Defence Research and Development Canada | Recherche et développement pour la défense Canada

DEFENCE R&D DÉFENSE

Complexity and chaos - State-of-the-art; Overview of theoretical concepts

M. Couture
DRDC Valcartier

Defence R&D Canada – Valcartier
Technical Memorandum
DRDC Valcartier TM 2006-453
August 2007

Canada

Complexity and chaos – State-of-the-art; Overview of theoretical concepts

M. Couture
DRDC Valcartier

Defence R&D Canada – Valcartier

Technical Memorandum
DRDC Valcartier TM 2006-453
August 2007

Principal Author

Mario Couture

Defence Scientist

Approved by

Guy Turcotte

Head/System of Systems Section

Approved for release by

Christian Carrier

Chief Scientist

This work is part of project 15bp01 – Defensive Software Design.

Abstract

Theoretical concepts used in the field of complexity theory are presented. Proposed definitions include the essential elements gleaned from the scientific literature. Key terms such as **system**, **complex system** and **complex adaptive system** and other preliminary notions for the study of complexity are first defined and described. Four classification criteria distilled from an extensive literature review (Couture, 2006a) are then described and used to classify and structure concepts, properties, mechanisms and emerging phenomena. The criteria incorporate concepts that are essential for the study of the above systems. For instance, they employ the concepts of **level** and **interrelationships** between levels, thus enabling researchers to describe level-dependent complex manifestations such as emergence. The criteria defined are then used in a review of complexity theory in the hope that the structured descriptions of the criteria will aid in elucidating the elements of this theory.

Finally, this document shows that complexity theory is in fact a rich set of interrelated theoretical concepts that already help us understand the increasingly complexity of our world. These concepts may also be used as guides to design specific properties or characteristics into information, communication and C2 systems to make them more efficient and effective in complex military operations.

Résumé

Les concepts utilisés dans le domaine de la théorie de la complexité sont présentés dans ce document. Les descriptions proposées intègrent l'essentiel de la littérature scientifique consacrée à cette science. Les mots clés tels que « **System** », « **Complex System** » et « **Complex Adaptive System** » et d'autres notions préliminaires à l'étude de la complexité sont d'abord définis et décrits. Un ensemble de quatre critères de classification déduit d'une revue de littérature étendue (Couture, 2006a) est ensuite décrit et utilisé pour regrouper et structurer concepts, propriétés, mécanismes et phénomènes émergents. Cet ensemble intègre les notions essentielles à l'étude de ces systèmes. Par exemple, il intègre la notion de **niveau** et les **interrelations** entre eux, permettant la description de manifestations complexes qui dépendent de niveaux comme l'émergence. Cet ensemble de critères est ensuite utilisé pour effectuer une revue de la théorie de complexité en espérant que les descriptions structurées impliquant ces critères vont contribuer à aider à la compréhension des éléments de cette théorie.

Ce document montre finalement que la théorie de la complexité est faite d'un riche ensemble de concepts théoriques qui sont interdépendants et contribuent déjà à aider à la compréhension de notre monde toujours plus complexe. Ces concepts peuvent également être utilisés comme guides pour munir les systèmes d'information, de communication et de C2 des propriétés et caractéristiques dont ils ont besoin pour améliorer leur capacité et leur efficacité lors d'opérations militaires complexes.

This page intentionally left blank.

Executive summary

Complexity and chaos – State-of-the-art; Overview of theoretical concepts:

Mario Couture; DRDC Valcartier TM 2006-453; Defence R&D Canada – Valcartier; August 2007.

Content of this document

Man-made systems are becoming more and more complex and harder to predict and control. They involve myriad combinations of individuals, organizations, data, hardware such as computers and network devices, software, and other technologies that are (and will continue to be) used in an increasingly intricate manner. These systems are considered as **complex adaptive systems** (Holland, 1996). Complexity theory already aids significantly in the study of these systems. Even if this theory has not reached its final level of maturity, interested parties in various disciplines may already incorporate these concepts and approaches into their work.

This document introduces notions related to complex systems and complexity theory. Basic concepts that are essential to the understanding of this theory are presented using a "medium" level of details.

Basically, the information following the Introduction (Chapter 1) are grouped into three chapters; Chapter 2: the preliminary concepts and basic tools of the theory, Chapter 3: important aspects of "complexity", and Chapter 4: the use of complexity theory. Chapter 2 contains definitions of basic key words that are used all along this document. For instance, the terms "system", "complex system" and "complex adaptive system" are defined at the beginning of this chapter. All other definitions that are proposed in this document are aligned with the semantic of these key words. A limited number of conceptual tools of complexity theory are then described in this chapter. The following concepts are, for instance, described: level, scale of resolution, phase space, power law of distribution, attractors, fitness landscape, and the possible types of systems' state evolution toward chaos. Chapter 3 is the core of this document. It groups and defines complex concepts, mechanisms, behaviors, and properties that are often found in the scientific literature. The structure of this chapter groups notions as a function of a system's hierarchical levels. Complex properties and mechanisms belonging to the "level of system's components" are first presented. Among others, independence, coherence, coupling, structure and aggregation of components are described. Complex phenomena and properties belonging to the "system level" (that depend on the first level) are then presented; emergence, resilience, robustness are, among others, described. Finally, the Chapter 4 proposes the preliminary skeleton of a process that, when completed and validated, will guide the use of concepts of complexity theory while studying and engineering complex systems. The conclusion lists important observations that were made all along the document and describes future potential work that could be achieved in this domain.

Sommaire

Complexity and chaos – State-of-the-art; Overview of theoretical concepts:

Mario Couture; DRDC Valcartier TM 2006-453; R & D pour la défense Canada – Valcartier; Août 2007.

Contenu de ce document

Les systèmes d'aujourd'hui faits par l'homme et utilisés dans nos sociétés deviennent de plus en plus complexes, difficiles à prévoir et à contrôler. Ils impliquent la combinaison de nombreuses personnes, organisations, données, appareils tels que les ordinateurs et dispositifs réseau, logiciels et autres technologies qui sont (et seront) utilisés de manière toujours plus complexe. Ces systèmes sont considérés comme étant des « Complex Adaptive Systems » (Holland, 1996). La théorie de la complexité apporte déjà une contribution significative à l'étude de ces systèmes. Même si cette théorie n'a pas encore atteint sa pleine maturité, les concepteurs, ingénieurs et autres responsables provenant de différents domaines et disciplines peuvent dès maintenant intégrer ces concepts et approches à leurs travaux.

Ce document présente les notions reliées aux systèmes complexes ainsi que la théorie de la complexité. Les concepts de base essentiels à la compréhension de cette théorie y sont énoncés et définis selon un degré de détail jugé « moyen ».

À la base, l'information suivant l'introduction (chapitre 1) est regroupée en trois chapitres ; chapitre 2 : les concepts préliminaires et outils de base de la théorie, chapitre 3 : les aspects importants de la « complexité », et chapitre 4 : l'utilisation de la théorie de la complexité. Le chapitre 2 contient les définitions des mots clefs de base qui sont utilisées tout au long du document. Par exemple, les termes « système », « système complexe » et « système complexe adaptatif » sont définis au tout début du chapitre. Toutes les autres définitions proposées ensuite dans ce document sont en conformité avec la sémantique de ces mots clefs. Un nombre limité « d'outils conceptuels » de la théorie de la complexité sont ensuite décrits dans ce chapitre. Les concepts suivants y sont par exemple décrits : niveau, échelle de résolution, espace de phase, la « Power Law of Distribution », les attracteurs, la « Fitness Landscape » et les différentes évolutions possibles de l'état d'un système vers le chaos. Le chapitre 3 constitue le cœur du document. Il regroupe et définit les concepts, mécanismes, comportements et propriétés complexes qui sont le plus souvent abordés dans la littérature scientifique. La structure de ce chapitre regroupe les notions en fonction des niveaux hiérarchiques des systèmes. Les propriétés et mécanismes complexes appartenant au « niveau des composants de système » sont d'abord présentés. On y retrouve entre autres la description de : l'indépendance, la cohérence, le couplage, la structure et l'agrégation des composants. Les phénomènes et propriétés complexes au « niveau du système » (qui dépendent du premier niveau) sont ensuite présentés ; l'émergence, la résilience et la robustesse notamment sont décrits. Finalement, le chapitre 4 propose l'ébauche préliminaire d'un processus qui, lorsqu'il sera achevé et validé, guidera l'utilisation des concepts de la théorie de la complexité dans l'étude et l'ingénierie des systèmes complexes. La conclusion répertorie les observations importantes faites tout au long du document et décrit les travaux potentiels futurs qui pourraient être réalisés dans ce domaine.

Table of contents

List of figures

List of tables

Acknowledgements

The author would like to thank Robert Charpentier from DRDC Valcartier for his support during all this work.

This page intentionally left blank.

1 Introduction

Man-made systems[1] like the Internet, the stock market, industries, cities and military command and control (C2) systems are becoming more and more complex and harder to predict and control[2]. They involve myriad combinations of individuals, organizations, data, hardware such as computers and network devices, software, and other technologies that are (and will continue to be) used in an increasingly intricate manner. Actually, these systems exhibit many of the properties of natural complex systems[3] (Holland, 1996). They are for instance composed of huge numbers of autonomous[4] sub-systems or agents (herein called **elements of complex systems**); these elements are able to communicate in different modes[5] through redundant network links[6]; and they use sets of shared and standardized communication protocols, values, rules and internal models. They show high levels of stability and coordination despite frequent environmental disruptions and the lack of central operational planning and control. Intricate interactions between the elements of these complex systems produce complex behaviours such as the emergence of self-organization, adaptation and long-term evolution. The elements' joint capabilities are greater than the sum of their individual capabilities.

Most of the time, complex systems are also highly dynamical and non-linear. Their composition, structure, internal interrelationships, shared rules, values, beliefs and internal models will evolve over time in response to their environment[7]; they will be modified in a non-linear manner.

Significant efforts have been made in the last decade (and are still being made) by the scientific community to find new approaches that would elucidate the basic principles of complexity theory. The Santa Fe Institute's (SFI, 2006) approach is one example. People at this institute use the term **complex adaptive system** (**CAS**). Their approach involves the use of modelling and simulation (M&S) and the study of similarities between different CASs to find underlying principles that would form the basis of a unified complexity theory, one that would be valid for all CASs. Holland (1996) describes the SFI approach in these terms: *The best way to compensate for this loss[8] is to make cross-disciplinary comparisons of CAS, in hopes of extracting common characteristics. With patience and insight we can shape those characteristics into building blocks for a general theory*. ([9])

[1] Words system, complex system and complex adaptive system are defined later in chapter 2.
[2] *Complexity is showing in many areas. Air travel now projects public health problems across the globe: witness the worldwide repercussions of the SARS epidemic, and the looming threat of Avian flu. More seriously still, we face complexity at the level of planet Earth, as we struggle to find a sustainable path into the future, learning to manage the Earth's climate and ecosystems* (ONCE-CS, 2006).
[3] Such as the human immune system, biological ecosystems, ant colonies, etc.
[4] Depending of the type of CAS, elements may for instance be autonomous managerially, financially and/or operationally.
[5] Modes can be for instance cooperation/collaboration, coalition, competition or conflict.
[6] As an example, the Internet's network systems communicate through redundant and loosely coupled links.
[7] Environment may be friendly, neutral and/or hostile.
[8] The fact that we are still looking for means that will allow the generalization of observations into a theory.
[9] All text reproductions originating from the scientific literature are written in *italic* in this document.

Similarities between CASs have revealed some hints regarding how man-made CAS composition, structure and operations can be improved (Holland, 1996). For instance, the human immune system suggests that an autonomous defensive system that is working in parallel with a CAS to protect the latter against unforeseen attacks may represent an effective and efficient solution. As an attack is detected by one element of the defensive system, a signal is transmitted to other relevant elements. They then form a countermeasure[10] that is immediately put into action. Actions posed by the countermeasure aim to 1. limit the effects of the attack, 2. neutralize or destroy the sources of attack, 3. learn[11] and 4. integrate the lessons learned[12]. The CAS would then capture the resulting knowledge and lessons learned and self-organize to adapt to this unforeseen situation. It would also concentrate its energy on self-repair and self-recovery tasks. This generic example depicts a whole system that is able to evolve through constant self-organization and adaptation. Its resilience is enhanced by the diversity and flexibility of its elements and their ability to re-deploy in a more focused manner. Actually, the defence system can attain its highest fitness level if its level of complexity[13] equals or surpasses that of the red systems (Bar-Yam, 2003d).

Complexity theory already provides a means of meeting today's complex challenges. It involves a shift in our way of thinking from purely logic-based rational design to a distributed design approach, harnessing a capacity for self-organization that is suited to the natural complexity and changeability of the real world, both natural and man-made (ONCE-CS, 2006). This document provides an overview of the concepts of this theory.

1.1 Objective of this document

Architects, engineers, commanders, operators and other stakeholders in various military domains or disciplines must now consider the concepts of complexity theory in order to better address current and future problems related to complex systems and operations. Even if this theory has not reached its final level of maturity, parts of it can already be used.

There is an abundance of scientific literature on complexity theory, but as the science is still the object of intense R&D, different interpretations of the component concepts can be found in the complexity community. Although a lack of consistency is normal in an evolving science, it does not contribute to uniform global understanding. The main goal of this document is to compile the appropriate information from the literature and build a global and integrated picture of complexity theory, chaos and complex systems. It is based on a review of literature that aimed at building a state-of-the-art on the subject (companion documents are Couture 2006a, 2006b, and 2006c). A moderate level of presentation was chosen to maintain the requisite minimum of rigour while producing a report that is easy to read. A glossary can be found in Couture (2006c).

---

[10] The composition and structure of the countermeasure should be tailored to defeat that specific type of attack.
[11] Learning new patterns of attack.
[12] Integration of lessons learned may take the form of mutations within the CAS to make it more robust for future attacks.
[13] For instance, the level of complexity of a CAS is raised when the diversity of its systems and complexity of collaboration between its systems are raised. The CAS has "more choices" on actions that can be taken to find solutions to complex problems.

This set of documents represents a point of reference for studying the complexity of systems. Its content and references will serve as both an introduction to these concepts and a conduit to more in-depth study. It is hoped that it will facilitate the understanding and re-utilization of these concepts in the military domain.

1.2 Contexts and scopes

This document is the fourth of a set of five DRDC Valcartier reports dedicated to the study of complexity theory, chaos and complex systems (Couture, 2006a, 2006b, 2006c, and one to be published in 2007). It is part of an overarching project being carried on at DRDC Valcartier, Project 15bp01 – Defensive Software Design. It focuses mainly on the presentation of concepts from this theory. There are only a few references to the architecting and engineering aspects of complexity. These aspects will be covered in another document.

1.3 Used methodology

Figure 1 depicts the general methodology used for this study. It is characterized by a main iterative and incremental loop (steps 1 and 2), which includes a number of sequential and parallel activities (steps 3, 4 and 5). This loop permits on-the-fly adjustment and optimization.

The five main activities or steps are:

1. **Search literature, projects, groups, etc**: Internet searches were made using Google and other search engines. A number of specialized databases were also searched (Dialog Database Catalog, 2005). These databases are listed in Annex C of Couture (2006a).

2. **Select potentially useful documents**: Documents were selected based on their potential applicability to the military context.

3. **Study selected documents**: Approximately 30–40% of the selected documents were read and studied in greater detail.

4. **Investigate in greater depth for the military context**: This involved finding elements that offer potential solutions in the military context.

5. **Update documents**: The content of each document was updated on the fly at each iteration.

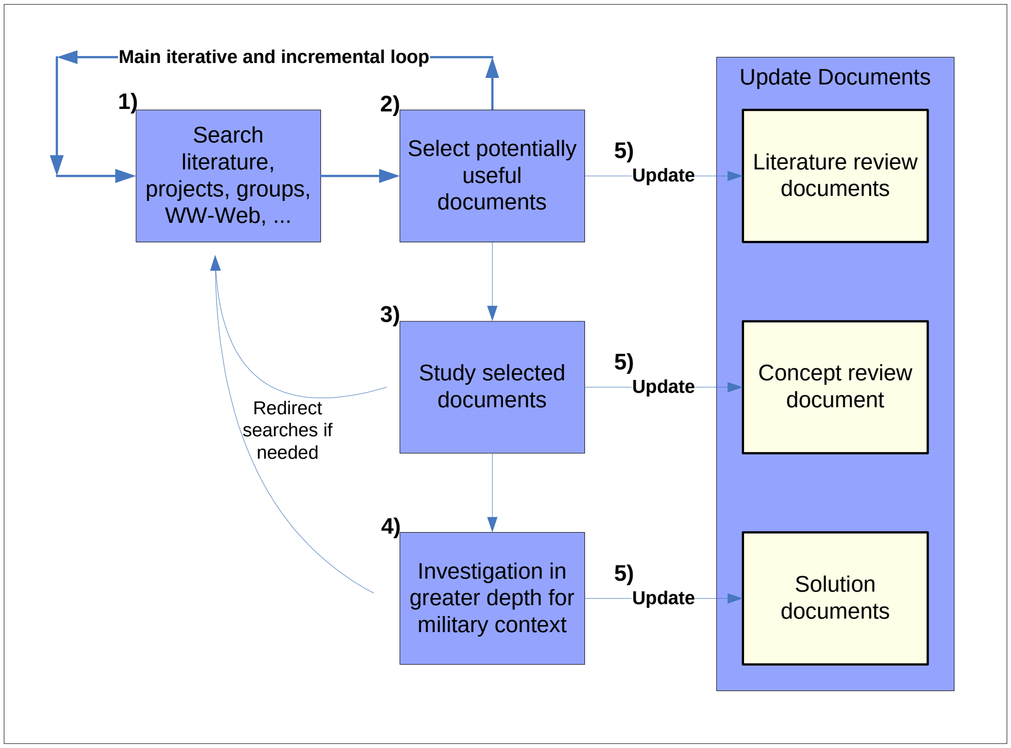


Figure 1 Methodology for this study.

The reports generated by this study are listed in Table 1. The first four reports will be published by the end of phase one of the study (by March 2007). The last will be published by the end of phase two (late 2007).

Table 1 List of documents to be published.

Title	Description
Complexity and chaos – State-of-the-art; List of works, experts, organizations, projects, journals, conferences and tools. (Couture, 2006a).	This Technical Note provides 471 references to scientific studies, organizations, scientific journals, conferences, experts and tools, plus 713 additional Internet addresses that are related to complexity theory, chaos and complex systems. Abstracts are included where available.
Complexity and chaos – State-of-the-art; Formulations and measures of complexity. (Couture, 2006b).	Different formulations and measures of system complexity are provided in this Technical Note. They were drawn from the scientific literature on complexity theory, chaos and complex systems.
Complexity and chaos – State-of-the-art; Glossary. (Couture, 2006c).	This Technical Note defines 335 key words related to complexity theory, chaos and complex systems. The definitions were extracted from the scientific literature.
Complexity and chaos – State-of-the-art; Overview of theoretical concepts. This document: Couture (2007).	This Technical Memorandum presents an overview of theoretical concepts pertaining to complexity theory.
Complexity and chaos – State-of-the-art; The Engineering of complex adaptive systems. (To be published in 2007).	Descriptions of the current approaches, methodologies and tools used to address problems related to the architecting, engineering and improvement of complex systems is included in this Technical Report.

1.4 Taxonomy for classifying properties and phenomena

Complexity theory involves concepts and principles derived from observation, logical deduction and more formal theoretical analysis. They must be named and classified using a unique set of descriptors in order to avoid confusion in interpretation.

A problem arises while reading the scientific literature on complexity theory: not all authors employ the same definitions for the key words, and they often use different descriptors. For instance, a specific complex phenomenon may be termed a **feature** by one author while another refers to the same phenomenon as a **fundamental principle**. Section 3.1 discusses some examples. These differences are an impediment to the understanding of complexity theory and the building of a global picture.

To avoid these differences in interpretation, a taxonomy is proposed and used throughout this document. It is made up of a structured list of descriptors, which are the underlined terms in Figure 2. The taxonomy has three main branches: 1. descriptions of observations; 2. validated or tested theoretical descriptions; and 3. theoretical descriptions that are not yet validated or tested. The observations of phenomena are presented in branch one using appropriate descriptors. More formal theoretical descriptions are presented in branches two or three, again using appropriate descriptors.

1.5 How to use this document

Chapter 2 presents a set of preliminary concepts that may be essential for the study of complex systems. First, some important definitions are given and a taxonomy for designating **systems** throughout this document is proposed. The current method of characterizing systems based on their state is then described. Finally, chapter 2 presents a basic toolkit that can be used to study complex systems. Notions presented in chapter 2 are used in subsequent chapters.

Chapter 3, the core of the document, provides a description of properties and phenomena related to complex systems. A set of four criteria are first described and then used to structure this description.

Chapter 4 describes some current trends in this science and some precautions to be taken when discussing concepts of complexity theory.

Finally, the Conclusion section lists some general observations on this study.

The notions presented in this document may originate from studies that were made in the 19th and 20th centuries, particularly those relating to systems and no-linearity. This document does not provide references to these original studies. It rather refers to the more recent studies of complexity theory, which gained in popularity in the last 10 to 15 years. The reader is referred to historical reviews of the literature[14] for more details on the original identification and description of these concepts.

The reader is encouraged to read all chapters in order, although a fast reading would involve the consecutive reading of chapter 2, chapter 3 and the Conclusion.

All reproductions of text originating from the scientific literature are written in *italic* in this document.

[14] Levine (1996) would be a good starting point; The Control Handbook by Williams S. Levine, CRC Press, ISBN: 0-8493-8570-9.

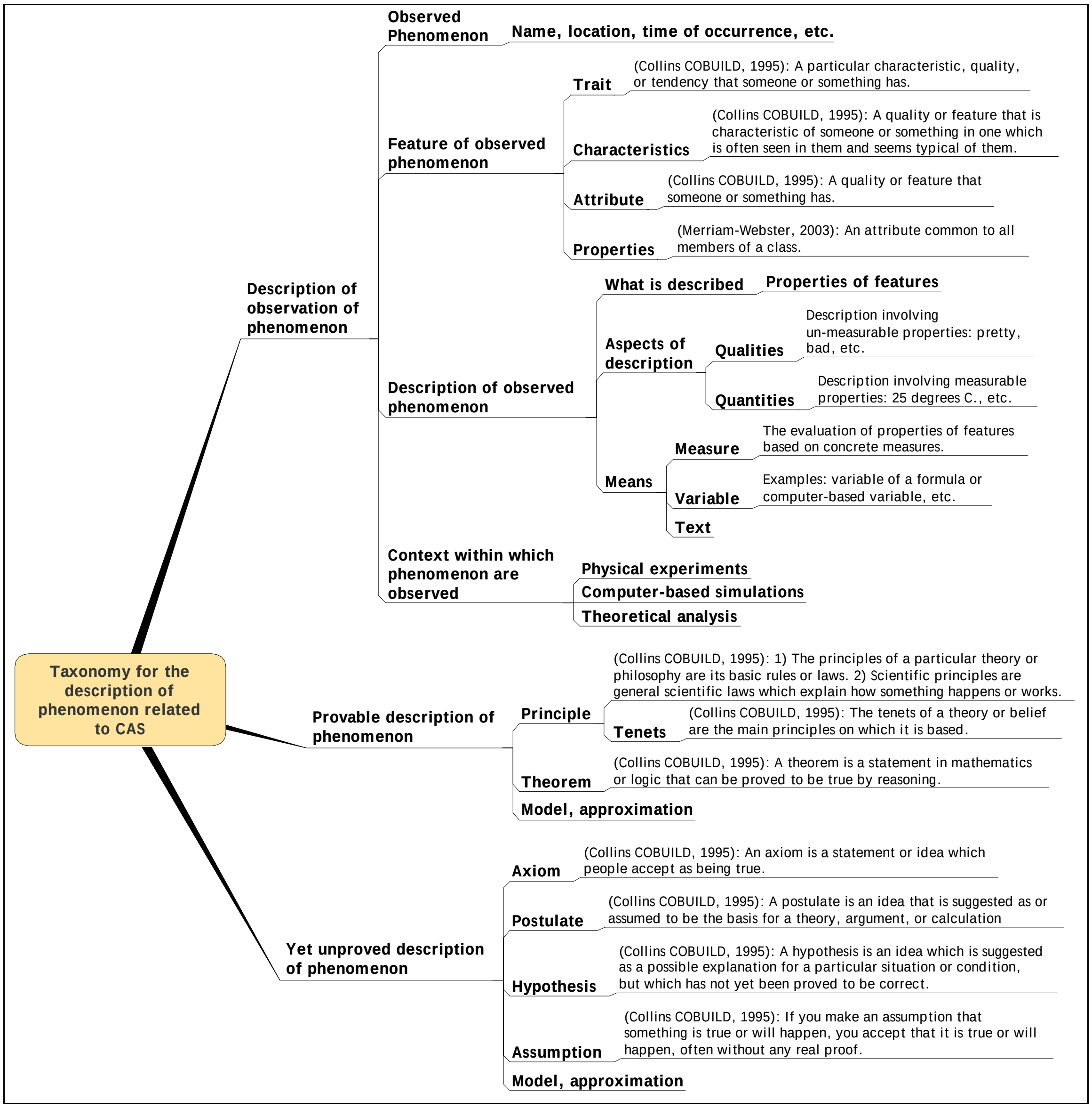


Figure 2 Taxonomy for classifying complex phenomena and properties.

2 Preliminary concepts

An overview of preliminary concepts for studying complexity theory and complex phenomena is presented in this Chapter. Important key words such as **system**, **complex system** and **complex adaptive system** are first defined and described. A scheme for grouping systems based on their state is then presented. Some basic conceptual tools of complexity theory such as phase spaces and attractors are finally briefly described.

2.1 System

The following definition of the term **system** is sufficiently generic to be used in any domains or disciplines. All other definitions of systems will be considered as specializations of this definition.

2.1.1 A Generic definition

Following points constitute the essence of the definition of the term **system** for this document:

- A system is made of any combination of interacting elements[15] like: **people** (person, group of people, organizations of people, etc), **intangible elements** (military doctrines, methods, approaches, theories, software, processes, concepts, ideas, etc), and **tangible elements** (computers, network devices, mechanical devices, radio, vehicles, etc.).
- Elements of a system interact dynamically; they are evolving in environments and contexts[16].
- Elements of a system aim at achieving one or many functions, goals and/or missions.
- The system's openness with its environment allows all necessary exchanges through the system's boundaries[17].
- Outputs to the environment result from transformation and production mechanisms that are internal to the system. They may take many different forms.
- Transformation and production mechanisms are influenced by internal rules, values, beliefs, constraints, culture and internal models (as defined by Holland, 1996) that in turn may be influenced by the environment.
- As it will be shown in Section 2.5, a system may evolve in linear, complex or chaotic states.
- The concept of system is recursive; a system may be composed of other sub-systems that are themselves systems.

[15] A system is made of interacting parts that are called elements, which are themselves systems.
[16] Examples of environments could be for instance: open field, urban cities, etc. Examples of contexts could be for instance: peace keeping, social reconstruction, etc.
[17] As it will be shown later in this document (Section 2.1.3), missions, goals or functions of a system may represent perspectives that can be used to identify and define boundaries.

Unless stated otherwise, studied systems in this work will have behaviours that originate exclusively from deterministic causes-effects relationships among their constituent elements; **there will be no random contribution**.

2.1.2 Description

Figure 3 shows a conceptual view of a system in its environment. Some elements of the IDEF0 notation (IDEF, 2006) are used in this figure. The system accepts and transform/produces input(s)/output(s) from/to the environment within which it evolves. Internal models, rules, beliefs, constraints, missions or goals and contexts guide this transformation/production processes.

Systems may also be controlled by other systems (Control in Figure 3), it may use external mechanism(s) (Mechanism) and it has the possibility to make calls to other systems (Call). External constraints and rules may also be imposed to the system by the environment.

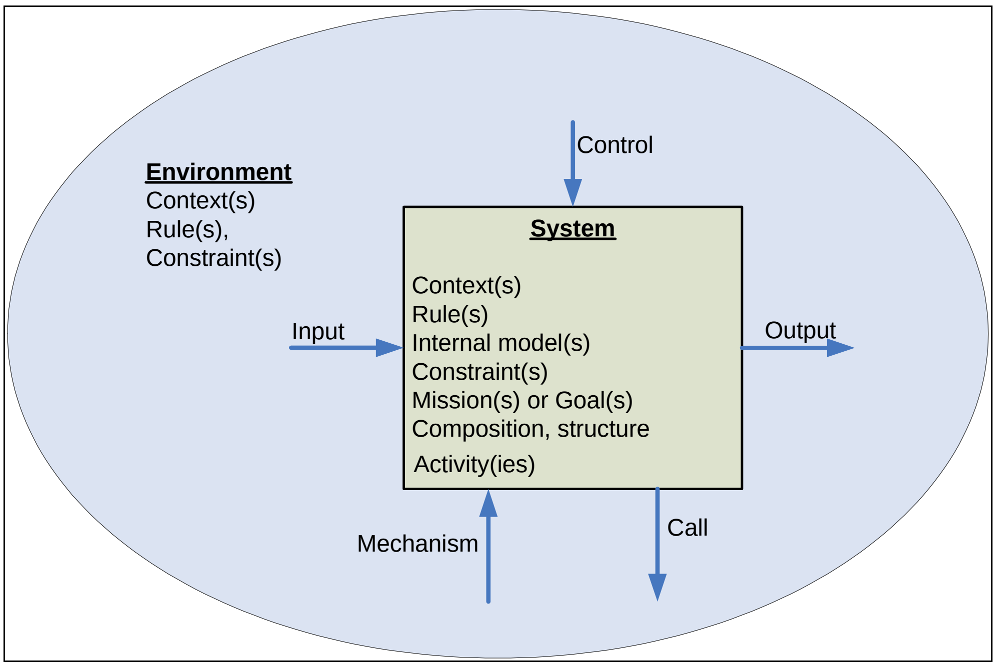


Figure 3 Conceptual view of a system in its environment.

Some important aspects of the definition may be emphasized here:

- **The dynamical aspect**. Systems are evolving with respect of time. **They are not considered as static sets of elements** because transformations/productions only happen if we consider systems as being **in action**. A static piece of hardware may potentially become a system when put in action with other elements such as human, processes, etc. This dynamical aspect is essential to support concepts of complexity theory. Beech (2004) mentions for complex adaptive systems (an instance of this generic definition of system): *Complex networks are referred to as "adaptive" or "**dynamic**", because they are **constantly changing their interrelationships** based upon the needs of individual agents and environmental impacts.*

- **Systems often include human.** For many systems, human is often considered as an integral part (or element) of the system. Senge (1994 (among many others) is clear on this point: *business and other human endeavors **are also systems***. A communication radio, a tank or a C2 software system for instance involve the active participation of human for the accomplishment of missions, goals and/or functions. Other kinds of system like autonomous software agents, automatic computers and network devices are less dependant on human's inputs, but they still need maintenance and restart (human interventions) for staying operational over long periods of time. Whether human is part or not of systems depends on the interactions of the former; Section 2.1.3.2 gives an example. At the opposite end, natural biological systems such as sets of natural growing biological cells[18] are human independent.
- **Systems may include intangible elements**. In this document, **intangible elements** represent all non-physical or non-palpable elements of a system. Examples are software[19] and processes. This differentiation between tangible and intangible elements allows for instance the differentiation between **intangible internal models** and the **tangible structure and composition** of a system. An example would be: the human mental models of understanding (intangible elements) within the human body (tangible element) that constitute a person (the whole system).

2.1.3 Boundaries of systems

In this document boundaries of systems are considered as **subjective to the observer** and they may evolve or change with respect of time. Two criteria are used to identify and define boundaries: the first one is based on **observer's perspectives** and the second one involves **time considerations**.

2.1.3.1 Boundaries identified out of observer's perspectives

In this document, systems are considered as demarcated by boundaries that are defined in function of the **observer's perspective**; boundaries are subjective to the observer. Perspectives are subjective ways of looking at systems; they may for instance be defined in function of **specific functionalities** of a system.

A car in action is used as an example of a global system to illustrate the identification of boundaries based on functionalities. One observer's perspective may be the propulsion functionality while another observer's perspective would be the security functionality. Both perspectives lead to two different boundaries defining two sets of elements or sub-systems (or systems) for the same car[20]: the **propulsion system** and the **security system**.

The two identified systems can be considered as elements of the whole system; the car. The study of these two systems is not sufficient to achieve a complete understanding of the whole car. Many

---

[18] For instance: *the embryogenesis of metazoans – the process whereby a fertilized egg progressively divides until it yields a mature many-celled organism that reproduces by producing another fertilized egg* (Holland, 1995).
[19] Software is made of an assembly of non-palpable executable machine code that is stored in palpable hardware devices. Software itself is intangible.
[20] Sub-systems are themselves systems.

other perspectives involving other functionalities must also be considered. Moreover, interactions between perspectives must be considered in a **global perspective** in order to take into account how elements are working together.

2.1.3.2 Evolution of boundaries with respect of time

The second criterion used to identify boundaries of systems is related to **time**. The composition and the structure of a system at one instant may not be the same at a later time; they may evolve.

For instance, one may ask the question: when is human considered as an element of a particular system? Figure 4 shows the state of this hypothetical system (Y-axis) in function of time (X-axis). For the laps of time lying between t_1 and t_2, human is involved in the starting of the system and between t_2 and t_3 s/he is not anymore involved in its operation. In this particular example, human is an element of the system for the time lying between t_1 and t_2. After t_2, human **may not** be considered as an element of the system.

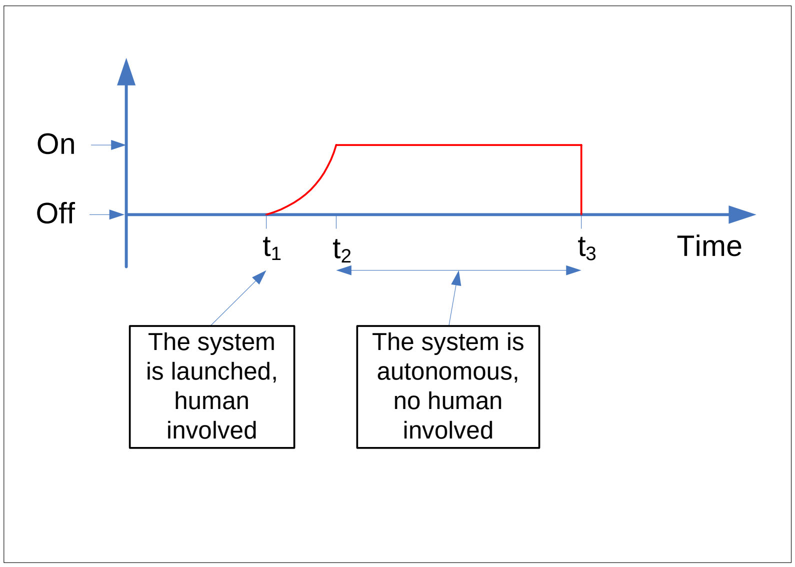


Figure 4 Identification of systems' boundaries out of time considerations.

This simple example suggests that boundaries defining the internal composition and possibly structure of a system at one instant will change with respect of time.

2.1.4 Types of system behaviour

According to Gharajedaghi (1999), systems' behaviours can be grouped into four main classes. They are: 1. passive; 2. reactive; 3. responsive; and 4. active (or pro-active). Corresponding classes of system are: 1. passive/linear; 2. reactive/self-maintaining; 3. responsive/goal-seeking; and 4. active/purposeful. Table 2 lists these four classes and specifies forms of means and finality for each.

Passive behaviours are exhibited by linear systems like **dedicated tools**. Associated **means** and **finality** of these systems are fixed at conception/development and described in specifications.

Systems that self-maintain themselves in changing environments show reactive behaviours; *they are able to react to changes in order to maintain their states under different environmental conditions*. If the finality of such systems remains the same, their means must vary according to pre-determined patterns (to adapt to changes).

Goal-seeking systems show responsive behaviours in response to changes in the environment. *They can respond differently to different events in the same or different environments until they produce particular outcomes (states)*. Their means vary according to on-the-fly system's choices that depend on observed changes. The finality may be adapted in function of observed changes but it must remain within pre-determined limits. *Such systems have the choice of means but not of ends; hence they are responsive rather than reactive.*

Table 2 Four types of system's behaviours (adapted from Gharajedaghi, 1999).

Behaviour	Means	Ends, Finality
Passive **(Linear tools)**	Fix	Fix
Reactive **(Self-maintaining systems)**	Variable and determined	Fix
Responsive **(Goal-seeking systems)**	Variable and chosen	Variable and determined
Active **(Purposeful systems)**	Variable and chosen	Variable and chosen

Purposeful systems demonstrate more intelligence and autonomy than any other kinds of systems. They are pro-active; their active behaviours demand choice and variability from means and finality. *They can produce not only the same outcomes in different ways in the same environment but different outcomes in both the same and different environment*. Purposeful systems have all the capability of goal-seeking and state-maintaining systems. *Controlling the behaviour of purposeful individuals in a multi-minded system by using supervision is less feasible or even desirable. To manage a multi-minded system with self-controlling members we need a new social calculus. This calculus should provide a new framework for creating vertical, horizontal, and temporal compatibility among the members of an organization*. This means among other things that complex systems are best operated when control is relatively distributed among elements instead of being rigid and strongly hierarchic (Section 3.3.11).

2.2 Complex systems

There is no agreement on a definition of **complex system** in the scientific literature. One reason for this is that complexity theory has not yet reached its final level of maturity.

