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A Model of Radio Wave Propagation in Ionospheric Irregularities for Prediction of High-Frequency Radar Performance

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Abstract

A new theory of radio wave propagation in ionospheric plasma density irregularities is presented. The theory is motivated by the need to quantify and understand the target-resolving capabilities and clutter characteristics of high-frequency radar systems. The theory is formulated in terms of path integrals of the ray tracing equations, which lead to expressions for the effects of the irregularities on the radar signal properties of skip distance, group delay, direction of arrival, and Doppler. The expressions are evaluated for the case of random density irregularities with a power-law wavenumber spectrum, which leads to predicted power spectra for the signal properties. The signal phase spatial-temporal autocorrelation function is also derived.

Résumé

Une nouvelle théorie de la propagation des ondes radio dans des conditions d'irrégularités de la densité du plasma ionosphérique est présentée. Cette théorie est née du besoin de quantifier et de comprendre les capacités de résolution des cibles et les caractéristiques d'élimination du clutter des radars haute fréquence. Elle est exprimée en termes d'intégrales de chemin des équations de tracé de rayon, qui donnent lieu à des expressions des effets des irrégularités sur les propriétés du signal radar, notamment la distance de saut, le temps de propagation de groupe, la direction du point d'origine et le décalage Doppler. Les expressions sont évaluées pour le cas des irrégularités de densité aléatoires avec un spectre loi de puissance en nombre d'ondes, ce qui donne lieu à des spectres de puissance prévisibles pour les propriétés du signal. De plus, la fonction d'autocorrélation spatio-temporelle de la phase du signal est dérivée.

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Executive summary

A Model of Radio Wave Propagation in Ionospheric Irregularities for Prediction of High-Frequency Radar Performance

R. J. Riddolls; DRDC Ottawa TM 2006-284; Defence R&D Canada – Ottawa; December 2006.

This memorandum develops a theory of High-Frequency (HF) radio wave propagation in the earth's ionosphere that accounts for the effects of ionospheric plasma density irregularities. Unlike previous isotropic plasma theories, this theory assumes an anisotropic ionospheric plasma that is inhomogeneous in the vertical direction. In this theory, a radio signal is transmitted from the ground toward the ionosphere, undergoes refraction in the ionosphere, and returns to the ground at a distance from the transmitter referred to as the skip distance, and with a time delay referred to as the group delay. The plasma density irregularities impart a randomization to the signal skip distance and group delay, as well as the signal Direction-Of-Arrival (DOA) and Doppler shift.

The theory is motivated by the need to quantify and understand the target-resolving capabilities of High-Frequency Over-The-Horizon Radar (HFOTHR) systems, as well as the ionospheric clutter characteristics of High-Frequency Surface-Wave Radar (HFSWR) systems. The formulation is in terms of path integrals of the ray tracing equations. A quiescent space-time signal trajectory is defined by the solution for the ray path in an ionosphere density profile without irregularities. The effects of the irregularities are modelled as small perturbations to the quiescent solution. The plasma density perturbations lead to predicted perturbations of the radar signal properties of skip distance, group delay, DOA, and Doppler. The perturbations are evaluated for the case of random plasma density irregularities with a power-law wavenumber spectrum, which leads to the predicted power spectra of the signal properties as a function of wavenumber and frequency.

Practical HF radar systems estimate signal properties by analyzing the space-time variation of the radar signal phase. Accordingly, the signal DOA and Doppler power spectra are used to derive the space-time autocorrelation of the signal phase. This autocorrelation is then recast as an autocorrelation of the signal complex amplitude for convenience of use in radar performance predictions. This autocorrelation has a width that depends on the outer scale length of the plasma density irregularities, the phase velocity of the irregularities, and the root mean square phase scintillation incurred during the signal transit of the ionosphere.

Future work will involve employing the theory presented in this memorandum to make quantitative predictions of the performance of HFOTHR and HFSWR systems. Of particular interest are the target-resolving capabilities of HFOTHR, and the physical limitations on useful radar waveform bandwidth, antenna aperture, and integration time. These results also carry over to hybrid HFOTHR-HFSWR systems that use an ionospheric transmit path and a surface wave receive path. Furthermore, the theory will be adapted to quantify the performance of HFSWR ionospheric clutter cancellation technologies, such as elevation angle-resolving planar arrays and Space-Time Adaptive Processing (STAP) algorithms.

