995), Janssen (1992) and Simos (1990). Today, decision aid science holds an increasingly larger part in the various international conventions, to the extent that some of these scientific events are exclusively devoted to this field. It is also important to note that the scientific contributions to the decision aid field are international in scope, and that the researchers have a multidisciplinary background.

This field of decision aid produced several methods and procedures. Each of the tools attempts to support the decision process and to ensure that *'good'* decisions are made. Each of these procedures attempts to help streamline the decision process and ensure that 'good' decisions are made. This diversity of tools is not only indicative of the high level of research activity, but also of the inherent weaknesses and difficulties of implementing these tools [Bouyssou et al., 1993].

In the MCDA field, two basic methodologies are generally recognised: i) multiple objective mathematical programming methods and ii) discrete multicriterion methods. This differentiation is based principally on the nature of the set of alternatives. In fact, if the cardinality of the set alternatives is large or infinite and could be represented by explicit constraints, then multiple objective mathematical programming would be more appropriate given its computing capacity. However, if the set is discrete or it is not possible to represent it by a set of explicit constraints, the discrete multicriterion methods are indicated. In principle, discrete multicriterion methods are grouped into three categories, based on the concept of "operational approach" [Roy, 1985]: i) unique criterion synthesising approach, ii) outranking synthesising approach and iii) interactive local judgment approach. Vincke (1992) has respectively designated the categories established by Roy as follows: i) multiattribute utility theory, ii) outranking synthesising methods, and iii) interactive methods. The first category uses a utility or value function to represent the decision maker's preferences that is decomposed according to each criterion (partial utility or value functions). The second category of methods tends to build an outranking relation by pairwise comparisons of the all the alternatives. Then this relation is exploited to rank, choose or sort the alternatives. Interactive methods evolve with the decision maker as he/she alternates between computations and dialogue [Vincke, 1989].

Most of the known MCDA methods introduce relative importance coefficients to balance the criteria. In the following section, we discuss the role of these parameters and their importance in the outcome of the decision analysis process.

3.0 Role of the Criteria Relative Importance Coefficient: the Problem

Multiple criteria methods are widely used to address different decision-making situations: select the best alternative from a finite set of decision alternatives, rank these alternatives or sort them in different categories with respect to multiple conflicting criteria (attributes). Most multiple criteria methods that generate a cardinal preference of the alternatives require the DM to provide information on the relative importance (weights) of the criteria [Chen and Hwang, 1992; Colson and de Bruyn, 1989; Hwang and Yoon, 1981; Keeney and Raiffa, 1976; Roy and Bouyssou, 1993; Vincke, 1992; Zeleny, 1982].

The relative importance coefficients of the criteria play a major role in major multiple criteria methods. These coefficients are in reality an estimate of the relative importance (π_j) that the DM (Commander) gives to each criterion in order to balance a decision. Even with graphical and comprehensive tools to set these relative importance coefficients, it is very hard to estimate these parameters with precision. It is impossible to eliminate completely the imprecision and vagueness of human judgments. Criteria weighting is a complex preference elicitation process. In practice, the vague nature of the criteria makes it difficult for the DM to assess precisely the criteria weights and their role in the outcome of the decision analysis process. As a result, inconsistent weights are often produced, which may lead to unreliable decision outcomes. It is evident that the development of a structured approach for assigning weights consistently with regard to decision-making context is desirable for solving practical multiple criteria decision-making problems. In those circumstances, it is helpful to determine to what extent the solution (ranking) obtained is sensitive to the relative importance coefficient variations.

A number of methods for determining criteria weights in multiple criteria methods have been developed. A good comparison of some weight assessment techniques is given in Hobbs (1980), Hwang and Yoon (1981), Schoemaker and Waid (1982), and Barron and Barrett (1996). Approaches to criteria weighting for multiple criteria models based on outranking methods (Roy, 1996) are well discussed by Voogd (1983), Vansnick (1986), and Solymosi and Dombi (1986). Guitouni *et al.* (1999) proposed a new graphical interactive method to determine the relative importance coefficients of the criteria that has been implemented in CASAP. This method requires that the Commander or the Chief-of-Staff sort the criteria from the most to the least important with possible *ex aequo*. The Commander can introduce gaps (units) between successive criteria representing the difference of importance. Then he will be asked to set a ratio of importance between the most and least important criteria. Relative importance coefficients will be then computed.

Keeney and Raiffaa (1976) present a value tradeoffs approach. This approach requires the DM to compare pairs of alternatives with respect to each pair of the criteria, with the assumption that both alternatives have identical values on the remaining criteria. The high value of one alternative is traded off for the low value of the other through a series of adjustments until an indifference value is achieved. The criteria weights are determined after numerous value tradeoffs processes.

Saaty (1980) develops a pairwise comparison approach based on the hierarchical structure of the problem. A reciprocal pairwise comparison matrix is constructed based on a subjective scale of 1-9. Criteria weights are obtained by synthesising various assessments in a systematic manner. This approach is generalised by Takeda et al. (1987) to reflect the decision maker's uncertainty about the estimates in the reciprocal matrix. Barzilai (1997) analyses properties of acceptable solutions of this approach. However, in certain situations this approach may cause the rank reversal phenomenon (Perez, 1995).

Von Winterfeldt and Edwards (1986) and Tabucanon (1988) propose a direct ranking and rating approach. The decision maker is required to first rank all criteria according to their importance, and then give each criterion an estimated numerical value to indicate its relative degree of importance. Criteria weights are obtained by normalizing these estimated values.

Mareschal (1988) uses a mathematical programming model with sensitivity analysis to determine the intervals of criteria weights, within which the same ranking result is produced. Fischer (1995) examined the sensitivity range of criteria weights using different weight assessment methods. The sensitivity analysis approach is also used by Bana e Costa (1988) to deal with the uncertainty associated with the criteria weights in a municipal management decision environment.

Sensitivity analysis provides decision maker the flexibility in judging criteria weights and helps him/her understand how criteria weights affect the decision outcome, thus reducing their cognitive burden in determining precise weights. However, this process may become tedious and difficult to manage as the number of criteria increases.

By recognising the fact that criteria weights are context-dependent, Ribeiro (1996) reviews and proposes preference elicitation techniques for use by the DM at run time to determine weights. In actual applications, the same DM may elicit different weights using different approaches, and no single approach can guarantee a more accurate result [Barron and Barrett, 1996]. This may be mainly due to the fact that the DM cannot always provide consistent value judgements under different quantifying procedures. Different DMs using the same approach may give different weights due to their subjective judgements [Diakoulaki et al.,1995]. As a result, inconsistent ranking outcomes may be produced, leading to ineffective decisions being made.

In addition, to solve the MCDA problem, the current approaches virtually require the DM to consider all requirements simultaneously for assessing criteria weights. This often places a heavy cognitive burden on the DM due to the limitations on the amount of information that humans can effectively handle [Miller, 1956; Morse, 1977]. The presence of imprecision and subjectivity further complicates the criteria weighting process.

In the context of the Canadian airspace protection, the Commander is presented with a set of feasible courses of action. He is asked to select the most appropriate one to the situation. In order to do so, he considers a set of evaluation criteria and balances these criteria. To help him, CASAP offers a suite of tools to balance the criteria and analyse the COAs. A multiple criteria method is then used to aggregate the information contained in the decision matrix and the preference parameters (including the weight) in order to produce a ranking of the alternatives. This ranking reflects the goodness of the COAs.

The multiple criteria method implemented in CASAP is PAMSSEM [Guitouni et al., 1999]. In order to improve the analysis of the COAs, it is required to develop a stability analysis method. The earlier proposals related to weight stability analysis in multiple criteria analysis go back to Mareschal (1988) who proposed weight stability intervals for specific multiple criteria methods. These intervals are obtained for one criterion at a time and do not represent an overall view of the stability of the result. Moreover, the analysis proposed is very limited to PROMETHEE methods. Msezaros and Rapcsak (1992) discussed the sensitivity analysis for a class of decision systems. Wolters and Mareschal (1995) proposed a new sensitivity analysis method that is also very limited. Triantaphyllou (1997) proposed a sensitivity analysis approach for deterministic methods. Yeh et al. (1999) proposed a weight analysis method for some methods based on multi-attribute and analytic hierarchy process based.