2.2.1 Definition

Complex systems are often defined in function of their characteristics. ONCE-CS (2006) mentions for instance that *they are made of a large number of component elements that are showing strong interactions between each other. They are having a rich dynamics with patterns and fluctuations on many scales of space and time along with the absence of equilibrium. What is most striking is that complex systems that apparently have little in common – a collection of machines in a manufacturing plant, nodes in a P2P system, a group of chemical receptors on a cell's surface or even a group of human agents in an economic setting –* ***often share remarkably similar structures and means of organisation****.*

The following points constitute the essence of the definition of complex systems for this document.

- A complex system is made of an assemblage of autonomous **elements**[21] that **work together** to achieve a common goal or mission.
- Each element behaves according to its own set of internal rules, beliefs, constraints and models in response to local interactions with other elements and its environment. *Agents* (elements) *are driven by local assessments, motivated by the necessity to couple with other, forming interdependent relationships to the mutual fulfillment of their individual requirements.* (…) *Agents are constantly reassessing their need preferences and degree to which they will compromise to bond with other* (Beech, 2004).
- Complex systems achieve missions, goals or functions through intricate interactions between elements. Taken separately, elements cannot achieve the same capabilities; the whole is greater than the sum of the parts.
- Intricate interactions between elements of a complex system trigger the emergence of complex phenomena.
- Even when rules within each element are simple and deterministic, the behaviour of the whole is often complex and hard to predict at mid and long time-scales.
- Complex systems are non-linear and feedback is present.
- Complex systems achieve a state of self-organized criticality without a blueprint or centralized control mechanism.

[21] Elements and complex systems are systems (see definition in Section 2.1). Elements of a CAS may for instance be autonomous managerially, financially and hierarchically; they have a relative independence.

- Complex systems evolve near what Kauffman called the edge-of-chaos. At the edge-of-chaos, the system is optimized for adaptation; the number of interactions is great enough that truly novel change can occur, but the system does not become totally unstable. *Change occurs according to a power law distribution* (Calhoun, 2004) (Section 2.6.4).
- Complex systems *exhibit coherence under changes, via conditional action and anticipation, and they do so without central direction or planning* (Holland, 1996).

Unless stated otherwise, studied complex systems in this work will have behaviours that originate exclusively from deterministic causes-effects relationships among their constituent elements; **there will be no random contribution**.

2.2.2 Description

Figure 5 shows a conceptual view of a complex system in its environment. It is made of twelve independent interacting elements or systems (S-1 to S-12)[22] and external elements (called S). In this Figure, some elements are controlled by others (S-3 and S-4), some others are working in cooperation or collaboration (S-5 and S-9) while others are working in competition (S-4 and S-9). All form of interrelationships can be found (Section 2.2.3). Feedback[23] is also present between some of the elements (between S-9 and S-11 and between S-10 and S-6). All these elements are concurrently evolving, making the whole system **highly dynamical**, **non-linear**, and **non decomposable** into its elements. Figure 5 also shows that complex systems may have their own contexts, constraints and missions.

As it will be shown later in this document, the degree of complexity of systems often refers to *the potential for emergent behaviour in complex and unpredictable phenomena* (Chan, 2001). Economy, ecosystems, the human brain, developing embryos and ant colonies are all examples of complex systems. When an economy is considered, the elements might be individuals or enterprises, in an ecosystem elements might be the species, in the human brain elements are nerve cells, in an embryo the elements are cells, etc.

Each complex system is made of a network of elements that are concurrently interacting with each other and with the environment. There are continual actions and reactions to what other agents are doing; nothing is essentially fixed.

Elements of complex systems have a certain degree of autonomy and their individual missions may not necessarily be perfectly aligned with the mission of the whole. The **ability** and/or the **willing** of each element to adapt or align its own missions (or will) with the global mission are factors (called attractors in Section 2.6.7) that will influence the effectiveness and efficiency of the whole complex system to achieve its global mission. It will also provide a degree of **order** within the system.

---

[22] Only a limited number of elements are shown in this generic example for clarity purposes. Some complex systems may involve large number of elements.
[23] Feedbacks shown in this Figure are internal to the complex system. Feedback may also be used at a higher level (Section 3.3.9).

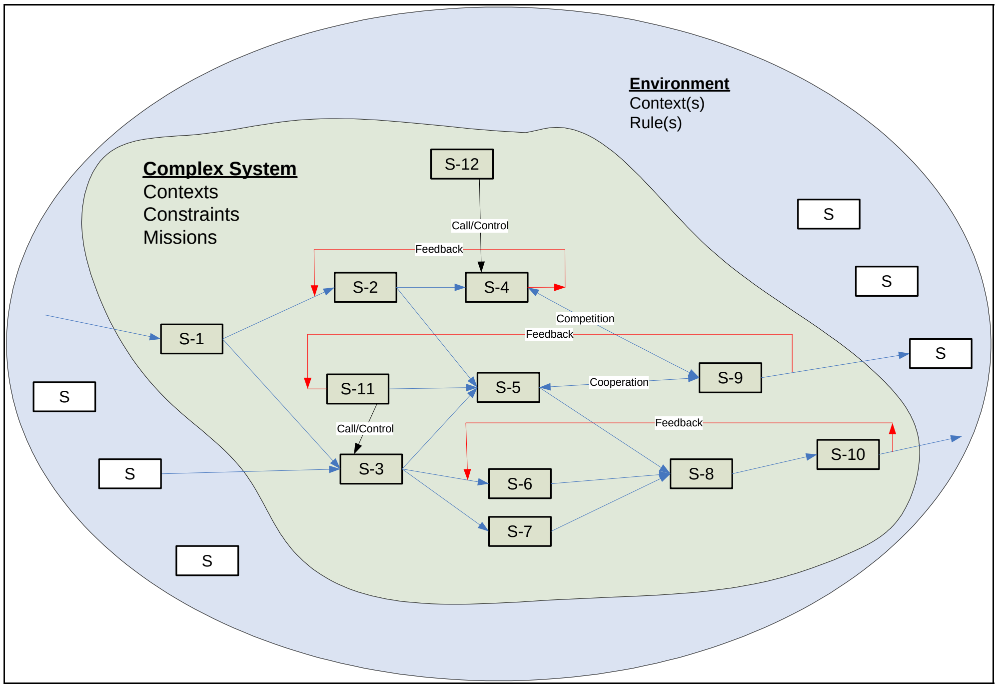


Figure 5 Conceptual view of a complex system in its environment.

2.2.3 Classes of relationships between elements of complex systems

According to Gharajedaghi (1999), at least four classes of interrelationships can be found between elements of complex systems. They are determined by the level of compatibility between their means and finalities. They are:

- **Cooperation**. *Cooperation* (or collaboration) *between elements is possible when means of elements are compatible and finalities of elements are compatible.*
- **Coalition**. *Coalition may happen when means of elements are compatible but finalities of elements are incompatible.* Coalition may involve two opposite situations; the first one involves two compatible systems that are working together to achieve a common and shared goal or mission. The second involves two compatible systems that are not working together but they are achieving a common goal or mission. Coalition may results thus in constructive or disinterested interactions.
- **Competition**. *Competition may happen when means of elements are incompatible but finalities of elements are compatible.* Systems that are in competition are working for themselves in order to get the most advantageous place or results. Both systems are doing their best to arrive at the best results as quickly as possible. Seen from a global perspective, it may thus generate constructive results because the finalities are compatible.
- **Conflict**. *Conflict may happen when means of elements are incompatible and finalities of elements are incompatible.* Conflict may result in destructive interactions and aggressions between systems. In this mode, systems will usually fight against each other with the intention to destroy.

Holland's (1996) types of relationships between elements of a complex system can be grouped within Gharajedaghi's classes:

- **Offence**: may fit in competition and conflict.
- **Defence**: may fit in cooperation, coalition and maybe competition.
- **Aggregation**: may fit in cooperation and coalition.
- **Adhesion**: may fit in cooperation, coalition and maybe competition.
- **Selection**: may fit in cooperation, coalition and maybe competition.
- **Replication**: may fit in cooperation, coalition and maybe competition.

The efficiency and effectiveness of a complex system to achieve its mission may concurrently involve any combination of these classes of relationships between elements (Gharajedaghi, 1999).

2.3 Complex adaptive systems (CAS)

The differences between complex systems and complex adaptive system (CAS) are not always clear in the scientific literature. The former often appears to be more generic then the latter (which is used at Santa Fe Institute). Sometimes, complex systems are also used for natural but non biological systems such as Per Bak's sand pile experiment. This Section provides a definition of CAS and Section 2.4 proposes taxonomy for designating any kinds of system in this document.

John Holland, a pioneer in the field of complexity, coined the term **complex adaptive system** (CAS) to describe *the constantly evolving nature of complex systems.* It is worth citing Holland (1998) on this subject: *Many natural systems (e.g., brains, immune systems, ecologies, societies) and increasingly, many artificial systems (parallel and distributed computing systems, artificial intelligence systems, artificial neural networks, evolutionary programs) are characterized by apparently complex behaviors that emerge as a result of often nonlinear spatio-temporal interactions among a large number of component systems at different levels of organization. These systems have recently become known as* ***Complex Adaptive Systems*** *(CAS). The theoretical framework is based on work in the natural sciences studying CAS, e.g., physics, chemistry, biology. The analysis of CAS is done by a combination of applied, theoretical and experimental methods (e.g., mathematics and computer simulation).*

An interesting definition of complex adaptive systems can be found in Dooley (1996): *A CAS is a system composed of many interacting semi-autonomous parts (usually called agents) where each part has a few simple individual behaviors which when aggregated with other parts can produce systems with emergent behaviors of high complexity*.

Inspired from the work from the Santa Fe Institute, Dodder and Dare (2000) adds more characteristics: *CASs are made of a network of many elements gathering information, learning and acting in parallel in an environment produced by the interactions of these agents*. *They are co-evolving with their environment and their states lye between order and anarchy at the edge-of-chaos. Order is always unfolding into transitional and new emergent phenomenon* (instead of pre-determined).

CASs tend to exist in many levels of organization in the sense that **elements at one level are the building blocks for elements at the next level**. An example is cells, which make up organisms, which in turn make up an ecosystem. Section 3.3.16 describes some structures of CASs.

Finally, CASs by their non-linear nature have a future that is hard to predict. Gell-Mann (1994) proposes some examples: *the origin of life on Earth; biological evolution; the behaviour of organisms in ecological systems; the mammalian immune system; learning and thinking in animals; the evolution of societies; and the behaviour of investors in financial markets*.

2.4 Proposed taxonomy for designating systems

Terms used to designated systems are often defined and used differently from one author to another in the scientific literature; actually, they are often defined in function of their domain of applicability. In order to avoid any confusion on semantic, it is worth defining the taxonomy that is used in this document for designating systems. Used terms are:

- **System**. This term will be used as a generic word for designating any kind of system (as defined in Section 2.1.1). As shown in Figure 6, this definition supports all other kinds of system.
- **Complex system**. Complex system **is a** system. Complex system will refer to any system that satisfies the conditions listed in Section 2.2.1.
- **Complex adaptive system**. Complex adaptive system (CAS) **is a** complex system and **it is also a**[24] system. The concept of CAS comes from the approach taken by the Santa Fe Institute (SFI, 2006) for studying complex systems (Section 2.3). Holland (1996) for instance suggests that immune systems, cities and ecosystems are examples of complex adaptive systems *that share certain properties that make it useful to consider them as instance of a class of phenomena*.
- **Complex system (definition issued from Per Bak's experiment)**. Using Per Bak's work (Bak, 1991; 1997) Lansing (2003) mentions a possible difference between complex systems and complex adaptive systems: *nonliving systems can also exhibit self-organizing properties that may take them to the edge of chaos (Bak & Chen 1991, Bak 1997). Bak's examples include phenomena like earthquakes, which are unlike biological systems because **no process of adaptation is involved**; these are known as complex systems rather than complex adaptive systems.* This definition or semantic is not used in this document.
- **Element of CAS or elements of complex systems**. In this document, the term **element** refers to the interacting parts forming complex adaptive systems. Elements are themselves systems.
- **Agent**. Elements of CAS can be called **agent** in the sense of economics by Holland (1996). Agents are systems. The term agent is not used in this document.

Taxonomy used in this document is illustrated in Figure 6; used terms are bolded and underlined.

[24] Using Unified Modeling Language terminology, one could say that CAS definition **inherits** from the complex system definition, which in turn **inherits** from system definition.

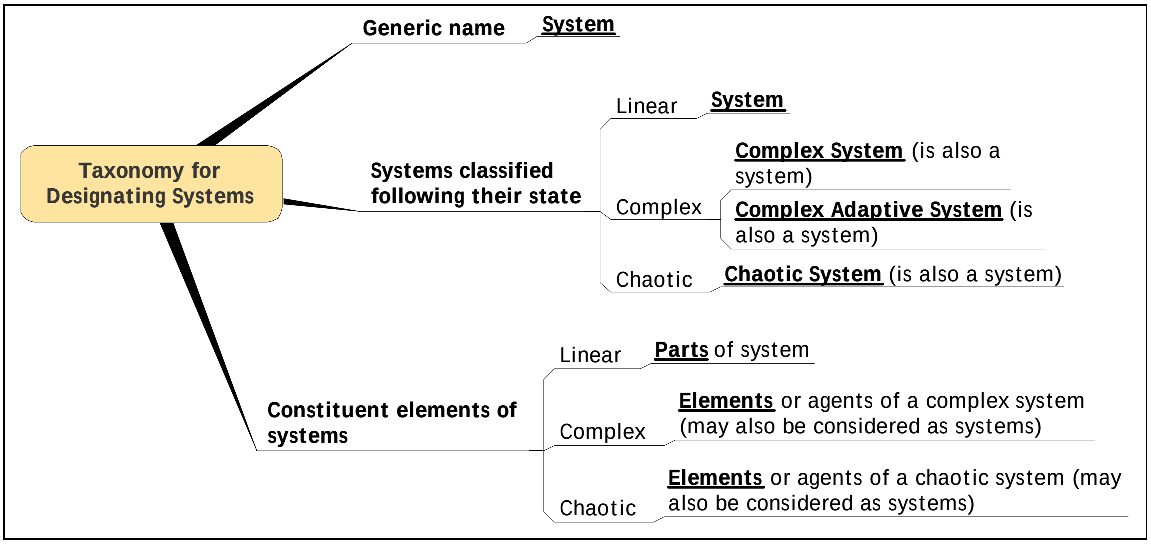


Figure 6 Proposed taxonomy for designating systems.

2.5 The States of systems

A scheme used to group systems is presented in this Section. It is based on their dynamical state and it is inspired from early works made in complexity theory by pioneers such as Dr. Kauffman from the Santa Fe Institute and Dr. Wolfram. Older historical works were not considered.

2.5.1 Kauffman's and Wolfram's historical experiments

Dr. Stuart A. Kauffman is a biologist associated with the Santa Fe Institute. Kauffman's research in the sixties aimed at understanding the evolution of biological systems at the level of genes. He studied questions such as: the existing relationship between the average connectedness of genes to the global ability of organisms to evolve. *Later, Kauffman observed that one can ask analogous questions about the connectedness of firms in an economy or species in an ecosystem*, and other kinds of CASs (Lansing, 2003).

Kauffman's experiments revealed dynamical regimes within which CASs may evolve. Lansing (2003) gives a short description of this experiment: *Imagine a collection of N Christmas tree lights. Each bulb has one of two possible states, on or off, and is wired up to K other bulbs. A simple rule tells each bulb what to do. For example, let K = 3, meaning that each bulb is wired to 3 other bulbs. From one moment to the next, each bulb decides whether to turn itself on or off in accordance with the state of these neighbors. A typical rule is majority wins, meaning that if 2 or 3 of its neighbors are on, the bulb will itself turn on; otherwise it will turn off. How will such a system behave when the electricity goes on?* Kauffman found that two patterns of behaviour are possible; ***ordered*** (linear) and ***disordered*** (chaotic).

Later on, a third regime was discovered by Langton (1990); a **transition point** between order and chaos. Behaviour at this point was different enough to be categorized as a third regime; they were among the firsts who called this transition point **edge-of-chaos** (Section 2.5.3 and Section 3.3.8). The three regimes are:

- ***(1) Frozen or periodic**. K is small (K = 1) in Kauffman's experiment. Some lights flip on and off a few times, but most of the array of lights will soon stop twinkling.*
- ***(2) Complex**. K is around 2. Complex patterns appear, in which twinkling islands of stability develop, changing shape at their borders.*
- ***(3) Chaotic**. K is large. Bulbs keep twinkling chaotically as they switch each other on and off.*

They found that networks of connected lights that are either frozen or chaotic cannot: 1. transmit information; and 2. they cannot adapt. But Langton discovered an interesting property of CASs operating at the edge-of-chaos; *a complex network—one that is near the "edge of chaos"—**can do both*** Langton (1990).

More recently, mathematician Dr. Stephen Wolfram carried out another type of experiment in an attempt to clarify the dynamic of complex behaviour in cellular automata[25] (Wolfram 2002). Lansing (2003) gives a description of this experiment: *A simple two-dimensional cellular automata begins with a line of different-colored cells on a grid or lattice. Each cell checks its own color and that of its immediate neighbors and decides on the basis of a rule whether to turn color in the next line of the grid. It is equivalent to a two-dimensional NK model where the K inputs are restricted to the cell's closest neighbors on the lattice.*

Wolfram identified four classes of behaviour out of his experiment; they are: Class 1: **Fixed**; Class 2: **Periodic**; Class 3: **Chaotic**; and Class 4: **Complex**. Langton (1990) then developed a measure (the lambda parameter is shown in Figure 7) which relates the nature of the governing rules (between cells) to the overall behaviour of the cellular automata. He identified that **Wolfram's Class 3 corresponded to Langton's complex regime**. It lies between the periodic and chaotic regimes, **at the edge-of-chaos**. Used methods to study complexity in cellular automata differ from those used to investigate NK models but it is interesting to see that each of them provides intuitive examples of similar complex behaviour at the transition point between linearity and chaos.

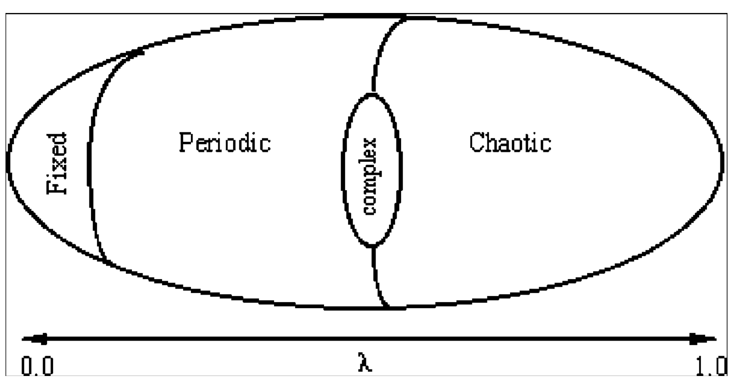


Figure 7 Langton's behaviour classification for cellular automata (Flake, 1998).

[25] The experiment is made of computer-based simulations of *a discrete dynamical system that is composed of an array of cells, each of which behaves like a finite-state automaton. All interactions are local, with the next state of a cell being a function of the current state of itself and its neighbors* (Flake, 1998).

Dynamical regimes are called s**tate domain** in this document. As shown in Figure 8, they are: **linear State Domain** (left of X axis); **complex state domain** (middle of X axis); and **chaotic state domain** (right of X axis).

Following lines present an overview of some differences between each domains (please refer to Figure 8).

- **State**. The states of systems evolving in linear state domain are pre-defined at their conception; they are finite and fully controlled. Systems evolving in the complex/chaotic State Domain have an/a increased/huge number of states; they are hard/impossible to predict at mid and long time scales.
- **Order**. Elements and structures of a system belonging to linear state domain are and remain ordered; they normally keep this state all along the operation of the system. Systems in the complex state domain are still ordered but these systems are able to self-organize their elements and then self-adapt in function of internal and external changes[26]. The order within systems belonging to chaotic state domain is hard to observe as it is continually and rapidly changing with respect of time.
- **Linearity**. The linearity of systems decreases rapidly from linear state domain to chaotic state domain. Systems evolving in the complex state domain already show strong non-linearity.
- **Input** versus **output**. Small perturbations of inputs to a system will have different effects depending on the state domain the system evolves. Linear systems will not be affected by slight variations of their inputs. At worst, they will stop working at the moment input variations exceed pre-defined specifications. The situation is different for systems belonging to complex and chaotic state domains. Small perturbations of inputs to a complex/chaotic system will show different/completely different behaviours. **The more the system evolves near the chaotic state domain the more it becomes sensible to perturbations**.
- **Control**. If linear systems are relatively easy to control, complex and chaotic systems are not because of their complex nature. Changing only one element within a complex system may for instance induce hard to predict global behaviour at mid and long terms. Chaotic systems are impossible to control, they are completely unpredictable.
- **Determinism**: as mentioned in the definition of system and complex systems, all systems that are studied in this work behave deterministically; random component are not considered[27].

[26] Possibly by changing their composition and/or structures.
[27] Actually, adding randomness does not help understand complexity aspects or complex phenomena.

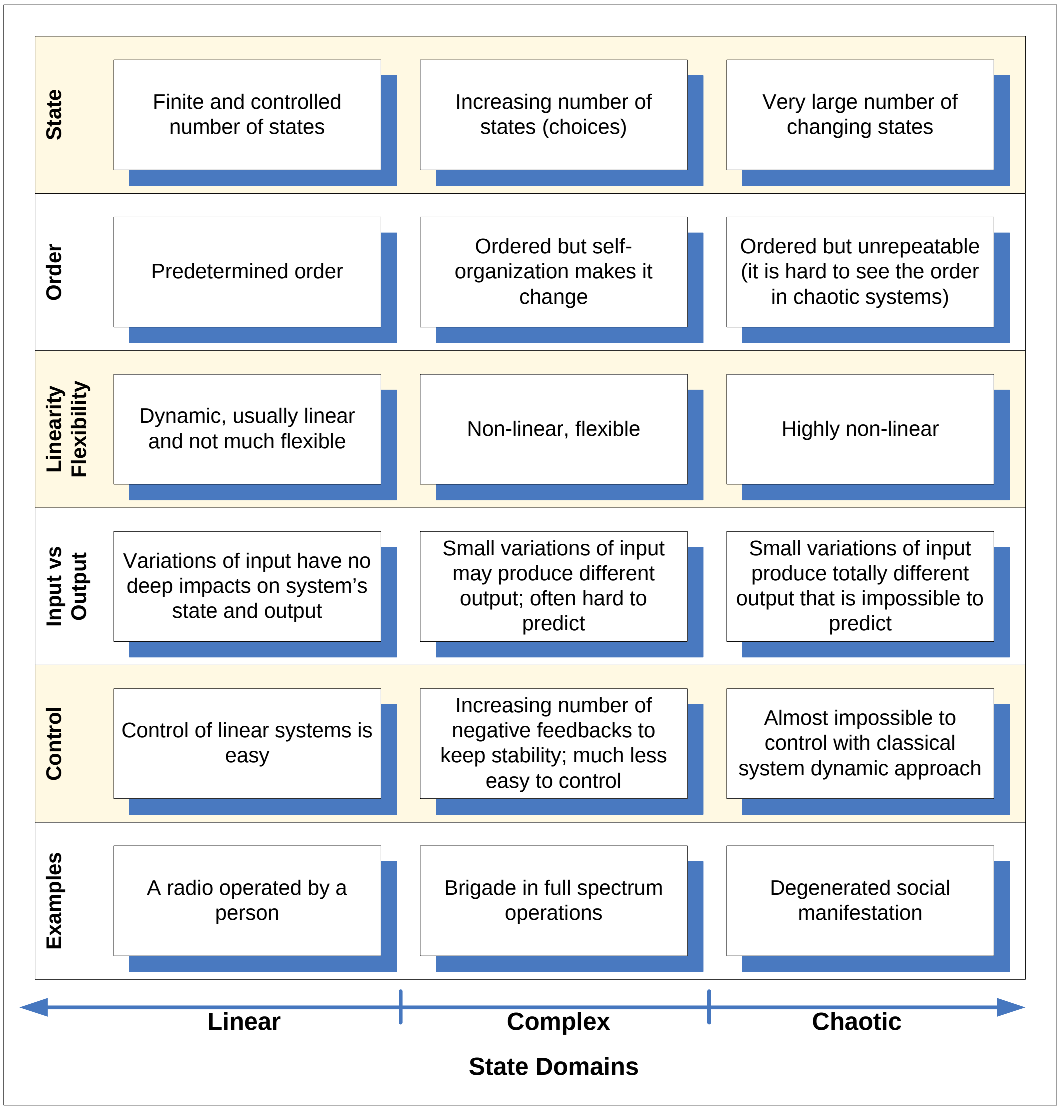


Figure 8 The Three state domains.

The following Sections give an overview of each state domain. More in depth descriptions of properties and phenomena related to complex and chaotic state domains are given in Chapter 3.

2.5.2 Linear state domain

A working civilian radio is an example of linear system when it is considered as a whole. It is designed to work and being managed in pre-determined number of ways[28], which can hardly be changed. Its elements always behave and work approximately in the same ways. It has a limited number of states and its elements are rigidly structured and have predefined roles. Linear systems behaviours are easy to predict at all time scales. For instance, raising the volume of the radio in

[28] They are usually defined and described in the system's specifications.

any proportion will always raise the number of decibels in the same proportion. They do not offer a great amount of flexibility to different contexts and they are not capable of self-organization and self-adaptation.

2.5.2.1 Approaches for studying linear systems

The theory and approaches that are often used to study and control linear systems originate from the general theory of systems. Broadly speaking, reductionism for instance consists in: 1. decomposing the system under study into its constituent elements; 2. studying each element separately and considering links and interrelationships between elements as pre-determined and fixed; and finally 3. gathering back of results from step 2 into a whole. This methodology is appropriately suited for linear systems because of their pre-fixed functions, composition, structure and interactions; elements are assembled linearly.

2.5.3 Complex state domain

Shelter (2002) calls the complex state domain a *zone of creative adaptability*. It is at the edge-of-chaos that emergence happens. It is specifically at this transition point that CASs are more robust, able to adapt, tolerant to faults, scalable, and flexible. Freniere et al (2003) give some examples of systems evolving in this domain: *between the extremes of complete linear simplicity and complete chaotic simplicity lies a wide range of complex systems, including those containing most targets of military significance. Examples include electrical distribution grids, transportation networks, communications architectures, command and control organizations, naval missile exchanges, and ground combat.*

2.5.3.1 Description

A shift of paradigm happens when systems get higher levels of complexity. This happens when: their number of elements is raised; when these elements have more choices regarding the actions they can take; when elements become more able to communicate and collaborate in a more intricate manner; when elements become able to dynamically self-organize in function of the environment; etc. This shift of state brings systems into the complex state domain; near or at the edge-of-chaos.

New global properties or phenomena (such as emergence) arise from CASs operating in this domain; they are hard to predict. These complex phenomena originate from intricate interactions between elements of CASs. For instance, the brain's consciousness is an emergent phenomenon. It comes from concurrent intricate interactions of a high number of brain cells. *Global properties result from the aggregate behaviour of individuals* (Chan, 2001).

Most of the time, there is not an operational central control or planning that dictates elements actions they must take; the control, intelligence and decision making tend to be distributed throughout the system. For instance, there *is no cell within a developing embryo, nor a master neuron in the brain. The overall behavior observed in the economy is a result of the countless decisions made by millions of individual people. Any coherent behavior in a system arises from competition and cooperation among the agents themselves* (Chan, 2001).

Near the edge-of-chaos, order often results from non-linear feedback interactions between elements of CASs; each element goes about its own business[29]. Ilya Prigogine's work on dissipative structures in 1977 showed for instance that the second law of thermodynamics - *systems tend toward disorder* - was not true for all systems. In another work, Kauffman showed that *it is possible for the order of new survival strategies to emerge from disorder through a process of spontaneous self-organization* (Chan. 2001).

Complex systems having their state lying within the complex state domain are showing enhanced robustness, adaptability, fault-tolerance, scalability, concurrency and flexibility. The price for such qualities is: low predictability, difficulty of control, harder engineering and design, possible accidents and errors (Fromm, 2005b) and maybe performance (Section 3.4; Figure 28).

2.5.3.2 Complexity theory

In the last two decades, scientists have come to the evidence that the world is made of complex, dynamical and non-linear systems that cannot be understood and resolved anymore through linear approaches like reductionism (Edmonds, 1999; Calhoun, 2004). These non-linear systems are showing behaviours and types of orders that are hard to predict, even when they are governed by simple rules. It can be seen in the scientific literature that the study of these systems is becoming the foundation of an entirely new conception of science: complexity theory[30] (Edmonds, 1999) or the complex systems science (Shetler, 2002).

According to De Wolf and Holvoet (2005), there are actually four central schools of research that influence the way complex behaviour of complex systems is studied:

- **(1) Complex adaptive systems theory (**Santa Fe Institute; SFI, 2006). Some important contributors to the institute are: George Cowan (founder); Murray Gell-Mann (winner of the Nobel Prize in physics); Stuart Kauffman; John Holland; and Kenneth Arrow (a Nobel laureate in economics). SFI’s members sought to pursue a common theoretical framework for complexity and a means of understanding the spontaneous, self-organizing dynamic of the world (Dodder and Dare, 2000). In this approach, complex systems are seen as having similarities that can be studied and exploited in order to ease the finding of underlying principles of a unified complexity theory (Holland, 1996). People at the SFI often call complex systems complex adaptive systems (CAS). *The **CAS movement** appears to be predominantly American, as opposed to the European “natural science” tradition in the area of cybernetics and systems. CAS is distinguished by the extensive use of computer simulations as a research tool, and an emphasis on systems, such as markets or ecologies, which are less integrated or “organized” than the ones studied by the older tradition* (Chan, 2001). Complexity theory has thus forged bonds between researchers from across the spectrum of disciplines in natural and social sciences, military and in engineering. For instance, Beech (2004) states that: *alternative theories that bring into focus networks and dynamic systems may help inform a US strategy to defeat global terrorism.* It is shown by this author (and other such as Marion and Uhl-Bien, 2002) that terrorist groups show strong evidences that they can be considered as CASs. They refer to *alternative theories*

[29] Elements have a relative autonomy and independence but also shared common interests.
[30] The reader is invited to see Waldrop (1993) or Lewin (1993) for an early popular introduction of Complexity Theory.

(complexity theory) as means to address features of terrorism problematic. The SFI approach will be used all along this document.

- **(2) Nonlinear dynamical systems theory and chaos theory**. This school promulgates the central concept of attractors. *One kind of attractor is the so called strange attractor that the philosopher of science David Newman classifies as an authentically emergent phenomenon* (Newman, 1996).
- **(3) The Synergetics school**. This school initiated the study of emergence in complex systems. *They describe the idea of an order parameter that influences which macro-level coherent phenomena a system exhibits* (Haken, 1981).
- **(4) Far-from-equilibrium thermodynamics**. This school was introduced by Ilya Prigogine. *It refers to emergent phenomena as dissipative structures arising at far-from-equilibrium conditions* (Nicolis, 1989).

The New England Complex Systems Institute (NECSI) is another important organization dedicated to advancing the study of complex systems. *NECSI joins faculty of New England academic institutions in an effort to collaborate "outside of institutional and departmental boundaries"* (Dodder and Dare, 2000).

CASs are fundamentally different from the kinds of systems with which science and engineering have traditionally dealt. Complexity theory states that *critically interacting components self-organize to form potentially evolving structures exhibiting a hierarchy of emergent system properties* (CALRESCO, 2006). It views CASs behaviour and actions as the result of intricate interrelationships between many elements and it refers to these interrelationships or systems as **complex**, *because it is impossible to fully understand these systems by reducing them to an examination of their constituent parts* (Beech, 2003).

Complexity theory is imposing thus a shift from the **traditional analytical thinking** (where variables are independent) to a more **holistic thinking** (where variables are interdependent). Complex systems must be considered and studied as wholes, *rejecting the traditional emphasis on simplification and reduction as inadequate techniques on which to base this sort of scientific work. Such techniques, whilst valuable in investigation and data collection, fail in their application at system level due to the inherent nonlinearity of strongly interconnected systems - the causes and effects are not separate and the whole is not the sum of the parts.* This does not mean that reductionism should be rejected; it means that it should be integrated into a more holistic approach. CASs' related phenomena must be characterised by holistic features (Muller, 1997; Holt, 2000).

Holism involves the concurrent consideration of the whole and the parts. Another approach that may be used is the **middle-out approach**. Middle-out is a combination of top-down and bottom-up approaches. It is useful for studying complex phenomena that involve both top-down and bottom-up cause/effect relationships. Emergence is an example of such complex phenomena. As it will be shown in Chapter 3, it originates from intricate interactions between elements (at a lower level) and it manifests at a higher level, at the level of CAS (bottom-up cause/effect). In turn, emergent phenomena (at the level of CAS) influence back elements located at a lower level (top-down cause/effect). The middle-out approach considers both bottom-up and top-down cause/effect interrelationships.

The study of complex systems often involves interdisciplinary works. This is particularly evident at the Santa Fe Institute (SFI, 2006). As mentioned earlier, their approach takes into account the fact that there exist commonalities between CASs pertaining to any domains or fields. Interdisciplinary works will favour the discovery of principles underlying a unified complexity theory that will be valid for all CASs (Holland, 1996).

Finally, it is worth mentioning that the use of concepts of complexity theory may trigger side effects. For instance, Shetler (2002) warns that *there is still considerable difficulty in identifying the right level at which to develop more precise theoretical generalizations with well-specified domains of applicability* (Cohen, Riolo, and Axelrod, 1998). The utilization of concepts of this theory may also pose other problems; resistance may be encountered while trying to apply the new theory at enterprise level. Shetler has made a literature review on this subject. She mentions that: *Members may be wary of unaccustomed CSS* (complex system sciences) *concepts such as self-organization that appear to make managerial control superfluous (Morgan, 1997); planners may object to seeing outcomes as unpredictable (Brown, and Eisenhardt, 1998); colleagues may feel uneasy that nonlinear outcomes make it hard to trace and assign credit or blame for performance (Holland, 1996, 1998) thus undermining expectations for mutual accountability (Axelrod and Cohen, 1999; Tetlock, 1985); leaders may think that edge of chaos/far-from-equilibrium operation threatens structural stability (Byeon, 1999), or fear that ongoing complex adaptive alterations are precarious (Brown and Eisenhardt, 1998), and so on. Such resistance to displacement, arising from embedded structural customs, shared beliefs, and collective habits can be seen in CSS terms of system* ***attractors*** (Section 2.6.7). This last citation suggests (like Senge, 1994) that organizations can be considered and studied as complex systems.

2.5.4 Chaotic state domain

Systems having their state lying in chaotic state domain are showing very high level of complexity; they have passed the edge-of-chaos threshold (Langton, 1990).

A degenerated social manifestation can be seen as an example of chaotic system, the weather is another example. The order within such systems is present but it evolves or changes constantly and rapidly. Chaotic systems are highly non-linear, and their behaviour cannot be predicted at mid and long time scales. Actually, the analysis of data representing chaotic behaviour shows that *they pass all tests of randomness* (Williams, 2001). Another important characteristic is that small variations of inputs to these systems lead to completely different outcomes or behaviour.

2.5.4.1 Description

Chaotic systems are briefly described in this Section by listing their main characteristics. The following lines come from Williams (2001) and CALRESCO (2006). Note that some of the characteristics refer to the logistic equation experiment (Section 2.6.9.1). The reader may refer to Williams' (2001) references for a more detailed description of chaotic systems.

- *Chaos results from a deterministic process.*
- *It happens only in nonlinear systems.*
- *The motion or pattern for the most part looks disorganized and erratic, although sustained. In fact, it can usually pass all statistical tests for randomness.*

- *It happens in feedback systems – systems in which past events affect today's events, and today's events affect the future.*
- *Systems governed by physical laws of deterministic equations can produce regular results under some conditions, but irregular or disorderly results under others.*
- *It can result from relatively simple systems. With discrete time, chaos can take place in a system that has only one variable. With continuous time, it can happen in systems with as few as three variables.*
- *For given conditions or control parameters, it's entirely self-generated. In other words, changes in other (i.e. external) variables or parameters aren't necessary.*
- *It isn't the result of data inaccuracies, such as sampling error or measurements error. Any particular value of* x_t *(right or wrong), as long as the control parameter is within an appropriate range, can lead to chaos* (Section 2.6.9.1).
- *In spite of its disjointed appearance, it includes one or more types of order or structure. Period-doubling (bifurcation) followed by irregular fluctuations in some case indicates that those fluctuations are chaotic* (Section 2.6.9.1).
- *The ranges of the variables have finite bounds. The bounds restrict the attractor to a certain finite region in phase space* (Section 2.6.6).
- *Details of the chaotic behaviour are hypersensitive to changes in initial conditions (minor changes in the starting values of the variables)* (Section 2.6.9.1).
- *A random-like or even chaotic evolution doesn't have to be the result of a random operation. Instead it can arise by design.*
- *Chaotic data is both random and deterministic.*
- *Forecasts of long-term behaviour are meaningless. The reasons are sensitivity to initial conditions and the impossibility of measuring a variable to infinite accuracy. Description of chaos as random-like behaviour is mostly justified. Where reliable long-term predictions are impossible, a statistical approach may be the only viable alternative.*
- *Short-term predictions, however, can be relatively accurate.*
- *The Fourier spectrum is "broad" (mostly uncorrelated noise) but with some periodicities sticking up here and there.*
- *Information about initial conditions is irretrievably lost. In the mathematician's jargon, the equation is "noninvertible". In other words, we can't determine a chaotic system's prior history.*
- *The phase space trajectory may have fractal properties.*
- *As a control parameter increases systematically, in initially non-chaotic system follows one of a select few typical scenarios, called routes, to chaos* (Section 2.6.9.1).
- *The transition to chaos is preceded by a very high number of bifurcations. These bifurcations preceding the transition to chaos are characterized by the Feigenbaum number.*
- *It is not possible yet to identify, in advance, the particular path that a dynamical process will follow in going to chaos.*

2.5.4.2 Chaos theory

A possible relation between complexity theory and chaos theory is given by Shetler (2002). This author unifies both theories into a unique *complex systems science (CSS)*. The CSS makes use of both the complex and chaotic models into **one science** because they both complement each other. *The CAS model epitomizes the integration of nonlinearity and emergence (Kauffman, 1993, 1995; Holland, 1996, 1998) and offers insights into organizational innovation and change (Poole, Van de Ven, Dooley, and Holmes (2000); Weick and Quinn (1999); Van de Ven and Poole (1995). Chaos theory models that are self-organized around constructs such as attractors offer useful insights into collective interactions as the generating mechanisms of self-organizing networks (e.g., Contractor, Whitbred, Fonti, Hyatt, O'Keefe, and Jones, 1998).*

Approaches and methods used for studying complex systems may be used with other more specialized means to study chaotic systems. The review of chaotic systems' characteristics suggests that additional means are needed for being able to address the random aspect of chaotic systems. Probability analysis, Fourier analysis, extended phase space and attractor analysis are few examples proposed by Williams (2001).