Sommaire

A Model of Radio Wave Propagation in Ionospheric Irregularities for Prediction of High-Frequency Radar Performance

R. J. Riddolls; DRDC Ottawa TM 2006-284; R & D pour la défense Canada – Ottawa; décembre 2006.

Le présent mémoire développe une théorie de la propagation des ondes radio haute fréquence (HF) dans l'ionosphère de la Terre qui tient compte des effets des irrégularités de la densité du plasma ionosphérique. Au contraire des théories antérieures du plasma isotrope, cette théorie présume la présence d'un plasma ionosphérique anisotrope qui est hétérogène dans la direction verticale. Selon cette théorie, un signal radio transmis du sol vers l'ionosphère subit de la réfraction dans l'ionosphère, puis retourne au sol à une certaine distance de l'émetteur, distance qu'on appelle distance de saut, et avec un retard qu'on appelle temps de propagation de groupe. Les irrégularités de la densité du plasma appliquent une randomisation à la distance de saut et au temps de propagation de groupe du signal, ainsi qu'à la direction du point d'origine (DOA) et au décalage Doppler.

La théorie est née du besoin de quantifier et de comprendre les capacités de résolution des cibles des radars haute fréquence transhorizon (HFOTHR), ainsi que les caractéristiques d'élimination du clutter ionosphérique des radars haute fréquence à onde de surface (HFSWR). La théorie est exprimée en termes d'intégrales de chemin des équations de tracé de rayon. Une trajectoire du signal spatio-temporel de repos est définie par la résolution du chemin de rayon dans un profil de densité ionosphérique sans irrégularités. Les effets des irrégularités sont modélisées comme de faibles perturbations de la résolution pour la condition de repos. Les perturbations de la densité du plasma donnent lieu aux perturbations prévisibles des propriétés du signal radar, notamment de la distance de saut, du temps de propagation de groupe, de la direction du point d'origine et du décalage Doppler. Les perturbations sont évaluées pour le cas des irrégularités de densité aléatoires avec un spectre loi de puissance en nombre d'ondes, ce qui donne lieu aux spectres de puissance prédits pour les propriétés du signal en fonction du nombre d'ondes et de la fréquence.

Des radars HF pratiques estiment les propriétés du signal en analysant la variation spatio temporelle de la phase du signal radar. Par conséquent, la DOA du signal et les spectres de puissance Doppler sont utilisés pour dériver l'autocorrélation spatio-temporelle de la phase du signal. Cette autocorrélation est ensuite remaniée comme autocorrélation de l'amplitude complexe du signal pour faciliter l'utilisation en prédiction des performances du radar. Cette autocorrélation a une largeur qui

dépend de la longueur de l'échelle externe des irrégularités de la densité du plasma, de la vitesse de phase des irrégularités et de la valeur efficace de la scintillation de phase subie pendant le passage du signal par l'ionosphère.

Les recherches futures consisteront à utiliser la théorie présentée dans ce mémoire pour faire des prédictions quantitatives des performances des radars HFOTHR et HFSWR. Ce qui est particulièrement intéressant, ce sont les capacités de résolution des cibles du HFOTHR et les limitations physiques de la largeur de bande utile du signal radar, de l'ouverture d'antenne et du temps d'intégration. Ces résultats s'appliquent également aux radars hybrides HFOTHR-HFSWR qui font appel à un trajet d'émission ionosphérique et à un trajet de réception par onde de surface. En outre, la théorie sera adaptée pour quantifier les performances des technologies d'élimination du clutter ionosphérique du HFSWR, axées, par exemple, sur des réseaux planaires de résolution de l'angle de site ou sur des algorithmes de traitement adaptatif de signaux spatio-temporels (STAP).

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1 Introduction

This memorandum develops a theory of High-Frequency (HF) radio wave propagation in the earth's ionosphere that accounts for the effects of ionospheric plasma density irregularities. Unlike previous isotropic plasma theories, this theory assumes an anisotropic ionospheric plasma that is inhomogeneous in the vertical direction. In this theory, a radio signal is transmitted from the ground toward the ionosphere, undergoes refraction in the ionosphere, and returns to the ground at a distance from the transmitter referred to as the skip distance. Typically, transmitters radiate a broad angular spectrum of signals, and thus these signals return to the earth over a continuum of skip distances. In this analysis, we focus our attention on the effects of the irregularities on a single angular mode. The effects will be described as a randomization of the properties of this mode, which include the mode's skip distance, group delay, Direction-Of-Arrival (DOA), and Doppler shift.