All these proposals are very limited. Moreover, there is no approach flexible enough to be applied to outranking methods like PAMSSEM. A novel type of stability analysis approach is then proposed in this document. This analysis leads to determine stability intervals for the relative importance coefficient of each criterion π_j. These intervals represent the limits of variations for each π_j without any order reversal. An order reversal means that no preference relation could be reversed. This analysis can help the Commander of the AOC in identifying the sensitive factors that can affect the outcomes of the decision analysis process.

Let

$$\begin{cases} \mathbf{A} = \{a_1, a_2, ..., a_m\} = \text{the set of COAs} \\ \mathbf{F} = \{g_1, g_2, ..., g_n\} = \text{the set of criteria (attributes)} \\ \mathbf{A} = \left[g_j(a_i) \middle| i = 1,...,m;\ j = 1,...,n\right] = \text{the decision matrix (Multiple Criteria Table)} \end{cases} \quad (1.)$$

PAMSSEM II leads to a total order of the alternatives (with possible *ex aequo*). The rank of each COA a_i is determined by computing its net flow $\Phi(a_i)$. Let Π^0 be the initial set of weights chosen by the decision maker using the CASAP interactive tool. Let $\Phi\left(a_i \middle| \Pi^0\right)$ be the net flow computed for each COA a_i:

$$\Phi(a_i|\Pi^0) = \sum_{j=1}^{n}\left[\pi_j^0 \cdot \sum_{k=1}^{m}\left(Q_{ik}^j - Q_{ki}^j\right)\right], \quad \forall i \in \{1,2,...,m\} \quad (2.)$$

where $Q_{ik}^j = \delta_j(a_i, a_k) \cdot X_{ik};\ Q_{ii}^j = 0\ \forall i, j$ and $X_{ik} = \prod_{j=1}^{n}\left(1 - \left[D_j(a_i, a_k)\right]^3\right)$. $\delta_j(a_i, a_k)$ is a local outranking index computed for each pair of COAs according to each criterion. The local discordance index $D_j(a_i,a_k)$ states the opposition of the criterion j to the assertion that a_i outranks a_k (see Guitouni et al. (1999) for more details). Under Π^0, we obtain an order *O* by ranking all $a_i \in \mathbf{A}$ by decreasing $\Phi(a_i)$. We assume in the following that all COAs have been rearranged in non-increasing order of $\Phi(a_i)$:

$$\Phi(a_i) \geq \Phi(a_{i+1}) \quad \forall i \in \{1,\ldots,m-1\} \tag{3.}$$

The problem is now to find all Π' that will preserve O:

$$\Phi(a_i|\Pi^0) \geq \Phi(a_{i+1}|\Pi^0) \quad \Rightarrow \Phi(a_i|\Pi') \geq \Phi(a_{i+1}|\Pi') \tag{4.}$$

Moreover, all Π' should constitute intervals and should include Π^0. This is explained by the fact that the DM (Commander) is interested in the analysis of stability zones around the assessed initial weights. The interpretation of these zones should help him identifying for instance the most sensitive criteria. In the following chapter, we present a novel formulation of an optimisation method to solve this problem.

4.0 Stability Analysis Formulation

Let $\mathbf{P} = \left\{ \Pi \in \Re^n \middle/ \Pi \geq 0, \sum_{j=1}^{n} \pi_j = 1 \right\}$ be the set of all possible weight vectors. We define a weight vector $\Pi \in \mathbf{P}$ as *order-preserving* if it causes no rank reversal with respect to the ranking obtained using Π^0:

$$\Phi\left(a_i \middle| \Pi^0\right) \geq \Phi\left(a_{i+1} \middle| \Pi^0\right) \Rightarrow \Phi\left(a_i \middle| \Pi\right) \geq \Phi\left(a_{i+1} \middle| \Pi\right) \tag{5.}$$

Given a Π^0, an order-preserving condition could be stated as a linear inequality:

$$\Phi(a_i) - \Phi(a_{i+1}) \geq 0 \Leftrightarrow \sum_{j=1}^{n} \left[\pi_j \cdot \sum_{k=1}^{m} \left(Q_{i+1,k}^{j} - Q_{k,i+1}^{j} - Q_{i,k}^{j} + Q_{k,i}^{j} \right) \right] \leq 0 \tag{6.}$$

Considering O, all order-preserving conditions could be stated as a set of linear inequalities:

$$\begin{cases} C\Pi \leq 0 \\ \text{where } C = \left[c_{ij}\right]; \text{and } c_{ij} = \sum_{k=1}^{m} (Q_{i+1,k}^{j} - Q_{k,i+1}^{j} - Q_{i,k}^{j} - Q_{k,i}^{j}) \\ \forall\, i \in \{1,\dots,m-i\}; \quad \forall\, j \in \{1,\dots,n\} \end{cases} \tag{7.}$$

A stability zone $\mathbf{S} \subseteq \mathbf{P}$ is a set of order-preserving weight vectors such as:

$$C\Pi \leq 0; \quad \forall \Pi \in \mathbf{S} \tag{8.}$$

Now, the objective is to find a zone **S**. It is very difficult to find all possible solution to this problem. Also, we require finding this zone around Π^0. We then try to find a *centroid* Π around Π^0 and a radius α such that:

$$\begin{cases} \mathbf{S} = \{\Pi\} + \alpha \cdot \mathbf{B} \\ \mathbf{B} = \left\{ x \in \Re^n \,\middle|\, \|x\| \leq 1 \right\}, \text{ where } \|.\| \text{ is any given norm} \\ C\Pi' \leq 0; \forall \Pi' \in \mathbf{S} \cap \mathbf{P} \\ \left\| \Pi - \Pi^{\mathbf{0}} \right\| \leq \theta\alpha \end{cases} \tag{9.}$$

A *centroid* is geometric shape built around Π^0. The shape of the *centroid* could be defined to capture the shape of the solution set or the search direction.

B represents the unit ball, and θ is a pre-determined "centrality-parameter" for the initial weight vector $\Pi^0 \left(0 \le \theta \le 1\right)$. In the context of CASAP, we are concerned with stability zones derived from multidimensional intervals:

$$\mathbf{S} = \mathbf{P} \cap \left[\Pi^-, \Pi^+\right] = \left\{\Pi' \in \mathbf{P} \middle| \Pi^- \le \Pi' \le \Pi^+\right\} \tag{10.}$$

Such intervals can be created through a scaled max (l_∞) norm:

$$\|x\| = \max\left\{\frac{|x_j|}{\beta_j} \middle| 1 \le j \le n\right\} \tag{11.}$$

The relative scale parameters β_j are chosen in advance $\left(\beta_j \in \Re_+\right)$. These parameters determine the shape of a rigid or fixed proportion box. Given a choice of $\beta=[\beta_j]$ $\left(\beta \in \Re_+^n\right)$, the problem amounts to placing a rigid box of maximal radius within the stability domain as shown in Figure 1.

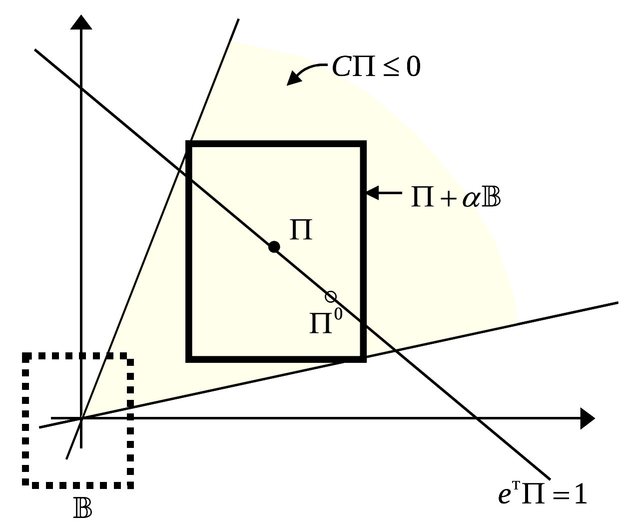

Figure 1. Stability box around the initial Π^0

Now the problem is to find a maximal stability zone. Given fixed-proportions parameters β_j, we seek values of Π (*centroid*), α (box radius, $\alpha \in \Re_+$) and *D* (permissible deviations from centroid, $D \in \Re_{+}^{n}$) that are optimal solutions of the following lexicographic mathematical problem:

$$\begin{cases} Max_{\Pi,\alpha,D} \quad \alpha - \varepsilon\, e^{T}(D - \beta\alpha) \\ \text{Subject to :} \\ C(\Pi + D') \le 0; \quad \forall D' \middle| 0 \le D' \le D \\ D \ge \alpha\beta \\ \left\| \Pi - \Pi^{0} \right\| \le \theta\alpha \\ \Pi \in \mathbf{P} \end{cases} \tag{12.}$$

where ε is a positive infinitesimal and the perturbation term $\varepsilon\, e^{T}(D - \beta\alpha)$ ensures optimality (maximum) of the constructed zone. e^{T} = a vector of ones $(e^{T} = [1,1 \ldots 1])$. In order to solve this problem, we need to formulate the linear programming problems. First let $\overline{C} = |C|$ = [$|c_{ij}|$] be the absolute value of the matrix C. We can reformulate the problem as follows:

$$\begin{cases} Max_{\Pi,\alpha,D} \quad (1 - \varepsilon e^{T}\beta)\alpha + \varepsilon e^{T} D \\ \text{Subject to:} \\ C\Pi + \overline{C}D \le 0 \\ \alpha\beta - D \le 0 \\ \Pi + \theta D \ge \Pi^{0} \\ \Pi - \theta D \le \Pi^{0} \\ e^{T}\Pi = 1 \\ \Pi, D \ge 0 \end{cases} \tag{13.}$$

The demonstration of this reformulation could be found in Guitouni and Lang (2003). The first part of the objective function ($(1 - \varepsilon e^{T}\beta)\alpha$) leads to maximise α. This means that the solution will maximize the expansion of the rigid box around the initial weight vector Π^0. The second part of this function ($\varepsilon e^{T} D$) will lead to augment the solution in those dimensions where still there are slacks. ε is a weighting parameter that will ensure that the first part of the objective function will be maximised first. The second part will be maximised if the first is at its maximum (lexicographical optimization). The resulting interval is $[\Pi^{-},\Pi^{+}]$ with:

$$\Pi^{-} = \Pi - D \text{ and } \Pi^{+} = \Pi + D \tag{14.}$$

Now how to calibrate the box? The problem is to find the β. The relative-size parameters are generally user-defined. However, to provide a sense of what is possible, we compute the largest feasible stability-preserving deviation in each dimension:

$$
\text{For } j = 1,\ldots,n \quad
\begin{cases}
\hat{\beta}_j = Max \quad d_j \\
\text{Subject to:} \\
C\Pi + \overline{C}D \le 0 \\
\Pi + \theta D \ge \Pi^0 \\
\Pi - \theta D \le \Pi^0 \\
e^T \Pi = 1 \\
\Pi, D \ge 0
\end{cases}
\tag{15.}
$$

Where D = [d_j] and $\hat{\beta}_j$ are computed estimation of β_j. Then we set the initial parameters proportional to these estimates:

$$
\beta_j = \frac{\hat{\beta}_j}{e^T \hat{\beta}}; \; j = 1,\ldots,n \tag{16.}
$$

β determines the shape of the rigid box around Π^0. The maximum expansion ratio could be found by solving the following problem:

$$
\begin{cases}
\alpha_{\max} = Max_{\alpha} \quad \alpha \\
\text{Subject to:} \\
C\Pi + \overline{C}D \le 0 \\
\alpha\beta - D \le 0 \\
\Pi + \theta D \ge \Pi^0 \\
\Pi - \theta D \le \Pi^0 \\
e^T \Pi = 1 \\
\Pi, D \ge 0
\end{cases}
\tag{17.}
$$

It is also possible to explore the problem by adding an expansion limitation factor $\rho \in (0,1)$ such that:

$$
\begin{cases}
Max_{\Pi,\alpha,D} \quad (1-\varepsilon e^T \beta)\alpha + \varepsilon e^T D \\
\text{Subject to}: \\
C\Pi + \overline{C}D \le 0 \\
\alpha\beta - D \le 0 \\
\Pi + \theta D \ge \Pi^0 \\
\Pi - \theta D \le \Pi^0 \\
\alpha \le \rho\alpha_{\max} \\
e^T \Pi = 1 \\
\Pi, D \ge 0
\end{cases}
\qquad (18.)
$$

The proposed formulation is also flexible enough to accommodate partial stability analysis in case the decision maker needs to analyse the stability of a sub-set of criteria. This could be done by introducing new constraints or by maximising only d_j for these criteria.

The linear mathematical program (15) provides, for each criterion, the maximal deviation from the actual weight. Problem (17) allows expanding the centroid found computing *m* times expression (16) to the maximum. Then, the linear program (18) allows finding the slack variation available on some criteria where the frontiers were not reached.

In the next chapter, we present the implementation of this approach in CASAP. Two illustrative examples will be presented. Computational results are also discussed.

5.0 Case study: Implementation

Knowledge acquisition sessions with the operational air force personnel led to the identification of five aspects to be considered while evaluating COAs for counter-drug scenarios in a peacetime context. These aspects were modelled as factors: *Flexibility*, *Complexity*, *Sustainability*, *Cost-of-Resources*, and *Risk*. These factors are evaluated by considering 14 evaluation criteria. The evaluations of the COAs according to these criteria are measured on scales ranging from cardinal deterministic, to ordinal, distributional, fuzzy, and probabilistic (see Table 1).

To illustrate the implementation of the stability analysis approach, let us consider six fictitious different COAs. The analysis of the COAs by the staff led to the following decision matrix (Table 2). Let us say that the decision maker (Commander) set the initial weight vector, as shown in Table 3. Based on this initial weight and all the other information, CASAP computed the net flow for each COA (Table 4) and ranked these COAs based on this net flow (Figure 2). The DM used CASAP stability analysis facility to produce the results shown in Figure 3. Figure 4 is a zoom out of the stability analysis result window show in Figure 3. The red zones represent the unconditional stability intervals around the initial weight vector Π^0. Any weight vector of these zones will preserve the order. However, the blue zones are conditional stability intervals. The weight vector picked in this zone should be a convex combination of the maximal solutions found by solving problem (15) in order to preserve the order. In other word, the blue zones are the union of all possible solutions to the problem. Table 5 shows another initial weight vector chosen to illustrate another example. The net flow for each COA computed by CASAP is shown in Table 6. Figure 5 shows the ranking of the COA based on that net flow. Note that in the new ranking COA "test1" and COA "Option D" are *ex aequo* (indifference situation). The stability analysis for this second illustrative example is shown in Figure 6.

Table 1. Criteria used to evaluate COAs in CASAP

Factor	Criterion	Optimisation	Scale	Evaluation
		Flexibility		
C_1:	Covering Operational Tasks	Maximise	Cardinal on [0,1]	Crisp, Deterministic, Continuous
C_2:	Covering Mission's locations	Maximise	Cardinal on [0,1]	Crisp, Probabilistic, Continuous
C_3:	Covering Enemy's CoA	Maximise	Cardinal on [0,1]	Crisp, Probabilistic, Continuous
		Complexity		
C_4:	Operations Complexity	Minimise	Ordinal, 5 echelons	Crisp, Deterministic, Discrete
C_5:	Logistics Complexity	Minimise	Ordinal, 5 echelons	Crisp, Deterministic, Discrete
C_6:	Command and Control Complexity	Minimise	Ordinal, 5 echelons	Distribution, Discrete
		Sustainability		
C_7:	Sustainability	Maximise	Cardinal, $\mathbf{R}^+$	Crisp, Deterministic, Continuous
		Cost of resources		
C_8:	Cost of Resources	Minimise	Cardinal, $\mathbf{R}^+$	Crisp, Deterministic, Continuous
		Risk		
C_9:	Impact of Sensors Coverage Gap	Minimise	Ordinal, 3 echelons	Distribution, Discrete
C_{10}:	Military personnel loss	Minimise	Ordinal, 7 echelons	Crisp, Probabilistic, Discrete
C_{11}:	Collateral damage	Minimise	Ordinal, 7 echelons	Crisp, Deterministic, Discrete
C_{12}:	Confrontation risk	Minimise	Ordinal, 7 echelons	Crisp, Probabilistic, Discrete
C_{13}:	Equipment reliability	Maximise	Cardinal on [0,1]	Crisp, Probabilistic, Discrete
C_{14}:	Personnel effectiveness	Maximise	Ordinal, 5 echelons	Fuzzy, Distribution, Continuous