2.6 Basic toolkit for studying complex systems

This Section introduces some notions, tools and other means that appear to be useful for the study of CASs. They are presented as a **toolkit** in this document.

2.6.1 The Concepts of level, scale and resolution

Concepts of level, scale and resolution are often used in complexity theory to study complex phenomena and systems (examples of studies involving the multi-scale complex systems analysis are (Section 2.6.3): Bar-Yam, 2003d; 2004b; and 2004d).

A brigade in operation is proposed as an example of complex system (Figure 9). The Brigade is made of battalions, each of which in turn is made of a number of companies, and so on. This system may be studied using different perspectives or points of view; **level**, **scale** and **resolution**. Each of them is described in the following lines.

- **The level**[31]. One may study the brigade at a specific level; for instance the **company level** as shown in Figure 9. In this example, companies will then be the **basic objects** used to study specific features pertaining to the brigade. The concept of levels allows for instance the study of complex phenomena, which are level dependent. Emergence is an example of these phenomena; using Figure 9, emergence may manifest at one specific level (company).
- **The scale**. The scale complements the level. It allows the possibility to specify a number of supplementary levels (up and/or down to the level) in order to study level dependant complex phenomena. In the example shown in Figure 9, the scale covers one level down to the company level; emergence at the level of company results from intricate interactions of elements at the immediate lower level (platoon).

[31] More detailed description can be found in Bahill et all (2005).

- **The resolution**. The resolution allows the specification of the **thinness of details** that is needed to appropriately study complex systems and phenomena at a specific level, using a specific scale. The resolution is different from the scale in that a study involving a fixed level and resolution may need a resolution that would take into account some essential details lying outside the range specified by the scale. In Figure 9 for instance, the **soldier resolution** was chosen to study the brigade at the level of company. The study of emergence that manifests at the level of company from interactions of elements at the level of platoon may involve the consideration of some details that have the thinness of soldier level.

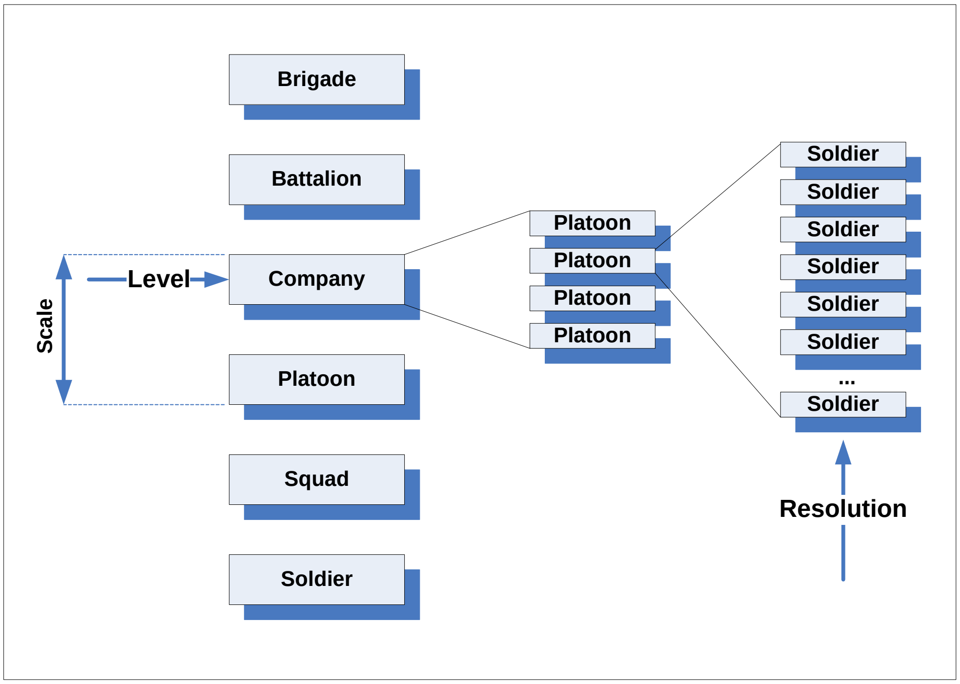


Figure 9 The Concepts of level, scale and resolution.

2.6.2 Time-scales within complex systems

Another important characteristic of CASs is noted in this short Section. The dynamical regimes at two adjacent levels of one CAS may be different; they may **operate at different time scales**.

Holland (1996) gives an example of this in his Two-tiered models and in his CCC (Civilian conservation corps) example. Activities at a specific level of a CAS that produce complex behaviour at the adjacent higher level show a faster dynamic or dynamical regime than the one found at the higher level, which show slower dynamic.

2.6.3 Multi-scale complex systems analysis (MCSA) and complexity profile

The concept of MCSA was introduced and used by Dr. Bar-Yam to study military complex systems in their environments and contexts (Bar-Yam, 2003d; 2004; 2004b; 2004c; 2005 among others). MCSA *provides a formal framework for understanding the interplay of scale and complexity in complex systems and their capabilities in the face of challenges*. It is based upon the concept of **complexity profile**, *which characterises the dependence of complexity on scale*.

A specific scale of a CAS is defined by Bar-Yam as the number of consecutive levels (Section 2.6.1) within which a number of elements are acting together *in a strictly coordinated way*. An observer studying elements of a **scale** might not be able to see or study elements pertaining to other scales due to observational limitations. Consecutive scales may show different degrees of complexity at each scale; the whole spectrum is called the **complexity profile** of the CAS. *This dependence of complexity as a function of scale reveals the capabilities of the* (military) *force at each scale of a potential encounter, from the smallest to the largest*.

As an example, for a military force in operation the complexity profile roughly corresponds to the number of elements at each level of command[32] and it is function of the degree of independence of elements in face of their immediate higher-level hierarchical commander[33]. The more this independence is high at a specific level, the more there are choices for the finding of solution to complex problems and thus, the more the complexity of the CAS at this level is high[34].

Dr. Bar-Yam uses these concepts to show that *the scale and complexity necessary to overcome a particular enemy force is dictated by the scale dependent structure of the enemy force itself (the degree to which its forces are aggregated), and the scale dependent structure of constraints in the battle space (terrain, etc.), as well as the scale dependent structure of objectives, including objective constraints (political, etc.).*

2.6.4 Power law distribution

Numerous natural and man-made phenomena are distributed according to a power-law distribution. A power-law implies that small occurrences are common, whereas large occurrences are rare; it applies to CASs when **large is rare** and **small is common**. The distribution of individual wealth is a good example where there are a very few rich men and a lots of poor people. A familiar way to think about power laws is the 80/20 rule: 80% of the wealth is controlled by 20% of the population (Wikipedia, 2006).

Per Pak's sandpile experiment is another example showing the occurrence of the power law for complex systems (Bak and Chen, 1991; Bak, 1997). Lansing (2003) describes the experiment: *If you patiently trickle grains of sand onto a flat surface, at first the sand will simply pile up; but eventually the pile will reach a critical state. At that point, Bak found that the size of the avalanches triggered by dropping another grain of sand follows a power law distribution: **The size of avalanches is inversely proportional to their frequency***. The power law states that there will be many little avalanches, and few large ones. Such sandpile's regime lies in the complex State Domain; at the edge-of-chaos.

---

[32] In this example, scales of the complexity profile correspond to levels of command and control.
[33] For instance: *how independent the individuals are within fire teams, how independent fire teams are within squads, how independent squads are within companies and how independent companies are within battalions*.
[34] This is related to Holland's concept of building blocks that may be recombined in a different manner in order to find novelty or new solutions (Sections 3.3.3).

Newman (2005) reviewed some of the empirical evidence for the existence of power-law forms and theories to explain them. He gives a more formal definition of the power law: *When the probability of measuring a particular value of some quantity varies inversely as a power of that value, the quantity is said to follow a power law, also known variously as Zipf's law or the Pareto distribution. Power laws appear widely in physics, biology, earth and planetary sciences, economics and finance, computer science, demography and the social sciences.* Examples are: the distributions of cities varies inversely of their size, the occurrence of earthquakes varies inversely of their magnitude, etc. The origin of power-law behaviour has been a topic of debate in the scientific community for more than a century.

2.6.5 The Law of requisite variety

Bar-Yam's (2003d) description of this law is integrally reproduced in this Section. *The Law of Requisite Variety provides a quantitative expression relating the complexity of the environment, the complexity of the system and the likelihood of success of the system in performing a particular function for which it is designed. It states:* ***The larger the variety of actions available to a control system, the larger the variety of perturbations it is able to compensate*** *[18]* ([35]). *Quantitatively, it specifies that the probability of success, P, of a well adapted system in the context of its environment is decreased by the complexity of the environment C(e) and increased by the complexity of its actions C(a) according to the expression:*

$-Log2(P) < C(e) – C(a).$

Qualitatively, this theorem specifies the conditions in which success is possible: a matching between the environmental complexity and the system complexity, where success implies regulation of the impact of the environment on the system. The implications of this theorem are widespread in relating the complexity of desired function to the complexity of the system that can succeed in the desired function. This is relevant to discussions of the limitations of specific engineered control system structures, to the limitations of human beings and of human organizational structures.

The Requisite Law Variety is related to Holland's concepts of building blocks that can be recombined in different ways in order to find new solutions (Section 3.3.3).

2.6.6 Phase space – The Playing field

One way to ease the understanding of dynamic of complex systems is to graph the chronological evolution of their features or properties. The first kind of graph that can be used is the two-dimensional time series. Time series consist of plotting the values of a variable (on the Y axis) in function of evolving time (the X axis). Depending on the length of the time plotted, they may produce wide graphs (long X axis); they may be not practical for analysing huge sets of data.

Another (more practical) way of studying behaviour of complex systems is to use **phase space graphs** (Williams, 2001). These graphs complement time series by providing a different view for

[35] Ashby, W. R., 1957. An Introduction to Cybernetics. Chapman and Hall, London.

understanding the evolution of systems. As it will be shown in the following sections, they also provide additional means for studying features and phenomena of CASs.

Phase space graphs may take two forms; **standard phase space** and **pseudo-phase space**.

2.6.6.1 Standard phase space

Williams (2001) defines standard phase space as: *an abstract space in which coordinates represent the variables needed to specify the state of a dynamical system as a particular time. On a graph, a plotted point neatly and compactly defines the system's condition for some measuring occasions, as indicated by the point's coordinates (values of the variables)*. He gives a simple illustration of a standard phase space by plotting a baby's height (on the Y axis) against its weight measured **at the same time** (on the X axis) at different times. This plot shows patterns that are function of the dynamical system (the growth of the baby). The linking of these chronological points shows temporal evolution of the system; the line is called **trajectory**. Trajectories are called **orbits** when they are closed. An important fact about trajectories is that each of their points is partly a result of the preceding point; the feedback (Section 3.3.9). This is an important feature for the study of CASs; their state at an instant (t) depends on past states (t-1, t-2, etc)[36]. *The trajectory is a neat, concise geometric picture that describes part of the system's history* (Williams, 2001). True random dynamical systems would show trajectories that would randomly cover the whole phase space (*erratic-like trajectories*; Williams, 2001).

Trajectories, trajectory pattern and their boundaries in the phase space depend on initial conditions, environment, contexts and other conditions. Williams (2001) mentions on this: *The phase space is a world that shows the trajectory and its development. Depending on various factors, different trajectories can evolve for the same system. The phase space plot and such a family of trajectories together are a phase space portrait, phase portrait, or phase diagram. The phase space for any given system isn't limitless. On the contrary, it has rigid boundaries. The minimum and the maximum possible values of each variable define the boundaries*.

The baby's example above has produced a two-dimensional phase space but other CASs showing high degrees of complexity may involve graphs having many dimensions. For instance, CASs may involve a high number of variables that should be concurrently taken into account in the phase space. This would produce phase spaces having a number of dimensions higher than three; making them hard to plot on a regular two-dimensional paper. In this case, one option consists in restricting phase space analysis to three dimensional graphs using different combinations of variables. The analysis of such n-dimensional patterns in phase space can also be made by mathematical methods (Williams, 2001).

2.6.6.2 Pseudo (lagged) phase space

One common term in literature related to complexity and chaos is the one of **map**. *In chaos, a map or function is an equation that specifies how a dynamical system evolves forward in time. It turns one number into another by specifying how x, usually (but not always) via a discrete step,*

[36] Recall one particularity of CASs: *It happens in feedback systems – systems in which past events affect today's events, and today's events affect the future*. Dynamical complex systems' *state at time (t) is dependent on the previous state, or that at time (t-1)* (Shetler, 2002).

goes to a new x (Williams, 2001). Actually, a map tells us how to pass from a value of a variable x at a given instant to its next value x at the next instant (one time step later). Pseudo phase spaces allow this.

Taking the baby's example of preceding Section, one might be interested by the baby's weight rates of change (or map) in function of evolving time. It is possible to study this system by using a **one-dimensional pseudo phase space**. Points (x, y) on this two-dimensional diagram would be the coordinates of baby's weights at two different times; the X axis would be weights at times **t** while Y axis would be weights at times **t + lag**. An example of such map is given in Figure 10. Each point on this diagram has coordinate of: (Weight(t), Weight(t + lag)); each plotted point represents sequential measurements rather than a concurrent measurement. *Hence, the graphical space for a one-dimensional map is really a pseudo phase space. Pseudo phase space is an imaginary graphical space in which the axes represent values of just one physical feature, taken at different times* (Williams, 2001).

In that comparison, we call the group Weight(t) the **basis series** and the group Weight(t + lag) **values of the sub-series**. **Lag** is a constant interval in time. It specifies the rule or basis for defining the sub-series. A pseudo phase space graph such as the one shown in Figure 10 can have two or three axes or dimensions. As for standard phase spaces, it is possible to extend the idea to more than three dimensions.

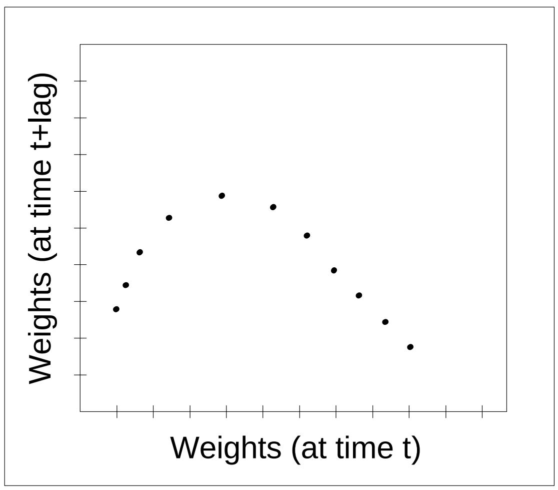


Figure 10 Pseudo phase space.

Standard and pseudo phase space diagrams are useful tools for studying CASs, no matter their state. Chaotic systems would for instance show random-like behaviours in time series diagrams while pseudo phase space diagrams would reveal hidden order in chaos (as it will be shown in Figure 23 of Section 3.3.7). Random systems would randomly cover the whole graph without any order because the states of a random system at two consecutive time steps are not related; this fact shows the main difference between random systems and chaotic systems.

The reader is invited to see Section 2.6.9.1 for a simple mathematical example involving the logistic equation. Interesting observations can be made when this simple system evolves from linear to complex, and then to chaotic state domains.

2.6.7 Attractors

Trajectories of complex systems in phase spaces show patterns that are dependent on factors like: initial conditions, the kinds and roles of CASs' elements, kinds of relationships between these elements, feedbacks, shared rules, values, beliefs and internal models, etc. Actually, these patterns have the tendency to converge toward specific areas in phase spaces with respect of time. These areas are called **attractors**.

The following lines list some characteristics of attractors.

- Systems are organized around attractors, which may be defined as: *A point to which a system tends to move, a goal, either deliberate or constrained by system parameters (laws)* (Shetler, 2002).
- These attractors *attract all trajectories emanating from some range of starting conditions* (Williams, 2001).
- *An attractor is a characterization of the long term behaviour of a dissipative dynamical system. Over long periods of time, the STATE SPACE* (or phase space) *of some dynamical systems will contract toward this region* (Flake, 1998).
- *Attractors determine those states that a dynamic system will tend to adopt over time. Depending on the system's initial conditions, it will proceed along a specific trajectory to the resulting stable state cycle. It is the presence of attractors in a system that enables its self-organization and orderly, rather than purely random, behaviour* (Calhoun, 2004).
- *Attractors are sometimes called basin of attraction* (Williams, 2001).
- Attractors *act like self-organizing magnets of behavior for agent interaction, analogous to the way that individual water molecules, collectively obeying the simple law of gravity, appear magically drawn to form a neat whirlpool circling down the bathtub drain* (Shetler, 2002).

A number of properties for trajectories and attractors have been proposed by Gharajedaghi (1999). They are:

- *Hidden in the apparent disorganization is a great deal of structure.* The order within CASs becomes apparent in phase spaces graphs (see for instance Figure 24 of Section 3.3.6).
- *The phase space trajectory may have fractal properties.*
- *The attractors of a system are uniquely determined by the state transition properties of the nodes* (their logic, rules, internal models, etc) *and the actual system interconnections.*
- *The range of the variables has finite bounds. The bounds restrict the attractor to a certain finite region in phase space.*
- *The ratio of the basin of attraction size to attractor size (called here the Self-Organizing Factor or SOF) varies from the size of the whole state space (totally ordered, point attractor) down to 1 (totally disordered, ergodic attractor).*
- *Single connectivity mutations can considerably alter the attractor structure of networks, allowing attractors to merge, split or change sequences. Basins of attraction are also altered and initial points may then flow to different attractors* (Section 2.6.8).

- *Single state mutations can move a system from one attractor to another within the system. The resultant behaviour can change between fixed, chaotic, periodic and complex in any combination of the available attractors and the effect can be predicted if the system details are fully known* (Section 2.6.8).

- Shetler (2002) adds about perturbation of CAS: *Systems* (CASs) *in equilibrium tend to return to the same state when perturbed, falling back into repetition of their accustomed patterns. This system propensity to return to its old habit is related by CSS theorists to resistance to change (e.g., Goldstein, 1994; Lewin and Regine, 2000).* (Resistance to change can be interpreted as potential (or gravitational) force surrounding a point attractor that keeps the CAS's trajectory within a specific basin of attraction.) *When perturbed beyond its ability to return to the original state, an equilibrium system will abruptly shift to a new equilibrium. Goldstein (1994) describes this stage-wise change behavior as analogous to the traditional organizational development concept of change, i.e., Kurt Lewin's model of force-field shift from one kind of equilibrium to another (Lewin, 1951).*

Williams (2001) describes attractors using the logistic equation experiment (Section 2.6.9.1). He adds:

- *If the initial condition* (x_min) *lies on the attractor, then the trajectory stays there forever (never leave the attractor);*

- *A trajectory* (of CAS in the chaotic state domain) *never gets completely end exactly all the way onto an attractor.*

Shetler (2002) gives a more concrete idea of the meaning of attractor by making direct links between this concept and social sciences systems (which are considered as CASs): *Talk of the attraction of values leads to the concept of culture* (…) *the dynamics of an attractor may be used a metaphor for the mechanism of spontaneous self-organization of collective behavior patterns in a social system. The attractor is primed by the same co-constructed beliefs, shared values, developed ideologies, learned stories, etc. that characterize culture formation (Lewin and Regine, 2000)*. The same author describes examples of sources of organizational attractors: *One way organizational attractors are generated is by a leader who creates attraction through communication with organization members.* It will be shown later in this document that shared rules, beliefs, values, internal models and ideologies within elements of CASs are attractor-like factors that contribute to create or influence specific patterns of behaviour of CASs. The reader may for instance refer to Marion and Uhl-Bien (2002) and Beech (2004) to see their importance for Al-Qaeda.

There exist only few forms or types of attractor. The number varies from one author to another but they can all be grouped in two broad classes: **non-chaotic attractors** and **chaotic attractors** (Williams, 2001). According to this author, non-chaotic attractors are:

- **Point attractors.** Static systems.

- **Periodic attractors.** Systems that are cycling between two extremes or limits at one frequency and that may correspond to Wolfram's class II of behaviour for cellular automata (Wolfram, 2002).

- **Torus attractors.** Multi-frequency systems.

For non-chaotic attractors, minor perturbations to CAS's generally do not have significant long term effects, *neighboring trajectories stay close to one another and their prediction are fairly meaningful and useful, in spite of errors or differences in starting conditions* (Williams, 2001).

Chaotic attractors are **strange attractors**[37]; a far-from-equilibrium system whose behaviour is chaotic, not random, but lies within boundaries or constraints. *Chaotic or strange attractors arise only after the onset of chaos. They take on many interesting and complex shapes in phase space. Unlike non-chaotic attractors, chaotic attractors' trajectories are highly sensitive to initial conditions and small perturbations will result in completely different outcomes* (Williams, 2001).

The following Sections introduce and briefly describe each type of attractor.

2.6.7.1 Point attractors

The simplest form of attractor is the point attractor. It is a single fixed point in phase space that represents the state of a system that comes to rest as time passes (Crutchfield, et al, 1986) or *that progress to a state where they no longer vary with time.* (…) *Once steady-state condition arrives, the point attractor is independent of time. It "stays fixed" and the system no longer evolves. In phase space, the system is static* (Williams, 2001). Examples of such systems involving gravity and friction are a bouncing ball that comes to rest, a pendulum and a marble rolling in a bowl and coming to rest at the bottom. The pendulum for instance will always reach the same point attractor no matter where it began swinging.

More concretely, Gharajedaghi (1999) mentions that *point attractors represent the behaviour of social beings in pursuit of their natural instincts – fears, love, hate, desire to share, or self-interest*. With the point attractor in play, a person for instance may invariably be drawn to one particular activity or person, or be repelled from another. This is similar to the positive or negative poles of electromagnetic energy.

Depending of the systems' dynamic, trajectories reach attractor points in three different ways; 1- directly to the point attractor; 2- in an alternating fashion, with damped oscillations converging to the point; and 3- spirally, with the center of the spiral corresponding to the point.

2.6.7.2 Periodic attractors or limit cycle

Periodic attractors represent the next simplest type of attractors. Systems showing periodic attractors in their phase spaces have behaviour that oscillate periodically and continuously between two attractor values. The duration of the cycle is its periodicity and both attractors act as limits for any trajectory that originate from the basin. Periodic attractors are often called **limit cycle**.

A periodic attractor is stable, it resists to changes or perturbations that are under acceptable limits[38]. *Theoretically, repetitions or oscillations can go on forever but in real world, they usually slow down and drop after a number of cycles, unless the system receives energy from some*

[37] So far, only one type of chaotic attractor has been identified. It is also important to say that there is not any universal agreement on a definition of a chaotic attractor.
[38] Huge perturbations may of course change the dynamics of the system, modifying its attractor.

source. Something else is important: even if we nudge or otherwise perturb the pendulum (the system)*, it tends to return to its standard cycle* (Williams, 2001).

Periodic attractors can be observed in the predator/prey system[39]; Figure 11 shows the corresponding trajectory in the phase space. The respective predator/prey populations cycle up and down in relation to each other.

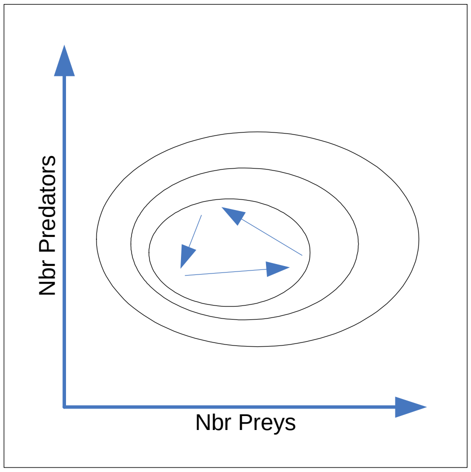


Figure 11 Phase space for predator–prey relationships (adapted from Flake, 1998).

Starting at some point in the cycle, a huge number of preys will eventually induce an increased number of predators because the latter have plenty of food (the former) to reproduce. After a certain amount of time, the increased number of predators will cause a diminution of the number of preys because more predators are hunting the same biomass at the same time (first limit; many predators and few preys).

As this cycle evolves over time, the number of predators will lower because there is not enough preys to feed all predators so they can procreate. As the number of predators lowers, the number of preys will then be raised because preys will be less hunted by predators and more preys will have more time to procreate (second limit; few predators and a lot of preys).

This cycle goes on forever unless a drastic external perturbation (energy) modifies the system beyond its limits, and changes its dynamic. In this case, if the external energy (for instance preys' food) would be missing, the whole cycle would quickly disappear.

Periodic attractor results thus in oscillations between two or more states (or attractors). More generally, it is the *pursuit of seemingly opposite but complementary tendencies: stability and change, security and freedom, and, in general, differentiation and integration* (Gharajedaghi, 1999).

[39] Vito Volterra and Alfred J. Lotka independently noticed in the early nineties the cyclic nature of population dynamics. This system is called the Lotka-Voltera system (Flake, 1998).

The example above deals with two variables or dimensions (predators and preys) in isolation[40]. In reality, other systems are affecting the predator-prey system. They should be taken into account altogether in order to get the whole complex picture of interacting systems. Possible examples of these may be: 1- the temperature-salinity system that affects both prey and predator systems; 2- the prey's food system; 3- human fishing activities that are affecting predators; etc. This CAS will thus have multi-dimensional phase spaces involving many interdependent variables, yielding multi-dimensional periodic attractors.

A concluding remark regarding the predictability CASs' behaviour is given by (Calhoun, 2004): *A network made up of a large number of interacting agents* (or elements) *can contain many attractors, but if they are point or limit cycle attractors they will lead to behavior that is simple and predictable, and they will cause the network to achieve an orderly state.*

2.6.7.3 Torus attractors

Torus attractor is a phase space shape that concurrently accommodates trajectories for more than one complex system at the same time. It consists of combining periodic attractors from different systems into one composite shape; the **torus** (Figure 12).

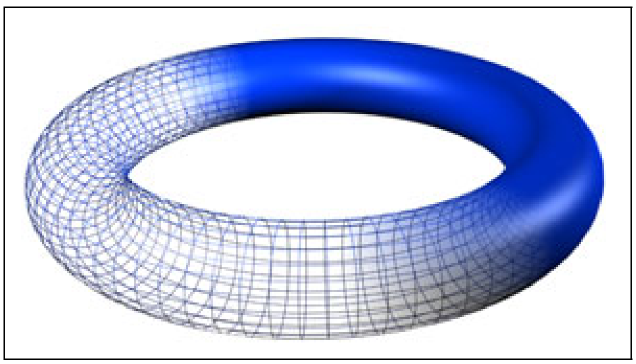


Figure 12 The Torus Attractor (From Wikipedia, 2006).

The torus is an object in the phase space that looks like the inner tube of a tire, or a doughnut. Mathematically the Torus is made up of a spiralling circle on many planes which may, or may not, eventually hook up with itself after completing one or more full revolutions (http://www.fractalwisdom.com). It is the result of the plotting of **two different but interrelated systems on the same phase space graph**; it brings them together into one compound system of four variables. Any single point on the torus is now the unique combination of four variables.

Such attractors allow for instance the representation of systems' states that show complex direct and indirect interactions between a numbers of interdependent species of an ecosystem. *All the action – all the dynamics – take place on the surface or shell of the doughnut*[41] (Williams, 2001). Examples of interdependent variables (Section 2.6.7.2) would be: number of predators, number of

[40] The isolation of a system from its environment is built upon arbitrary boundaries that are often subjective to human perspectives (Section 2.1.3).

[41] The reader is invited to refer to Williams (2001) for a description of the construction of such attractor involving two different systems.

preys, water temperature, and water salinity. The Torus attractor can be associated with organized complexity that repeats itself. *Torus attractors exemplify the behaviour of open systems. These systems are guided by image (DNA) of what they ought to be, as growth patterns of biological systems* (Gharajedaghi, 1999). Responsive or goal-seeking systems (Section 2.1.4) show behaviour with torus attractors in phase spaces.

2.6.7.4 Strange attractors

As mentioned by Williams (2001), some authors like Gregobi et al. (1984) and others make a clear distinction between strange and chaotic attractors. In this document, **chaotic attractor** and **strange attractor** are used synonymously.

Systems that are evolving within the chaotic State Domain show trajectories that are strange; they do not behave like other kinds of attractors. They take on many interesting and complex shapes in phase space. Williams (2001) lists a number of features that strange attractors have in common with non-chaotic attractors. They are:

- *It's still the set of points (but in this case an infinite number of points) that the system settles down to in phase space.*
- *It occupies only certain zones (and is therefore still a shape) within the bounded phase space. All data points are confined to that shape. That is, all possible trajectories still arrive and stay "on" the attractor.*
- *A chaotic attractor shows zones of recurrent behaviour in the form of orderly periodicity.*
- *It's quite reproductible* (the shape).
- *It has an invariant probability distribution.*

One of the most famous strange attractor was first discovered in 1963 by Edward Lorenz[42]. The attractor itself and the equations from which it is derived were introduced by this author in 1963. While working on a method for modeling and simulating atmospheric conditions, he *derived it* (the strange attractor) *from the simplified equations of convection rolls arising in the equations of the atmosphere* (Wikipedia, 2006). Lorenz plotted the possible configurations of weather system variables in three dimensional phase space, ending up with a butterfly-like shape (Figure 13); each wing of the shape corresponding to one meteorological mode or system.

Studies around the Lorenz's attractor revealed several key characteristics of strange attractors. First, strange attractors lead to a much more complex form of order than point, periodic and torus attractors. Second and more importantly, dynamical systems containing strange attractors are highly sensitive to initial conditions. This sensitivity is a central feature of chaotic systems and strange attractors.

[42] A good description of Lorenz experiment can be found in Lorenz (1993) and Gleik (1989).

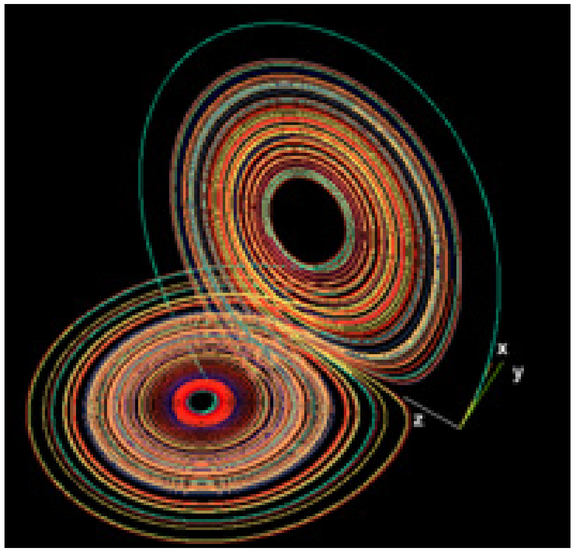


Figure 13 Three dimensional view of Lorenz's strange attractor (Wikipedia, 2006).

Referring to Lorenz strange attractor, Calhoun (2004) provides some interesting subtleties: *one need only imagine pairs of adjacent points beginning to move along trajectories on the Lorenz attractor. It can be readily seen that depending on where they begin their movement, adjacent points can follow trajectories that will take them both to a pattern of activity on the left wing of the attractor, both to a pattern of activity on the right wing of the attractor, or each to separate patterns of activity on opposite wings of the attractor. These patterns of activity are not limit cycles. The wings of the butterfly are thin, but they each contain an infinite number of points. The weather characteristics get "trapped" on one wing of the attractor or the other, therefore displaying order, but each can occupy any of an infinite number of states on the wing where it is trapped, resulting in unpredictability: "This infinite of complex surfaces-- the strange attractor-- embodies a new kind of order. Though the trajectory's motion is unpredictable in detail, it always stays on the attractor, always moves through the same subset of states. That narrowness of repertoire accounts for the order hidden in chaos and explains why its essence never changes." This key characteristic of strange attractors explains a fundamental trait of complex systems-- despite the fact that they are governed by only a few simple, deterministic rules, they display behavior that is orderly, yet unpredictable.*

These subtleties are not aligned with traditional deterministic, time-reversible Newtonian view in which natural processes can be explained by a linear theory that provides accurate predictions as long as precise information is available regarding initial conditions. The popularized terms **Butterfly Effect**[43] are often used to emphasize the importance of initial conditions or small perturbations on chaotic systems.

One may ask the question: based on current and past experiments on chaos, how can we define chaotic (strange) attractor? Based on recent studies on chaos, Williams (2001) proposes two of the best definitions found in scientific literature. They are:

[43] Note that the Butterfly Effect idea has nothing to do with the butterfly form of Lorenz strange attractor.

- *A chaotic attractor is a complex phase space surface to which the trajectory is asymptotic in time and on which it wanders chaotically (Grebogi et al. 1982).*

- *A chaotic attractor is an attractor that shows extreme sensitivity to initial conditions (Eckmann and Ruelle, 1985; Holden and Muhammad, 1986).*

Williams (2001) proposes a list of distinctive properties of chaotic attractors. They are listed in the following lines with additional comments from Shetler (2002):

- *A trajectory within the chaotic regime is usually more complex than just a simple, regular loop. At some values of the control parameter*[44]*, it supposedly never repeats itself (never stabilizes).* It can be read from Shetler's (2002) literature review on this subject: *This strange attractor behavior never quite repeats itself, but its novel expression, nevertheless, stays within certain bounds, so it is sometimes called unpredictable but intelligible (Solé and Goodwin, 2000), or nonrepetitively repetitive (Marion, 1999; Pascale, Milleman, & Gioja, 2000).*

- *Trajectories on chaotic attractor do not cross. If they did, then the system could behave in very different ways whenever the conditions at the crossing point recur.*

- *Two trajectories that at one time are quite close together diverge and eventually follow very different paths. That's because of the sensitivity to initial conditions that characterizes the chaotic regime.*

- *The phase space path of a chaotic trajectory also does a folding maneuver. That occurs when the trajectory reaches its phase space boundary and rebounds or deflects back in its plotted pattern.*

- *A chaotic attractor has a complex, many-layered internal structure. The reason is that "folding" happens over and over again. That internal structure is usually (but not always) fractal.*

- *The external appearance is elaborate and variable compared to the loops or smooth-surface tori of the nonchaotic attractor. To date, many chaotic attractors have been found. Many more probably will be discovered.*

- *Its dimension doesn't have to be an integer, such as 2 or 3. The noninteger and usually fractal nature of chaotic attractors led Mandelbrot (1983: 197) to recommend calling them fractal attractors rather then chaotic or strange attractors.*

Pro-active purposeful systems (Section 2.1.4) may show the emergence of strange attractors over time in phase space. More concretely, *Strange attractors reflect the behaviour of sociocultural systems with choice of ends and means; unpredictable patterns emerge out of stylistic preferences of purposeful actors* (Gharajedaghi, 1999). In its literature review, Shetler (2002) mentions that: *the strange attractor is an obvious metaphor for social phenomena" (Mario, 1999, p. 18). According to McMaster (1996), individual agents' actions self-organize themselves into collective relationships to attractors, i.e., around the mutual* (or shared) *rules or values* (and/or internal models; Holland, 1996) *by which an organization operates. Collective stories, shared symbols like flags, or even communal schedules, like Daylight Savings Time, or the factory whistle,* (or Holland's concept of tag; Section 3.3.15) *can create basins of attraction around which tangible*

[44] Refer to the Logistic equation experiment (Section 2.6.9.1).

behavioral patterns (such as rush hour) can be seen to self organize (Herman, 1982; Lissack and Roos, 1999). Leaders can supply heroic images, develop symbols, logos, etc., ***but can neither force the formation of attraction****, nor easily counteract it once it forms (Lissack and Roos, 1999). This dilemma was recognized long before the development of CSS* (Complex System Science)*. In a non-CSS discussion of how leaders can lead in a time of chaos, O'Toole (1995) says, "To be effective, leaders must begin by setting aside the culturally conditioned 'natural' instinct to lead by push, particularly when times are tough. Leaders must instead adopt the unnatural behavior of always* ***leading by the pull of inspiring values****" (p. 11). The recommendation of leading by the pull of values resonates with the metaphor of an attractor for organizing behavior. In this way, attractors can be seen as underlying the self-organization of culture in complex systems.*

Two important points can be drawn from the last citation. The first one is that CASs' global coherent behaviour is the result of internal self-organizations of their elements. The main drivers that guide this self-organization are attractors, which are driven or conditioned by the presence of rules, values, beliefs and internal models within each element (and the effect of the environment). The more elements are sharing the same set of rules, values, beliefs and internal models, the more ability they will have to self-organize and show coherent global behaviour at the level of CASs. Actually, their presence within CASs will have the tendency to simplify the patterns of attractors found within phase spaces. Broadly speaking, attractors can be seen as one of the factors underlying the self-organization within complex systems.