The motivation for this work is twofold. First, there is a need to quantify the performance of High-Frequency Over-The-Horizon Radar (HFOTHR) systems, sometimes referred to as HF sky-wave radar systems. In the HFOTHR configuration, the ionosphere is used as a reflecting surface to permit illumination of targets at distances beyond the line-of-sight horizon of the earth. In other words, the transmitter beams a radar signal at an oblique angle to the ionosphere, the signal reflects from the ionosphere, and the target, such as an airplane or a ship, is illuminated by the signal. The target then scatters the signal back to the transmitting location along a similar propagation path. Since the radar uses the ionosphere as a propagation medium, irregularities in the medium can affect the radar's ability to resolve targets downrange of the radar. Thus, there is a need to quantify the effects of irregularities to understand the resolution limits of these radars in a realistic environment.

The second motivation for this work is to develop an understanding of ionospheric clutter observed by High-Frequency Surface-Wave Radar (HFSWR) systems. In the HFSWR configuration, a radar waveform is transmitted along the surface of the earth, typically over the ocean, and the signal refracts around the curvature of the earth to illuminate ship and low-flying aircraft targets beyond the line-of-sight horizon. However, during transmission, a certain amount of radiation is inevitably produced in the vertical direction, and the reflection from the ionosphere is observed as intense radar clutter. This clutter can impose detection range limitations in practical long-range HFSWR systems. A quantitative model of the effects of irregularities on signal skip distance, group delay, DOA, and Doppler, is useful to quantify and understand the performance of clutter mitigation techniques applied to HFSWR systems.

This memorandum is divided into the following sections. Section 2 reviews the topic of HF propagation in the ionosphere. The theory for propagation in a homogeneous anisotropic medium is presented first, and then extended to the case of inhomogeneous anisotropic media using the concepts of ray optics. The ray optics formulation is then applied in Section 3 to the problem of the propagation of radar signals through a plasma medium containing density irregularities. One can determine the impact of the irregularities on the signal properties of skip distance, group delay, DOA, and Doppler, by evaluating path integrals along the ray trajectories determined in Section 2. Section 4 then turns to the problem of estimating signal properties using standard radar techniques, namely the analysis of the variation of signal phase over space and time. The phase space-time autocorrelation function is computed, along with the autocorrelation of the complex signal amplitude, this latter function being useful for evaluation of the performance of the HF radar configurations mentioned previously. A conclusion is made in Section 5.

2 HF wave propagation in the ionosphere

This section summarizes high-frequency wave propagation in the ionospheric plasma and derives results needed for Section 3. For more details on the derivations below, the reader is referred to standard references [1, 2, 3]. The first subsection considers propagation in anisotropic homogeneous media. The second subsection derives the ray tracing equations for propagation in inhomogeneous media.

2.1 Modes in a homogeneous plasma

The ionosphere consists predominantly of atomic oxygen ions mixed with free electrons. Accordingly, we describe electromagnetic wave propagation in the ionosphere by the wave equation coupled with conservation equations for ion and electron fluids. The wave equation for the electric field $\mathbf{E}$ is given by

$$\nabla \times (\nabla \times \mathbf{E}) + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_0 \frac{\partial \mathbf{J}}{\partial t} = -\mu_0 \frac{\partial \mathbf{J}_e}{\partial t}, \tag{1}$$

where $1/c^2 = \mu_0 \epsilon_0$. Here, the electric current consists of external sources $\mathbf{J}_e$ imposed on the plasma, and internal self-consistent currents $\mathbf{J}$ in the plasma. The self-consistent currents are related to the electric field by an anisotropic conductivity tensor:

$$\mathbf{J} = \boldsymbol{\sigma} \cdot \mathbf{E}. \tag{2}$$

If we assume that $\mathbf{E} \propto \exp(-i\omega t)$, the wave equation is

$$\nabla \times (\nabla \times \mathbf{E}) - \omega^2 \mu_0 \boldsymbol{\epsilon} \cdot \mathbf{E} = -\mu_0 \frac{\partial \mathbf{J}_e}{\partial t}, \tag{3}$$

where the permittivity tensor is given by

$$\boldsymbol{\epsilon} = \epsilon_0 (\mathbf{I} + i\boldsymbol{\sigma}/\epsilon_0 \omega), \tag{4}$$

and $\mathbf{I}$ is the identity tensor. To determine the permittivity tensor for the ionospheric plasma, we consider conservation of momentum for the sth charged species (oxygen ions or electrons):

$$m_s \frac{d\mathbf{v}_s}{dt} = q_s(\mathbf{E} + \mathbf{v}_s \times \mathbf{B}). \tag{5}$$