Table 2. Decision matrix (illustrative example)

Criteria	Option A	Option B	Option C	Option D	Test 1	Test 2
Covering Operational Tasks	79%	73%	70%	68%	73%	23%
Covering Mission's locations	100%	100%	100%	90%	100%	50%
Covering Enemy's COA	100%	100%	95%	78%	100%	94%
Operations Complexity	Low	Medium	Very Low	Low	Very High	Very Low
Logistics Complexity	Very Low	Very Low	Very Low	Very Low	Very Low	Very Low
Command and Control Complexity	(0,0,0.5,0.5,0)	(0,0,1,0,0)	(1,0,0,0,0)	(0,1,0,0,0)	(1,0,0,0,0)	(1,0,0,0,0)
Sustainability	0%	72%	67%	0%	111%	0%
Cost of Resources (K$)	474.00 $	684.70 $	375.70 $	252.40 $	368.70 $	149.30 $
Impact of Sensors Coverage Gap	(1,0,0)	(1,0,0)	(1,0,0)	(0.667,0.333,0)	(0.2,0.8,0)	(1,0,0)
Military personnel loss	Very Very Low	Very Very Low	Low	Very Very Low	Very Very Low	Very Very Low
Collateral damage	Low	Very Low	Low	Very Very Low	Very Very Low	Very Very Low
Confrontation risk	Low	Low	Very High	Very Very Low	Very Low	Very Very Low
Equipment reliability	82%	88%	66%	70%	63%	96%
Personnel effectiveness	Very Low	Very Low	Very Low	Very Low	Very Low	Missing Evaluation

Table 3. Initial weight set by the Commander (example 1)

Criteria	0j	Criteria	0j
C1	10%	C8	5%
C2	5%	C9	7%
C3	10%	C10	6%
C4	5%	C11	10%
C5	8%	C12	10%
C6	9%	C13	5%
C7	5%	C14	5%

Table 4. Computed evaluation of the COAs (example 1)

a_i	$\Phi(a_i)$
Ideal	4.15
Test2	2.24
OptionD	0.74
Test1	0.72
Option C	-0.31
Option B	-1.58
Anti Ideal	-5.96

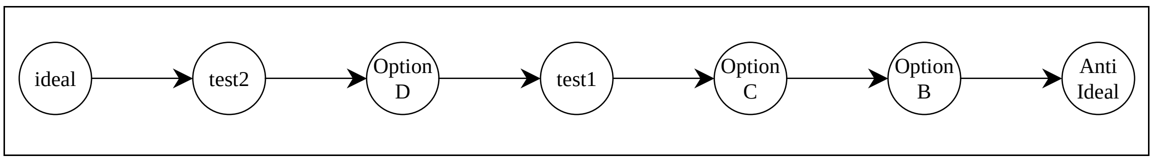

Figure 2. Ranking of the COAs (example 1)

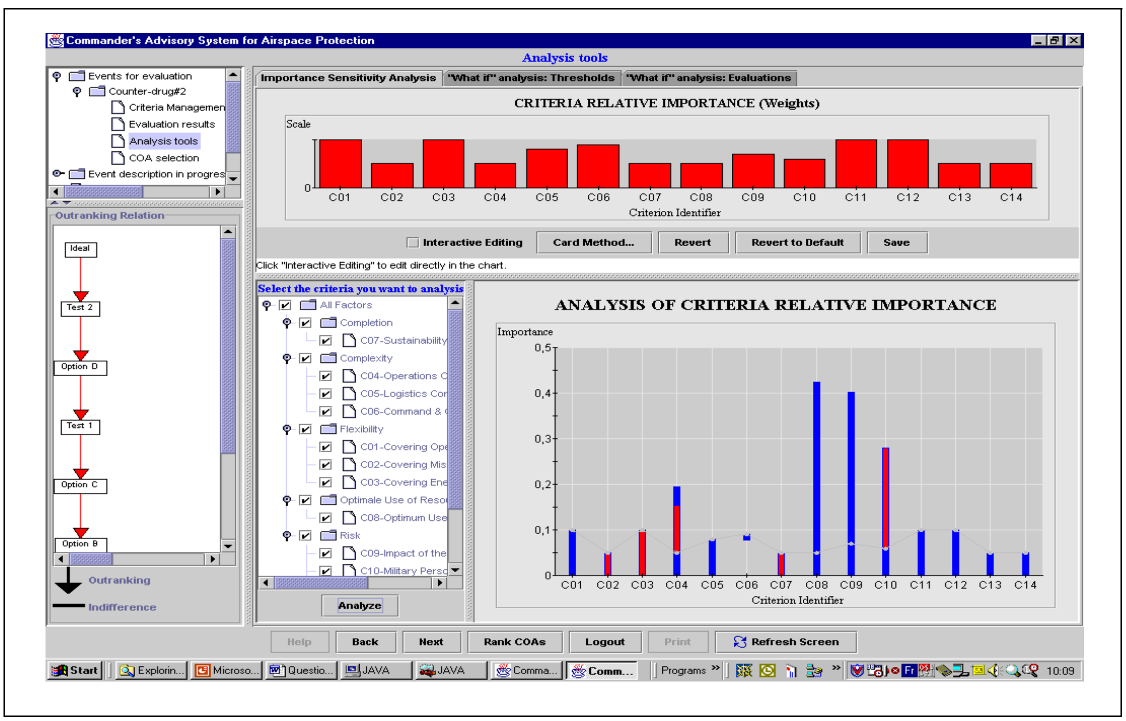

Figure 3. CASAP stability analysis interface: Results for example 1

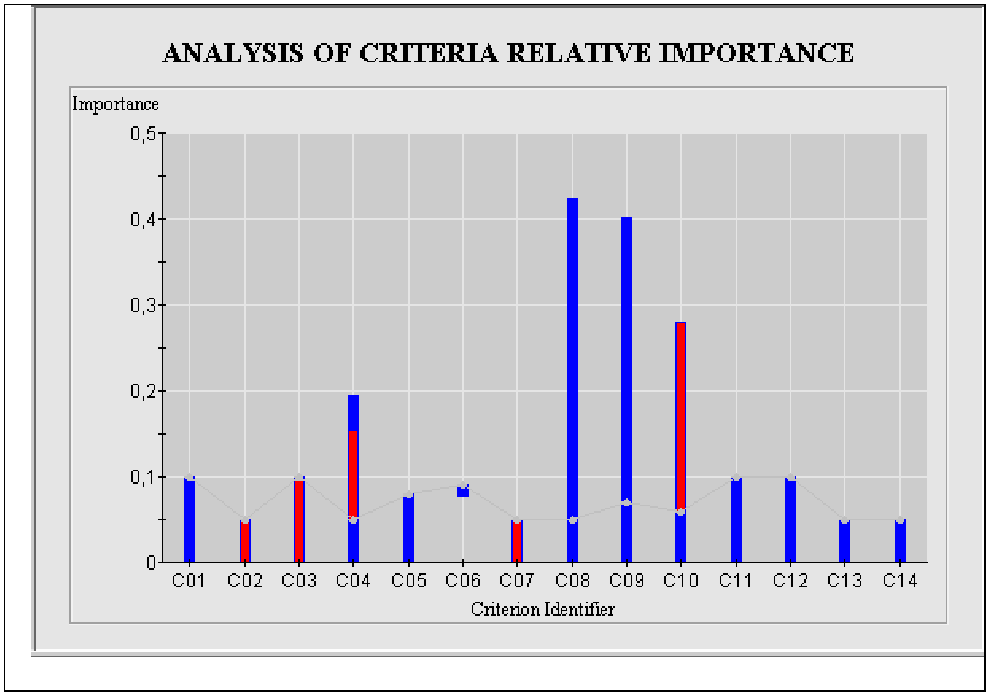

Figure 4. Zoom out of part of Figure 3 (illustrative example 1)