The second point is that the control of such CASs might not be as easy or direct as it was thought. In a complex organization, a leader cannot for instance blindly add or remove one element to the structure of a CAS without changing the whole dynamic and raising the risk of global instabilities. The reason is that cause/effect interrelationships are highly non-linear and behaviour are often hard (when not impossible) to predict. Instead, s/he must use a global strategy that will encourage inspiration and willing of each elements of the CAS to work in specific ways. This global strategy involves the identification and the understanding of attractors and their sources or causes prior to any modification. The organization's phase space is the playing field for team leaders.

2.6.8 Fitness landscape

Dr Kauffman was the first to describe the co-evolution of CASs using the concept of **fitness landscape**. The fitness landscape is an n-dimensional function made of many maxima/minima. Each of them corresponds to a potential of fitness/unfitness. The higher a maximum/minimum is, the greater the fitness/unfitness it represents. Figure 14/15 shows a bi-dimensional/three-dimensional fitness landscape.

Chan (2001) compares the time evolution of a CAS with a voyage across a fitness landscape with the goal of locating the highest peaks. The system can get stuck on the first peak it approaches if the strategy represents a non-negligible incremental improvement. In the case where the system changes its strategy, the landscape will undergo some changes. In biology for instance, fitness landscapes are made of peaks (maxima) and valleys (minima). Populations typically are **looking for maxima of fitness**. Once a maxima has been found, populations **climb** until the maxima is reached (red trajectory in Figure 14) and remain there unless genetic changes (mutations) open a path to a new higher fitness maxima (Wikipedia, 2006). Biological CASs will seek optimal adaptation through the navigation of this landscape *climbing the various "peaks" in search of the most beneficial adaptive state or highest peak* (Calhoun, 2004).

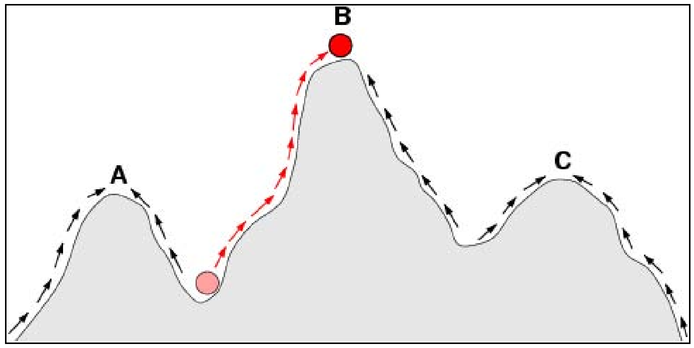


Figure 14 Two dimensional sketch of a fitness landscape (Wikipedia, 2006).

Using Kauffman's NK model, the fitness landscape becomes more rugged when the number of elements and the number of interactions between them increase for a CAS. A fitness landscape with a lot of maxima offers the global system more choices during its **voyage** (or many opportunities for self-organization or for the finding of new solutions), enhancing its flexibility.

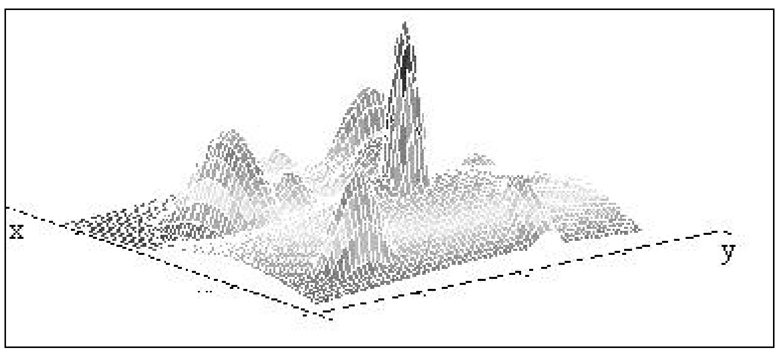


Figure 15 Three dimensional sketch of a fitness landscape (CALRESCO, 2006).

The concept of a fitness landscape has also gained importance in evolutionary optimization methods such as genetic algorithms or evolutionary strategies. *In evolutionary optimization, one tries to solve real-world problems (e.g., engineering or logistics problems) by imitating the dynamics of biological evolution. For example, a delivery truck with a number of destination addresses can take a large variety of different routes, but only very few will result in a short driving time. In order to use evolutionary optimization, one has to define for every possible solution "s" to the problem of interest how "good" it is. This is done by introducing a scalar-valued function "f(s)", which is called the fitness landscape. A high f(s) implies that s is a good solution. The best, or at least a very good, solution is then found in the following way. Initially, a population of random solutions is created. Then, the solutions are mutated and selected for those with higher fitness, until a satisfying solution has been found* (Wikipedia, 2006).

CASs that are able to change their number of connections (by mutation) are found to move from the chaotic (K high; Section 2.5.1) or linear-static (K low) regions spontaneously to that of the phase transition and stability - the **self-organizing criticality**. *The maximum fitness is found to peak at this point* CALRESCO (2006).

2.6.9 Routes to chaos

Williams (2001) calls **route to chaos** the *transition of systems' state from orderly to chaotic behavior*. For dynamical systems, this transition may take place in many ways; *roads to chaos differ from one another by the way in which the periodic regime loses its stability* (Williams, 2001). Roads to chaos are often grouped by type and they *make up a rich and intricate landscape between order and chaos* (Percival, 1989).

Routes to chaos are important *for greater understanding of chaos and for practical purposes. For example, identifying pre-chaotic patterns or behaviour might help us anticipate the occurrence of chaos*. But, **these transitions are not easy to detect**, **nor is it easy to identify, in advance, the particular path of the system's evolution toward chaos**. In the worse situation, *a system might follow one route on one occasion and another on the next* (Williams, 2001).

All the routes to chaos have not yet been discovered. Nonetheless, three of them (1. period doubling; 2. intermittency; and 3. quasi-periodicity) are introduced and briefly described in the next sections. As the reader will remark, more emphasis has been put on the first one. The reason is that it is the object of many studies and it is a good introduction for the others (Swinney, 1986).

2.6.9.1 Period-doubling

As mentioned earlier, period-doubling is *the most extensively studied type of system transition; it is the first road to chaos* (Williams, 2001). It happens in fluid convection, water waves, biology, electricity, acoustics, chemistry, and optics, to name a few (Swinney, 1986). It shows up for instance in transition from stability to turbulence in pot a liquid being heated. The logistic equation is used in this Section to show the manifestation of period-doubling. This theoretical example is often used to demonstrate how linear systems may become complex and then chaotic only by changing its key parameters. The simulation of logistic equation has also the advantage of showing many other interesting properties and phenomena of complex systems.

The logistic equation is a one-dimensional feedback system designed to model the long-term change in a species population (May, 1976; Briggs and Peat, 1989). *The population is assumed to change at discrete time intervals, rather than continuously. Typical time-intervals are a year or the time from one breeding season to the next* (Williams, 2001). Equation 1 defines the model.

$$x_t = Kx_{t-1}(1 - x_{t-1}) \tag{1}$$

This system is simple and **there is no random component**. Equation 1 shows that the multiplying factor $\mathbf{Kx_{t-1}}$ represents the growth of the system while the factor $\mathbf{(1\text{-}x_{t-1})}$ is a limiting one, which prevents infinite growths. The population at one specific moment (x_t) is determined by some fixed proportion of the previous moment's population (x_{t-1}), where **K** is a constant called the **control parameter**; it may reflect the net birth/death rate[45]. Equation 1 is normalized, data is ranging from zero (the minimum possible population) to one (the maximum possible population). *The word "logistic" has many meanings. One is "provision of personnel and material" (as in logistics, the military meaning). Another is "skilled in computation". In our case Equation (1), "logistic" has mathematical meaning and refers to a particular type of so-called growth curve (an expression that specifies how the size of a population varies with time)* (Williams, 2001).

A number of simulations of Equation 1 using different values for K were made in order to study the route to chaos for this particular system. Figures 16 to 20 show selected results. Left graphs (labelled A) are time series diagrams while right graphs (labelled B) are pseudo phase spaces diagrams involving plots of x_t against x_{t-1}. Simulations are always carried out the same way:

- **Initial conditions**. At the beginning of each simulation, the parameter K is given a value that stays constant during this simulation. The system is also given an initial value of x (X_min). Each simulation involves the same time increment[46].
- **Simulation**. The simulation consists in evolving Equation 1 with respect of time using the specified initial conditions.
- **Computing results**. Successive values of x_t are calculated using Equation 1. For each time increment, the old value of x (x_{t-1}) is reintroduced in Equation 1 (feedback) in order to find the value of x for the current time increment.
- **No random component** is introduced in the computation, the system is totally deterministic. All calculated values are kept for further drawings and analysis.

A number of observations can be made from the comparison of Figures 16 to 20. Some of them are described in the following lines with particular emphasis on the way the system becomes more and more complex, and then chaotic. As it will be shown, route to chaos of this system is **period-doubling**.

A first important observation can be made from the two first simulations (Figures 16A and 17A). Varying the initial condition (X_min) from one simulation to another will produce different results. The more the system evolves near the chaotic domain (with increased values of K), the more small differences of initial condition will produce huge differences in outcomes (results not shown). This is aligned with one of the characteristics of complex systems stating the sensitivity of complex/chaotic systems to initial conditions.

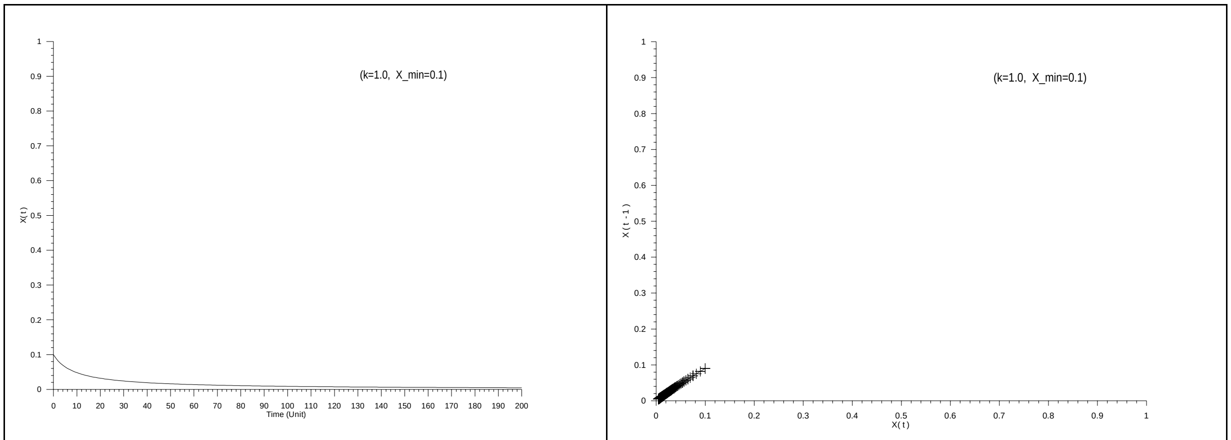
Another important observation can be made from these simulations. Keeping the initial condition the same (X_min) and simulating the system with increased value of K shows that results are progressively becoming more and more complex, and eventually chaotic (Figures 17A, 18A, 19A and 20A). With K = 1, the system's behaviour converge to a single value or attractor (Figure 17A). With K = 3.0 (or greater) the trajectory no longer converges to a single value. In this last case, Figure 20A shows that it is not possible to count the number of attractors. Behaviour of the system appears to be erratic or random.

---

[45] The parameter *K is often an environmental or control parameter. It can take on any realistic value, such as 0.5, 1.0, or 1.87* (Williams, 1997).

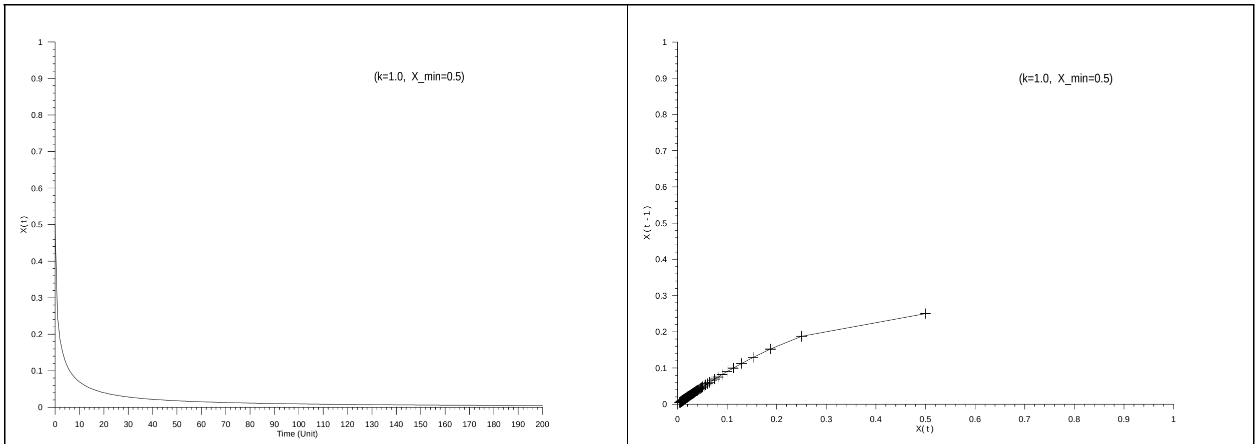
[46] The same value of time increment has been used for all simulations.

In another perspective, Figures 17B, 18B, 19B and 20B show that the augmentation of the K value (from one simulation to another) progressively details the pattern[47] in the phase space. For instance, Figure 20B shows a certain amount of **order** in the phase space, which is not apparent in time series diagram (Figure 20A).



A) Time series.	B) Pseudo Phase Space.

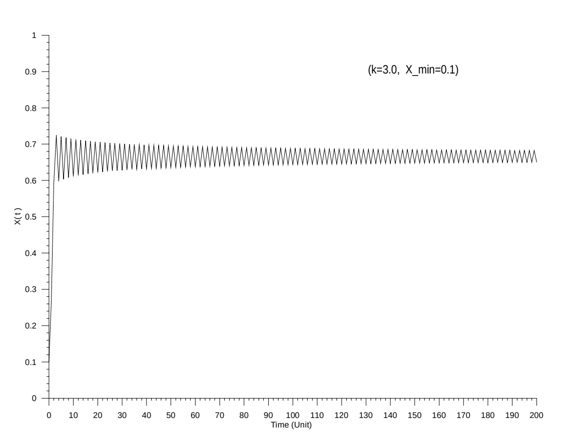
Figure 16 Discrete simulation of the logistic equation (parameters: K=1.0, X_min=0.1).



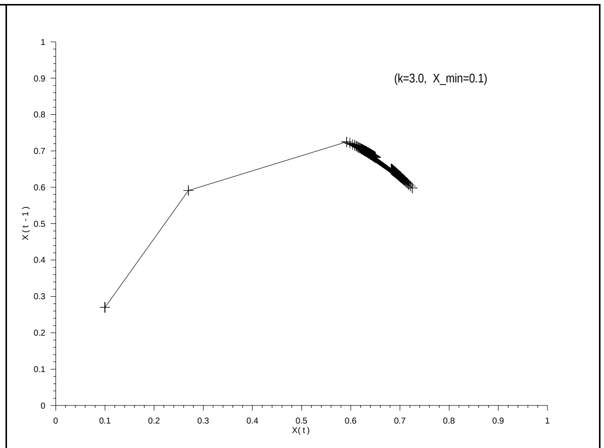
A) Time series.	B) Pseudo Phase Space.

Figure 17 Discrete simulation of the logistic equation (parameters: K=1.0, X_min=0.5).

[47] This pattern is typical for this system using the same value of X_min.

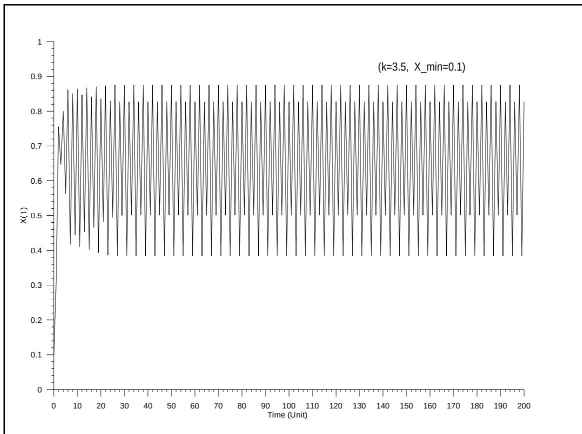


A) Time series.

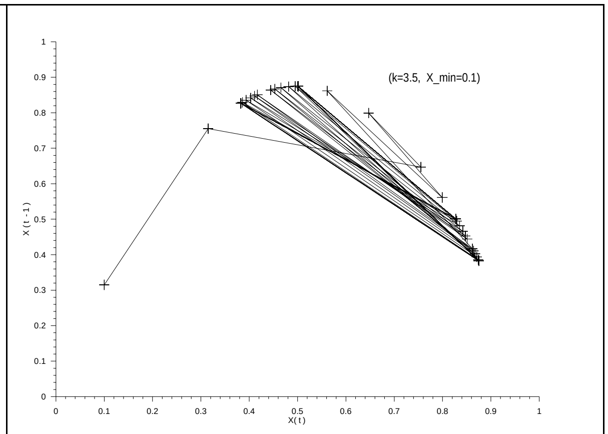


B) Pseudo Phase Space.

Figure 18 Discrete simulation of the logistic equation (parameters: K=3.0, X_min=0.1).



A) Time series.



B) Pseudo Phase Space.

Figure 19 Discrete simulation of the logistic equation (parameters: K=3.5, X_min=0.1).

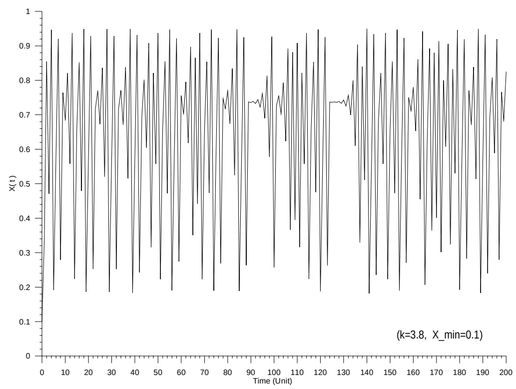
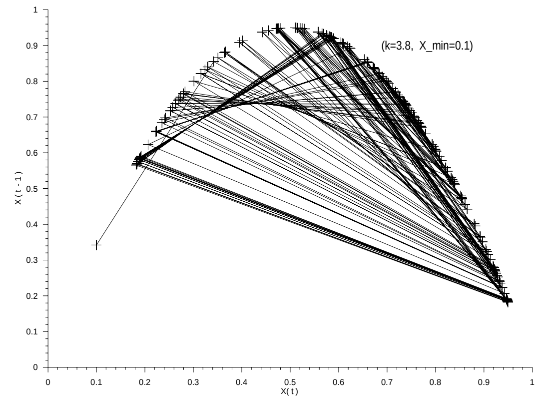
<table>
<tr>
<td>
</td>
<td>
</td>
</tr>
<tr>
<td>A) Time series.</td>
<td>B) Pseudo Phase Space.</td>
</tr>
</table>

Figure 20 Discrete simulation of the logistic equation (parameters: K=3.8, X_min=0.1).

Results from all the simulations that were made in this experiment are summarized in Table 3. This Table shows that there exist critical values for K (or thresholds) that delimitate **modes** for this system. Each mode has a predefined number of attractors and the system is able to evolve in only one mode at a same time.

For instance, the condition **K<3** will always show one attractor (mode 1); the condition **3<K<3.449499** will always show 2 attractors (mode 2) and so on. The passage from one mode to another is called **period-doubling**[48] because the number of attractors is always doubled. Another interesting observation is that period-doubling involves smaller increases of the K value as the number of attractors gets higher (or K gets higher).

Figure 21 shows a different view of these results. It plots the possible values of attractors in function of K. In the case of K<3 for instance, only one value is possible, it is the attractor associated with mode 1. The number of attractors is raised exponentially with the raise of of K. At some value of K (for instance 3.569946), the system reaches the chaos state domain; the number of attractors is still finite but dramatically high. Some general observations for this experiment are:

- initial conditions (X_min) have strong impacts on the system's evolution;
- the value of K has decisive impacts on the system's state and on onset of chaos;
- the route to chaos for this system is period-doubling.

[48] Other names for period-doubling are: flip bifurcation or sub-harmonic bifurcation.

Table 3 Period-doubling for the logistic equation (adapted from Williams, 2001).

Critical value of K	Number of corresponding attractors	Comment
K < 3	**1**	All trajectories for a given K lead to the same point attractor
3 <= K < 3.449499	**2**	The population keeps fluctuating between two point attractors
3.4449499 <= K < 3.544090	**4**	The population keeps fluctuating between four point attractors
3.544090 <= K < 3.564407	**8**	The population keeps fluctuating between eight point attractors
3.564407 <= K < 3.5568759	**16**	…
…	**32**	…
…	**…**	…
Higher than 3.569946	**Very high**	System (Logistic Equation) is in the **chaos state**. Trajectory's route looks erratic, with no apparent order.

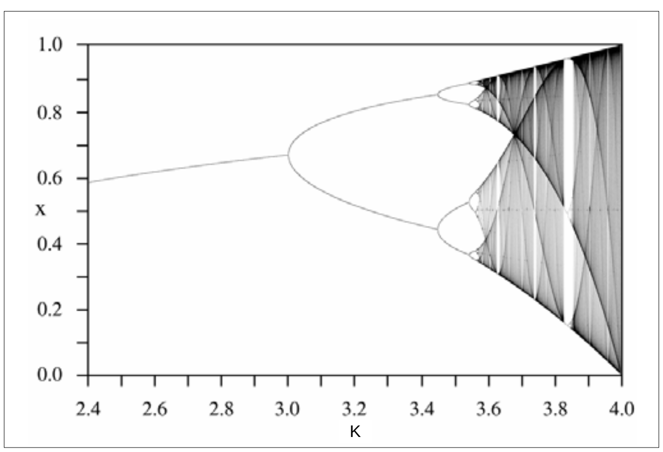


Figure 21 Bifurcation diagram for the logistic equation (Wikipedia, 2006).

It should be noted that these simulations have shown a transition to chaos that is entirely self-generated only from this simple system; no external forces were involved.

2.6.9.2 Other routes to chaos

Other routes to chaos are not described in details because they are beyond the scope of this document. However, a brief introduction of **intermittency** and **quasi-periodicity** inspired from Williams (2001) is presented. The reader is invited to refer to this book for a more detailed description and further references.

Intermittency is the second possible route to chaos. It manifests in systems showing periodic motion, it does not involve period-doubling. *Regular oscillations are interrupted by occasional bursts of chaos or noise at irregular intervals. In mathematical modeling, the periodic motion (limit cycle) typically shows up under relatively low values of the control parameter. Gradually increasing the control parameter brings infrequent chaotic bursts in the time series. These bursts set in abruptly, rather than gradually. With further increase of the control parameter, chaotic bursts are more frequent and last longer, until the pattern eventually becomes completely chaotic.*

Quasi-periodicity is the third possible route to chaos. Systems showing chaos manifestations through quasi-periodicities involve *motion or behaviour caused by two or more simultaneous periodicities whose different frequencies are out of phase (not commensurate) with one another. Since the frequencies are independent and lack a common denominator, the motion never repeats itself exactly. However, it can almost repeat itself, or seem at first glance to repeat itself. Hence, the name "quasiperiodicity". It doesn't show up readily on a time-series graph and usually requires more sophisticated mathematical techniques to be seen.*

2.6.10 Similarity between complex systems

Similarity is a *property that is both innate and accumulative to humans* (Holt, 2000). It is used by human to group, interpret and understand real life objects[49], their structures and behaviour.

The comparison of similar phenomena appears to be fundamental for learning, knowledge and thought. It allows the ordering of things into categories and stimulates logical deductions based on the assumption that similar causes will have similar effects. Holt (2000)'s article emphasises on the fact that *similarity can be used to discover patterns in the complexities of the natural world*. It portrays similarity assessment as *a generative technique for retrieving and analysing complex environmental and spatial information*. It may help researchers *describe and explore certain phenomena, its immediate environment and its relationships to other phenomena*; *patterns could be unearthed if the cached information which alludes such similarities was analysed*.

[49] Complex systems would be one.

3 Aspects of complexity

An overview of concepts, properties, mechanisms and phenomena related to complexity theory and complex adaptive systems (**CAS**) is presented in this Chapter. For clarity purposes, only a limited number of references are cited all along this text. The reader is invited to refer to Couture (2006a) for a more complete list of references regarding presented subjects.

3.1 Criteria for grouping and structuring notions of complexity theory

Generally speaking, descriptors are key words used to identify, differentiate or describe items in an information storage and retrieval system. In this document, descriptors define criteria that will be used to group and structure concepts, properties, mechanisms (etc.) of complexity theory into a coherent integrated picture.

The reading of the scientific literature dedicated to complexity theory, chaos and complex systems often shows that used descriptors are not exactly the same from one author to another; when they are, they are not always used in the same way. This contributes to harden the understanding of complexity theory.

Descriptors listed in Tables 4, 5, 6, 7 and 8 (Annex A) were extracted from the scientific literature; they are used to illustrate this problem. Descriptors are: **fundamental elements**; **characteristics**; **typical features**; **basics of CAS**; **key concepts**; and **basic complexity parameters**. At first glance, these descriptors define criteria that appear to be relatively complementary and sufficient for grouping/structuring notions of complexity theory. Nevertheless, differences of interpretation of concepts may appear when one investigates how these descriptors are used; the semantic of descriptors is not exactly the same from one author to another. Some examples from these tables are:

- The concept of **correlation** is considered as a **fundamental element** in Beech (2004) while it is considered as a **characteristic** in CALRESCO (2006).
- The concept of **adaptation** is considered as a **fundamental element** in Beech (2004) while it is considered as a **typical feature** in CALRESCO (2006).
- The concept of **aggregation** is considered as a **fundamental element** in Beech (2004) while it is considered as one of the **basics of CAS** in Ilachinski (1996) and Axelrod and Cohen (2001) and as a **basic complexity parameter** in Holland (1996).

Another observation can be made from the scientific literature dedicated to complexity theory. Often, described concepts, properties, mechanisms (etc.) and complex phenomena do not make reference to their **domain of applicability**. For instance, these descriptions do not make reference to CASs' logical levels (as defined in Section 2.6.1). For instance, the **Typical features** listed in Table 4 to 8 do not make this distinction; terms like **adaptation** or **resilience** should refer to the system as **a whole** (a higher level description of the CAS) while terms like **redundancy** or **hierarchy** should refer to its **elements** (a lower level description).

An effort has been made in this work to identify a structure that would ease the understanding of concepts of complexity theory. As it will be shown later in this text, the proposed structure is made of a set of four criteria, which take into account important commonalities that can be found in the scientific literature. It is based upon an extensive review of literature that was made on complexity theory, chaos and complex systems (Couture, 2006a). Figure 22 summarizes some of these important commonalities. Emergence of complex phenomena[50] is a central topic; it results from interactions between CASs' elements and its description (or modeling) involves the notion of **level**. Two levels are shown in Figure 22: rectangles R1 and R2 correspond to the Level 1 and rectangles R3 and R4 to Level 2. **Descriptions of CASs** are represented by rectangles R1 and R4 while **Manifestations of CASs** are represented by R2 and R3.

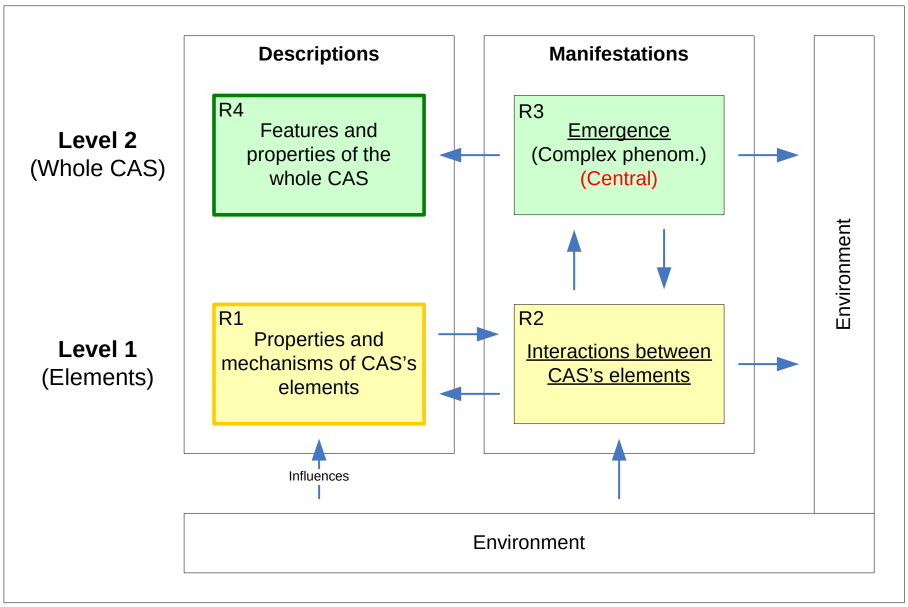


Figure 22 Commonalities from the literature dedicated to complexity theory.

Based on Figure 22, some commonalities are listed in the following points.

- Boundaries defining CASs are subjective to the observer (Section 2.1.3).
- CASs should be studied using multi-levelled structures (Holland, 1996).
- CASs can be described at both levels (Figure 22).
  - Level 1 gives **internal descriptions** of CASs (R1: elements, internal interrelationships, rules, values, beliefs, models, etc).
  - Level 2 gives **global descriptions** of CASs (R4: performance, fitness, resilience, etc.).
- Properties and mechanisms of CASs at Level 1 (R1) determine the types of interaction that happen between elements at Level 1 (R2).

---

[50] Emergence can be considered as "supervenience"; a term borrowed from Psychology (Section 3.4.1.3). Description of emergence is given in Section 3.4.1.

- Properties, mechanisms and manifestations of CASs at Level 1 (R1, R2) are determinant for observed phenomena at Level 2 (R3, R4).
- Interactions between elements at Level 1 (R2) trigger emergences at Level 2 (R3).
- Interactions between elements at Level 1 (R2) may also influence mechanisms, features and properties at Level 1 (R1).
- Complex phenomenon at Level 2 (R3) may influence back interactions at Level 1 (R2). For instance, long term evolution at Level 2 will influence short term activities at Level 1 (Holland, 1996).
- The description of the CAS's environment is not part of the CAS's description; it lies outside the CAS[51].
- Environment influences CASs at Level 1 (through its elements and interrelationships; R1 and R2).
- Both levels of CAS may influence its environment (R2 and R3).

A choice has been made regarding the use of descriptors and criteria in this document[52]. This choice takes into account all described commonalities. A number of four descriptors defining four criteria that will be used for grouping and structuring concepts, properties, mechanisms and complex phenomenon are proposed in the following lines. They will be called **Criterion**.

- **Criterion 1: The basic conditions for CASs to exhibit emergence of complex phenomena** (Level 1; Figure 22). This criterion gathers the minimal set of primary conditions that must be satisfied for CASs to show emergence of complex behaviour. Three have been identified, they described in Section 3.2.
- **Criterion 2: Properties and mechanisms at the level of interacting elements** (Level 1; Figure 22)**.** This criterion gathers together properties and mechanisms that are related to interacting elements of CASs. Some are described in Section 3.3.
- **Criterion 3: Complex phenomena at the level of CASs** (Level 2; Figure 22). This criterion gathers together complex behaviour originating from the operation of CASs. Some are described in Section 3.4.
- **Criterion 4: Properties at the level of CASs** (Level 2; Figure 22)**.** This criterion gathers together global properties of CASs. Some are described in Section 3.5.

Two levels (Figure 22) are used to describe CASs in this document. It is expected that this set of criteria will contribute to ease the building of understanding pictures of complexity theory and the identification and understanding of interdependencies between theoretical concepts.

Figure 23 is a conceptual view showing how concepts, properties, mechanisms and complex phenomena can be grouped and structured using this set of criteria. Criterion 1 gathers three basic conditions[53] for the occurrence of complex phenomena; it is central to the operation of all CASs (Criterion 1 is represented by inner yellow circle). These three conditions are related to elements,

[51] CASs' boundaries depend on observers' perspectives (Section 2.1.3).
[52] Details related to this set of classification criteria and descriptors will be given in another publication.
[53] Section 3.2 describes Conditions 1, 2 and 3 of Criterion 1.

their types, roles, the interrelationships between them, internal rules, values, beliefs, models, etc. Not satisfying this set of three conditions will limit or even prevent the emergence of complex phenomena during operations.

Criterion 2 gathers internal descriptions of CASs; it refers to the description of properties, features, mechanisms at the level of elements (Level 1 in Figure 22). Properties and mechanisms of elements may influence: 1- interrelationships between elements (Level 1); 2- the generation of emergences (Level 2); and 3- global properties of CASs (R4; Figure 22). Criterion 2 is represented by outer yellow circle in Figure 23.

Criterion 1 and Criterion 2 are intimately interrelated and they are not orthogonal. The set of two-sided arrows crossing the limit between the two yellow circles in Figure 23 depicts interrelationships. Some examples of these interrelationships are listed in the following lines.

- The connectivity between elements of a CAS may have determinant influences on Conditions 2 of Criterion 1;
- Redundancy among elements is related to Condition 1 of Criterion 1;
- Decentralization of control, intelligence and decision making within CASs is related to Condition 3 of Criterion 1.

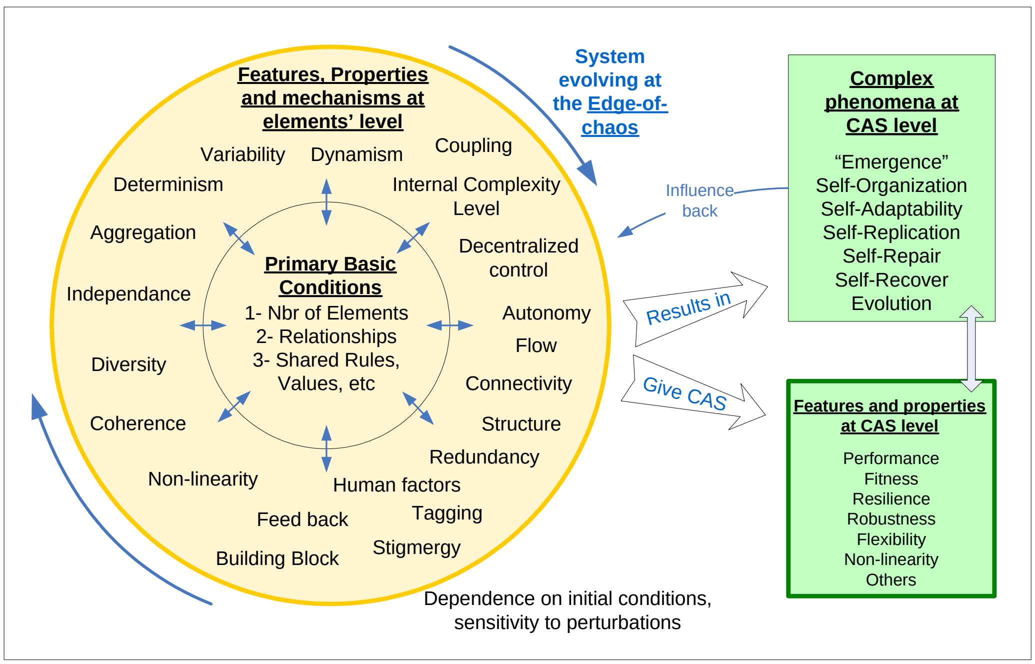


Figure 23 Conceptual view of the set of criteria.

The operation of CASs at the edge-of-chaos is represented by the two cycling arrows in Figure 23. CASs' specificities will be determined by conditions and descriptions gathered in Criteria 1 and 2. As mentioned in sections 2.3, 2.4, 2.5.3.2, CASs are considered similar (at least from a theoretical point of view; see SFI). It is the different conditions and descriptions (gathered in Criteria 1 and 2) that make CASs different from one another. It seams logical that metrics should capture all important aspects of these conditions and descriptions during operation; the latter are at the source of all manifestations of CASs.

Operations under these specific conditions will make CASs exhibit emergence of complex phenomena. Criterion 3 refers to the types of complex phenomena CASs will exhibit (the upper green rectangle in Figure 23 lists some of them). They will also provide CASs global properties that are observable at Level 2 (R4 in Figure 22); Criterion 4 gathers these global properties (the lower green rectangle in Figure 23 lists some of them).

To our knowledge, this set of criteria supports all past and current works in complexity theory; it takes into account concepts, properties, mechanisms and observations made on CASs. Consider for instance the following classical text from Waldrop (1992). Each concept (bolded in the reproduced text) may be grouped in one of proposed four criteria[54]:

Many perplexing questions from a diverse range of disciplines shared four common characteristics.

- *First, these questions concerned systems that could be described as complex, meaning they have a* ***large number of agents*** (Criterion 1, Condition 1) *that* ***interact with each other*** (1, Condition 2) *in a* ***large number of ways*** (1, Condition 2).
- *Second, in addition to being complex, these systems demonstrate the* ***ability*** *to* ***self-organize*** (3 and 4)*, meaning that in the* ***absence of a managing or controlling function*** (1, Conditions 1, 2, and 3) *they spontaneously develop* ***collective properties*** (4) *and* ***elaborate organizations*** (2 and/or 3)*.*
- *Third, these complex self-organizing systems demonstrate the* ***ability to adapt*** (3 and 4)*, or actively* ***evolve in response to their environment*** (3 and 4)*.*
- *Fourth, they demonstrate the* ***ability*** (4) *to avoid either excessive* ***stability*** (4) *or* ***disorder*** (4)*, existing instead at the "edge of chaos," a* ***balancing point between stability and change*** (1, Conditions 1, 2 and 3) *where the system does not remain* ***static*** (2, and 4)*, but also does not devolve into complete* ***disorder*** (2 and 4)*. It is at the edge of chaos that a complex system can achieve a* ***paradoxical kind of harmony*** (3 and 4) *in which both* ***self-organization*** (3 and 4) *and* ***truly novel change*** (3 and 4) *can occur.*

Properties, features, mechanisms and complex phenomena related to CASs are described in the following sections using this set of criteria.