Here, m_s is the mass of the species, $\mathbf{v}_s$ is the velocity of the species fluid, and q_s denotes the species charge. This equation relates the change in momentum $m_s \mathbf{v}_s(\mathbf{r}, t)$ of a species at the location $\mathbf{r}$ of a fluid element, to the Lorentz force acting on the species. However, it is more practical to evaluate the left side at a fixed point in space rather than at the location of the moving fluid element. The transformation from

Lagrangian (fluid element-based) coordinates to Eulerian (fixed) coordinates can be done by expanding the left side as follows:

$$\frac{d\mathbf{v}_s}{dt} = \frac{\partial \mathbf{v}_s}{\partial t} + \frac{\partial \mathbf{v}_s}{\partial x}\frac{dx}{dt} + \frac{\partial \mathbf{v}_s}{\partial y}\frac{dy}{dt} + \frac{\partial \mathbf{v}_s}{\partial z}\frac{dz}{dt} \tag{6}$$

$$= \frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla)\mathbf{v}_s. \tag{7}$$

The implication of the $(\mathbf{v}_s \cdot \nabla)\mathbf{v}_s$ term is that the momentum conservation equation is nonlinear in $\mathbf{v}_s$. To find a first-order approximation for $\boldsymbol{\sigma}$, we consider an equilibrium consisting of zero background electric field, zero fluid flow, and a constant magnetic field, and then impose small perturbations about this equilibrium. Using the subscript 0 to denote equilibrium and 1 to denote the perturbation, the momentum equation to first order in the perturbed quantities is

$$-i\omega m_s \mathbf{v}_{s1} = q_s(\mathbf{E}_1 + \mathbf{v}_{s1} \times \mathbf{B}_0). \tag{8}$$

We assume without loss of generality that the magnetic field points in the $\hat{\mathbf{z}}$ direction ($\mathbf{B}_0 = B_0\hat{\mathbf{z}}$), which allows us to solve for the Cartesian components of $\mathbf{v}_{s1}$:

$$v_{s1x} = \frac{iq_s}{m_s\omega}\frac{[E_{1x} \pm i(\omega_{cs}/\omega)E_{1y}]}{1 - (\omega_{cs}/\omega)^2} \tag{9}$$

$$v_{s1y} = \frac{iq_s}{m_s\omega}\frac{[E_{1y} \mp i(\omega_{cs}/\omega)E_{1x}]}{1 - (\omega_{cs}/\omega)^2} \tag{10}$$

$$v_{s1z} = \frac{iq_s}{m_s\omega}E_{1z}, \tag{11}$$

where $\omega_{cs} = |q_s|B_0/m_s$ is the cyclotron frequency for species s, and the upper and lower signs refer to the cases of positive charge (ions) and negative charge (electrons), respectively. The plasma current is the summation of the species contributions:

$$\mathbf{J}_1 = \sum_s q_s N_s \mathbf{v}_{s1} \equiv \boldsymbol{\sigma} \cdot \mathbf{E}_1, \tag{12}$$

where N_s is the species density. This expression for $\mathbf{J}_1$ defines the first-order conductivity tensor $\boldsymbol{\sigma}$. Inserting $\boldsymbol{\sigma}$ into Equation (4), the plasma dielectric permittivity tensor is given by

$$\boldsymbol{\epsilon} = \epsilon_0 \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}, \tag{13}$$

where

$$
\begin{aligned}
S &= (R+L)/2 && (14)\\
D &= (R-L)/2 && (15)\\
P &= 1-\sum_s \frac{\omega_{ps}^2}{\omega^2} && (16)\\
R &= 1-\sum_s \frac{\omega_{ps}^2}{\omega^2}\left(\frac{\omega}{\omega \pm \omega_{cs}}\right) && (17)\\
L &= 1-\sum_s \frac{\omega_{ps}^2}{\omega^2}\left(\frac{\omega}{\omega \mp \omega_{cs}}\right), && (18)
\end{aligned}
$$

and $\omega_{ps}^2 = q_s^2 N_s/(\epsilon_0 m_s)$ is referred to as the plasma frequency of species s.