Table 5. Initial weight set by the Commander

Criteria	π^0_j	Criteria	RIC
C1	7%	C8	7%
C2	7%	C9	7%
C3	7%	C10	7%
C4	7%	C11	7%
C5	7%	C12	7%
C6	7%	C13	7%
C7	7%	C14	7%

Table 6. Evaluation of the COAs

a_i	$\Phi(a_i)$
Ideal	4.19
Test2	2.17
OptionD	0.56
Test1	0.56
Option C	-0.16
Option B	-1.49
AntiIdeal	-5.82

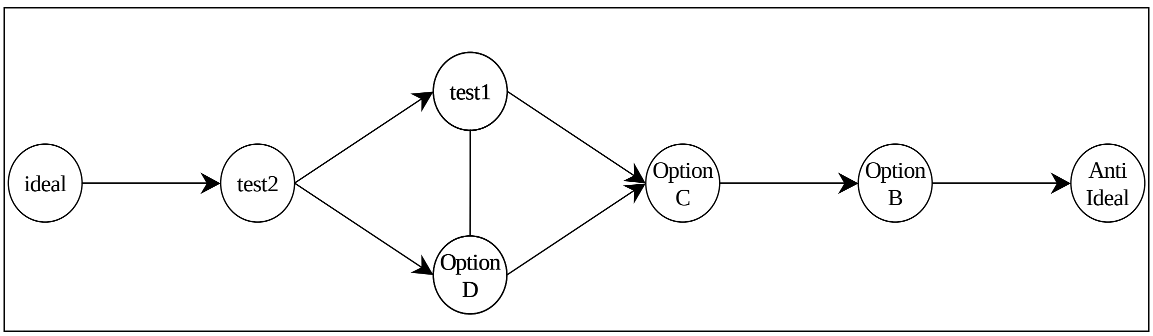

Figure 5. Ranking of the COAs (Illustrative example 2)

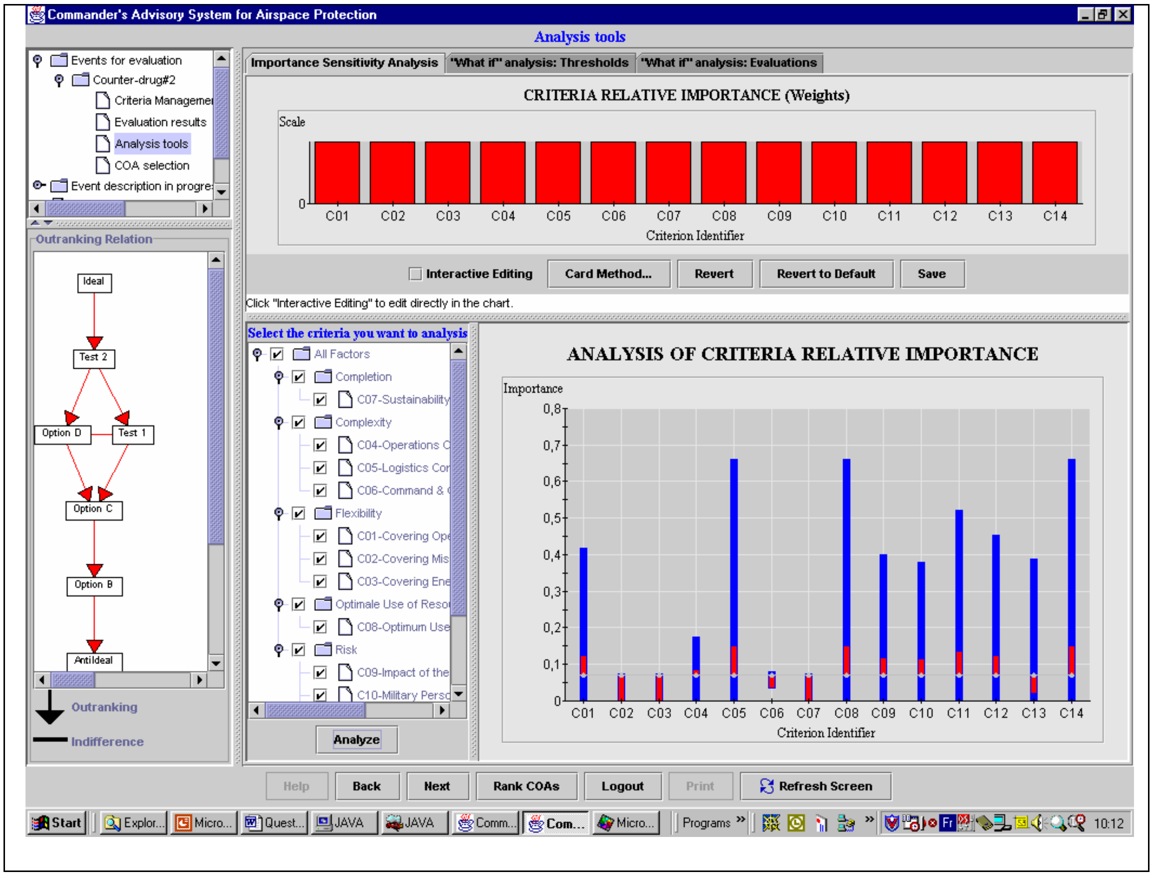

Figure 6. CASAP Stability Analysis Interface (Results for illustrative example 2)

We generated 30 random computational tests for the illustrative example 1. Given the initial weight vector, we performed a first stability analysis, which produced a first stability zone. Then, we randomly set a new weight vector within the red stability zone and then we computed the new net flows and obtained the new ranking. Then, we repeated the process 30 times. The results of these 30 tests are show in Table 7. We also performed 10 random tests for the illustrative example 2. The results for this example are shown in table 8.

The first part of these two tables shows the variation of the weights of the different criteria. The second part shows the net flows computed for each tests. In the case of both illustrative examples, no order reversing was observed as shown in Figures 7 and 8. The percentage of variations shown in the bottom of each table (7&8) should be interpreted with caution. Let us consider the variation of weight stability red zones in case of criteria C5, as shown in Figure 9. The zone obtained in Test 1 is more constrained than the one obtained in test 2. Why? The explanation is simple: when selecting the new initial weight vector Π^0, a translation in the solution space occurred. In the case of C5, this translation led to a less constrained space on π_5, but more constraints on other criteria. Then by selecting a new initial vector it creates another translation. This is clearly shown in Figure 9. Then the variation computed in Tables 7 and 8 is an aggregation of all the variations.

It is clear that from the different random tests that the stability analysis produces the expected results. All randomly selected solutions in the red zones produced no rank reversal. Moreover, from these zones it was possible to identify the most constrained criteria. These criteria are the most conflicting ones. It is easy to verify this conclusion by building a GAIA plan (see Guitouni et al. (1999).

In the following chapter, we discuss the extension of this approach to other MCDA methods like PROMETHEE and MAUT. We also discuss how this approach could be used to learn the weight vector from past similar examples.

Table 7. Random tests (illustrative example 1)