3.2 Basic conditions for CASs to exhibit emergence of complex phenomena

This Section gathers together conditions related to Criterion 1 (Section 3.1). Criterion 1 is made of a minimal set of essential and orthogonal conditions for CASs to exhibit emergence of complex phenomena[55]. All conditions having influences on emergence that are considered as

[54] Numbers in parenthesis refer to our classification criterion numbers.
[55] Many works have inspired the building of this set. Holland's works had determinant influences (Holland, 1992; 1995).

secondary importance in this document[56] have been removed from this list; they are instead captured by Criterion 2 (Section 3.3). This set of primary conditions is:

- **Condition 1**. The presence of a number of independent and interrelated elements forming CASs. Condition 1 includes composition, structures, roles, etc. Elements may evolve with respect of time in function of internal and/or external factors.
- **Condition 2**. The presence of intricate interrelationships between elements of CASs. They often involve variable links and knowledge of each other. Interrelationships may evolve with respect of time in function of internal and/or external factors.
- **Condition 3**. The presence of shared[57] rules, values, beliefs, and internal models within each element of CASs. A set of rules that is shared by many elements of a CAS may represent an important attractor (Section 2.6.7) that will act as a driver for the whole CAS. Shared rules, values, beliefs and internal models will contribute to provide CASs global coherent and oriented behaviour. They may also evolve with respect of time.

These three conditions appear in many major works dedicated to complexity theory but it seems that there is not an agreement on one definitive list of conditions. Holland (1992) for instance lists some common of properties that are shared by all CASs. They are reproduced in the following lines with references to the set of three conditions[58].

- CASs incorporate **large numbers of parts** (1) that are *undergoing a kaleidoscopic array of simultaneous* ***nonlinear interactions*** (2 and maybe 3). The non-linear interactions between elements of CASs make the whole greater (more important or useful) than the sum of parts.
- *It is the* ***aggregate behaviour*** *of the whole system that is of interest* (1 and 2). Using the examples of Government economic statistics influencing the plans of individual businesses in an economy to note that *the aggregate behaviour often feeds back to the individual parts modifying their behaviour* (3).
- ***Interaction between the parts*** *of the system evolves over time* (2 and maybe 3) *as the parts adapt in an attempt to survive in the environment provided by the other parts*. Elements are facing perpetual novelty, and the CAS *typically operates far from a global optimum or equilibrium.*
- *Complex adaptive systems anticipate* (Criterion 3). Holland sees elements as **developing rules** (2 and 3) that become components of a model that anticipates the consequence of response. Description of this process is also described in Holland (1996).

The three conditions of Criterion 1 also enclose Williams' *minimal set of ingredients for complex adaptive emergent systems to have complex dynamic behaviours* (Williams, 2001). They are reproduced in the following lines with references to the set of three conditions.

- *A* ***large number*** (1) *of somewhat* ***similar*** (1) *but* ***independent*** (3) ***items, particles, members, components or agents*** (1).

---

[56] **Secondary importance** properties means here: **properties that influence up to a certain point the emergence**. In this document they are considered **less primary** for the manifestation of emergence but they must be taken into account in the description (or modeling) of emergence.
[57] **Shared** means that some of these rules for instance are present and used by many elements of the CAS.
[58] Numbers in parenthesis correspond to Conditions of Criterion 1.

- *Dynamism – the particles' persistent movement and readjustment. Each agent continually* ***acts on and responds*** (2 and possibly 3) ***to its fellow agents in perpetually novel ways*** (2 and possibly 3).
- *Adaptiveness:* (Criterion 3; a complex behaviour) *the system conforms or adjusts to new situations so as to insure survival or to bring about some advantageous realignment.*
- ***Self-organization***, (Criterion 3; a complex behaviour) *whereby some order inevitably and spontaneously forms.*
- ***Local rules*** (3) *that* **govern each cell or agent**[59].
- ***Hierarchical progression in the evolution*** *(complex behaviour) of* **rules** (2 and 3) ***and structures*** (1 and 2). *As evolution goes on, the rules become more efficient and sophisticated, and the structure becomes more complex and larger.*

As mentioned in CALRESCO (2006): *The richness of possible behaviour increases rapidly with the number of interconnections* (Condition 2) *and the level of feedback. For small systems we are able to analyse the state possibilities and discover the attractor structure* (Condition 3). *Larger systems* (Condition 1, 2 and 3) *however require a more statistical approach where we sample the system by* ***simulation*** *to discover the emergent properties.*

Primary conditions for CAS to exhibit complex phenomena in an environment and context are included in the three conditions of Criterion 1. These three orthogonal conditions are interrelated. Drastic changes or modifications to the composition or structure of this trio of conditions at Level 1 {elements, interrelationships, and rules/values/beliefs/internal models} may potentially have hard to predict global consequences at Level 2. If the CAS's dynamical stability has been modified by a perturbation, precise readjustments of the whole may be hard to achieve due to the presence of strong non-linearity. For instance, eliminating a shared rule within elements of an organization may trigger the evolution of its state toward the chaos state domain. Reinserting this rule in elements of the system may not re-establish the original state.

3.3 Properties and mechanisms at the level of interacting elements

This Section gathers properties and mechanisms that are related to Criterion 2 (Section 3.1). Properties and mechanisms described in this Section are lying at Level 1 of Figure 22.

3.3.1 Aggregation

The aggregation property was queued by Holland (1996). It enters the study of CAS in two senses. First it refers to a standard way of simplifying the understanding of **complicated** systems *by aggregating similar things into* ***categories*** *and then treat them as equivalent*. One possible use of categories is as following: categories are well understood building blocks (Section 3.3.3); they can be re-assembled in a different manner to understand a new complicated scene, to find new solutions to complex problems or to generate novelty.

[59] Holland (1995) also describes how rules, values and internal models are used in elements of CASs. By evolving "rules" in his model ECHO, he reproduces long term evolution of CASs.

Holland's second sense of aggregation *is closely related to the first, but it is more a matter of what CASs do, rather than how they are modelled*. It concerns the emergence of *global complex large-scale behaviour from the aggregate interactions of less complex elements*. Holland uses the ant example to explain this second sense of aggregation. Individual ants have highly stereotyped behaviour, and they often die when circumstances do not fit with stereotypes. *On the other hand, the ant aggregate – the ant nest (or CAS) – is highly adaptive; it is able to face a wide variety of hazards.*

Aggregates (or CASs) may in turn act as elements and form a higher-level meta-CAS. At a higher level, meta-CASs may in turn aggregate and form a meta-meta-CAS, and so on. *When this process is repeated several times, we get the hierarchical organization so typical of CAS* (Section 3.3.15).

3.3.2 Autonomy and independence of elements

Loosely speaking, an autonomous and independent element is an entity that interacts with its environment and acts independently from all other elements. *It does not take commands from some seen or unseen leader, nor does an independent element have some idea of a global plan it should be following* (Flake, 1998). It may be for instance independent financially, managerially, hierarchically, etc. An independent element does its own things and may be willing (or not) to collaborate/cooperate or compete with other elements of a CAS.

Autonomy of elements is closely related to the decentralization of control that is typical of CASs (Section 3.3.11). Taking the example of Internet as a CAS, there is not an operational centralized control centre that dictate network devices what actions they must take. Such centers would have needed huge amount of resources for being able to achieve its mission; performance would be dramatically lowered. Instead, network devices are autonomous and they have the ability to decide where to send arriving packets of information. The control (and load of work) is distributed among devices of the network. This contributes to increase the performance of the whole network (in this example).

3.3.3 Building blocks

The concept of building block was queued by Holland; it is related to the concept of aggregation (Section 3.3.1). Building blocks are well understood entities that can be recombined into new larger-scale entities to favour the discovery of new ideas, solutions, situations, scenes or objects. Well known elements of solution may for instance be recombined in many different patterns in search of new **higher-level solutions** to complex problems. This recombination process is at the origin of the process of scientific discoveries (Holland, 1996); known and well mastered solutions are used in different combinations to find novelties.

For instance, biological CASs such as the immune system cannot have a complete **list of models** of all possible invaders of the human body. A minimal set of relevant building blocks[60] that takes into account past evolutions or experiences is needed. When an attack happens, immune system recombines known building blocks (or **knowledge** from past experiences) to find models that best

[60] Small-scale models expressing some commonalities between invaders for the human body.

fit the unforeseen invader. Recombining relevant building blocks in different compositions and structures will eventually result in a model of the invader that is close to the real one. The higher is the number of choices of recombination, the higher is the possibility of generating the emergence of novelties. *This use of building blocks to generate internal models is a pervasive feature of CASs* (Holland, 1996).

Elements of a loosely coupled CAS (Section 3.3.5) can be seen as building blocks that can be recombined in different ways to find new solutions, exhibit emergence of new phenomena. This process of building blocks recombination does not necessarily involve a centralized control or intelligence (Section 3.3.11) and may result in different types of internal structure of CASs (Section 3.3.15).

3.3.4 Coherence

Coherence among element of a CAS refers to a logical and consistent correlation of its elements. *Coherence spans and correlates the separate lower level components into a higher level unity* (De Wolf and Holvoet, 2005).

The correlation of elements (Holland, 1996) may be enhanced by rules, values, beliefs and internal models (Section 3.2) if they are shared by many elements of a CAS. For instance, the beliefs "American Society is bad" is shared by all members (the elements) of terrorist organizations (the CASs). Values, beliefs, and rules make the global behaviour of the CAS more homogeneous or coherent (Beech, 2004).

3.3.5 Coupling of elements

The coupling of elements of CASs is related to Holland's (1996) concepts of **correlation** and **aggregate** (Section 3.3.1).

Elements of a CAS need to interact for being able to exhibit emergence; *parallelism is not enough* (De Wolf and Holvoet, 2005). Relationships between elements of CASs can be categorized as: 1. tightly coupled; 2. moderately coupled; and 3. loosely coupled (Beech, 2004; Marion and Uhl-Bien, 2002). Tightly coupled elements display high degrees of interdependence such as in linear systems. Moderately and loosely coupled elements display low degrees of interdependence (such as for CASs evolving near the edge-of-chaos; Section 2.5.3).

The number of connections between elements is another factor that influences the type of coupling between elements of CASs (Section 2.5.1). For self-organization to occur, *the system must be neither too sparsely connected (so most units are independent) nor too richly connected (so that every unit affects every other). Most studies of Boolean Networks suggest that having about two connections for each unit leads to optimum organisational and adaptive properties* (CALRESCO, 2006).

3.3.6 Determinism

As mentioned in Chapter 2, systems studied in this document show behaviour that is exclusively driven by deterministic cause/effect relationships[61]. Systems near chaotic state domain show behaviour that appears random; *actually they often pass random tests*. *One way to differentiate random data from chaotic data is to plot it on difference plots* (Williams, 2001). *They may help distinguish highly deterministic chaotic (or near chaotic) data from negligibly deterministic ("random") data.*

Figure 24 shows the difference between random and chaotic behaviour. Both time series diagrams show the appearance of randomness while phase spaces show **order** for chaotic data and randomness for random data. Feedback is one of the factors that contribute to explain the presence of order in Figure 24. Simulations of the logistic equation have shown similar results (Section 2.6.9.1).

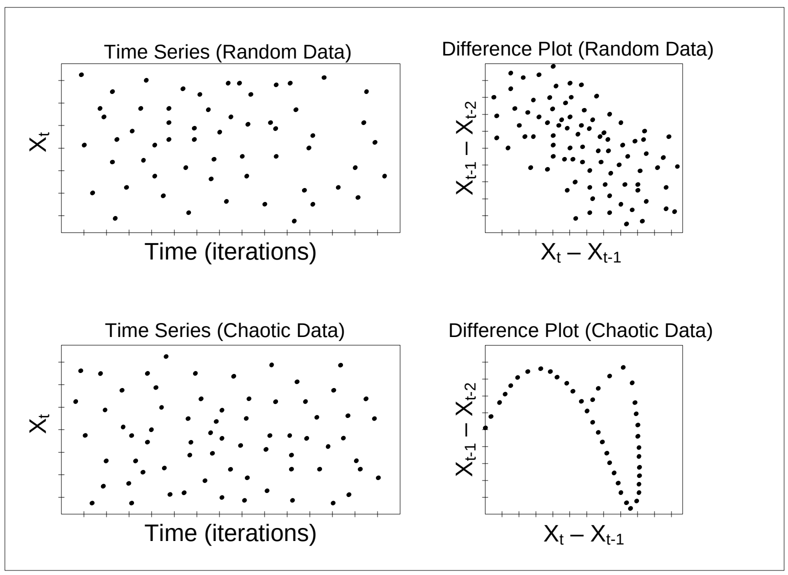


Figure 24 Data from random and chaotic systems (adapted from Williams, 2001).

3.3.7 Diversity and redundancy

Diversity may mean: the number of **kinds of elements** (defined by their roles, expertises, etc.) that is found in a CAS. A high diversity of elements of a CAS will provide the latter more choices (Section 3.3.3) or flexibility that can be explored. Redundancy within a CAS will contribute to raise the number of multi-way chains of causality (building blocks can be recombined in many new different ways to find new solutions or novelty).

It is neither accidental nor random in CASs. *If we remove one kind of agent* (or element) *from the system* (CAS), *creating a "hole", the system typically responds with a cascade of adaptations resulting in a new agent that "fill" the hole. The new agent typically occupies the same niche as*

[61] Eliminating any random component eases the study of complex systems by removing noise.

the deleted agent and provides most of the missing interactions. This process is akin to the phenomenon called convergence in biology (Holland, 1996).

3.3.8 Dynamic, equilibrium and edge-of-chaos

There are many studies that describe the dynamic of CASs at or near the edge-of-chaos. Edge-of-chaos state is *characterized by* ***stability and instability****,* ***competition and cooperation****,* ***order and disorder****, etc* (Chan, 2001). CASs evolving at the edge-of-chaos are not in equilibrium; they exhibit emergence of complex phenomena in search of new possibilities. In 1989, Nicolis and Prigogine said on this subject: *when a physical or chemical system is pushed away from equilibrium, it could survive and thrive. If the system remains at equilibrium it will die*. *The "far from equilibrium" phenomenon illustrates how systems that are forced to explore their space of possibilities will create different structures and new patterns of relationships*.

In another study, Kaufman's computer simulations demonstrate that: *it is possible for the order of new survival strategies to emerge from disorder through a process of spontaneous self-organization*. Order may result from non-linear dynamical feedback interactions between elements of a CAS where each element **goes about its own business** (Chan, 2001).

Chan states another interesting example from medical cardiology; the study of normal and abnormal heartbeat patterns. *The rhythmic beating of the heart is very orderly but there exists a subtle but apparently fundamental irregularity. The interval between heartbeats varies in a disorderly and unpredictable manner in healthy individuals, particularly in young children. Regularity of the heartbeat interval is a sign of dinger –* ***order in heart dynamics indicates insensitivity and inflexibility****. Therefore, it can be said that complex adaptive systems function best when they combine order and chaos in an appropriate measure.*

The combination of both order and disorder within CASs contributes to improve their flexibility to find **new solutions** or **new ways to operate** (Section 3.3.3). As opposed to rigid traditional linear and hierarchical structures, this flexibility contributes to enhance CASs' ability to re-organize and self-adapt to changing environment. The ability of CASs to learn from past experiences and to integrate lessons learned for future uses (new found combinations) will condition its long term evolution.

3.3.9 Feedback

Feedback loops have been known since the time of Cybernetics in the 1950s and 60s (Fromm, 2005b). *Already W.R. Ashby noticed the importance of feedback in coupled systems for forms of self-organization which are more than transitions from 'parts separated' to 'parts joined' [Ash62]*.

Fromm identifies two purposes of feedback:

- (1) The feedback loops are used to control CASs. *Feedback loops across different levels and complicated causal relationships can be found in stigmergy* (Section 3.3.14) *and swarm-intelligence [Cam03]. Both are linked to causal relations across the system-environment boundary. The agents of the system affect the environment, which in turn influences the behavior of the agents* (Section 3.6.2).

- (2) The feedback signal indicates at the same time the current state of the controlled element. An example involving stigmergy is given by Fromm (2005b): *Pheromone trails control the movement of ants, but they also signal the place of the food and the "foraging" state of the colony.*

Flake (1998) describes feedbacks between a CAS and its environment and makes the difference between adaptive and non-adaptive systems. *In a strictly non adaptive control system there is either no extra reinforcement information or the reinforcement is somehow trivially bundled into the observable state.* (…) *In a truly adaptive system, actions that were successful in previous but similar states may be discarded in favor of actions that were more successful. In a sense, feedback allows for a complex adaptive system to reprogram itself.* This is completely in line with Holland's model that uses **feedback**, **credit assignment** and **building blocks recombination concepts** to describe CAS adaptation through the discovery of new rules (Holland, 1996).

Feedback can happen inside CASs (as shown in Figure 23) or between CASs and their environments.

3.3.10 Flows

This is one of Holland's basics; it is considered as a property by this author (Holland, 1996). It is related to *flows over a network of nodes and connectors*. The nodes refer to elements of CASs and connectors refer to links allowing exchanges of resources between them.

The triad {node, connector, and resources} exists for all CASs and none of them can be considered as fixed with respect of time. *They are patterns that reflect changing adaptations as time elapses and experience accumulate.*

3.3.11 Hierarchical versus distributed control

Bar-Yam's concept of **complexity profile** (Section 2.6.3) is used to explain some limitations of rigid hierarchical command and control (Bar-Yam, 2003d). *The key to this understanding is that each individual has a limited complexity. In particular, an individual is limited in ability to process information and to communicate with others (bandwidth) [12-15]. In an idealized hierarchy* (Figure 25)*, only the single leader of the organization can coordinate the largest organizational units whose commanders are directly under his/her command.* ***The coordination between these units cannot be of greater complexity than the leader****. More generally, we can state that to the extent that any single human being is responsible for coordinating parts of an organization, the coordinated behaviors of the organization will be limited to the complexity of a single individual. Since coordinated behaviors are relatively large scale behaviors, this implies that* ***there is a limit to the complexity of larger scale behaviors of the organization****. Thus,* ***<u>using a command hierarchy is effective at amplifying the scale of behavior, but not its complexity</u>****. By contrast, a network structure (like the human brain)* (Figure 25) *can have a complexity greater than that of an individual element (neuron). While an arbitrary network is not guaranteed to have a complexity higher than that of an individual component, it is possible for such a network to exist.* ***For high complexity tasks, we therefore consider hierarchical systems inadequate and look to networked systems for effective performance****.*

Consider the same example as in Section 3.3.2; packets of data that are traveling over Internet from one point to another through multiple network domains. There is not a centralized controller that decides in advance (for the whole Internet) the path the packet will take. To make good decisions this controller would needs to gather and maintain knowledge on traffic throughout the entire network; this is impossible (Fromm, 2005b; De Wolf and Holvoet, 2005). The path is rather built or decided as the packet encounter new routers along its route through network domains. Each router **decides** where to send packets based on a number of pre-identified local/regional parameters that change over time. *Decentralised control is using only local mechanisms to influence the global behaviour. There is no central control, i.e.* ***no single part of the system directs the macro-level behaviour*** (De Wolf and Holvoet, 2005). The actions of elements are controllable but the whole is not directly controllable. *Today's networks have overwhelmed this "top down" approach, and the industry has had to move toward decentralized control within networks that make decisions for themselves* (ONCE-CS, 2006).

Figure 25 shows three possible types of network topology: 1. Hierarchy; 2. Hybrid; and 3. Network (Bar-Yam, 2005). The exchange of information in hierarchic structures may be less effective than in network-like structures for finding novelty. **Hierarchies** are relatively rigid and permanent structures while **network** allows more choices to find solutions (Section 3.3.3); usually, their topology and composition are more variable with time.

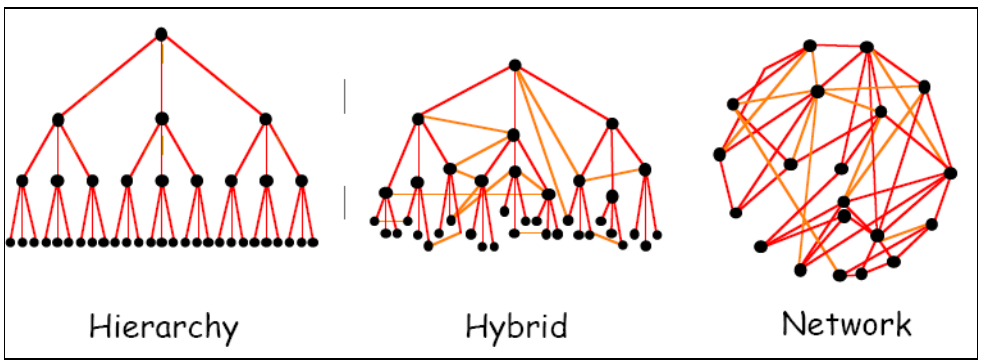


Figure 25 Types of organizational structure (Bar-Yam, 2005).

Complex systems are often qualified as **heterarchies** as opposed to hierarchies (Jen, 2003). Heterarchy means: *interconnected, overlapping, often hierarchical networks with individual components simultaneously belonging to and acting in multiple networks, and with the overall dynamics of the system both emerging and governing the interactions of these networks*. Human societies in which individuals act simultaneously as members of numerous networks familial, political, economic, and professional (among others) are one example of heterarchies.

It is worth ending this Section by reproducing Bar-Yam's (2003d) comment on the decentralization of command and control: *Distributed control is often discussed today as a panacea for problems of hierarchical control. While distributed control can help, it must be recognized that the concept of "distributed control" does not correspond to a specific control structure. Distributing control in and of itself does not lead to effective systems or solve problems with hierarchical control. It is the design of specific distributed control structures that are effective in specific types of tasks that provides a functional advantage. Still,* ***we now recognize that there are many ways to achieve effectively functioning systems where functional behavior***

and control is distributed and can be said to arise by self-organization, and that the traditional perspective that the only alternative to hierarchical control is anarchy is not correct.

3.3.12 Internal complexity levels

The internal complexity of CASs may take many complementary but interrelated aspects. Couture (Couture, 2006b) lists some formulations and measures that may be used for evaluating many aspects of this complexity. The following lines list some examples[62].

- The concept of internal complexity is well described by Jost (2003)[63]. The complexity of a CAS's internal models depends among other things on the level of complexity of its environment. The complexity of CASs' internal models should be equal or higher than the level of complexity of its environment (Bar-Yam, 2005).
- The number of elements within a CAS and their intricate relationships using more or less complicated rules, beliefs and internal models (Section 3.2) are other factors that may influence CASs' internal complexity.
- The types of coupling between elements of a CAS and their variety may also influence CASs' internal complexity (Section 3.3.7). The complexity increases *when the variety (distinction) and dependency (connection) of parts or aspects increase in at least one of many possible dimensions, including the three ordinary spatial dimensions as well as the dimensions of geometrical structure, spatial scale, time or dynamics, or temporal or dynamical scale* (Heylighen, 1996).

It should be noted here that there is not an agreement on the list of metrics that should be used to measure aspects of CASs' complexity. This difficulty originates from the fact that this science in still in evolution. It may also be caused by our reductionism ways of thinking and addressing problems. Some important questions to be answered are:

- Do we, at this moment, use the best approaches to study complexity aspects of CASs?
- What is really meant by holism?
- How should we change our scientific approaches in order to better address complex problems?
- What are the effects on metrics?

These are important questions (among others) that remain to be answered and this author did not find scientific papers that provide complete answers. Once discovered, they will provide strong hints on how to measure complexity.

3.3.13 Human aspects

Human aspects are of considerable importance for military CASs and complex operations. They are factors that contribute to add internal complexity to CASs.

[62] Please see also Section 3.6.2 and Section 4.2.1.1.
[63] Section 3.6.2 gives a description of Jost's definition of "internal complexity".

It is beyond the scope of this document to describe these aspects. The reader is invited to refer to Dr. Thagard's extensive work for supplementary information and links (http://cogsci.uwaterloo.ca/Biographies/pault.html). Some aspects that are considered are human cognition processes such as: analysis, inference, learning, decision making, etc.

3.3.14 Non-linearity

Wikipedia's definitions of linearity and non-linearity is as follow: *a linear relationship is simply one whose graph is a straight line, so a linear connection between two things is one in which change on one side of the connection induces proportional change in the other. A nonlinear connection means that change on one side is not proportional to change on the other* (Wikipedia, 2006).

Whole branches of mathematics are devoted to finding linear functions that are reasonable approximations when linearity cannot be directly established in problem analysis. *Most of our mathematical tools, from simple arithmetic through differential calculus to algebraic topology, rely on the assumption of linearity*. (…) *Unfortunately, none of this work well for CASs* (Holland, 1996). Holland uses the Lotka-Volterra model as an example to show the importance of considering non-linearity in the predator-prey system.

Non-linearity is a common property to all CASs operating at or near the edge-of-chaos. It reflects the fact that **CASs are more then the sum of their parts**[64]. Non-linearity is present among elements and interrelationships of a CAS; it contributes to make the whole harder to predict.

3.3.15 Stigmergy

Stigmergy is a mechanism for **indirect communications** between elements of CASs. Elements modify their environment in such ways that other elements can interpret these modifications (or marks) as messages. Wikipedia (2006) mentions on stigmergy: *Stigmergy was first observed in nature - ants communicate to one another by laying down pheromones along their trails, so where ants go within and around their ant colony is a stigmergic system.* (…) *Stigmergy is not restricted to eusocial creatures, or even to physical systems. On the internet there are many emergent phenomena that arise from users interacting only by modifying local parts of their shared virtual environment.* (…) *The term is also employed in experimental research in robotics, multi-agent systems and communication in computer networks. In these fields there exist two types of stigmergy: active and passive. The first kind occurs when a robotic or otherwise intelligent "agent" alters its environment so as to affect the sensory input of another agent. The second occurs when an agent's action alters its environment such that the environmental changes made by a different agent are also modified.*

The importance of the stigmergy mechanism is becoming critical in our contemporary complex systems and society (Poussart, 2006a; 2006b).

[64] *A linear system is subject to the principle of superposition, and hence is literally the sum of its parts, while a nonlinear system is not* (Wikipedia, 2006).

3.3.16 Structure

CASs tend to exist in many levels of organization, forming structures in which elements at one level are the building blocks for elements at the next higher level (Section 3.3.3). *An example is cells, which make up organisms, which in turn make up an ecosystem* (Dodder and Dare, 2000). Structure is closely related to the concept of boundary and level (Section 2.1.3 and 2.6.1). Boundaries help determine which elements form a CAS, their internal organization defines its structure.

What is an element and what is a CAS is function of the level considered (the perspective). In Dodder and Dare's example, elements are the cells and the CAS is the organ, which contains the cells. Using another higher-level perspective, organs may be considered as elements of the organism (the CAS). The internal structure of a CAS depends among other things on its type and on the type of problems it has to solve.

Holland (1996) uses the embryogenesis of metazoans[65] example to describe typical structures of biological CASs. The organism (CAS-L2 in Figure 26) is made of many different organs (CAS-L1) that in turn are made of many kinds of cells (Units). As cells multiply with respect of time, new organisms are formed (CAS L1), which in turn make up an ecosystem at a higher level (CAS L2), and so on.

The structure of biological CASs varies or evolves with respect of time, but also in function of the type of organism considered (internal rules; Section 3.2); multiplying cells may become a tiger or an elephant depending on the genetic information lying within each cell. Numerical simulations of the long term evolution of biological CASs' involve crossovers and mutations of this genetic data (Holland, 1996).

Man-made CASs have structures that may vary and evolve differently from biological organisms. Figure 26B shows an example of such structures. In this figure, blue arrows represent relatively permanent interrelationships between elements (the ones that stay rigidly structured or strongly coupled) while green arrows represent ephemeron interrelationships (moderately to loosely coupled elements). The latter are able to recombine rapidly in function of unforeseen situations or problems to solve for instance. Elements linked with green arrows offer more combination possibilities (Section 3.3.3) than the ones linked with blue arrows; they offer more choices. This raises the chances of finding new solutions to new unforeseen complex problems.

A hockey coach for instance may use different combinations of players in his trios in order to adapt to different adverse teams. Only some combinations will be effective against players of a particular team. Using the same combinations of player against all adverse teams would probably not be the best strategy. A good coach would instead quickly find and use on-the-fly the optimal combinations of players. Optimum combinations correspond to higher potentials in the fitness landscape (Section 2.6.8). Figure 26B may correspond to Bar-Yam's Hybrid Structure (Figure 25).

[65] *The process whereby a fertilized egg progressively divides until it yields a mature many-celled organism that reproduces by producing another fertilized egg.*

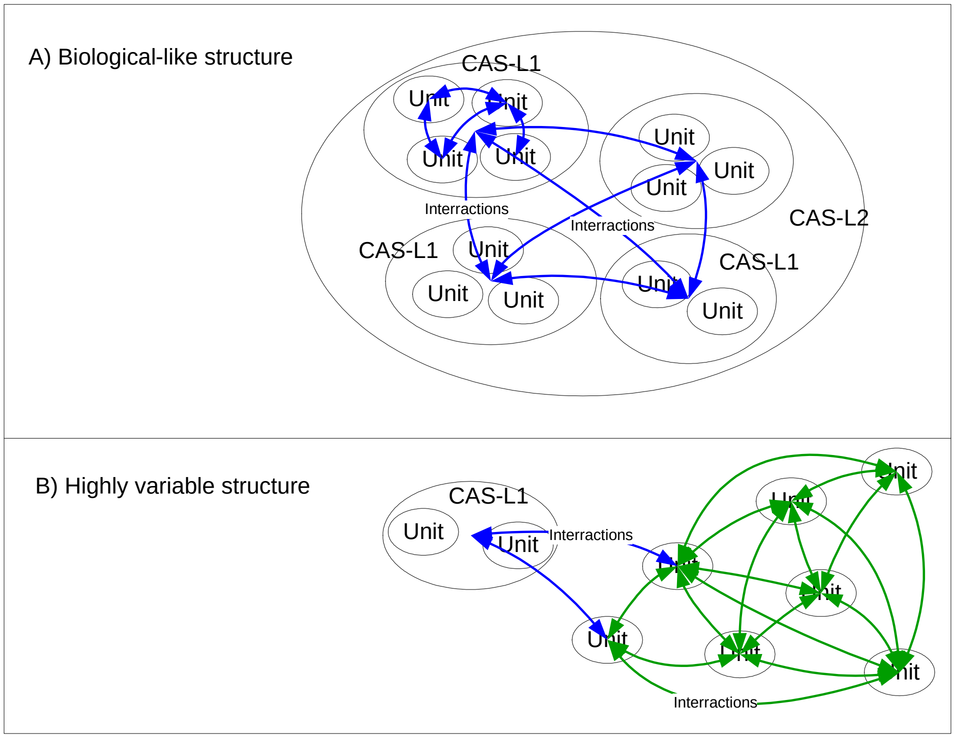


Figure 26 Two types of internal structures of CASs.

3.3.17 Tagging

Tagging is one of Holland's basics of CASs. It is considered as a *pervasive mechanism for aggregation and boundary formation in CASs; CASs use tags to manipulate symmetries* (Holland, 1996). This author uses the familiar example of a banner or flag (as tags) to *rally members of an army or people of similar political persuasion. The header on a message that knits together members of a bulletin board or conference group* is another example.

Tags facilitate selective interactions. Holland's description of tag give a very good idea of the inherent mechanism and consequences: Tags *allow agents to select among agents or objects that would otherwise be indistinguishable. Well-established tag-based interactions provide a sound basis for filtering, specialization, and cooperation. This, in turn, leads to the emergence of meta-agents and organizations that persist even though their component are continually changing. Ultimately, tags are the mechanism behind hierarchical organization – the agent / meta-agent / meta-meta-agent / … organization so common in CASs* (Section 3.3.15).

3.4 Complex phenomena at the level of CASs

This Section describes emergence of complex phenomena (Criterion 3; Section 3.1). Emergence described in this Section manifests at Level 2 of Figure 22.

It is worth beginning this Section by saying that the emergence[66] of complex phenomena may take many forms. Its manifestation may potentially have strong impacts on CASs' identity, appearance, behaviour and long term evolution. Figure 27 proposes, for this Section only, four arbitrary descriptors[67] that can be used to group complex phenomena; **identity, intentionality**, **evolution** and **maintenance**.

Al Qaeda can be seen as a CASs that shows complex phenomena such as the ones depicted in Figure 27 (Beech, 2004; Marion and Uhl-Bien, 2002). Its elements (individuals and cells of people) have the consciousness of being part of this organization; the whole has an identity through its elements. This consciousness combined with shared values, beliefs, culture, and mental models contribute to insure a level of coherence of the whole (through interactions).

This CAS has intents, it is able to learn, infer, and innovate in order to pose actions that are aligned with common interests and intent. It also has the ability to evolve by self-organizing and self-adapting its elements to different situations and environments. Its non-hierarchical interlinked distributed structure and internal flexibility contribute to ease its self-maintenance, self-recovery and self-repair in case of attacks.

Terms depicted in Figure 27 appear to be aligned with DeWolf and Holvoet's (2005) definition of emergence. As it will be shown in Section 3.4.1, it is the dynamical interactions[68] between elements of a CAS (Level 1; Figure 22) that trigger the emergence of global complex phenomena (Level 2; Figure 22). Emergence evolves with respect of time and in function of many internal and external factors that concurrently act on (or influence) CASs' elements and interrelationships between them.

[66] Used definition in this document is the one of DeWolf and Holvoet (2005): *A system exhibits emergence when there are coherent emergents at the macro-level that dynamically arise from the interactions between the parts (or elements) at the micro-level. Such emergents are novel w.r.t. the individual parts of the system.*

[67] More work is needed to identify the set of orthogonal descriptors that would be sufficient to classify all aspects of any type of CASs.

[68] Beech (2004) mentions on interactions between elements of a CAS: *Complexity Theory views* (complex) *behaviors as the* (result of) *constantly changing interdependent interactions.* De Wolf and Holvoet (2005) add: *The parts* (elements of CASs) *need to interact – parallelism is not enough. Without interactions, interesting macro-level behaviours* (emergence) *will never arise.*

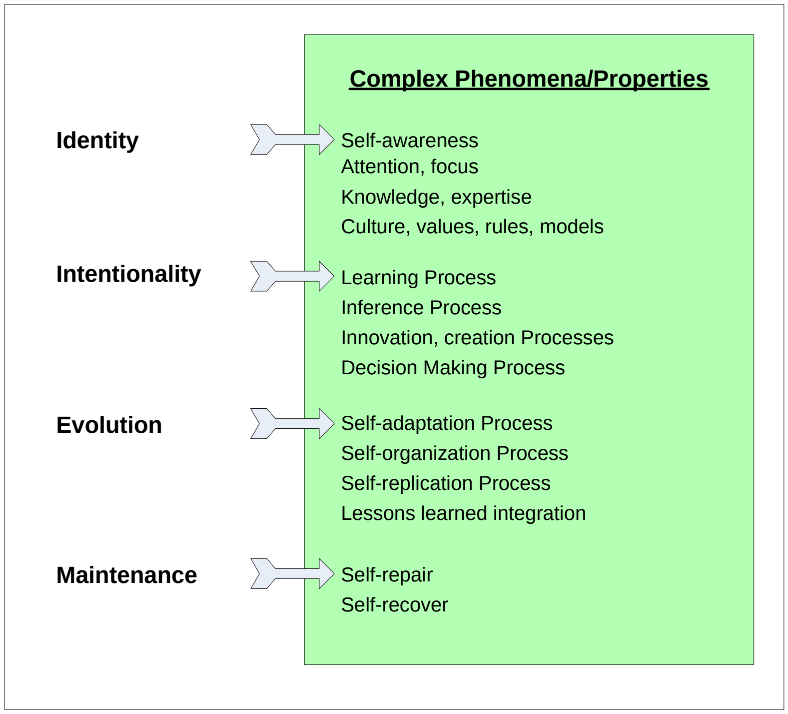


Figure 27 Types of complex phenomena exhibited by CASs.

Figure 28 shows some interrelationships between selected CASs' properties and complex phenomena[69]. Green rectangles (black outlines) represent CASs' complex phenomena, yellow rectangles (yellow outlines) represent properties of CASs' elements (at Level 1; Figure 22) and green rectangles (green outlines) represent features or properties of CASs (at Level 2; Figure 22). Arrows with the positive/negative (+/-) signs represent positive/negative **contributions** of the originating rectangle to the destination rectangle.

It can be seen in this figure that emergence is a core principle for self-organization (Fromm, 2005b), which in turn favour self-adaptation. Loosely coupled elements within a CAS contribute to increase the number of choices the latter has to solve problems and being more resilient to attacks for instance. The reason for this is that elements of a loosely coupled CAS form building blocks (Section 3.3.3) that can be re-combined in many ways, enhancing the probability of finding appropriate new solutions to unforeseen problems. Complex networks are said to be recursive. Through the process of *aggregation and correlation* (Holland, 1996) the network develops *redundant multi-way chains of causality to accomplish its collective interests and contribute to the network's resilience* (Beech, 2004).

The raised number of choices also contributes to increase the fitness of the whole in its environment because the number of available configurations is also increased. This increase of flexibility is often made at the price of a diminution of global performance; chances are that loosely coupled elements will encounter interoperability or communication limitations for instance, lowering performances of the whole.

[69] Only a limited number of concepts are shown for clarity purposes.