The wave equation can now be solved for arbitrary sinusoidal excitations. Of particular interest are the natural modes of the ionosphere, namely electric field oscillations that propagate outside the region of the excitation. In particular, we investigate plane wave solutions, where the field quantities vary with $\exp(i\mathbf{k}\cdot\mathbf{r})$, where $\mathbf{k}$ is the wavenumber. In this case, the wave equation outside the source region can be written

$$
\mathbf{n}\times(\mathbf{n}\times\mathbf{E}_1)+\frac{\boldsymbol{\epsilon}}{\epsilon_0}\cdot\mathbf{E}_1 = 0, \tag{19}
$$

where $\mathbf{n} = c\mathbf{k}/\omega$ is a refractive index vector. Using a well-known vector identity, the wave equation can be written as

$$
\left(\mathbf{nn} - n^2\mathbf{I} + \boldsymbol{\epsilon}/\epsilon_0\right)\cdot\mathbf{E}_1 = 0, \tag{20}
$$

where $n = |\mathbf{n}|$. Without loss of generality, let us assume that $\mathbf{n}$ lies in the x-z plane at an angle θ with respect to $\hat{\mathbf{z}}$. The wave equation can then be written

$$
\begin{pmatrix}
S - n^2\cos^2\theta & -iD & n^2\sin\theta\cos\theta\\
iD & S-n^2 & 0\\
n^2\sin\theta\cos\theta & 0 & P - n^2\sin^2\theta
\end{pmatrix}\cdot\mathbf{E}_1 = 0. \tag{21}
$$

Nonzero solutions for $\mathbf{E}_1$ occur when the determinant of the matrix is zero. This condition defines a dispersion relation governing wave propagation in the plasma:

$$
An^4 - Bn^2 + C = 0, \tag{22}
$$

where

$$
\begin{aligned}
A &= S\sin^2\theta + P\cos^2\theta && (23)\\
B &= RL\sin^2\theta + PS(1+\cos^2\theta) && (24)\\
C &= PRL. && (25)
\end{aligned}
$$

2.2 Modes in a slowly varying inhomogeneous plasma

Let us write the plasma dispersion relation, Equation (22), as an implicit function $G(\mathbf{r}, t, \mathbf{k}, \omega) = 0$ that accounts for slow spatial-temporal variations in ω_{ps} and ω_{cs}. As the wave propagates through the plasma, it must always satisfy the plasma dispersion relation such that $G = 0$ along the entire wave trajectory in $(\mathbf{r}, t, \mathbf{k}, \omega)$-space. If τ is a variable parameterizing this trajectory, and G is always identically zero along this trajectory, then $dG/d\tau$ is zero:

$$\frac{d}{d\tau}G(\mathbf{r}, t, \mathbf{k}, \omega) = \frac{\partial G}{\partial \mathbf{r}}\frac{d\mathbf{r}}{d\tau} + \frac{\partial G}{\partial t}\frac{dt}{d\tau} + \frac{\partial G}{\partial \mathbf{k}}\frac{d\mathbf{k}}{d\tau} + \frac{\partial G}{\partial \omega}\frac{d\omega}{d\tau} = 0. \tag{26}$$

To understand the propagation, we want to find the derivatives with respect to τ. To do this, we view the radar pulse as a wave packet in the form

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{(2\pi)^4}\iiiint d\mathbf{k}\, d\omega\, \mathbf{E}(\mathbf{k}, \omega) \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t). \tag{27}$$

If the spatial variation of $\mathbf{E}(\mathbf{k}, \omega)$ is slow compared to $2\pi/|\mathbf{k}|$, and the temporal variation is slow compared to $2\pi/\omega$, then constructive interference occurs when the integrand phase is a constant. Differentiating this phase with respect to t and $\mathbf{k}$, and equating to zero, yields:

$$\frac{d\mathbf{r}}{dt} = \frac{\partial \omega}{\partial \mathbf{k}} = -\frac{\partial G/\partial \mathbf{k}}{\partial G/\partial \omega} = \frac{d\mathbf{r}/d\tau}{dt/d\tau}. \tag{28}$$