Test #	π_j														COAs Evaluation						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	ideal	test2	optionD	test1	option C	option B	antiideal
1	0.1	0.05	0.1	0.05	0.08	0.09	0.05	0.05	0.07	0.06	0.1	0.1	0.05	0.05	4.15	2.24	0.74	0.72	-0.31	-1.58	-5.96
2	**0.09**	0.03	0.07	0.11	0.08	0.09	0.05	0.05	0.07	0.06	0.1	0.1	0.05	0.05	4.1	2.43	0.91	0.4	-0.21	-1.67	-5.96
3	0.08	0.03	0.07	0.11	0.1	0.09	0.05	0.05	0.05	0.04	0.13	0.1	0.05	0.05	4.07	2.49	0.91	0.45	-0.26	-1.7	-5.96
4	0.05	0.03	0.07	0.11	0.16	0.09	0.01	0.05	0.05	0.04	0.13	0.1	0.01	0.1	3.93	2.65	1.14	0.38	-0.22	-1.97	-5.91
5	0.03	0.03	0.07	0.12	0.12	0.09	0.01	0.05	**0.06**	0.05	0.13	0.1	0.01	0.13	3.95	2.73	1.15	0.31	-0.29	-2	-5.88
6	0.03	0.03	0.07	0.12	0.06	0.09	0.01	0.13	0.04	0.05	0.13	0.1	0.01	0.13	4.15	2.93	1.18	0.19	-0.41	-2.16	-5.88
7	0.05	0.03	0.04	0.12	0.04	0.09	0.01	0.13	0.04	0.05	0.13	0.1	0.04	0.13	4.25	2.87	1.22	0.11	-0.48	-2.09	-5.88
8	0.06	0.03	0.03	0.1	0.03	0.12	0.02	0.13	0.04	0.04	0.13	0.11	0.03	0.13	4.28	2.75	1.23	0.28	-0.48	-2.18	-5.88
9	0.06	0.03	0.03	0.08	0.06	0.12	0.02	0.13	0.03	0.04	0.13	0.11	0.03	0.13	4.25	2.73	1.23	0.39	-0.53	-2.19	-5.88
10	0.06	0.03	0.03	0.1	0.03	0.12	0.02	0.13	0.02	0.04	0.16	0.11	0.02	0.13	4.26	2.74	1.3	0.38	-0.57	-2.23	-5.88
11	0.06	0.03	0.03	0.07	0.08	0.12	0.02	0.13	0.04	0.02	0.13	0.11	0.03	0.13	4.23	2.71	1.2	0.39	-0.45	-2.2	-5.88
12	0.06	0.03	0.03	0.07	0.04	0.14	0.02	0.13	0.04	0.02	0.15	0.11	0.01	0.15	4.27	2.67	1.25	0.48	-0.48	-2.33	-5.86
13	0.05	0.03	0.03	0.09	0.02	0.12	0.02	0.13	0.02	0.01	0.2	0.12	0.01	0.15	4.29	2.76	1.33	0.46	-0.65	-2.33	-5.86
14	0.06	0.01	0.02	0.09	0.02	0.21	0.02	0.13	0.02	0.01	0.13	0.12	0.01	0.15	4.27	2.79	1.2	0.42	-0.3	-2.52	-5.86
15	0.04	0.02	0.02	0.09	0.02	0.13	0.02	0.13	0.02	0.01	0.13	0.12	0.01	0.24	4.48	2.71	1.23	0.27	-0.46	-2.45	-5.78
16	0.01	0.03	0.03	0.09	0.02	0.13	0.02	0.13	0.04	0.04	0.13	0.12	0.04	0.17	4.28	2.99	1.15	0.27	-0.6	-2.24	-5.85
17	0.04	0.04	0.04	0.07	0.05	0.11	0.01	0.13	0.06	0.03	0.11	0.12	0.03	0.16	4.27	2.78	1.15	0.31	-0.48	-2.17	-5.86
18	0.01	0.04	0.04	0.1	0.04	0.1	0.02	0.13	0.06	0.04	0.11	0.12	0.04	**0.15**	4.23	2.96	1.12	0.17	-0.48	-2.13	-5.87
19	0.03	0.01	0.03	0.08	0.08	0.12	0.01	0.13	0.03	0.02	0.12	0.14	0.05	0.15	4.2	3.05	1.29	0.22	-0.64	-2.26	-5.86
20	0.1	0.04	0.08	0.07	0.08	0.09	0.05	0.05	0.07	0.07	0.1	0.1	0.05	0.05	4.13	2.3	0.85	0.62	-0.33	-1.61	-5.96
21	0.07	0.04	0.06	0.07	0.1	0.09	0.05	0.08	0.07	0.05	0.1	0.1	0.05	0.07	4.14	2.48	0.91	0.51	-0.32	-1.78	-5.94
22	0.07	0.04	0.05	0.07	0.13	0.09	0.05	0.09	0.05	0.05	0.1	0.1	0.05	0.06	4.11	2.5	0.98	0.53	-0.35	-1.82	-5.95
23	0.07	0.04	0.05	0.06	0.2	0.07	0.05	0.1	0.05	0.05	0.1	0.1	0.03	0.03	3.95	2.47	1.1	0.62	-0.34	-1.83	-5.97

Table 7. Random tests (illustrative example 1)

	π_j														COAs Evaluation						
Test #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**ideal**	**test2**	**optionD**	**test1**	**option C**	**option B**	**antiideal**
24	0.07	0.02	0.02	0.07	0.2	0.07	0.05	0.17	0.05	0.02	0.1	0.1	0.03	0.03	4.05	2.73	1.29	0.35	-0.37	-2.08	-5.97
25	0.06	0.02	0.02	0.07	0.1	0.07	0.05	0.21	0.05	0.02	0.1	0.1	0.03	0.1	4.34	2.8	1.25	0.24	-0.48	-2.24	-5.91
26	0.02	0.02	0.02	0.07	0.01	0.07	0.05	0.28	0.01	0.02	0.1	0.1	0.02	0.21	4.69	3.02	1.21	0.13	-0.65	-2.59	-5.81
27	0.01	0.02	0.02	0.07	0.01	0.14	0.02	0.28	0.01	0.02	0.1	0.1	0.02	0.18	4.49	3.27	1.23	0.13	-0.56	-2.72	-5.84
28	0.07	0.03	0.03	0.07	0.1	0.08	0.04	0.28	0.01	0.02	0.1	0.1	0.02	0.05	4.33	2.93	1.33	0.25	-0.49	-2.4	-5.95
29	0.06	0.03	0.03	0.07	0.07	0.08	0.04	0.28	0.01	0.02	0.11	0.12	0.02	0.06	4.37	2.99	1.36	0.27	-0.64	-2.4	-5.95
30	0.06	0.03	0.03	0.07	0.06	0.08	0.04	0.28	0.01	0.02	0.12	0.14	0.02	0.04	4.33	3.03	1.45	0.28	-0.74	-2.39	-5.96
Max	**0.10**	**0.05**	**0.10**	**0.12**	**0.20**	**0.21**	**0.05**	**0.28**	**0.07**	**0.07**	**0.20**	**0.14**	**0.05**	**0.24**							
Min	**0.01**	**0.01**	**0.02**	**0.05**	**0.01**	**0.07**	**0.01**	**0.05**	**0.01**	**0.01**	**0.10**	**0.10**	**0.01**	**0.03**							
Variation	**90%**	**80%**	**80%**	**140%**	**238%**	**156%**	**80%**	**460%**	**86%**	**100%**	**100%**	**40%**	**80%**	**420%**							
Mean	**0.05**	**0.03**	**0.04**	**0.08**	**0.07**	**0.10**	**0.03**	**0.14**	**0.04**	**0.03**	**0.12**	**0.11**	**0.03**	**0.11**							
σ	**0.02**	**0.01**	**0.02**	**0.02**	**0.05**	**0.03**	**0.02**	**0.07**	**0.02**	**0.02**	**0.02**	**0.01**	**0.01**	**0.06**							

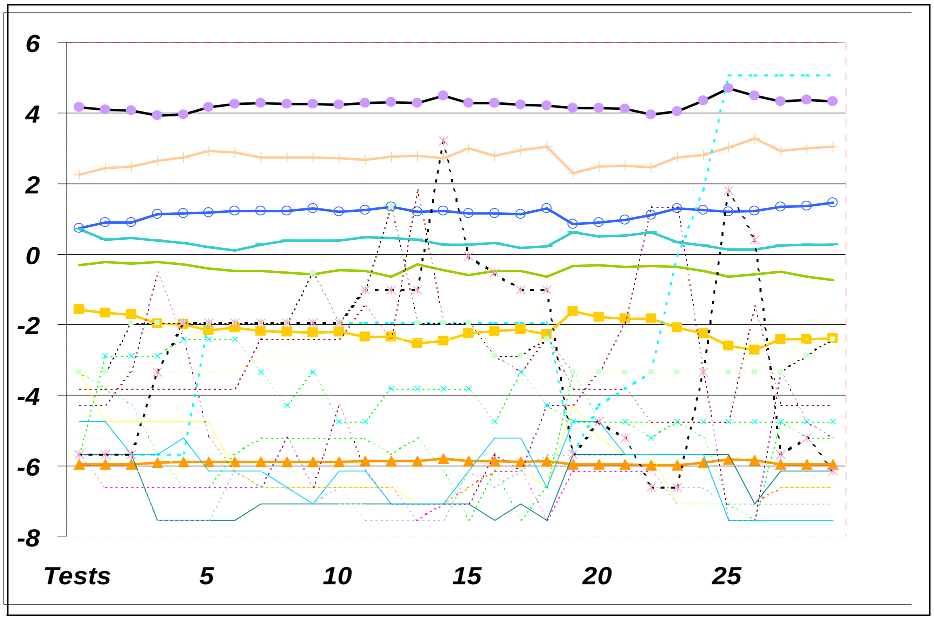

Figure 7. Results for illustrative example 1

Table 8. Random tests (illustrative example 2)