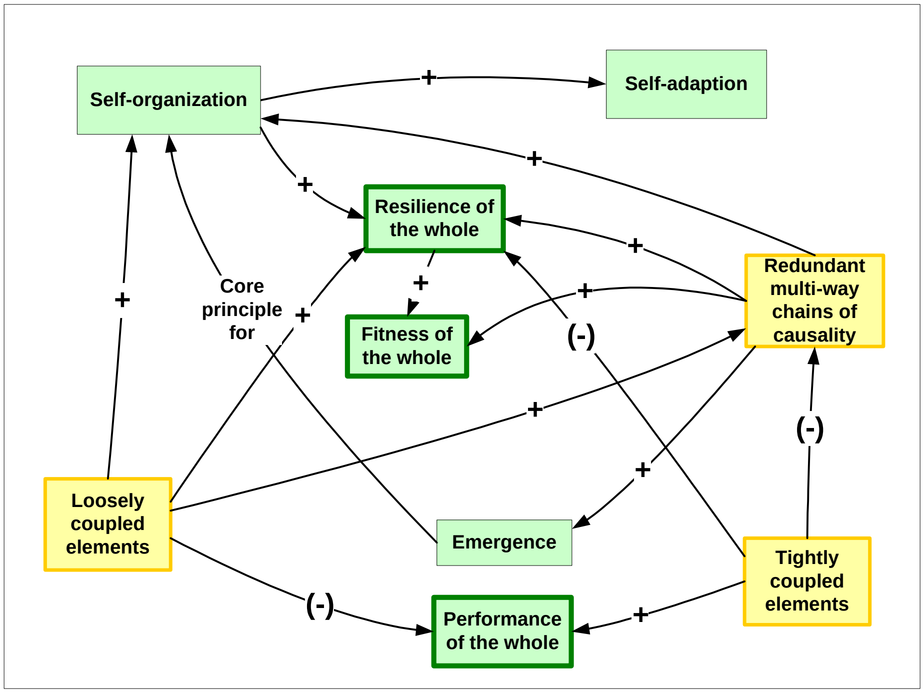


Figure 28 Interrelationships between CASs' phenomena, features and properties.

On the opposite, tightly coupled elements often involve rigid structures (like in linear systems; Section 2.5.2). Their performance is increased because their elements are made to work always in the same ways and interoperability problems were solved at conception. This rigidness contributes to lower the degree of resilience and the flexibility of the whole CAS; it has limited redundant multi-way chains of causality. Linear systems are less able to re-combine in different configurations when unforeseen situations happen.

3.4.1 Emergence – A Fundamental phenomenon of CASs

The concept of emergence is neither a new concept (Ablowitz, 1939; Morgan, 1923; Peper, 1926) nor it is exclusive to any scientific field or domain. It first appeared in philosophy (Mill, 1843; Lewes, 1874) and is still the object of intense researches in the domain of CASs[70]. Economy (enterprises), biological ecosystems (species), the human brain (groups of neuron cells), developing embryos (groups of cells) and ant colonies (groups of ants) are all examples of complex systems (CASs) that manifest emergence.

Emergence is a complex phenomenon that distinguishes CASs from other complicated multi-component systems (Prokopenko and Wang, 2004b); *multiple links among the components may achieve efficient interaction and control with fairly predictable and often*

[70] See for instance: Steels (1992); Deneubourg et al. (1992); Gilbert (1995); Bonabeau and Dessales (1997); Bonabeau et al. (1998); Barabasi abd Albert (1999); and Deneubourg et al. (2002). Kubic (2003) lists some other papers that can be read as introduction to emergence. Some of them are: Cariani (1989); Bonabeau et al. (1995a and 1995b); Brooks (1991); Hillis (1988); Kelemen (2000); Klee (1984); Langton (1989); Matarié (1994); and Minsky (1986).

preoptimised properties. It is the intricate interactions between elements that make CASs able to exhibit emergence. Elements alone cannot produce emergence; again, the whole is greater than sum of its parts. Two main movements dedicated to the study of emergence were identified by DeWolf and Holvoet (2005)[71]. They are:

- **(1) The emergent evolutionism or proto-emergentism**. The term emergence was taken up in the 1920s by *a loosely joined movement in the sciences, philosophy and theology known as emergent evolutionism*. (But the process of emergence itself remained hardly knowable.) *The concept of emergence was hotly debated and mainly used against reductionism, which stated that a system can be reduced to the sum of its parts.*
- **(2) The neo-emergence or complexity theory**. This second movement tries to address the lack of understanding of emergence. Experiments with computer programs known as cellular automata showed that simple interactions between simple agents could give rise to surprisingly complex behaviour (Langton 1986, Holland 1996, Kauffman 1995, and others).

There are many reasons why the concept of emergence gathers so much attention in the scientific literature. Some of them are given by Boschetti's et al. (2005) state-of-the-art:

- *Emergent behaviour seems to be ubiquitous in Nature.*
- *Standard analytical tools used in physics do not seem to be able to describe the generation of 'novelty''.*
- *Computational tools have allowed us to model examples of emergence and shown that it is 'easy' to generate emergent features. Nevertheless, how emergence works and what is it is still not clear.*
- *Fundamental physical equations (theory of everything) still are of no use in describing macroscopic phenomena and the world as we see it. (Reference to the Theory of everything being the theory of nothing). The understanding emergence is a crucial missing component in our understanding of the world.*
- *Because we believe the certain basic emergent properties are shared by very different systems, steps forward in our understanding and modeling of emergence would have huge practical implication for disparate applications.*

3.4.1.1 Definitions of emergence

Currently, there is not an agreement on a definition of emergence. Some definitions are proposed in this Section, they were extracted from Boschetti et al. (2005a, 2005b). The one used in this document is from DeWolf and Holvoet's (2005).

- ***A property is emergent if it cannot be explained from the properties and interactions of the lower level entities****. Kubik (2003), Shalizi (2001) and Crutchfield (1994) criticize this definition; mostly on the basis that such definition simply implies that we are currently unable to explain its relation to lower level entities. One day, with better scientific knowledge, we may be able to. Consequently such statement is based on a temporary state of knowledge of the observer rather than on an intrinsic property of a system.* (…) *This*

[71] Based on Goldstein's (1999) paper.

point is very important and should be addresses in the framework of the paper on engineering control over agent based modeling.

- ***A property is emergent if it is not displayed by the lower level entities****. A problem with this definition is that basically everything can be seen as emergent. All macroscopic matter is made of atoms, and no individual atoms display the features of the macroscopic material (see Bickhard, 2000). It may be useful, though, if used within the context of specific features we may be interested of. Another reason why this definition should not be disregarded is that a considerable body of work has been done here, in particular in the CA literature.* (…)
- ***A feature is emergent if it can provide better predictability on the system behaviour, compared to the lower level entities****. Shalizi (2001) and Crutchfield (1994) give a formal, information-theoretic definition of the above concept. Interestingly, while the idea of predictability naturally involves an agent (observer), the 'measure' of emergence they provide is observer independent, and thus an intrinsic property of a system. A practical limitation of this approach is that, while a measure is proposed in theory, it is quite hard to actually compute such measure in real systems. Still, the concept has obvious appeal from a practical perspective.*
- *Another approach to emergence* ***involves the concept of 'downward causation'*** *(See Goldstein 2002, Bickhard 2000, Heylighen, 1991). Roughly,* ***a feature is emergent if it has some sort of causal power on lower level entities****. While we assume that lower level entities must have an 'upward' causation on the emergent features, this approach assumes a 2-way causal relation. As an example, we can imagine individuals organising into a city. Their actions affect how the city develops. And the development of the city itself affects the behaviour and interaction of the individuals living in it.* (…) *Nevertheless, this seems to be a quite workable definition, since it clearly goes beyond the reductionist approach to the analysis of complex system. We have so far been unable to find any example of how such approach can be tackled numerically or analytically though.*
- *A few other approaches to emergence, although very different from each other, may somehow be* ***related to the concept of 'logical depth'****, and we consequently will (arbitrarily) group them together. 'Logical depth' is an information-theoretic measure of the time a (universal) computer takes to perform a task (for a more formal definition see Bennett, 1988). A system is then considered complex, and/or certain features emergent, if it exhibits high logical depth, or if a feature can not be computed any faster than the time Nature took to produce it, or if it can be modelled only via simulation on a computer. The arguments are discussed at different level of depth (from very trivial to extremely insightful) in Bedau (1997), Darley (1994), Bennett (1988, 1990), Kauffman (1996), D'Abramo (2002). On one side they can be trivialized to limiting emergent to what needs to be simulated on a computer, while on another level it discusses the limits of computability and modelling and the role of natural evolution as computation device*.
- *Another broad category (which we also probably join arbitrarily)* ***relates emergence to semantics and meaning****. This emphases the importance of the context the features are analysed in. Obviously, this approach is observer-dependent. Belonging to this category is Edmonds 1999 (who refers more explicitly to linguists), Pattee 1997a,b (who analyses of the meaning of language and dynamics in biological evolution) and Kubik (2003). Vaguely related to this is also the hierarchical epsilon machine analysis of Crutchfield (1994). I*

believe this approach, going past information theoretical measures and exploring semantics, would be extremely useful, especially for social sciences application.

- *Finally, we have the view of emergence* ***linked to the concept of evolution and its computational simplification in the form of Genetic Algorithms and related ideas*** *(Holland 1998, Boden 1994). This can be applied to biology as well as to social science, memes, ect. Views that Darwinian-type evolution is the only engine for emergence are stated in Heylighen (1991) and Pattee (1995, 1997) among others.*

Another interesting definition of emergence is proposed in the following lines. It is based on DeWolf and Holvoet's (2005) review of literature on this subject and it is the one used in this document.

A system exhibits emergence when there are coherent emergents at the macro-level that dynamically arise from the interactions between the parts (or elements) at the micro-level. Such emergents are novel w.r.t. the individual parts of the system.

Some other ideas from other authors can be added to this definition: *Emergent phenomena are typically persistent patterns with changing components (Holland, 1998). Fromm (2005a)* mentions on these components: *they are changeless and changing, constant and fluctuating, persistent and shifting, inevitable and unpredictable. Moreover an emergent property is a part of the system and at the same time it is not a part of the system, it depends on a system because it appears in it and is yet independent from it to a certain degree.*

To our knowledge, these two last paragraphs represent a good definition of emergence because they put in relation all relevant concepts in a concise and complete formulation.

3.4.1.2 The Flock of birds classical example

Examples of emergence and self-organization are numerous in nature and in computational sciences (Flake, 1998). The numerical simulation of flock of birds is a classical CAS example showing emergence. In this example, flying birds are independent computational entities that are showing spatial arrangements that are similar to the ones of migrating gooses. The simulation *consists of numerically simulating the formation of structures within a flock of flying birds by specifying a set of simple rules that are shared by all birds* (Flake, 2003). Typical **V structures** emerge with respect of time.

These simulations do not impose rules at the level of the flock; there is not for instance a bird that controls all other birds. Each individual computational entity has its own set of simple rules (called *tricks* by Fromm, 2005a), which for instance takes into account the distance and angle between birds. Emergence results from the interactions between these computational entities. *If the flock forms, it does so from* ***bottom-up*** *as an emergent phenomenon, not top-down* (Flake, 1998).

Once the emergent V structure has emerged, some external changes from the environment may induce the emergence of new spatial arrangements. The detection of a flying predator by one of the flying birds will for instance alter this V structure. This bird will naturally change its own flying direction in order to get rid of the predator (based on the same set of internal rules). All other birds will react accordingly, even if they did not see the predator; knowing that something is happening. Neighbours of neighbours will do the same and so on; a new structure will then emerge. The emergence of this new structure is also the result of interactions between flying birds that are guided by the same shared set of internal rules.

Similar evolutions of spatial arrangement can be seen in shoals of fishes when a predator suddenly arises nearby. This disturbance does not break the order in the spatial arrangement; it only modifies it, triggering the emergence of new patterns. Emergent patterns are broken if disturbance effects overtake CAS's ability to adapt based on internal rules. This would happen for instance if the predator actions (flying toward and within the flock) are much faster then the flock global harmonious reactions.

3.4.1.3 Description and characteristics of emergence

There are at least three characteristics that summarize the concept of emergence (CALRESCO, 2006). They are reproduced in the following lines.

- ***First is the idea of 'supervenience'*** *([72]), this means that the emergent properties will no longer exist if the lower level is removed (i.e. no 'mystically' disjoint properties are involved).*
- ***Secondly the new properties are not aggregates***. Emergence is directly related to the idea that the whole is greater than the sum of all its parts taken separately. Emergence of a phenomenon cannot be decomposed into parts; it is said to be irreducible.
- ***Thirdly there should be causality** - emergent properties are not epiphenomenal (either illusions or descriptive simplifications only). This means that the higher level properties should have causal effects on the lower level ones - called 'downward causation'* (Figure 22)*, e.g. an amoeba can move, causing all its constituent molecules to change their environmental positions (none of which however are themselves capable of such autonomous trajectories). This implies also that the emergent properties 'canalize' (restrict) the freedom of the parts (by changing the 'fitness landscape', i.e. by imposing boundary conditions or constraints).*

Irreducibility represents a serious problem for architects and engineers trying to conceive CASs that would deliver pre-determined emergences or capabilities. Standish (2001) mentions on this: *Of considerable interest is, given a system specified in its micro language, does it have emergent properties? There is no general procedure for answering this question. One has to construct a macro description of the system. If this macro description contains atomic concepts that are not*

---

[72] Supervenience is a term borrowed from Psychology. It is defined as: *a dependency relation between 'higher-level' (e.g. mental) and 'lower-level' (e.g. physical) properties. Informally, a group of properties X supervenes on (alternatively, is supervenient on) a group of properties Y exactly when the X-group properties are determined by the Y-group properties, where "determined by" is taken non-specifically. Formally, X-group properties supervene on Y-group properties if and only if either of the following holds for all objects a and b: a and b cannot differ in their X-group properties without also differing in their Y-group properties. If a and b have identical Y-group properties, then they also have identical X-group properties. If a and b do not have identical X-group properties, then they also do not have identical Y-group properties. (All of these formulations are logically equivalent, so if one of them holds, all of them do.)* (Wikipedia, 2006)

simple compounds of micro concepts, then one has emergent properties. (…) *emergence is not due to the failure of the micro description as a modeling effort. An emergent phenomenon is simply one that is described by atomic concepts available in the macro language, but cannot be so described in the micro language.*

DeWolf and Holvoet (2005) give another set of characteristics related to emergence. They are reproduced in the following lines:

- ***Micro-Macro effect****: This is the most important characteristic and is mentioned explicitly in most literature. A micro-macro effect refers to properties, behaviours, structures, or patterns that are situated at a higher macro-level and arise from the (inter)actions at the lower micro-level of the system.*
- ***Radical Novelty****: The global behaviour is novel w.r.t. the individual behaviours at the micro-level, i.e. the individuals at the micro-level have no explicit representation of the global behaviour. In terms of reductionism this is formulated as: the macro-level emergents are not reducible to the micro-level parts of the system (= non-reductionism).*
- ***Coherence****: Coherence refers to a logical and consistent correlation of parts. Emergents appear as integrated wholes that tend to maintain some sense of identity over time (i.e. a persistent pattern).*
- ***Interacting Parts****: The parts need to interact – parallelism is not enough. Without interactions, interesting macro-level behaviours will never arise.*
- ***Two-Way Link****: In emergent systems there is a bidirectional link between the macro-level and the micro-level. From the micro-level to the macrolevel, the parts give rise to an emergent structure (see 'micro-macro effect' above). In the other direction, the emergent structure influences its parts.*
- ***Dynamical****: In systems with emergence, emergents arise as the system evolves in time.*
- ***Decentralised Control****: Decentralised control is using only local mechanisms to influence the global behaviour. There is no central control, i.e. no single part of the system directs the macro-level behaviour. The actions of the parts are controllable. The whole is not directly controllable.*
- ***Robustness and Flexibility****: The need for decentralised control and the fact that no single entity can have a representation of the global emergent, implies that such a single entity cannot be a single point of failure. Emergents are relatively insensitive to perturbations or errors. Increasing damage will decrease performance, but degradation will be 'graceful': the quality of the output will decrease gradually, without sudden loss of function. The failure or replacement of a single entity will not cause a complete failure of the emergent. This flexibility makes that the individual entities can be replaced, yet the emergent structure can remain.*
- ***Diversity:*** *Diversity of elements, relationships and/or rules or values can be added to the list. The greater the diversity in an organisation, the greater the 'possibility space' (or emergence) which it can explore.*

Emergence also manifests when the system evolves in the *zone of creative adaptability between stable and unstable states of complex dynamical systems, dynamically maintained in a situation* *sometimes referred to as self-organized criticality (Bak, 1996), far from equilibrium (FFE), or the edge of chaos (Holland, 1996, 1998; Kauffman, 1993, 1995; Langton, 1989)* (Shetler, 2002).

Fromm (2005a) used results from other authors[73] to build a new taxonomy for emergence. It is based on *different feedback types and the overall structure of causality or cause-and-effect relationships, which fits perfectly to the classification of Eppstein for CA.* Using this taxonomy, the different types of emergence can be grouped roughly through the four following types or classes:

- ***Type I** contains no feedback at all, only "feed-forward".*
- *The major characteristic of **Type II** is simple feedback: (a) positive or (b) negative.*
- *Multiple feedbacks, learning and adaptation are important for **Type III**. John H. Holland said about emergence in adaptive systems: "Any serious study of emergence must confront learning" (Holland, 1998). Type III appears in very complex systems with many feedback loops or complex adaptive systems with intelligent agents. It is the class with a large amount of external influence during the process of emergence (the internality/ externality dimension of Heylighen and Bar-Yam's system to environment relational property).*
- ***Type IV** emergence is characterized in the words of Heylighen [Heylighen91] by multi-level emergence and a huge amount of variety in the created system, i.e. the number of possible states of the emergent system is astronomical due to combinatorial explosion. It is the form of emergence which is responsible for structures on a higher level of complexity which cannot be reduced, even in principle, to the direct effect of the properties and laws of the elementary components.*

In terms of constrained generating processes or roles, Type I corresponds to fixed roles, Type II to flexible roles, Type III to the appearance of new roles and the disappearance of old ones and Type IV corresponds to the opening of a whole new world of new roles.

Intentional emergence of Type I is predictable, weak emergence of Type II is predictable in principle, multiple emergence of Type III is not predictable at all, and strong emergence of Type IV is not predictable in principle (Fromm, 2005a).

3.4.2 Self-organization and self-adaptation

A description of the essence of self-organization is given for network systems in CALRESCO (2006). Elements[74] of CASs are constantly *reassessing their need preferences and the degree to which they will compromise to bond*[75] with other elements. The CAS adapts through the process of *compromise and competition, called correlation, in which each entity* (element of the CAS) *accepts, rejects or changes its relationship with other agents* (elements) *based upon its needs and the changing environment.* (…) *function emerges when system components self-organise into highly versatile organisational structures that try to react to external constraints or an external environment.* In complex networks adaptation is spontaneous, because *innovation emerges from*

---

[73] Chalmers (2002); Bedau (2002); and Bar-Yam's (2004).
[74] Recall that elements of CASs are autonomous.
[75] Kauffman referred to the interdependent bonding of system's elements as "coupling" (Beech, 2004).

the constituent parts rather than a single directing intelligence; for many types of CAS, control is decentralized among elements (Section 3.3.12).

What is usually referred to as self-organization is *the spontaneous formation of well organized structures, patterns, or behaviors, from random initial condition.* Systems that self-organize *possess a large number of elements or variables, and thus very large state space* (Rocha, 1998). Most of the time, the purpose of a CAS *is not explicitly designed, programmed, or controlled*. Its elements interact freely with each other and with the environment, *mutually adapting so as to reach an intrinsically "preferable" or "fit" configuration* (attractor and fitness landscape; Sections 2.6.7 and 2.6.8)*, thus defining the purpose of the system in an emergent way.*

A given CAS *is always bound to the complexity its attractor allows* (Rocha, 1998). The study of self-organization of real-life CASs is equivalent to investigating their attractors, their form and dynamic (CALRESCO, 2006). *When started with some initial conditions they* (CASs) *tend to converge to small areas of this space (attractor basins) which can be interpreted as a form of self-organization. This process of self-organization is often interpreted as the evolution of order from a disorder start* (Rocha, 1998).

Rocha (1998) also mentions that the process of self-organization is often interpreted as *the evolution of order from disorder start*; it manifests by an apparent increase of order. Extropy is the term used to denote *the tendency of systems to grow more organised, in opposition to the entropy expectation* (CALRESCO, 2006). Apparently, this fact contradicts *the second law of thermodynamics that captures the tendency of systems to disorder* (Prokopenko and Wang, 2004b). Actually, *self-organisation and the loss of entropy occur at the macro level, while the system dynamics on the micro level generates increasing disorder* (Rocha, 1998).

Self-organization may be carried on in different ways depending on the type of CAS considered. Fromm (2005b) lists some of them.

- **Multi-agent systems:** *Self-organization in Multi-Agent Systems (MASs) is closely connected to the phenomenon of "emergence", contrary to other systems.*
- **Physical systems:** *In physical systems with many particles, self-organization is associated with self-organized criticality [Bak96], critical points and phase transitions.*
- **Network systems:** *In net systems with connected nodes, self-organization is realized through rewiring (small-world nets) or "preferential attachment" (scale-free nets).*
- **Living systems:** *Self-organization in living systems is related to self-regeneration, metabolism and autopoiesis.*

3.5 Properties at the level of CASs

This Section gathers together properties of CASs as **global entities**. They are related to Criterion 4 (Section 3.1). Properties described in this Section are lying at Level 2 of Figure 22.

3.5.1 Ability of CASs to generate novelty

Novelty arises from the operation of CASs because their behaviour are not readily understood from the behaviour of their elements. Intricate non-linear interactions between elements of a CAS trigger the emergence of complex behaviour (at Level 2; Figure 22) that is more than the sum of each element's individual behaviour.

The global behaviour is *novel with respect to the individual behaviours at the micro-level; the individuals at the micro-level have no explicit representation of the global behaviour* (De Wolf and Holvoet, 2005).

3.5.2 Dependency on initial conditions and perturbations

CASs are sensitive to initial conditions and perturbations (see Section 2.6.9.1 for an example), *variations of inputs characteristics or rules are not correlated in a linear fashion with outcomes* (Chan, 2001). Two identical CASs operating in the same environment, which input conditions are slightly different will not behave the same way. This is particularly true if they operate near the edge-of-chaos (Williams, 2001). *Chaologists refer to such trait as sensitive dependence on initial conditions, or simply sensitivity to initial conditions. ("Initial" in this sense means any time at which we begin comparing system's behaviours).*

Edward Lorenz was among the first to study this dependency in the 1960s. Using computers he simulated the long term evolution of weather by using a simplified version of the Navier Stokes equations (Section 2.6.7.4). By slightly varying initial values of temperature, pressures and other parameters, he found solutions showing new type of behaviour patterns. Very small variations in initial conditions in the weather system (the CAS) led to **unpredictable behaviours**, even if all elements in the complex system were causally connected, in a deterministic way[76]. The current state of the weather is not a predictor of what it will be at mid and long time scales, because tiny disturbances can produce exponentially divergent behaviour.

Lorentz found that to improve weather predictions at these time scales, he had to **significantly raise the precision of numeric variables** (the number of bits of each numeric variable of the Navier Stokes equation).

The consequences of Lorentz's mathematical discovery are profound; **precise long term predictions of weather would demand an infinite precision of computer variables**.

More generally, real systems, especially living organisms, are fundamentally unpredictable in their behaviours (Chan, 2001). Long-term prediction and control of CASs are therefore believed to be very hard or impossible. *Unpredictability is the most common interpretation of sensitive dependence on initial conditions* (for systems at the edge-of-chaos). *"Unpredictable" here refers to the future state, condition, or behaviour. State more formally, extreme sensitivity to initial conditions, combined with the inevitable measuring errors, round off errors, and computer precision, imposes limits on how accurately we can predict the long-term temporal behaviour of any chaotic process. Beyond a certain time, long-term behaviour looks random, is indeterminable, and cannot be reliably predicted* (Williams, 2001).

[76] The simulation of the Logistic Equation shows similar results (Section 2.6.9.1).

There are two direct implications to this limitation:

- (1) Measurements of any variable or parameter over time will not improve long term forecasts of system's behaviours operating near the chaos state domain.
- (2) Tiny variations will probably cause strong unpredictable variations of system's behaviour at mid to long time scale.

Another important fact about initial conditions is described by Williams (2001): *initial conditions are irretrievably lost. The equation or systems behaviours are not reversible. Attractors result in the merging of historical positions. Thus irreversibility is inherent in the concept. Many scenarios can result in the same outcome; therefore a unique logical reduction that a state arose from a particular predecessor (backward causality) is impossible, even in theory.*

3.5.3 Evolution

The long term evolution of CASs is the result of the sum of all internal modifications over time. For instance, elements, interrelationships, rules, values, beliefs and internal models may experience permanent and determinant changes over time.

The simulation of Holland's (1996) model reproduces long term evolution of CASs.

3.5.4 Fitness

The fitness of a network or CAS is proportional to its degree of emergence and resilience; *said another way, its ability to self-propagate and recuperate. A fit network has to have three main elements: first it must have a multitude of individual entities; second those entities must be compelled by a need to interact; and third the network must possess a balance of loose, moderate and tight coupling appropriate to its needs* (Beech, 2003).

Tightly coupled networks are vulnerable for disruption *because damage to one part of the network can easily surge across numerous linkages causing network wide damage. A multitude of loose and moderately coupled interrelationships allows network to dissipate the impact of assaults or environmental changes* (Beech, 2003).

3.5.5 Resilience

Beech (2004) defines the resilience of complex networks as: their capability *to absorb or recuperate from assaults on its constituent parts.* The resilience of CASs can be attributed primarily to their self-organizing characteristics. He mentions that strong hierarchical organizations for instance cannot be as resilient as CASs *because the power of complex networks resides not within its leadership or a few capabilities, but within its ability to spontaneously adapt to changes in the surrounding environment. Consequently, multidirectional and redundant pathways of interdependent relationships allow networks to survive assaults on its constituent parts.*

Coupling between elements of CASs may also influence its resilience.

3.5.6 Robustness

There is not an agreement on definition of robustness; it is still the object of discussion among the scientific community. The discussion group from the Santa Fe Institute (http://discuss.santafe.edu/robustness/) proposes many variants of this definition.

In another study, Jen (2003) makes the difference between stability and robustness and then provides more precisions on what is meant by robustness. Her definitions are reproduced in the following lines.

Robustness (I): *Robustness is a measure of feature persistence for systems, or for features of systems, that are difficult to quantify, or to parametrize (i.e., to describe the dependence on quantitative variables); and with which it is therefore difficult to associate a metric or norm.*

Robustness (II): *Robustness is a measure of feature persistence in systems where the perturbations to be considered are not fluctuations in external inputs or internal system parameters, but instead represent changes in system composition, system topology, or in the fundamental assumptions regarding the environment in which the system operates.*

Robustness (III): *Robustness moreover is especially appropriate for systems whose behavior results from the interplay of dynamics with a definite organizational architecture. Examples of organizational architectures include those based on modularity, redundancy, degeneracy, or hierarchy,* (and heterarchy) *among other possibilities, together with the linkages among organizational units.*

Note that robustness is meaningful for heterarchical and hierarchical systems only when accompanied by specification of the "level" of the system being so characterized. In other words, presence or absence of robustness at one level does not imply presence or absence at another level, and perhaps the most interesting cases are those in which the interconnections among components not themselves robust give rise to robustness at the aggregate level [20, 34].

3.5.7 Self-organized criticality

The definition of self-organized criticality used at the Santa Fe Institute[77] is reproduced in the following lines.

- *In physics, a critical point is a point at which a system changes radically its behavior or structure, for instance, from solid to liquid. In standard critical phenomena, there is a control parameter which an experimenter can vary to obtain this radical change in behavior. In the case of melting, the control parameter is temperature.*
- *Self-organized critical phenomena, by contrast, is exhibited by driven systems which* ***reach a critical state by their intrinsic dynamics****,* ***independently of the value of any control parameter****.*

They give the example of Bak's sand pile: *The archetype of a self-organized critical system is a sand pile* (Bak, 1991; 1997)*. Sand is slowly dropped onto a surface, forming a pile. As the pile*

[77] See: http://www.santafe.edu/~hag/internet/node9.html.

grows, avalanches occur which carry sand from the top to the bottom of the pile. At least in model systems, ***the slope of the pile becomes independent of the rate at which the system is driven by dropping sand****. This is the (self-organized) critical slope.*

Critical states of a system are signaled by a power-law distribution (Section 2.6.4) *in some observable.*

- *In the case of a solid-liquid transition, one can measure the heat-capacity of the system.*
- *In the case of sand-piles, one can measure the distribution of avalanche sizes.*
- *In the present case of internet access, curiosity is measured.*

The analogy with sand piles is clear: a grain dropped onto the pile corresponds to an initial access to the document. The size of an avalanche corresponds to depth of reading of a document. In order to maintain a critical slope in a sand pile in a finite geometry, sand is removed at the edges of the pile. One can think of the sand pile as sitting on a table. Sand falls off as it reaches the edge of the table. The same process could be operating in the case of hypertext access to a document: once readers have achieved a certain depth in the document, they may decide that the document is sufficiently useful to them that they should obtain a hardcopy. At that point, they will stop issuing http requests and then issue a ftp request to retrieve the full document.

3.6 The Perception and understanding of complexity

The text contained in this Section is inspired from following works (among others): Xing and Manning (2005); Fioretti and Visser (2004); Flynn et al. (2996); Rocco et al. (2002); Hilburn (2004); Koros et al. (2003); and Histon and Hansman (2002).

3.6.1 Objective formulation and subjective perception of complexity

A clear distinction is made in this Section between: 1. the intrinsic complexities of a system (**the objective formulation of complexity**), 2. its perception by human (**the subjective perception of complexity**) and 3. its understanding by human (**the subjective understanding of complexity**). The example used to make this distinction is depicted in Figure 29.

Figure 29 shows an external observer (the stick man), which is observing a CAS that is evolving in its environment. The observer uses some means (represented by the blue lens) to acquire the information from the CAS and its environment[78]. These means may possibly transform the information all along its path toward the observer[79]. Three points of view are presented in this figure; 1. the observer looking at the CAS; 2. the observer looking at the CAS's environment; and 3. the CAS looking[80] at its environment. Only the first point of view is considered in the following description.

[78] The information may take the form of visual signs, sound, electronic signs, data, etc.
[79] Color changing arrows represent this transformation.
[80] Let us assume that this CAS is made of people that have the ability to examine their environment.

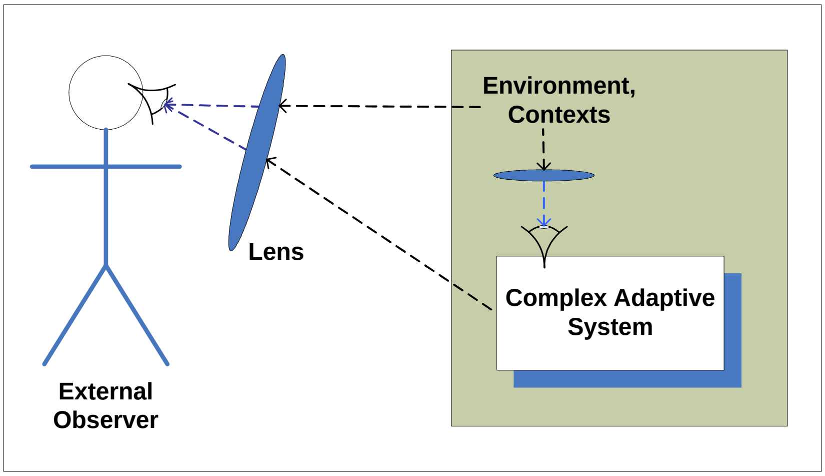


Figure 29 External observation of a system in its environment.

The **intrinsic complexity** is a property that is inherent to the observed complex system; it is **objective** as it is normally independent of any observer or observation. It starts to loose some of its objectivity when it is expressed using current ways of describing (or formulating) the complexity of systems (Couture, 2006b lists and describes some of them). The main reason for this is that there is not a general consensus in the scientific literature on which aspects of complexity should be described and how. Complexity theory has not yet reached the necessary level of maturity and many questions remain to be addressed. For instance, two descriptions[81] of the same complex problem would probably present notable differences, especially when these descriptions involve not yet mature theoretical concepts.

The observer in Figure 29 makes **subjective choice of the means**[82] (the lens) for observing the CAS and its evolving complexity. The observer's **perception** of the system's complexity is thus subjective to this choice. Edmonds (1999) for instance notes that: *complexity necessarily depends on the **language used** to model a system. He argues that effective complexity **depends on the framework** chosen from which to view/model the system of study*. Edmonds' language and framework for viewing the complexity is equivalent to subjective means (lens)[83] in our example. Two observers will probably have different perspectives of the same system if they are observing its complexity aspects through two different sets of means. Their perceptions depend on chosen language, devices and other technologies.

Once the information has been acquired through means, human's cognitive processes start. They **build an understanding** of the captured data. This process involves cognitive mental models (often referred to as **internal models** in this document), knowledge from past experiences and analysis abilities to interpret perceived information and to build an understanding of the system's complexity. Fioretti and Visser (2004) mention about interpretation of organizational complexity: *in order to further our understanding, complexity should be understood in terms of the human cognition of a structure or behavior. (…) complexity as numerosity, diversity, and*

[81] If they are made by two people using concepts from complexity theory.
[82] The lens may for instance represent: metrics, language, protocols, sensors, computers-based software, hardware tools for presentation and interactions, etc.
[83] The framework may also refer to the "objective formulation" of complexity.

unpredictability matters because of the increasing demands it imposes on decision makers concerned with attaining overall organizational effectiveness. But such demands are cognitive in nature.

Human cognitive models are not exactly the same from one person to another. For this reason, the understanding of one complex situation or problem might not be exactly the same from one observer to another, even if the means for acquiring the information are exactly the same. In this sense, human understanding is subjective to used mental models of understanding.

Figure 30 generalizes the above example by showing that any CAS in operation will acquire information through sets of means in order to understand complex aspects of other CASs or environments. Actually, environments may be made of other blue, red and brown CASs, and other natural or man made or natural complex systems. The understanding of the environment is necessary to *improve decision making process, operations planning and inferences* (Fioretti and Visser, 2004).

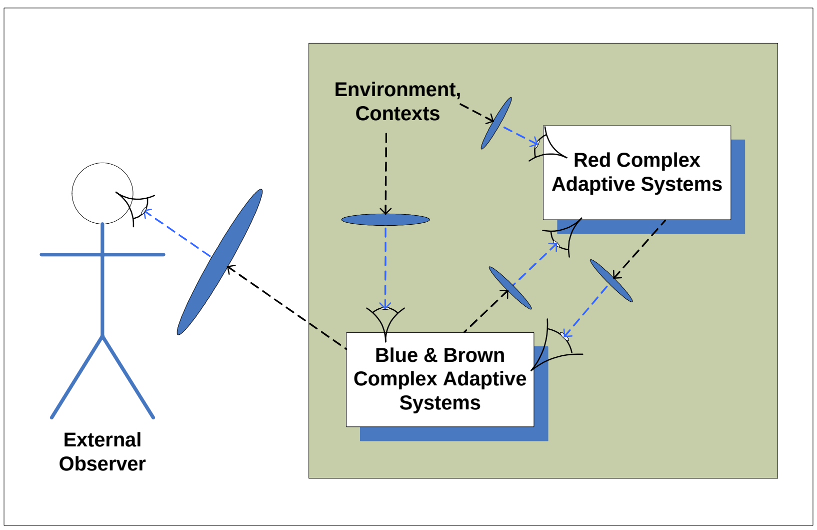


Figure 30 CASs' mutual observations.

The use of concepts of complexity theory has proven to be useful for building human understanding of complex military situations (Marion and Uhl-Bien, 2002; Bar-Yam, 2003d; and Beech, 2004 are good examples). In some cases they help identify vulnerable parts of red CASs and they guide neutralization operations.

R&D efforts are currently deployed in air traffic control (ATC) domain to improve ways operators perceive and understand complex situations. These works aim at studying ATC through the lens of complexity theory in order to find the most appropriate bi-dimensional visual screens that would best improve the subjective perception and understanding of complex situations. They are finding solutions that contribute to ease and faster the building of understanding of complex situations in contexts of high stress. Military complex operations using complex systems present similar problems and needs.

Figure 31 involves the same set of symbols as the ones depicted in Figure 29. It shows an example of a ATC controller dealing with three air planes (CASs[84]). The operator is part of the complex situation because s/he has deterministic effects on the whole system; her/his constant security guidance and orders strongly influence the whole.

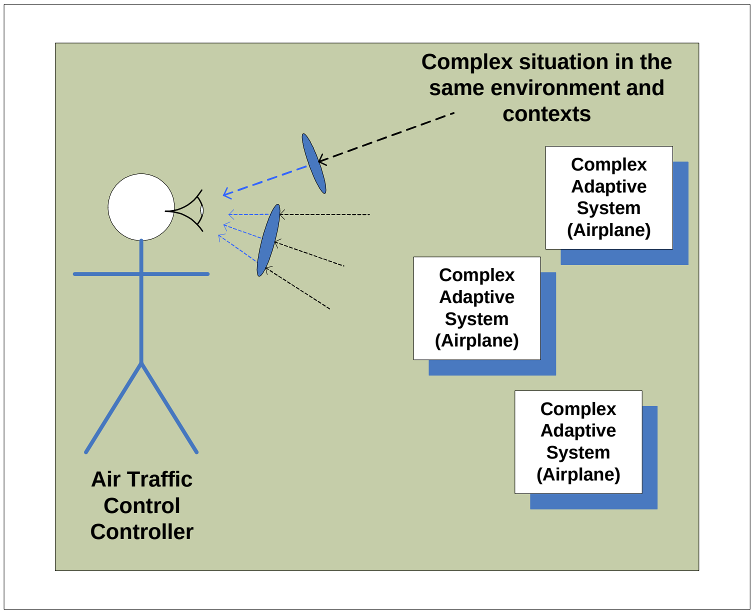


Figure 31 Observation and control of a complex adaptive system in air traffic control.