	π_j														**COAs Evaluation**						
Test #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**ideal**	**test2**	**optionD**	**test1**	**option C**	**option B**	**antiIdeal**
1	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	4.19	2.17	0.56	0.55	-0.16	-1.49	-5.82
2	0.07	0.04	0.04	0.07	0.12	0.07	0.04	0.1	0.07	0.07	0.07	0.07	0.07	0.1	4.22	2.51	0.92	0.31	-0.3	-1.75	-5.91
3	0.05	0.04	0.04	0.07	0.08	0.07	0.04	0.1	0.07	0.09	0.11	0.07	0.07	0.1	4.2	2.66	0.99	0.38	-0.56	-1.76	-5.91
4	0.04	0.04	0.04	0.07	0.06	0.07	0.04	0.1	0.05	0.09	0.11	0.14	0.05	0.1	4.18	2.73	1.18	0.47	-0.82	-1.83	-5.91
5	0.04	0.04	0.03	0.09	0.07	0.07	0.04	0.1	0.05	0.09	0.11	0.12	0.05	0.1	4.18	2.73	1.19	0.38	-0.72	-1.85	-5.91
6	0.04	0.02	0.03	0.09	0.13	0.07	0.04	0.1	0.05	0.04	0.14	0.1	0.05	0.1	4.1	2.81	1.18	0.32	-0.56	-1.94	-5.91
7	0.03	0.02	0.03	0.09	0.06	0.07	0.04	0.11	0.05	0.04	0.17	0.11	0.05	0.13	4.22	2.87	1.23	0.33	-0.77	-2	-5.88
8	0.03	0.03	0.03	0.08	0.06	0.1	0.03	0.11	0.03	0.04	0.17	0.11	0.05	0.13	4.2	2.86	1.21	0.42	-0.74	-2.07	-5.88
9	0.02	0.03	0.03	0.11	0.03	0.13	0.03	0.11	0.03	0.04	0.17	0.11	0.03	0.13	4.17	2.9	1.21	0.38	-0.58	-2.2	-5.88
10	0.04	0.03	0.03	0.11	0.03	0.13	0.03	0.11	0.01	0.04	0.15	0.11	0.03	0.15	4.29	2.77	1.17	0.37	-0.54	-2.19	-5.87
Max	0.07	0.07	0.07	0.11	0.13	0.13	0.07	0.11	0.07	0.09	0.17	0.14	0.07	0.15							
Min	0.02	0.02	0.03	0.07	0.03	0.07	0.03	0.07	0.01	0.04	0.07	0.07	0.03	0.07							
Variation	71%	71%	57%	57%	143%	86%	57%	57%	86%	71%	143%	100%	57%	114%							
Mean	0.04	0.04	0.04	0.09	0.07	0.09	0.04	0.10	0.05	0.06	0.13	0.10	0.05	0.11							
σ	0.02	0.01	0.01	0.02	0.03	0.03	0.01	0.01	0.02	0.02	0.04	0.02	0.01	0.02							

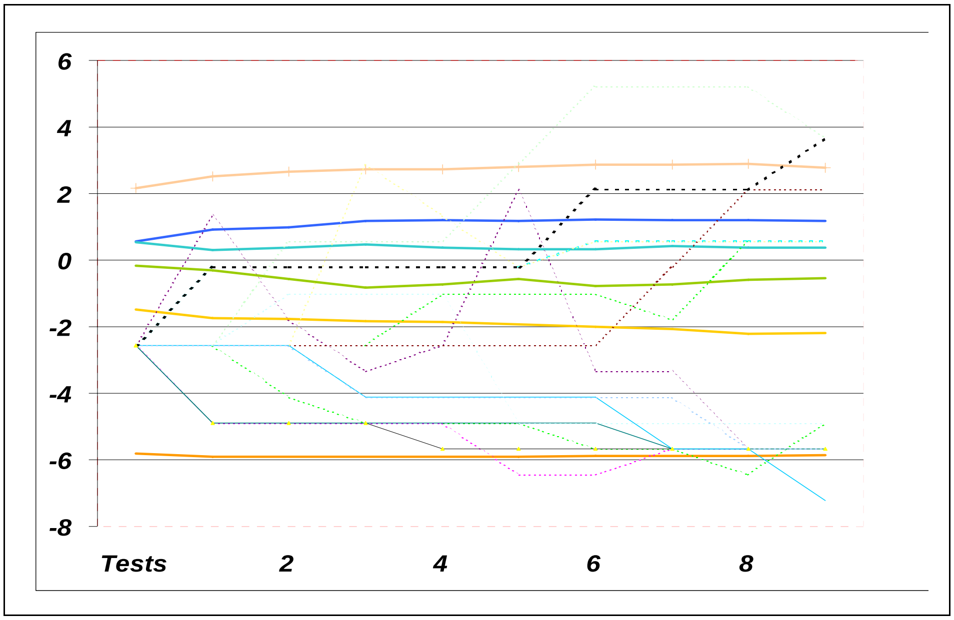

Figure 8. Results for illustrative example 2

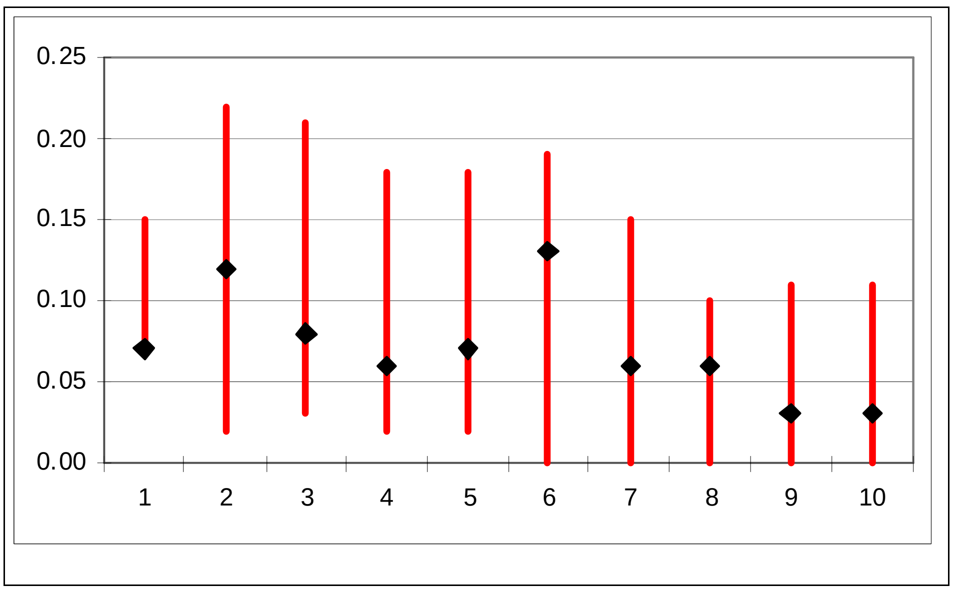

Figure 9. Variation of the stability zones in case of criteria C5 (illustrative example 2)

6.0 Discussion and extensions

The proposed formulation has been illustrated in the case of PAMSSEM, the MCDA method implemented in CASAP. In this chapter, we show how it could be extended for many ranking MCDA methods like the weighted sum, MAUT/MAVT and PROMETHEE.