3.6.2 Internal and external complexity, and adaptation

The environment within which CASs evolve is always more complex than CASs themselves. As shown in Figure 32[85], the input a CAS receives or extracts from its environment has regularities as well as aspects that appear random to the system (Jost, 2003). *Only the regularities are useful for the system because by its very nature, a system will itself be defined by regularities that it constructs from its input and that are maintained through and expressed by internal processes. So the system needs external regularities that it can translate into these internal ones while random input at best is useless and at worst detrimental for the system.* It is the CAS's internal models (used to represent or understand aspects of its environment) that determine *which part of potential input is meaningful and regular and which part is devoid of meaning and structure, and random. In that situation,* ***adaptation consists in increasing the former at the expense of the latter, under the capacity constraints imposed by the system's internal structure****.*

The CAS will use appropriate strategies to increase the amount of meaningful inputs in order to better adapt to its environment. As mentioned by Jost (2003), CASs *try to increase their external*

[84] The content of green rectangle including the controller can be considered as a huge complex adaptive system. From their point of view, airplanes are also complex systems but in the whole's perspective, they are considered as "complex adaptive sub-systems" (Section 2.1.3).
[85] Figure 32 is a conceptual view showing a CAS that is separated from its environment. This "error" was intentionally made for this description only; CAS operates "in" its environment.

complexity ([86]) *and to reduce their internal complexity* ([87]) meaning that *when the system wants to handle additional, new* (and more complex) *input, to increase its external complexity***,** *it may then also first increase its internal complexity* (through the creation of internal redundancy and new combinations of building blocks for instance) *and thereby create the potential for a subsequent reduction of the internal complexity on another time scale perhaps.* (…) *Each of these processes will operate on its own time scale(s), but they are also intricately linked and mutually dependent upon each other*.

Strategies can be used to increase the quantity and quality of meaningful information CASs capture. As external complexity is evaluated by what can be processed by internal models, and as these models are constructed according to which the CAS operates and is modulated by external input (Jost, 2003), it can be said that:

- The CAS may modify or re-adjust on-the-fly the means (the lens in Figure 32) to improve the capture and the transformation of the information related to its environment. The aim will be to improve the subjective perception of external complexity (Section 3.6.1). The pre-selection of what[88] is transmitted to the CAS can be made at this level.
- How this information is presented to human belonging to the CAS can be improved in many ways. Listed references at the beginning of this Section give good examples of such improvements for Air Traffic Control.
- The CAS may recompose, restructure or recombine its teams of data interpreters. Internal redundancy, flexibility, complementarities of expertises and knowledge are factors that may contribute to help this process.
- The subjective understanding of external complexity may be enhanced if human's internal models of understanding are improved. This will contribute to make external data less random and more useful to the CAS.

In military operations, the environment is often dangerous and complex. It influences the CAS in many ways (Figure 32). The CAS must adapt by generating on-the-fly appropriate functions that will in turn influence elements of this environment. Rocha (1998) states similar ideas for cognitive systems: *the self-organizing system must be structurally coupled to some external system* (external to its boundaries) *which acts on structural changes of the first and induces some form of explicit or implicit selection of its dynamic representation: selected self-organization.*

[86] Jost (2003) defines external and internal complexity as: *External complexity measures the amount of input, information, energy obtained from the environment that the system is capable of handling, processing.* (…) *Internal complexity measures the complexity of the* <u>*representation of this input*</u> *by the system* (or internal models).

[87] The aim of the system then is to handle as much input, as many data as possible with as simple a model as possible.

[88] Ideally, the aim would be to reduce the amount of information and to enhance its quality and ease its interpretation and understanding by human.

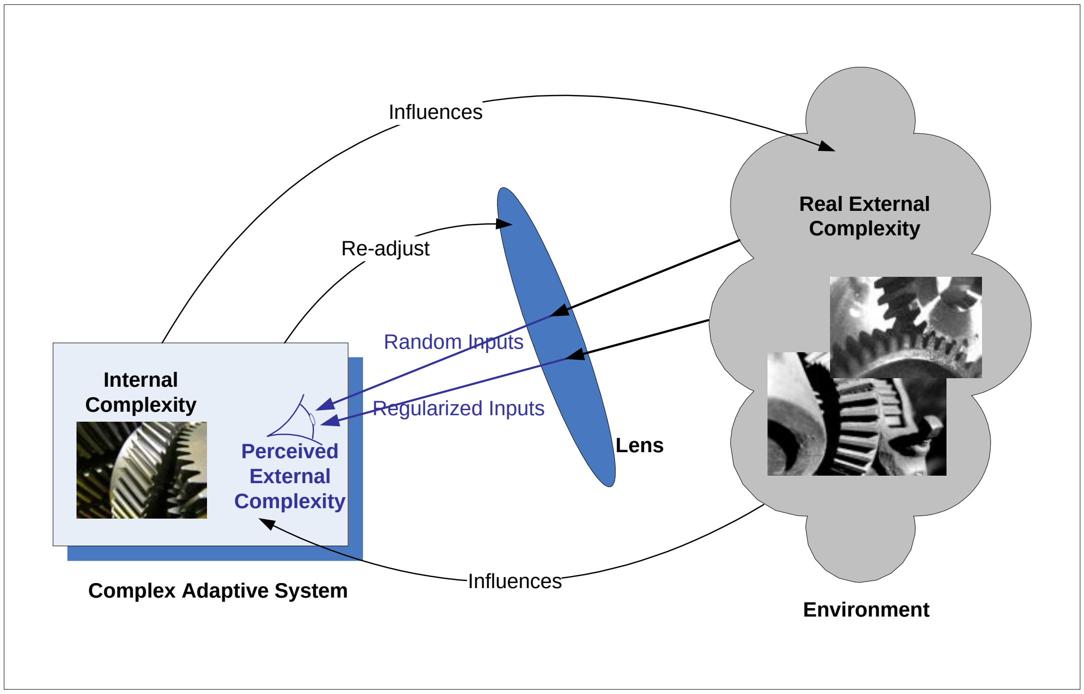


Figure 32 Internal and external complexity.

In summary, Figure 32 shows that the global dynamic the CAS in its environment reflects permanent mutual influences between both of them, perpetual re-adjustments of means (the lens) for the perception of complexity and the perpetual improvement of internal models for the understanding of this complexity; *its potential for adaptations, however, is determined essentially by its internal form* (Jost, 2003).

4 Using complexity theory

It is not possible to catalogue the huge amount of problems and solutions that have been addressed so far by scientists engaged in the complexity theory[89]. Nevertheless, a brief overview of trends and suggestions are provided in this Chapter.

The reader should note that the content of this Chapter does not pretend to be complete. The fifth document of this work to be published in 2007 (see Section 1.3) will provide a more complete review with examples.

4.1 Overview of current trends

4.1.1 Trends in research activities

In there overview of works related to complexity theory, Dodder and Dare (2000) find that research activities in this domain tend to fall under three categories:

- **(1) *Recognizing Patterns of Complexity*.** Santa Fe Institute (SFI, 2006) is a good example of organization that promotes the idea of exploiting the commonalities between complex CASs in order to find underlying principles of complexity theory (Holland, 1996). Some examples of concerns are: *To what extent can we draw comparisons across systems? How do we improve our analytical methods for recognizing and describing the patterns in these systems, in terms of both their structure and dynamic behaviour? Why do some networks or systems persist, despite the continual changeover in the components of the systems? To what extent can we separate a common set of "systems" features or properties that are not context dependent?*
- **(2) *Measuring Complexity*.** Using SFI approach, is it possible to find measures of complexity that are not sensitive to contexts? How CASs originating from different domains or disciplines can be compared? Edmonds (1999) provides a good review of literature.
- **(3) *Modeling Complexity*.** The issue of modeling encompasses two important endeavours.
  - **First**: *how can computer-based modeling be developed further? What are the limitations of these tools?*
  - **Second**: *in studying complexity and phenomena such as emergence, how does one create a "model" without resorting to either extreme of reduction (dissecting and studying the parts in isolation) or abstraction (describing the emergent patterns and macro-structures without much of the detail). A model cannot contain all of the details and complexity of the real life system, but how can one ensure that the model* ***has captured*** (all) ***the critical aspects****?*

Work made at SFI has fostered a unique dialogue across a broad range of disciplines pertaining to both the natural and social sciences. Commonalities of patterns and complex phenomena from

[89]Couture (2006a) provides an incomplete but sufficient list of scientific documents, experts, organizations and journals that helps the finding of information that is related to this theory as well as complex systems.

CASs have contributed to the building of bridges between independent disciplines and resulted in an increase of exchanges of concepts and methodologies between them. *Consequently, as complexity theory develops, it has also provoked thought as to the existence of "universal laws of complexity". Yet, the diversity and constantly evolving nature of complex systems seems to place a limit to the amount of generalizable "laws" that can be derived through complexity. As summarized by Goldenfeld and Kadanoff (1999): "Up to now, physicists looked for fundamental laws true for all times and all places. But each complex system is different; apparently there are no general laws of complexity. Instead,* ***one must reach for "lessons" that might, with insight and understanding, be learned in one system and applied to another****."* (Dodder and Dare, 2000).

The R&D community on complexity is establishing *a forum for interaction, as well as a language, terminology and set of methods for describing and analyzing complex systems.* Couture (2006a) lists for instance many experts, organizations, projects, journals, and conferences on this field and Couture (2006c) provides a Glossary of most used terms. Most references in Couture (2006a) provide indications that *the ideas have increasingly gained acceptance in disciplinary circles, although there continues to be some skepticism as to how far complexity science can progress in providing answers to the burgeoning set of questions and issues it has generated*. (Dodder and Dare, 2000).

4.1.2 The Current types of complexity

Dodder and Dare (2000) identified three categories (or types) that are often used to group *characterizations of system's "complexity"*. They are:

- **(1) Static complexity.** This type of complexity includes notions of *hierarchy, connectivity, detail, intricacy, variety, and levels/strength of interactions*; it refers to structural or physical aspects of a system's complexity. *Static complexity is to a certain extent context dependent, since the structural complexity would appear much differently on the micro versus macro-level scale, and would change as one redefines the scope and boundaries of the system*.
- **(2) Dynamic complexity.** This type of complexity includes notions of *behaviour, processes of cause and effect, feedback, fluctuations and stability, cycles and time scales*. It deals with the temporal evolution of complex phenomena that may *result from both the adaptation of the systems, as well as adaptation of the individual agents in the system*.
- **(3) Informational complexity.** This type of complexity is related to the measurement of complexity, *which can be thought of as the complexity involved in describing or evaluating a complex system. It can reflect both the static complexity, e.g. the intricacy of a network, as well as the dynamic complexity, e.g. the complexity of the processes involved in the creation of a system.*

Other types have been identified. Chris Lucas (CALRESCO, 2006) adds for instance: the Evolution complexity[90] and the Self-organization complexity[91]; the ways these types are orthogonal with the ones of Dodder and Dare (2000) is not clear. Nevertheless, they must be considered because they add new aspects to the description of complexity.

[90] This type appears to be related to the long term evolution of CASs.
[91] This type describes the system functions in terms of how they relate to the wider outside world.

4.1.3 The Current formulations and measures of complexity

The questions "How complex is a system and to what degree?" are the objects of preoccupation for many scientists from many fields[92]; it is still a difficult subject (Edmonds, 1999). Answers to these questions will enhance our capability to understand complex systems.

Dodder and Dare (2000) mentions that *the approaches to the measurement of complexity have tended to couple two complementary aspects:* ***knowledge*** *and* ***ignorance*** *of the system. With respect to the latter, the degree of entropy or ignorance provides one measure of complexity by determining the disorder of the system, which in turns establishes a measure of our ignorance about a system. With respect to the knowledge of a systems, "one apparently crucial element in any reasonable measure of complexity is the information processed or exchanged by the system under study" (Lloyd, 1990). Shannon's information theory uses this quantity of information as an indicator of complexity. Another widely explored measure is the Algorithmic Information Content (AIC), which relates complexity to the minimum amount of information needed to describe the system, as measured by the shortest computer program that can generate that system.*

An ideal set of metrics for measuring the complexity of systems would capture all the information in order to address all aspects of the complexity. As mentioned in Section 3.1, Criteria 1 and 2 provide good hints for the identification of basic metrics. Some domains of measurement are also proposed by CALRESCO (2006) in the following lines.

- Metrics to *explain emergent structures* (for instance self-organization).
- Metrics to *evaluate relative complexity* (hierarchical multi-parameter).
- Metrics to *allow control of complex systems* (steering points).
- Metrics that *lead to the generation of effective models* (abstractions).
- Metrics that *lead to statistical predictors* (constraints).
- Metrics to *solve outstanding problems* (breakthroughs).
- Metrics that *allow the demonstration of possible new applications* (novelty).
- Metrics to *quantify the laws of order and information* (if any).

This ideal set would also *combines how much information is required to describe a system's regularities, as well as the magnitude of the irregularities - i.e. what is the combination of deterministic and chaotic behaviour that gives rise to the complexity of the system?* (Dodder and Dare, 2000).

As mentioned by CALRESCO (2006), aspects to be measured cannot always be represented with numerical parameters; such measure may involve qualifiers like **good**, **better**, **cooperative**, etc. Measures may also involve interrelated aspects of complexity that happen at different time scales; measures are not static entities. Their intrinsic composition and structure may change or evolve with respect of time: *Measures should evolve with respect of time to follow the evolution of complexity in a system over time, in order to follow its path of increasing or decreasing*

[92] Including for instance Thermodynamics, Information Theory, Statistical Mechanics, Control Theory, Applied Mathematics, and Operations Research.

complexity (Dodder and Dare, 2000). The ultimate goal of measures and measurement is to allow for a *mathematics that can distinguish systems as easily as can humans, recognizing and classifying the patterns found therein* (CALRESCO, 2006).

The reader is invited to refer to Couture (2006b) and Edmonds (1999) for an overview on how complexity is currently formulated and measured.

4.2 The Use of complexity theory – Some suggestions

The structure of this Section is being inspired from the traditional scientific methodology. It proposes some suggestions for integrating complexity concepts at each of its step. Figure 33 depicts the proposed methodology for studying complex problems[93].

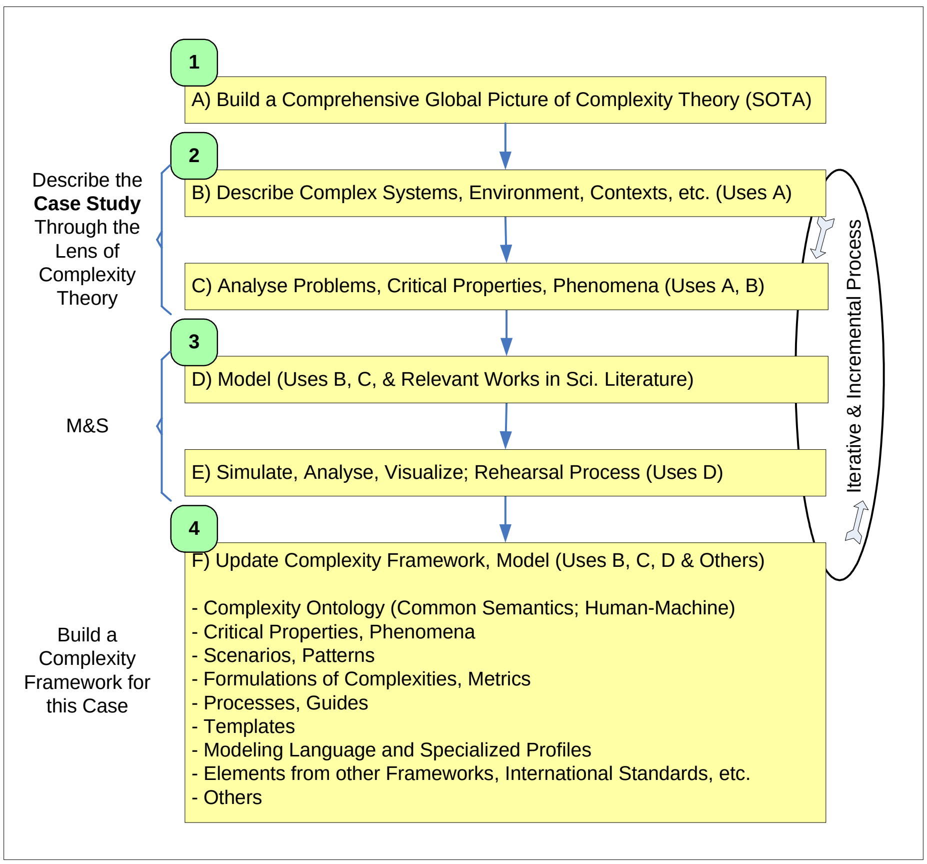


Figure 33 The Proposed methodology for studying complex problems or systems.

The methodology contains four steps (the numbers in the green rounded squares of Figure 33). They are:

1. Build a comprehensive integrated picture of all concepts of complexity theory (represented by the rectangle labelled A in Figure 33). The goal is to understand all aspects of this theory that

[93] A detailed description of this methodology, tools and framework will be given in another document.

will be relevant for the military domain. This step was already made in Couture (2006a, 2006b, 2006c) and this document.

2. Select, describe and analyse a case study using this theory (rectangles B and C in Figure 33).

3. Model and simulate the case study using appropriate approaches, techniques and tools (rectangles D and E in Figure 33).

4. Update and improve a complexity framework with all this information and lessons learned (rectangle F in Figure 33).

Steps 2, 3 and 4 constitute an **iterative and incremental process**. It should be made for every case study. Next sections provide more details for steps 2, 3 and 4.

4.2.1 Step 2B: Description and problem formulation

The identification and description of all critical aspects of complex problems should be achieved prior to the building of models or simulations. Some concerns that might have to be addressed are proposed in the following lines.

- Identify and describe all relevant aspects of systems and discover all interdependencies between them. Identify and describe boundaries (Section 2.1.3), structure (Section 3.3.15), dynamic, command, control, elements, their types, interrelationships, behaviour, properties, types and degrees of complexity, etc.
- Identify and describe all relevant aspects of environments and discover all interdependencies between them. Examples of environments are: cities, crowds, open fields, mountains, bad weather, bad terrain, etc.
- Identify and describe all relevant aspects of contexts and discover all interdependencies between them. Examples of contexts are: maintenance of peace, traditional war, humanitarian missions, etc.
- Identify and describe potential critical aspects and discover all interdependencies between them. These are parts of the problematic that must not be forgotten during this process because of their major impacts or importance on systems or operations.
- Formulate (model) and describe all aspects of internal and external complexity and complex phenomena and formulate (model) all interdependencies between them.
- Identify the relative importance of each aspect of complexity and all cause/effect relationships.
- Identify and describe interrelationships between the CAS and its environment; which elements are involved, which communication links are used, how, when, etc.
- Identify and describe needed emergences (capabilities) and start the identification of potential sources, mechanisms for the manifestation of emergence.
- Pose the problems using this information and all relevant theoretical concepts, past lessons learned, study cases, etc.

- Build a complete and coherent description of the problem. When possible, integrate this description in architecture description.
- Find measures of complexity aspects that could be used during operation. Find how and when to use them.
- Identify and understand all potential effects of simplifications or approximations during modeling. The lost of critical aspects may have dramatic effects on end results of analysis.

4.2.1.1 The Level of complexity of systems

It is necessary to evaluate the level of complexity of systems. Some criteria that can be used for this task are proposed by CALRESCO (2006) (See also Couture (2006b)). This list *does not pretend to be complete nor is it definitive* but it constitutes a good starting point.

- ***Connectivity Profile*** (Section 3.3.5). Elements of a system have many inputs and many outputs but as described in Kauffman's experiment, elements are not too linked for not being into the chaos State Domain. *This is the fan-in and fan-out structure.*
- ***Learning Availability***. Elements of a system have the ability to learn from experience and to integrate lessons learned. They are able to evolve or *change their rule sets and optimise (canalize) transitions.*
- ***Operation Parallelism***. Parallelism and autonomy of elements of a system are good indicators of adaptability and speed response.
- ***Interaction Variability***: Links allowing interrelationships between elements of a system can be modified with respect of time in function of evolving needs. This may allow for instance data prioritization.
- ***Feedback Loops*** (Section 3.3.9). *Outputs have a way to feed back into the beginning of the process, so the results of actions early in chains can be monitored.*
- ***Control Ability*** (Section 3.3.11). *All variables have control paths for stability (uncontrolled variables could indicate chaos potential). But control does not prevent change, just acts to limit runaway effects.*
- ***Basins of Attraction*** (Section 2.6.7). Many trajectories are available to the system for attaining its attractors (goals), *giving flexibility of response and creative freedom.*
- ***External Boundaries*** (Section 2.1.3). *System boundaries are neither closed nor totally open. The first is stagnation, the second panic. Filtering of information is necessary*. The definition of the boundary of a system may also change with respect of time in function of evolving needs.
- ***System Function***. *Multiple objectives or functions exist, giving a multi-dimensional fitness and resilience to single dimensional fluctuations* (Section 3.5.4, 3.5.5).
- ***Building Blocks*** (Section 3.3.3). *Sub-systems at various sizes are found, giving a modular, fractal, structure, each with higher level characteristics that reflect those seen in the parts* (elements of the system).

- ***Emergent Properties*** (Section 3.4). Unplanned phenomena emerge during operations. For instance, the systems self-organize from intricate interactions between elements. There is not an external control that drives this self-organization.

- ***System Resilience*** (Section 3.5.5). *Most internal or external perturbations leave overall function intact, but some show unexpected global effects. A power law spread of fluctuation size and duration is found.*

- ***Distributed Control*** (Section 3.3.11). *Control is distributed throughout the system; local decisions are made by parts or modules* (elements) *within overall constraints.*

- ***Information Flow*** (Section 3.3.10). *Increasing information flows can indicate a move from stability to chaos. Introducing information technology tends to do this naturally in social systems, unless checked.*

- ***Output Variability***. *Increasing swings* (increasing instabilities) *in results (e.g. sales for a company) can indicate a move towards chaos* (Section 2.5, 3.5.2).

- Many others.

A system that shows some of these properties is likely to be better analysable using complexity theory than by linear determinism or reductionism methods. Bar-Yam's MSCA approach and complexity profile should also be considered (Section 2.6.3).

4.2.2 Step 2C: Problem analysis

The analysis of the described problem should be achieved. Some aspects that might be investigated are proposed in the following lines.

- Concurrently use theory of systems, complexity theory and middle-out approach, available past lessons learned and case studies at all phases of the problem analysis and resolution.

- Recombine measures, knowledge, observations (Section 3.3.3) in order to find novelties. The diversity and complementarities of a high number of measures and observations are factors that may ease the discovery of novelties through the recombination process (Holland, 1996).

- Identify from this analysis other potential critical aspects related to this problem. Pose hypotheses on potential sources, causes, mechanisms, solutions, approximations, etc.

- Integrate this information and start modeling.

4.2.2.1 Potential approaches and techniques for problem analysis

Once a system has been identified as complex it must be studied using appropriate approaches and techniques. This Section reproduces CALRESCO's (2006) propositions on potential approaches and techniques. It is not a comprehensive review *since almost every researcher in this new field has their own emphasis.*

- ***Entropy*** (Couture, 2006c adds extropy). *This golden oldie is a good place to start, since it traditionally measures the opposite of order (or information in Shannon's formulation). Unfortunately there are so many types of entropy that the concept proves less than useful in* *practice. The main problem is that a single figure does not distinguish symmetrical or otherwise equally complex systems, and it says nothing about the actual structure present.*

- ***Algorithmic Information Theory*** (Couture, 2006b). *This technique, developed by both Kolmogorov and Chaitin, looks to describing complex systems by using the shortest computer program which can generate the system. Thus the length becomes a measure of the complexity. The drawback is that this has a high value for random noise (which we don't think complex). Such an approach also takes little account of the time needed to execute the program. Work is ongoing to address these issues, but again it reflects a single parameter.*
- ***Phase Transitions*** (Section 2.6.9). *Self-organizing systems are found to move from static or chaotic states to a semi-stable balance between. This property relates to the physics idea of phase transitions (e.g. the state change from ice to water), pioneered by Wilson. Attempts to quantify this point are seen in Langton's work on lambda and similar measures. Chief disadvantage is that such analysis is so far restricted to low dimensional systems (few variables).*
- ***Self-Organized Criticality*** (Section 3.5.7). *This technique, due to Bak, has much in common with Phase Transitions, but concentrates on the characteristic power law distribution of events (seen around the phase boundary) as an indication of self-organization. This allows the treatment of higher dimensional systems, but still gives little information about their inherent nature.*
- ***Algorithmic Chemistry***. *Another approach takes account of the fact that system parts interact freely, thus can be thought of as chemical elements, their reactions form compounds and eventually an autocatalytic set results (forming the system), which is self-maintaining. The mathematical analysis of such systems is largely due to Fontana. This treatment of the parts, whilst allowing innovation, doesn't quantify any emergent structure that results, just concentrations of components.*
- ***Attractors*** (Section 2.6.7). *Identifying the possible stable structures in connected systems requires the concept of attractors, and this idea is employed in work on neural networks by Hopfield, feature maps by Kohonen, and discrete networks by Wuensche. This is the best current technique for analysing internal network structure, but is difficult to do for realistic, high dimensional, systems.*
- ***Coevolution***. *Using the biology concept of fitness allows us to model systems as ecologies, where the parts coevolve with each other. This can be extended to model multiple systems, as in Kauffman's NKCS model, and we can derive system wide fitness measures. A drawback is that there are so many possible models that practical work can only sample them, generating purely statistical indicators.*
- ***Symbolic Dynamics*** (Holland, 1996). *Derived from linguistics, this treats systems as grammars and investigates rules of combination and structure. It is possible to include context in this formulation and thus this can be applied to environmentally situated systems, as seen in the classifier work of Holland. This is promising, but identifying the rules of existing systems is a major problem.*
- ***Far From Equilibrium*** (Section 2.5.3). *The analysis of non-equilibrium systems is also at an early stage, and is characterised by work on dissipative systems in physics (Prigogine) and autopoiesis in biology (Maturana/Varela). These self-maintaining systems are self-organizing structures, but again little direct attention is paid to pattern.*

- Dr. Bar-Yam's MSCA approach and complexity profile should also be considered (Section 2.6.3).

4.2.3 Step3D: Modelling

Wikipedia (2006) defines model as: *An abstract model (or conceptual model) is a theoretical construct that represents something, with a set of variables and a set of logical and quantitative relationships between them. Models in this sense are constructed to enable reasoning within an idealized logical framework about these processes and are an important component of scientific theories. Idealized here means that the model may make explicit assumptions that are known to be false in some detail. Such assumptions may be justified on the grounds that they simplify the model while, at the same time, allowing the production of acceptably accurate solutions.* Models are thus approximate representations of real life objects or concepts.

This same reference defines the process of modeling as: *the process of generating a model as a conceptual representation of some phenomenon. Typically a model will refer only to some aspects of the phenomenon in question, and two models of the same phenomenon may be essentially different, that is in which the difference is more than just a simple renaming. This may be due to differing requirements of the model's end users or to conceptual or esthetic differences by the modellers and decisions made during the modelling process. Esthetic considerations that may influence the structure of a model might be the modeller's preference for a reduced ontology, preferences regarding probabilistic models vis-a-vis deterministic ones, discrete vs continuous time etc. For this reason* ***users of a model need to understand the model's original purpose and the assumptions of its validity****.*

The building of models should make intensive use of results of Section 4.2.1 and 4.2.2. Some aspects that should be investigated are listed in the following lines.

- Decide which aspects of real life operations, systems, environment and contexts will and will not be integrated in models. This process defines used approximations. They may have profound effects on behaviour resulting from simulations.
- When appropriate, identify levels, resolutions, and scales (Section 2.6.1). The modeling and simulation of behaviour of complex oceanic currents for instance often involves models that are well defined at mid and large scales; at micro scales they only approximate physics laws of friction.
- When appropriate, conceive mechanisms allowing models to integrate interrelationships between levels. This may help the linking between sources and manifestation of emergence for instance.
- Find how to integrate and link these models in traditional architecture descriptions (when available).
- Avoid one-size-fits-all types of models or solutions.

4.2.4 Step 3E: Experimentation through simulation

Wikipedia (2006) defines simulation as: *an imitation of some real thing, state of affairs, or process. The act of simulating something generally entails representing certain key characteristics or behaviors of a selected physical or abstract system.*

This same reference defines computer simulation as: *A computer simulation is an attempt to model a real-life situation on a computer so that it can be studied to see how the system works. By changing variables, predictions may be made about the behaviour of the system.*

The simulation of models means: the **running** of numerical models over time.

The form of simulation to be made should be identified. Is it for instance a **one shot** simulation or a **batch** of simulations, which is made of many similar but slightly different simulations that provide huge amount of data? Is it prognostic or diagnostic? Is it virtual or constructive? How results from these simulations will be interpreted? What are the possible methods or techniques?

What can be expected (and not) from these simulations should also be identified and understood. The choice of models, simulators, environments, contexts and other parameters must be made based on the types of analysis that need to be achieved; what types of results are expected from these simulations? As an example, two simulations using two different models of the same CAS in the same environment and context will generally give results showing different aspects of the same complex phenomenon. Aspects that need to be studied will influence choices.

Modeling and simulation (M&S) appears to be an important tool in complexity theory. Nevertheless, precautions should be taken all along the process to keep models and simulators coherent and close to the reality. This Section proposes some of them that might have to be taken into consideration.

- Use appropriate types of simulator (Section 4.2.4.1 and Couture (2006a) lists some of them). The type of simulator used to simulate models over time should be chosen in function of the type of CAS and also in function of the type of analysis to be made.
- Use appropriate space scales. Will the models to be simulated evolve in one, two or three dimensional space? What is the spatial resolution of this space? Will this space be regularly divided (cells) or not? What are the effects on results? For instance, choosing low resolution in space may remove small scales complex effects from results.
- Use appropriate time scales. Is the time evolution of simulations are constant (constant delta-time) or is it variable (event based)? In the case of a constant delta-time, what are the effects of this value on results? Choosing a high value of delta-time may remove small time scales complex effects from results.
- Understand all effects of simplifications or approximations.
- Do prognostic and diagnostic types of simulations.
- Avoid black box types of simulators and models; the ones that do not allow the study and modification of their internal structure and composition. Favour the ones that can be opened, studied, merged with others and enhanced.

- Avoid one-size-fits-all types of simulator. Instead, use many different models with many simulators and compare results to detect different aspects of complex phenomenon and complexity.

4.2.4.1 Potential types of simulation

Some types of simulation and techniques are proposed by CALRESCO's (2006). They are reproduced in the following lines.

- ***Game Theory**. From political science, we have the theory of interactions based on decisions and relative advantage, usually associated in our field with Axelrod. This quantifies decision fitness at an individual pair level, but is harder to apply to more diffuse systems. The important aspect here is the distinguishing of positive from negative evolutionary paths - goal directed behaviours.*

- ***Spin Glasses**. This technique, again from physics, uses a lattice of interacting points and is chiefly seen in complexity work under the guise of cellular automata, which can be used to model many physical phenomena (as in the work of Rucker). The technique, whilst excellent for simulation, proves mathematically difficult, but is important in relation to the demonstration of emergence, higher level structure.*

- ***Time Series Analysis**. Based on communications theory, we look here to identifying regularities in the behaviour of a system over time, trying to quantify cyclic or chaotic (strange) attractors. It is often applied to financial systems, e.g. at Santa Fe. Chief drawback is that the system must have a lot of data available for analysis, but the advantage is that limits can be placed on the system behaviour.*

- ***Fuzzy Logic**. In the analysis of nonlinear systems we need a way of quantifying many interacting variables and fuzzy logic provides this, generating a result that maps all possible interactions of the inputs. This technique, due to Zadeh, has yet to be applied widely to complexity ideas, but has importance in the potential to treat multiple conflicting variables in decision systems.*

- ***Multiobjective Optimization**. This idea, from operational research, recognises the inter-dependency of multiple values in real world cases, and when combined with evolutionary computation allows us to study the dynamics of epistatic systems and the multiple global optima (Pareto fronts) common to such systems. There are many techniques involved here, some involving synergic considerations, for an overview see our introduction PMO.*

- ***System Dynamics**. Largely due to Forrester, this computer modelling technique looks to quantify how the dynamics of systems, based upon our assumptions of how the parts/variables are interconnected (their dependency structure), differs from our beliefs about such dynamics. It highlights the difficulties of predicting actual complex systems behaviour when our views are constrained by the results of over-simplified reductionist experiments.*

- ***Evolutionary Dynamics**. By statistically measuring the diversity, cumulative activity and innovations of evolving systems it becomes possible to classify these in terms of their open-ended evolutionary potential. The technique, due to Bedau, Snyder & Packard, allows the emergent behaviour of artificial and natural systems to be determined, but does require an* *historical record of their component activity to be available. Few, if any, artificial systems currently show any unbounded emergent potential however.*

- ***Multi-Agent Systems**. This technique, based upon artificial life ideas, studies the dynamics of collections of interacting autonomous agents. The self-organization that results from different initial assumptions and sets of agent values helps quantify how different features of real systems can arise, and evaluates their stability to perturbations caused by changes to internal structure and goals. It can be applied to many levels of reality and was pioneered in the social sciences by Epstein and Axtell.*

4.2.5 Step 3E: Interpretation and validation of results

Interpretation and validation of results from simulations are then achieved. Appropriate theoretical concepts, methodologies and techniques should be used.

An important problem that may arise while analysing and interpreting this data is related to the dependency of interpretations on used theoretical concepts, means and tools. Section 3.6 describes this dependency in terms of **subjective perception** and **subjective understanding**. The means used to perceive data and the mental models used to build understandings of this data may have strong influences on interpretations.

The analysis and interpretation of results from these simulations may ease the building of a framework for addressing the problematic.

4.2.6 Step 4: Improvements and optimizations

Once results from simulations have been interpreted, improvements to models, simulators and frameworks can be achieved. New found specificities of the simulated CAS may for instance be compared with the ones of real life observations and then used as basis for further improvements and optimizations.

4.2.7 Iterate

The whole process should be iterated many times, using previous iterations' results, knowledge and lessons learned as a basis for next iterations' improvements.

5 Concluding remarks

An effort has been made to present concepts of complexity theory in a simple but structured manner. This study does not purport to be comprehensive. The principal goal of this document and companions (Couture, 2006a, 2006b and 2006c) is to aid understanding of these concepts and to promote their utilization for practical purposes.

5.1 Main observations

Many high-level observations can be made from this work. Some key observations are briefly described below.

- Complexity theory is still evolving, and is the object of intensive R&D all around the world. It has not reached its final point of maturity. The basic principles and concepts are not always interpreted the same way by different authors.
- Complexity theory appears to be an overarching science, and its concepts may be used in different disciplines or domains.
- Theoretical concepts are not easy to understand. Some reasons are:
  - There is no single text book that defines all the concepts of complexity theory in a consistent and structured manner. Unfortunately, the reader must read numerous books and papers in order to fully perceive the current picture or feeling of this science. Many authors promote their own approach, methodologies, tools and solutions while ignoring those of others.
  - Authors often refer to CAS as a generic entity without providing illustrative examples.
  - Many concepts are abstract and their subtleties are hard to grasp. Moreover, our mental models based on the linear reductionist approach are ill suited for understanding such highly interrelated and interdependent concepts.
  - As mentioned above, authors from different disciplines often interpret key concepts differently.
  - Some concepts and phenomena are still not fully understood. One of them is emergence.
- It is hard to identify the underlying principles of complexity theory. The task of formulating a unifying theory for CASs is particularly difficult because the behaviour of the latter is more than a simple sum of the behaviours of its parts. CASs are highly non-linear and *our usual tools for generalizing observations into theory are badly blunted* (Holland, 1996).
- The traditional, linear reductionist approach alone cannot address complex problems. A holistic strategy encompassing both the top-down and bottom-up (middle-out) approaches appears to be better suited to the study of CASs. New techniques are needed for investigating and visualizing complex phenomena, particularly the design, engineering and maintenance of CASs.

- Complex phenomena are not easy to model. Emergence is one example. It originates from the interaction of CASs elements and manifests at the next higher level of CASs. No models have yet been designed to show how and why emergence takes place in a CAS.
- Security concerns: The rate of threat propagation has followed the rate of penetration of computer, software and network technologies around the world. We have no reason to think that the threat propagation rate will decrease in the near to medium term. This arms race suggests that security measures associated with CASs will have to be improved continuously, making use of complexity concepts at all levels; local, regional, national and international. Some concerns that need to be addressed are vulnerability of data, systems and software; threat awareness, prevention and response; and training.
- It is possible to use theoretical concepts of complexity theory in current military operations, systems, design, engineering, maintenance and training. A framework for the military application of the concepts of complexity theory is currently under development at DRDC Valcartier.
- Transformations and changes in military operations and systems are no longer considered *a clearly definable*, (static) *final objective*, but *an ongoing* (dynamical) *process* (Calhoun, 2004).
- The challenges of developing and sustaining large and complex military systems have grown significantly in the last decade. Today's complex systems and systems-of-systems require architecting and engineering for maximum flexibility and robustness. Some needs are expressed by Rhodes (2004):
    - Engineered solutions must be *capable of adapting to changes in mission and requirements*;
    - They must be *expandable/scalable, and designed to accommodate growth in capability*;
    - They must be *able to reliably function given changes in threats and environment*;
    - They should be *effectively/affordably sustainable over their lifecycle*;
    - They should be *developed using products designed for use in various platforms/systems*;
    - Finally, they should be *easily modified to leverage new technologies*.
- Technology improvements alone are not enough: *Reliance on technology to gain advantage over an enemy is expensive, and typically only results in a temporary advantage before the enemy finds some creative way to regain parity. Even the most significant technologic advances are only truly effective as a complement to doctrinal or conceptual innovation* (Calhoun, 2004).

5.2 Future Works

The aim of this document was to present key theoretical concepts of complexity theory, not including the architecting and engineering aspects.

The next logical steps to be made in thie overarching project are the following:

- **Present complexity theory concepts to CF clients**.
- **With CF clients, assess the potential for addressing current and future military problems**.
- **Formulate a few test cases to model and/or simulate and/or implement**.
- **Evaluate the results with CF clients**.
- **Capture the lessons learned in a complexity framework**.
- **Iterate**.

The following are some important questions that will be considered during this process.

- How are the concepts of complexity theory perceived and understood by humans? Which aspects should be presented to operators? How and when? Should allowance be made (and how) for situations of high stress. What factors should be considered?
- How should the military use systems and conduct operations to deal with increasingly complex situations such as a three-block war?
- What patterns of system behaviour should we anticipate with the use of new and complex military systems? How can we exploit such patterns?
- What are the processes and other factors that keep complex systems coordinated and stable despite highly variable environments?
- How can lessons learned be saved and used on the fly as building blocks to face unforeseen situations?
- How should we improve the ability of complex systems to learn and infer?
- How should complex systems be controlled? What are the levels of control?
- Which tools (with what capabilities) could potentially be used to improve the control of systems and operations?
- How can military systems be designed and/or improved to facilitate the emergence of capabilities that will fit best in unforeseen situations? What are the potential sources of emergence? How and when can we use them?
- What are the patterns that could potentially be used as building blocks at all stages of the military system procurement?
- How can complex systems be made more secure? What are the key vulnerable points of complex military systems in specific situations, environments and contexts? How do the systems evolve over time?
- How should complex systems be designed and specialized in order to enhance robustness, system fit with environment, performance and their ability to quickly self-repair, self-recover, self-organize and self-adapt to current and future unforeseen situations?
- How can we equip military systems with the necessary resilience to attack?
- Many others that were not yet considered.

References

Ablowitz, R., 1939. The Theory of Emergence. Philosophy of Science, Volume 6.

Axelrod, Robert and Michael D. Cohen, 2001. Harnessing Complexity – Organizational Implications of a Scientific Frontier. New York: Basic Books.

Bahill, Terry, Rick Botta and Eric Smith, 2005. What are Levels? From the Proceeding of the INCOSE 15th Annual International Symposium on Systems Engineering, Rochester, NY, July 10-14, 2005.

Bak P, Chen K., 1991. Self-organized criticality. Scientific American, 264(1):46–53.

Bak P., 1997. How Nature Works: the Science of Self-Organized Criticality. Oxford: Oxford Univ. Press.

Bar-Yam, Yaneer and Ali Minai, 1997. Unifying Themes in Complex Systems: Proceedings of the First International Conference on Complex Systems. HarperCollins Canada / Westview S/Dis ISBN: 0813341248, number of pages: 648.

Bar-Yam, Yaneer, 1999. Dynamics of Complex Systems. HarperCollins Canada / Perseus Books ISBN: 0201557487, number of pages: 848.

Bar-Yam, Yaneer, 2002. Complexity rising: From human beings to human civilization, a complexity profile. In Encyclopedia of Life Support Systems (EOLSS), developed under the Auspices of the UNESCO, EOLSS Publishers, Oxford, UK, http://www.eolss.net, 2002; also NECSI Technical Report 1997-12-01 (December 1997).

Bar-Yam, Yaneer, 2002b. Large Scale Engineering and Evolutionary Change: Useful Concepts for Implementation of FORCEnet. Report for Contract: F30602-02-C-0158, Multiscale Representations Phase II: Task 1: Implementation of Innovation in FORCEnet, can be found at: http://necsi.org/projects/yaneer/SSG_NECSI_2_E3_2.pdf.

Bar-Yam, Yaneer, 2003a. Unifying Themes in Complex Systems Volume II. HarperCollins Canada / Westview S/Dis ISBN: 081334123X, number of pages: 644.

Bar-Yam, Yaneer, 2003b. Dynamics of Complex Systems. HarperCollins Canada / Westview S/Dis ISBN: 0813341213, number of pages: 864.

Bar-Yam, Yaneer, 2003c. Unifying Themes in Complex Systems. HarperCollins Canada / Westview S/Dis ISBN: 0813341221, number of pages: 696.

Bar-Yam, Yaneer, 2003d. Complexity of Military Conflict: Multiscale Complex Systems Analysis of Littoral Warfare. Report for Contract: F30602-02-C-0158, Multiscale Representations Phase II: Task 2: Multiscale Analysis of Littoral Warfare, number of pages: 27, could be found in 2006 at: http://necsi.org/projects/yaneer.

Bar-Yam, Yaneer, 2003e. When Systems Engineering Fails --- Toward Complex Systems Engineering. Written for the 2003 IEEE International Conference on Systems, Man & Cybernetics, October 5–8, 2003, Washington, DC, U.S., can be found at: http://necsi.org/projects/yaneer/E3-IEEE_final.pdf.

Bar-Yam, Yaneer, 2004. A Mathematical Theory of Strong Emergence using Multiscale Variety. Complexity, Volume 9, Issue 6.

Bar-Yam, Yaneer, 2004b. Multiscale Variety in Complex Systems. NECSI Technical Report Number 2003-11-01. Or in: Complexity, Volume 9, Number 4. Can be found at: http://necsi.org/faculty/bar-yam.html.

Bar-Yam, Yanner, 2004c. Multiscale Complexity – Entropy. This report can be found at: http://necsi.org/projects/yaneer.

Bar-Yam, Yaneer, 2005. Making Things Work: Solving Complex Problems In A Complex World. Knowledge Press, ISBN: 0965632822, Number of pages: 306.

Bar-Yam, Yaneer, 2006a. Unifying Principles in Complex Systems. In Converging Technology (NBIC) for Improving Human Performance M. C. Roco and W. S. Bainbridge, Eds, in press.

Bar-Yam, Yaneer, 2006b. Introducing Complex Systems. Number of pages: 56, this report can be found at: http://necsi.org/projects/yaneer.

Barabasi A.L. and R. Albert, 1999. Emergence of scaling in random networks. Science, Volume 286, October 1999.

Beckerman, Linda, 2000. Application of Complex Systems Science to Systems Engineering. Systems Engineering, Volume 3, Number 2, could be found in 2006 at: http://www.incose.org.

Bedau, M. A., 2002. Downward causation and the autonomy of weak emergence. Principia 6 (2002): 5-50, special issue on Emergences and Downward Causation.

Beech, F. Michael, 2004. Observing Al Qaeda through the Lens of Complexity Theory : Recommendations for the National Strategy to Defeat Terrorism. Center for Strategic Leadership, Student Issue Paper, Volume S04-01, can be found at: http://www.carlisle.army.mil/usacsl/Publications/.

Bonabeau, E., J. L. Dessalles and A. Grumbach, 1995a. Characterizing emergent phenomena (1): A critical review. Revue Internationale de Systémique, Volume 9.

Bonabeau, E., J. L. Dessalles and A. Grumbach, 1995b. Characterizing emergent phenomena (2): A critical review. Revue Internationale de Systémique, Volume 9.

Bonabeau E., J.L. Dessalles, 1997. Detection and Emergence. Intellectica, Volume 2, Number 25.

Bonabeau E., G. Theraulaz, J.L. Deneubourg, N.R. Franks, O. Rafelsberger, J.L. Joly and S. Blanco, 1998. A model for the emergence of pillars, walls and royal chambers in termite nests. Philosophical Transactions: Biological Sciences, Volume 353, Number 1375, (1561-1576).

Boschetti, F., M. Prokopenko, I. Macreadie, A.-M. Grisogono, 2005a. Defining and detecting emergence in complex networks. The 9th International Conference on Knowledge Based and Intelligent Information and Engineering Systems (KES-2005).

Boschetti, F., M. Prokopenko, I Macreadie, A. Grisogono, J. Finnigan, P. Valencia, I. Enting, G German, D Newth, Dave Winkler, 2005b. Does anything emerge?. Draft document and MS Power Point Presentation. CSIRO-CSS Interaction Task, can be found at: http://www.per.marine.csiro.au/staff/Fabio.Boschetti/3054CO/papers/emergence_draft.pdf and http://www.cmar.csiro.au/ar/css/Aug%20Workshop%2005/Presentations/Boscetti_CSSwKshp_05.ppt.

Briggs, J. and F. D. Peat, 1989. The turbulent mirror. Harper and Row, New York.

Brooks, R. A., 1991. Intelligence without reason. In Proceeedings of the 21th International Joint Conference on Artificial Intelligence (IJCAI '91), San Fransisco: Morgan Kaufmann.

Calhoun, T. Mark, 2004. Complexity and Innovation: Army Transformation and the Reality of War. Fort Leavenworth, KS: School of Advanced Military Studies, Army Command and General Staff College, can be found at: http://handle.dtic.mil/100.2/ADA429156.

CALRESCO, 2006. Please see following web sites: http://www.calresco.org/; http://www.calresco.org/sos/sosfaq.htm

Cariani, P., 1989. On the design of devices with emergent semantic functions. Ph. D. dissertation, State University of New York at Birghamton.

CCRP, 2006. CCRP Publications Series. Please see following web site: http://www.dodccrp.org/

Chalmers, J. David, 2002. Varieties of Emergence, Preprint, 2002, http://consc.net/papers/granada.html.

Chan, Serena, 2001. Complex Adaptive Systems. ESD.83 Research Seminar in Engineering Systems, Massachusetts Institute of Technology, October 31, 2001/November 6, 2001, can be found at: http://web.mit.edu/esd.83/www/notebook/.

Chassin, P. David, Joel Malard, Christian Posse, 2004. Managing Complexity. DOI: nlin. AO/0408051, arXiv, 2004/08/27, can be found at: http://arxiv.org/ftp/nlin/papers/0408/0408051.pdf.

Collins COBUILD, 2003. English Dictionary. The Cobuild Series from the Bank of English, The University of Birmingham, Collins Cobuild, Harper Collins Publishers.

Couture, Mario, 2006a. Complexity and Chaos – State-of-the-Art; List of Works, Experts, Organizations, Projects, Journals, Conferences and Tools. DRDC/RDDC Valcartier Technical Note Number TN 2006-450. Defence R&D Canada, Valcartier, Quebec, Canada.

Couture, Mario, 2006b. Complexity and Chaos – State-of-the-Art; Formulations and Measures of Complexity. DRDC/RDDC Valcartier Technical Note Number TN 2006-451. Defence R&D Canada, Valcartier, Quebec, Canada.

Couture, Mario, 2006c. Complexity and Chaos – State-of-the-Art; Glossary. DRDC/RDDC Valcartier Technical Note Number TN 2006-452. Defence R&D Canada, Valcartier, Quebec, Canada.

Crutchfield, J. P., J. D. Farmer, N. H. Packard and R. S. Shaw, 1986. Chaos. Scientific American, Volume 255, Number 6.

Crutchfield, J. P., 1994. The Calculi of Emergence: Computation, Dynamics, and Induction. Physica D 1994. Special issue on the Proceedings of the Oji International Seminar Complex Systems — from Complex Dynamics to Artificial Reality held 5 - 9 April 1993, Numazu, Japan.

Deneubourg, J.L., G. Theraulaz and R. Beckers, 1992. Swarm-made architectures. In: Toward a practice of autonomous systems, ISBN: 0-262-72019-1, MIT Press.

Deneubourg J.L., A. Lioni and C. Detrain, 2002. Dynamics of aggregation and emergence of cooperation. The Biological Bulletin, Volume 2002, Number 3, (262-267).

De Wolf, Tom and Tom Holvoet, 2005. Emergence Versus Self-Organization: Different Concepts but Promising When Combined. Engineering Self Organizing Systems: Methodologies and Applications, Lecture Notes in Computer Science, Volume 3464.

Dodder, Rebecca and Robert Dare, 2000. Complex Adaptive Systems and Complexity Theory: Intere-related Knowledge Domains. ESD.83 Research Seminar in Engineering Systems, Massachusetts Institute of Technology, October 31, 2000, can be found at: http://web.mit.edu/esd.83/www/notebook/.

Dooley, K., 1996. A Nominal Definition of Complex Adaptive Systems. The Chaos Network, Volume 8, Number 1.

DuPreez, J. Lukas and Abraham J. Smith, 2004. The Application of Complexity Theory in the Development if Large Scale ICT Systems. In Proceedings of the 14th Annual INCOSE International Symposium, "Systems Engineering: Managing Complexity and Change", Toulouse, France, 20-24 June, 2004, (http://www.incose.org).

Edmonds, Bruce, 1999. Syntactic Measures of Complexity. PhD Thesis. University of Manchester, number of pages: 254.

ERCIM, 2006. European Research Consortium for Informatics and Mathematics, ERCIM News, Number 64, can be found at: http://www.ercim.org/.

Fioretti G. and B. Visser, 2004. A cognitive interpretation of organizational complexity. E:CO. Emergence, Special Double Issue, Volume 6, Numbers 1-2, can be found at: http://emergence.org/ECO_site/ECO_Archive/Issue_6_1-2/Fioretti_Visser.pdf.

Flake, W. Gary, 1998. The Computational Beauty of Nature: Computer Explorations of Fractals, Chaos, Complex Systems, and Adaptation. The MIT Press, ISBN: 0262561271, number of pages: 514.

Flynn, M. Geraldine, Claire Leleu, and Brian Hilburn, 2006. A Complexity Study of THE Maastricht Upper Airspace Centre. EEC Report No. 403, Project COCA, Eurocontrol Experimental Centre, 91 pages.

Forrest, S. and T. Jones. 1994. Modeling complex adaptive systems with Echo. pp. 3--21 in R.J. Stonier and X.H. Yu, eds. Complex Systems: Mechanisms of Adaptation. IOS Press, Amsterdam.

Freniere, W. Robert, John Q. Dickmann, Jeffrey R. Cares, 2003. Complexity-Based Targeting, New Sciences Provide Effects. Air & Space Power Journal, March, 2003, can be found at: http://www.airpower.maxwell.af.mil/airchronicles/apje.html.

Fromm, Jochen, 2005a. Types and Forms of Emergence. This paper can be found at: http://arxiv.org/abs/nlin.AO/0506028.

Fromm, Jochen, 2005b. Ten Questions about Emergence. This paper can be found at: http://arxiv.org/ftp/nlin/papers/0509/0509049.pdf.

Fromm, Jochen, 2006. On Engineering and Emergence. This paper can be found at: http://arxiv.org/abs/nlin.AO/0601002.

Hilburn, B., 2004. Cognitive Complexity in Air Traffic Control – A Literature Review. EEC Note No. 04/04, Project COCA, Eurocontrol Experimental Centre, 69 pages.

Holland, J.H., 1992. Adaptation in Natural and Artificial Systems. MIT Press.

Holland, J.H., 1996. Hidden Order: How Adaptation Builds Complexity. Harper Collins Canada, Perseus Books, ISBN: 0201442302.

Holland, J.H., 1998. Emergence: from chaos to order. Oxford University Press or Harper Collins Canada, Perseus Books, ASIN: 0201149435.

Jones, T. and S. Forrest. 1993. An Introduction to SFI Echo. Technical Report 93-12-074, Santa Fe Institute, Santa Fe NM. Available via anonymous ftp from ftp.santafe.edu:pub/echo/how-to.ps.Z.

Jost, Juergen, 2003. External and Internal Complexity of Complex Adaptive Systems. Santa Fe Institute Working Paper Number 03-12-070, could be found in 2006 at: http://www.santafe.edu/research/publications/wpabstract/200312070.

Gell-Mann, M., 1994. The quark and the jaguar. New York: WH Freeman.

Gershenson, Carlos and Francis Heylighen, 2003. When Can we Call a System Self-organizing?. Advances in Artificial Life, 7th European Conference, ECAL 2003, Dortmund, Germany, pp. 606-614. LNAI 2801. Springer, (http://www.vub.ac.be/CLEA).

Gharajedaghi, Jamshid, 1999. Systems Thinking: Managing Chaos and Complexity: A Platform for Designing Business Architecture. Butterworth-Heinemann, Reed Elsevier Group, ISBN: 0750671637, number of pages: 302.

Gilbert, N., 1995. Emergence in social simulation. In: Artificial Societies: The Computer Simulation of Social Life, UCL Press, London.

Gleick, James, 1989. Chaos. Penguin Paperbacks, ISBN: 0140092501, number of pages: 368.

Goldstein, Jeffrey, 1999. Emergence as a construct: History and issues. Emergence, Volume 1.

Gregobi, C. E., S. Pelikan and J. A. Yorke, 1984. Strange attractors are not chaotic. Physica D, Volume 13.

Haken, H., 1981. The Science of Structure: Synergetics. Van Nostran Reinhold, NY.

Heylighen, F., 1996. The Growth of Structural and Functional Complexity during Evolution. In F. Heylighen and D. Aerts, eds., The Evolution of Complexity (Kluwer Academic Publishers, 1996).

Hillis, D., 1988. Intelligence as an emergent behaviour; or the songs of Eden. Daedalus, Winter (175-189).

Histon, M. Jonathan and R. John Hansman , 2002. The Impact of Structure on Cognitive Complexity in Air Traffic Control . Report number: ICAT-2002-4, MIT International Center for Air Transportation, Department of Aeronautics & Astronautics, Massachusetts Institute of Technology, Cambridge, MA 02139 USA, 95 pages.

Holland, H. John, 1992. Adaptation in Natural and Artificial Systems. MIT Press.

Holland, H. John, 1996. Hidden Order: How Adaptation Builds Complexity. HarperCollins Canada, Perseus Books ISBN: 0201442302.

Holland, H. John, 1998. Emergence: from chaos to order. Oxford University Press or HarperCollins Canada / Perseus Books, ASIN: 0201149435, number of pages: 258.

Holt, A., 2000. Understanding environmental and geographical complexities through similarity matching. Complexity International, Volume 7, can be found at: http://www.complexity.org.au/ci/vol07/holt01/holt01.pdf.

IDEF, 2006: Integrated Definition Methods, http://www.idef.com/

Ilachinski, Andrew, 1996. Land Warfare and Complexity. Part I: Mathematical Backgorund and Technical Sourcebook. Virginia: Center for Naval Analyses.

Ilachinski, Andrew, 1996. Land Warfare and Complexity. Part II: An Assessment of the Applicability of Nonlinear Dynamics and Complex Systems Theory to the Study of Land Warfare. Virginia: Center for Naval Analyses.

Ilachinski, A., 2001. Cellular Automata: A Discrete Universe. World Scientific.

Jen, Erica, 2003. Stable or Robust? What's the Difference? Santa Fe Institute Working Paper Number 02-12-069, can be found at: http://www.santafe.edu/research/publications/wplist/2002.

Kauffman, Stuart, 1995. At Home in the Universe: The Search for the Laws of Self-Organization and Complexity. Oxford University Press, New York, NY, US, ISBN: 0195111303, number of pages: 336.

Kelemen, J., 2000. A note on colonies as post-modular systems with emergent behaviour. In R. Freund & A. Kelemenová (Eds), Grammar Systems 2000. Proceedings of the 4th International Workshop on Grammar Systems (203-213). Opava, Czech Republic: Silesian University.

Klee, R. I., 1984. Micro-determinism and concepts of emergence. Philosophy of Sciences, Volume 51, (44-63).

Koros, Anton, Pamela S. Della Rocco, Gulshan Panjwani, Victor Ingurgio, Jean-François D'Arcy, 2003. Complexity in Air Traffic Control Towers: A Field Study Part 1. Report number: DOT/FAA/CT-TN03/14, Federal Aviation Administration, Document is available to the public through the National Technical Information Service, Springfield, Virginia 22161, 125 pages.

Kubík, Aleš, 2003. Toward a formalization of emergence. Artificial Life, Volume 9, Issue 1.

Langton, C., 1986. Studying artificial life with cellular automata. In Farmer, D., Lapedes, A., Packard, N., Wendroff, B., eds.: Evolution, Games, and Learning: Models for Adaptation in Machines and Nature, Proceedings of the Fifth Annual Conference of the Center for Nonlinear Studies.

Langton, C. G., 1989. Artificial life. In C.G. Langton (Ed.), Artificial Life, Santa Fe Institute Studies in the Sciences of Complexity, Proc. Vol. VI, (1-48), Addison Wesley, Redwood City, CA.

Langton CG., 1990. Computation at the edge of chaos: phase transitions and emergent computation. Physica D 42:12–37

Langton CG., 1991. Computation at the edge of chaos: phase transitions and emergent computation. PhD diss. University of Michigan.

Lewin, Roger 1993, Complexity: Life on the Edge of Chaos, London: Phoenix.

Lorenz, Edward, 1996. Essence of Chaos. University of Washington Press ISBN: 0295975148.

Marion, Russ and Mary Uhl-Bien, 2002. Complexity Theory and Al-Qaeda: Examining Complex Leadership. Presented at Managing the Complex IV: A Conference on Complex Systems and the Management of Organizations, Fort Meyers, Fl, December, 2002.

Matarié, M. 1994. Interaction and intelligent behaviour. Ph.D. thesis, Combridge, MA, MIT Press.

May, R. M. 1976. Simple mathematical models with very complicated dynamics. Nature, Volume 261, Number 10.

Merriam-Webster, 2003. Merriam-Webster's 11th Collegiate Dictionary. MS-Windows software.

Minsky, M., 1986. The Cociety of Mind. New York, Simon and Schuster.

Morgan, C. L., 1923. Emergent Evolution. London: Northgate and Williams.

Muller, F., 1997. State-of-the-art in ecosystem theory. Ecological Modelling, Volume 100.

Newman, D., 1996. Emergence and strange attractors. Philisophy of Science, Volume 36.

Newman, M. E. J., 2005. Power laws, Pareto distribution and Zipf's law. Contemporary Physics, Volume 46.

Nicolis, G., 1989. Physics of far-from-equilibrium and self-organization. In Davies, P., ed., The New Physics, Cambridge University Press.

ONCE-CS, 2006. Complex Systems: Challenges and Opportunities. An orientation paper for Complex Systems research in FP7. The Open Network of Center of Excellence in Complex Systems (ONCE-CS), number of pages: 37, can be found at: http://complexsystems.lri.fr/Portal/.

Peper, S., 1926. Emergence. Journal of Philosophy, Volume 23.

Percival, I., 1989. Chaos, a science for real world. New Scientist, Volume 124, Number 1687.

Poussart (2006a). Complexity – An Overview of its Nature, Manifestations, and Links to Science & Technology Convergence, with Comments on its Relevance to Canadian Defence. DRDC-RDDC- Valcartier report, Defence R&D Canada, Valcartier, Quebec, number of pages: 88.

Poussart (2006b). Dr. Denis Poussart, Université Laval, Personal communication.

Prokopenko M. and Peter Wang, 2004. On Self-referential Shape Replication in Robust Aerospace Vehicles. In Proceedings of the 9th International Conference on the Simulation and Synthesis of Living Systems (ALIFE9), Boston, USA, 12-15 September 2004, pages 27-32, September 2004, can be found at: http://www.ict.csiro.au/staff/mikhail.prokopenko/Publications/Agents/shape9-final.pdf.

Rocha, Luis Mateus, 1998. Selected Self-Organization and the Semiotics of Evolutionary Systems. Evolutionary Systems: Biological and Epistemological Perspectives on Selection and Self-Organization. S. Salthe, G. Van de Vijver, and M. Delpos (eds.). Kluwer Academic Publishers, pp. 341-358, can be found at: http://informatics.indiana.edu/rocha/ps/ises.pdf.

Rocco, S. Della, Ferne Friedman-Berg, and Carol A. Manning, 2002. Application of Color to Reduce Complexity in Air Traffic Control. Technical Note number: DOT/FAA/CT-TN03/01, Federal Aviation Administration, Human Factors Division, Document is available to the public through the National Technical Information Service, Springfield, Virginia 22161, 54 pages.

Rhodes, H. Donna, 2004. Workshop Report Air Force/LAI Workshop on Systems Engineering for Robustness. Lean Aerospace Initiative, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 01239, 617-324-0473, 22 pages, rhodes@mit.edu.

SFI, 2006. Santa Fe Institute. Please see following web sites: http://www.santafe.edu/.

Senge, M. Peter, 1994. The Fifth Discipline: The Art and Practice of the Learning Organization. 1st edition in 1990, London: Random House. ISBN: 0-385-26095-4, number of pages: 424.

Sheard, A. Sarah, 2005. Practical Applications of Complexity Theory for Systems Engineers. In Proceedings of the 15th Annual INCOSE International Symposium, "Systems Engineering: Bridging Industry, Government, and Academia", Rochester, NY, 10-25 July, 2005, (http://www.incose.org).

Shetler C. Judith, 2002. Complex Adaptive Systems, Attractors, and Patching : a Complex Systems Science Analysis of Organizational Change. Dissertation Presented to the Faculty of the Graduate School of the University of Texas at Austin in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy, December, 2002, can be found at: http://www.lib.utexas.edu/etd/d/2002/shetlerjc029/shetlerjc029.pdf.

Sommerer, Christa and Laurent Mignonneau, 2002. Modeling the Emergence of Complexity: Complex Systems, the Origin of Life and Interactive On-Line Art. LEONARDO, Volume 35, Number 2.

Standish, K. Russell, 2001. On Complexity and Emergence. Complexity International, Volume 09, can be found at: http://www.complexity.org.au/.

Steels, L., 1992. Towards a theory of emergent functionality. In: From Animals to Animats SAB.

Swinney, H. L., 1986. Experimental observations of order and chaos. In Nonlinear Dynamics and Chaos: Geomarical methods for Engineers and Scientists, J. M. T. Thompson & H. B. Stewart (eds), John Wiley, New York.

Waldrop, Mitchell, 1992. Complexity: The Emerging Science at the Edge of Order and Chaos. Simon & Schuster: ISBN: 0671872346, number of pages: 384.

Wikipedia, 2006: http://en.wikipedia.org/wiki/.

Williams, P. Garnett, 2001. Chaos Theory Tamed. National Academy Press (Trade): ISBN: 0309063515, number of pages: 499.

Wolfram S., 2002. A New Kind of Science. Champaign, IL: Wolfram Media.

Xing, Jing and Carol A. Manning, 2005. Complexity and Automation. Display of Air Traffic Control: Literature Review and Analysis. Final Report. Civil Aerospace Medical Institute, Federal Aviation Administration, Oklahoma City, OK 73125, Number of pages: 20.

This page intentionally left blank.

Annex A Examples of Descriptors and Criteria

Table 4 Descriptors used in complexity theory (Beech, 2004).

Descriptor	Descriptor's features
Fundamental elements of self-organizing networks.	**Adaptation**. Complex networks are referred to as "adaptive" or "dynamic", because they are constantly changing their interrelationships based upon the needs of individual agents and environmental impacts.
	Correlation. Individual agents within the network are constantly reassessing their need preferences and the degree to which they will compromise to bond with other agents. Consequently, the network adapts through the process of compromise and competition, called correlation, in which each entity accepts, rejects or changes its relationship with other agents based upon its needs and the changing environment.
	Coupling. Kauffman referred to the interdependent bonding of agents as "coupling," and Marion categorized these relationships as **loose**, **moderate** or **tight**.
	Aggregation and recursion. Holland referred that these sets of agents bond through the process of correlation and are united by **shared purpose or interest** as "aggregates". Aggregates may accumulate with many other sets of agents or structures to form meta-aggregates and further connect with yet other structures that accomplish diverse functions or roles to then form meta-meta-aggregates.

Table 5 Descriptors used in complexity theory (CALRESCO, 2006).

Descriptor	Descriptor's features
Network characteristics.	The **evolution of a system into an organized form** in the absence of external pressures.
	A move from a large region of state space to a persistent smaller one (attractor), under the control of the system itself.
	The introduction of correlations (pattern) over time or space for previously independent variables operating under local rules.
Typical features.	**Absence of external control**. Autonomy.
	Dynamic operation. Time evolution.
	Fluctuations. Noise/searches through options.
	Symmetry breaking. Loss of freedom/heterogeneity.

	Global order. Emergence from local interactions.
	Dissipation. Energy usage/far-from-equilibrium.
	Instability. Self-reinforcing choices/nonlinearity.
	Multiple equilibria. Many possible attractors.
	Criticality. Threshold effects/phase changes.
	Redundancy. Insensitivity to damage.
	Self-maintenance. Repair/reproduction metabolisms.
	Adaptation. Functionality/tracking of external variations.
	♦ **Complexity**. Multiple concurrent values or objectives.
	Hierarchies. Multiple nested self-organized levels.

Table 6 Descriptors used in complexity theory (Ilachinski, 1996; Axelrod and Cohen, 2001).

Descriptor	Descriptor's features
Basics of CAS.	**Aggregation**.
	Building blocks.
	Diversity.
	Nonlinearity.
	Tagging.
	Flows.
	Internal models.

Table 7 Descriptors used in complexity theory (Ilachinski, 1996).

Descriptor	Descriptor's features
Key concepts.	**Variation**.
	Interaction.
	Selection.

Table 8 Descriptors used in complexity theory (Holland, 1995).

Descriptor	Descriptor's features
Basics or characteristics.	**Aggregation (seen as a property).**
	Tagging (seen as a mechanism).
	Nonlinearity (seen as a property).
	Flows (seen as a property).
	Diversity (seen as a property).
	Internal model (seen as a mechanism).
	Building Blocks (seen as a mechanism).

Bibliography

Mr. Couture received a B.Sc. degree in Physics and a M.Sc. in Physical Oceanography at the Université du Québec à Rimouski, Qc, Canada. After 8 years of M&S work at Fisheries and Ocean Canada, he completed a M.Sc. in Electrical Engineering at Laval University, Qc, Canada. In 2002, he joined Defence R&D Canada - Valcartier as a Defence Scientist in the Systems Engineering and Architecture (SEA) Group, which is part of the System of Systems (SoS) Section. His research interests are oriented toward the study, design and engineering of military complex systems through the lens of Complexity Theory.

This page intentionally left blank.

Distribution list

Document No.: DRDC Valcartier TM 2006-453

	LIST PART 1: Internal Distribution by Centre:
1	Director General
3	Document Library
1	Head/Information and Knowledge Management Section
1	Head/Decision Support Systems Section
1	Head/System of Systems Section
1	Mr Claude Roy (SDA)
1	Mr Mario Couture (author)
1	Mr Robert Charpentier
1	Mr Michel Ducharme
1	Dr Marielle Mokhtari
1	Mr François Bernier
1	Mr Michel Lizotte
1	Mr Éric Dorion
1	Mr Jean-Claude St-Jacques
1	LCol Michel Gareau
1	Mrs Micheline Bélanger
1	Dr Anne-Claire Boury-Brisset
1	Dr Abderezak Benaskeur
1	Dr Pierre Valin
1	LCdr Elizabeth Woodliffe
1	Mr Alexandre Bergeron-Guyard
1	Mr Marc-André Morin
1	Mr François Lemieux
1	Mr Frédéric Michaud
1	Mr Frédéric Painchaud
1	Mr Alain Bergeron
1	Dr Luc Pigeon
1	Mr Alain Bouchard
1	Mr Denis Gouin
1	Mr François Létourneau
1	Dr Michel Gagnon
1	Mr Yaïve Ferland
1	Mrs Régine Lecocq
1	Dr Alain Auger
1	Mr Jean Roy
1	Martin Salois
1	Luminita Stemate
1	Nawel Chefai
40	**TOTAL LIST PART 1**

LIST PART 2: External Distribution by DRDKIM
NDHQ, 101 Colonet Drive, Ottawa, On, K1A 0K2

1 Col Rick Williams (Associate DG RDP)

Directorate Science and Technoilogies (Policy)

1 Mr Clément Laforce
1 Dr Affa Ahmad

Directorate Science and Technology (C4ISR)

1 Dr Chris McMillan (DSTC4ISR)
1 Mr Klaus Kollenberg (DSTC4ISR 3)
1 Mrs Donna Wood (DSTC4ISR 4)

DRDC Centre for Operational Research and Analysis (CORA)

1 Mr Paul Comeau
1 Mr Jocelyn Tremblay
1 Mr Paul Massel

Military Technology Analysis

1 Len Goodman
1 Steven Hughes

Strategic Joint Staff

1 David Connel

DMP Strat.

1 John Verdon

13 TOTAL LIST PART 2

53 TOTAL COPIES REQUIRED

DOCUMENT CONTROL DATA

(Security classification of title, body of abstract and indexing annotation must be entered when the overall document is classified)

1. ORIGINATOR (The name and address of the organization preparing the document. Organizations for whom the document was prepared, e.g. Centre sponsoring a contractor's report, or tasking agency, are entered in section 8.)

   Defence R&D Canada – Valcartier
   2459 Pie-XI Blvd North
   Quebec (Quebec)
   G3J 1X5 Canada

2. SECURITY CLASSIFICATION (Overall security classification of the document including special warning terms if applicable.)

   UNCLASSIFIED

3. TITLE (The complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S, C or U) in parentheses after the title.)

   Complexity and chaos – State-of-the-art; Overview of theoretical concepts:

4. AUTHORS (last name, followed by initials – ranks, titles, etc. not to be used)

   Couture, M.

5. DATE OF PUBLICATION (Month and year of publication of document.)

   August 2007

6a. NO. OF PAGES (Total containing information, including Annexes, Appendices, etc.)

   1186

6b. NO. OF REFS (Total cited in document.)

   130

7. DESCRIPTIVE NOTES (The category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.)

   Technical Memorandum

8. SPONSORING ACTIVITY (The name of the department project office or laboratory sponsoring the research and development – include address.)

   Section: Information and Knowledge Management. Address: Defense R&D Canada, DRDC-Valcartier, 2459, boul. Pie-XI nord, Québec, Canada, G3J 1X5 (Robert Charpentier)

9a. PROJECT OR GRANT NO. (If appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant.)

   15bp01 – Defensive Software Design

9b. CONTRACT NO. (If appropriate, the applicable number under which the document was written.)

10a. ORIGINATOR'S DOCUMENT NUMBER (The official document number by which the document is identified by the originating activity. This number must be unique to this document.)

   DRDC Valcartier TM 2006-453

10b. OTHER DOCUMENT NO(s). (Any other numbers which may be assigned this document either by the originator or by the sponsor.)

11. DOCUMENT AVAILABILITY (Any limitations on further dissemination of the document, other than those imposed by security classification.)

   Unlimited

12. DOCUMENT ANNOUNCEMENT (Any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availability (11). However, where further distribution (beyond the audience specified in (11) is possible, a wider announcement audience may be selected.))

   Couture, Mario, 2006. Complexity and chaos – State-of-the-art; Overview of theoretical concepts. DRDC/RDDC Valcartier Technical Memorandum Number TM 2006-453. Defence R&D Canada, Valcartier, Québec, Canada.

13. ABSTRACT (A brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), (R), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual.)

Theoretical concepts used in the field of complexity theory are presented. Proposed definitions include the essential elements gleaned from the scientific literature. Key terms such as **system**, **complex system** and **complex adaptive system** and other preliminary notions for the study of complexity are first defined and described. Four classification criteria distilled from an extensive literature review (Couture, 2006a) are then described and used to classify and structure concepts, properties, mechanisms and emerging phenomena. The criteria incorporate concepts that are essential for the study of the above systems. For instance, they employ the concepts of **level** and **interrelationships** between levels, thus enabling researchers to describe level-dependent complex manifestations such as emergence. The criteria defined are then used in a review of complexity theory in the hope that the structured descriptions of the criteria will aid in elucidating the elements of this theory. Finally, this document shows that complexity theory is in fact a rich set of interrelated theoretical concepts that already help us understand the increasingly complexity of our world. These concepts may also be used as guides to design specific properties or characteristics into information, communication and C2 systems to make them more efficient and effective in complex military operations.

Les concepts utilisés dans le domaine de la théorie de la complexité sont présentés dans ce document. Les descriptions proposées intègrent l'essentiel de la littérature scientifique consacrée à cette science. Les mots clés tels que « **System** », « **Complex System** » et « **Complex Adaptive System** » et d'autres notions préliminaires à l'étude de la complexité sont d'abord définis et décrits. Un ensemble de quatre critères de classification déduit d'une revue de littérature étendue (Couture, 2006a) est ensuite décrit et utilisé pour regrouper et structurer concepts, propriétés, mécanismes et phénomènes émergents. Cet ensemble intègre les notions essentielles à l'étude de ces systèmes. Par exemple, il intègre la notion de **niveau** et les **interrelations** entre eux, permettant la description de manifestations complexes qui dépendent de niveaux comme l'émergence. Cet ensemble de critères est ensuite utilisé pour effectuer une revue de la théorie de complexité en espérant que les descriptions structurées impliquant ces critères vont contribuer à aider à la compréhension des éléments de cette théorie. Ce document montre finalement que la théorie de la complexité est faite d'un riche ensemble de concepts théoriques qui sont interdépendants et contribuent déjà à aider à la compréhension de notre monde toujours plus complexe. Ces concepts peuvent également être utilisés comme guides pour munir les systèmes d'information, de communication et de C2 des propriétés et caractéristiques dont ils ont besoin pour améliorer leur capacité et leur efficacité lors d'opérations militaires complexes.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (Technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus, e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

complexity; complex; theory; theories; concepts; systems; complex adaptive systems; CAS.

Defence R&D Canada

Canada's Leader in Defence
and National Security
Science and Technology

R & D pour la défense Canada

Chef de file au Canada en matière
de science et de technologie pour
la défense et la sécurité nationale

DEFENCE R&D DÉFENSE

WWW.drdc-rddc.gc.ca