The weighted sum is the most used aggregation method. Without discussing its weaknesses, the weighting sum computes a global value $\mathrm{V}(a_i)$ for each alternative:

$$\mathrm{V}(a_i) = \sum_{j=1}^{n} \pi_j \cdot g_j(a_i) \tag{19.}$$

The alternatives are therefore ranked in a decreasing manner of $\mathrm{V}(a_i)$. The MAVT (Multiattribute Value Theory) and the MAUT (Multiattribute Utility Theory) are two MCDA methods developed by Keeney and Raiffa (1976). The basic idea consists in building partial utility/value $u_j(X_j)/v_j(X_j)$ functions on each attribute X_j (utility in the context of uncertainty and value in a deterministic context). Many aggregation models have been proposed (see Vincke (1992) for more details) to build the global utility or value function $U(X_1,..,X_n)$ / $V(X_1,..,X_n)$. Once again, of this model is the additive model:

$$U(X_1,...,X_n) = \sum_{j=1}^{n} \pi_j \cdot u_j(X_j) \tag{20.}$$

Each attribute $X_j \in (x_{j*}; x_j^*)$, then $u_j(x_{j*}) = 0$ and $u_j(x_j^*) = 1, \forall j.$

There are many PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations) methods [Brans et al., 1984; Brans et al., 1986; Brans and Vincke, 1985]. For each couple of alternatives (a_i, a_k), PROMETHEE computes a "degree of preference":

$$P(a_i, a_k) = \sum_{j=1}^{n} \pi_j \cdot F_j(a_i, a_k) \tag{21.}$$

where

$$F_j(a_i, a_k) = \begin{cases} 0 & \text{if } g_j(a_i) \le g_j(a_k) \\ \mathrm{H}(g_j(a_i) - g_j(a_k)) & \text{if } g_j(a_i) \ge g_j(a_k) \end{cases} \tag{22.}$$

H is increasing function (predefined by PROMETHEE). Then, PROMETHEE computes the leaving and entering flows, which represents respectively the strength and the weakness of an alternative:

$$\begin{cases} \Phi^+(a_i) = \sum_{a_k \neq a_i} P(a_i, a_k) & \approx \text{"Streng of } a_i \text{"} \\ \Phi^-(a_i) = \sum_{a_k \neq a_i} P(a_k, a_i) & \approx \text{"Weakness of } a_i \text{"} \end{cases} \quad (23.)$$

PROMETHEE II computes also a global flow $\Phi(a_i) = \Phi^+(a_i) - \Phi^-(a_i)$, and then ranks the alternatives in a decreasing manner of this flow.

It is clear that in all the above MCDA methods, the ranking is based on the computation a global value for each alternative (COA). The computation of this value is obtained by a weighted sum:

$$\psi(a_i) = \sum_{j=1}^{n} \pi_j \cdot \mu_j(a_i) \quad (24.)$$

ψ could represent a value, utility or a net flow. μ_j represents a partial utility or value function, or a PROMETHEE preference function. Let suppose that all alternatives are rearranged such:

$$\psi\left(a_i \middle| \Pi^0\right) \geq \psi\left(a_{i+1} \middle| \Pi^0\right) \quad (25.)$$

Given a Π^0, then we have:

$$\psi(a_i) - \psi(a_{i+1}) \geq 0 \Leftrightarrow \sum_{j=1}^{n} \left[\pi_j \cdot \left(\mu_j(a_{i+1}) - \mu_j(a_i)\right)\right] \leq 0 \quad (26.)$$

It follows that order-preserving conditions could be stated as a set of linear inequalities:

$$\begin{cases} C\Pi \leq 0 \\ \text{where } C = \left[c_{ij}\right] \text{is and } c_{ij} = \pi_j \cdot \left(\mu_j(a_{i+1}) - \mu_j(a_i)\right) \end{cases} \quad (27.)$$

The proposed stability analysis approach is then generalised to all weighted sum based MCDA methods.

It is also possible to extend this approach to learn the weight from past similar cases or from initial examples. As discussed above, even with sophisticated weighting methods, it is difficult sometimes for the decision maker to set the initial weight vector. Instead, the decision maker could rank some known alternatives from the best to the worst. Or, we can use past similar cases where satisfactory rankings have been obtained to infer the initial weight vector.

Let **A*** be the set of initial alternatives such that their ranking is known *a priori*; the DM knows how to rank them or the ranking is driven from past event:

$$\mathbf{A}^{*} = \{a_1, a_2, ..., a_i, a_m \,|\, a_i \succ a_{i+1} \text{ or } a_i \sim a_{i+1}\} = \text{A given set of ranked COAs} \quad (28.)$$

Then

$$\Phi(a_i) - \Phi(a_{i+1}) \geq 0, \forall a_i \in \mathbf{A}^{*} \Leftrightarrow \sum_{j=1}^{n}\left[\pi_j \cdot \sum_{k=1}^{m}\left(Q_{i+1,k}^{j} - Q_{k,i+1}^{j} - Q_{i,k}^{j} + Q_{k,i}^{j}\right)\right] \leq 0, \forall a_i \in \mathbf{A}^{*} \quad (29.)$$

The problem now is to find a single weight vector Π that respects the constraints (29). To achieve that, it is recommended to find Π with maximal distance from the frontiers of the feasible data set. The problem becomes a search for the centre of a biggest feasible *centroid*. This centre will be labelled the robust weight vector Π. The formulation of this problem could be found in Guitouni and Lang (2003).

7.0 Conclusion

Multiple criteria methods are widely used to address different decision making situation: select the best alternative from a finite set of decision alternatives, rank these alternatives or sort them in different categories with respect to multiple conflicting criteria (attributes). Most multiple criteria methods require the decision maker to provide information on the relative importance (weights) of the criteria. The relative importance coefficients of the criteria or weights play a major role in determining the outcomes. These coefficients represent an estimate of the relative importance that the Commander gives to each criterion in order to balance his/her decision.

In CASAP, the Commander is provided with several comprehensive tools to set these parameters. Nevertheless, it is impossible to eliminate completely the imprecision and vagueness of human judgment. Criteria weighting is a complex preference elicitation process. In practice, the vague nature of the criteria makes it difficult for the Commander to assess precisely the criteria weights and their role in the outcome of the decision analysis process. As a result, inconsistent weights are often produced, which may lead to unreliable decision outcomes.

In those circumstances, it is helpful to determine to what extent the solution (ranking) obtained is sensitive to the relative importance coefficient variations. Stability analysis provides the decision maker with a precise idea about the sensitivity of his/her decision to any change in the weighting parameters.

In this report we developed, implemented and tested a novel type of stability analysis for MCDA net flow based methods. It is developed precisely for those methods producing a ranking of the alternatives. The formulation used a mathematical lexicographical program. The implementation of the approach and the experimental results are very significant. Using this approach, it is possible to identify the most sensitive criteria in the decision outcome. It is also possible to identify the most conflicting ones, for example.

We also discussed how we think to extend such approach to other MCDA methods and also how to use it for learning. We showed it is applicable for PROMETHEE, MAUT/MAVT and weighted sum type MCDA methods. It is possible to extend it in the case of partial orders. Another extension of this work will consider other types of mathematical norms to define other *centroids*.

In conclusion, we consider this approach as a valuable tool to be implemented in any MCDA-based decision support system. It is clear for us that proposed method has a value added to CASAP. In this later, we think we can improve the visualisation methods.

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This report presents a novel type of approach for weight stability analysis developed for multiple criteria methods, which is implemented in the Commander's Advisory System for Airspace Protection (CASAP) prototype. Criteria weighting is a complex preference elicitation process. In these circumstances, it is helpful to determine to what extent the solution (ranking) obtained is sensitive to variations in the relative importance coefficients. Stability analysis provides the decision maker (DM) with a precise idea about the sensitivity of a decision to any change in the weighting parameters.

In this report we developed, implemented and tested a novel type of stability analysis approach for MCDA net flow based methods. This approach is based on the concept of searching for a maximal geometrical box around a centric point (centroid). Then, a multi-objective problem is formulated to find the maximal stability zones around the weight vector fixed by the DM. The problem is then transformed into a lexicographical mathematical program. The implementation of the approach and some experimental results are presented. The empirical results are very significant: It is possible to identify the most sensitive criteria in the decision outcome. It is also possible to identify the most conflicting ones.

This report also discusses how we think such an approach could be extended to other MCDA methods as well as how it could be used for machine learning. For instance, we show its applicability in the case of PROMETHEE, MAUT/MAVT and weighted sum type MCDA methods. Another extension of this work will consider other types of mathematical norms to define other centroid.

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