We therefore find that

$$\frac{d\mathbf{r}}{d\tau} = \frac{\partial G}{\partial \mathbf{k}}, \tag{29}$$

if and only if the arbitrary parameter τ is defined such that

$$\frac{dt}{d\tau} = -\frac{\partial G}{\partial \omega}. \tag{30}$$

It then follows from Equation (26) that $\mathbf{k}$ and ω will evolve such that

$$\frac{d\mathbf{k}}{d\tau} = -\frac{\partial G}{\partial \mathbf{r}} \tag{31}$$

$$\frac{d\omega}{d\tau} = \frac{\partial G}{\partial t}. \tag{32}$$

Equations (29)–(32) form a very general description of the wavepacket trajectory in $(\mathbf{r}, t, \mathbf{k}, \omega)$-space, referred to as the “ray” or “ray path”. A more explicit description can be achieved by assuming that the medium is plane-stratified, such that ω_{ps} and ω_{cs} vary only with altitude. This makes it possible to evaluate the ray path $\mathbf{r}(\tau)$ as a simple one-dimensional integral over the altitude coordinate (denoted z):

$$\mathbf{r}(\tau) - \mathbf{r}(0) = \int_0^z dz\, \frac{d\mathbf{r}/d\tau}{dz/d\tau} = \int_0^z dz\, \frac{\partial G/\partial \mathbf{k}}{\partial G/\partial k_z} = -\int_0^z dz\, \frac{\partial k_z}{\partial \mathbf{k}} = -\int_0^z dz\, \frac{\partial n_z}{\partial \mathbf{n}}, \tag{33}$$

where k_z and n_z are the z components of $\mathbf{k}$ and $\mathbf{n}$, respectively. If we view the ionosphere as a continuum of layers, then an application of Snell's law at the boundary of each layer implies that n_x and n_y, the horizontal components of $\mathbf{n}$, are conserved. Thus $\partial n_z/\partial \mathbf{n}$ is well-defined and Equation (33) can be integrated easily. To illustrate, we suppose that the magnetic field of the earth follows a unit vector $\hat{\mathbf{l}} = (l_x, l_y, l_z)$ which is at an angle to the vertical z direction. The dot product of $\hat{\mathbf{l}}$ and $\mathbf{n}$ is given by

$$n \cos\theta = l_x n_x + l_y n_y + l_z n_z. \tag{34}$$

Inserting this equation and $n^2 = n_x^2 + n_y^2 + n_z^2$ into Equation (22) yields a quartic equation for n_z, usually referred to as the Booker quartic [4]:

$$F(n_z) \equiv \alpha n_z^4 + \beta n_z^3 + \gamma n_z^2 + \delta n_z + \epsilon = 0. \tag{35}$$

At HF frequencies, the dielectric response of the plasma ions may be neglected compared to the electrons due to ion inertia. Letting $\omega_{pi} \to 0$, and adopting the definitions $X = \omega_{pe}^2/\omega^2$ and $Y = \omega_{ce}/\omega$, the coefficients are

$$\alpha = 1 - X - Y^2 + XY^2 l_z^2 \tag{36}$$
$$\beta = 2 l_x l_z n_x X Y^2 \tag{37}$$
$$\gamma = [2Y^2 - 2(1-X)](1 - n_x^2 - X) + XY^2[1 - l_z^2(1 - n_x^2) + l_x^2 n_x^2] \tag{38}$$
$$\delta = -2 l_x l_z n_x (1 - n_x^2) X Y^2 \tag{39}$$
$$\epsilon = (1-X)(1 - n_x^2 - X)^2 - Y^2(1 - n_x^2)(1 - n_x^2 - X) - l_x^2 n_x^2 (1 - n_x^2) X Y^2. \tag{40}$$

To use the quartic in the ray tracing equations, we note that the quartic holds for all values of n_x and n_y, thus

$$\frac{dF(n_z)}{dn_x} = \frac{\partial F(n_z)}{\partial n_z}\frac{\partial n_z}{\partial n_x} + \frac{\partial \alpha}{\partial n_x} n_z^4 + \frac{\partial \beta}{\partial n_x} n_z^3 + \frac{\partial \gamma}{\partial n_x} n_z^2 + \frac{\partial \delta}{\partial n_x} n_z + \frac{\partial \epsilon}{\partial n_x} = 0 \tag{41}$$
$$\frac{dF(n_z)}{dn_y} = \frac{\partial F(n_z)}{\partial n_z}\frac{\partial n_z}{\partial n_y} + \frac{\partial \alpha}{\partial n_y} n_z^4 + \frac{\partial \beta}{\partial n_y} n_z^3 + \frac{\partial \gamma}{\partial n_y} n_z^2 + \frac{\partial \delta}{\partial n_y} n_z + \frac{\partial \epsilon}{\partial n_y} = 0. \tag{42}$$

Solving these equations for $\partial n_z/\partial n_x$ and $\partial n_z/\partial n_y$, and inserting into Equation (33) provides explicit expressions for the ray path in a plane-stratified ionosphere [5]:

$$x(z) = x(0) - \int_0^z dz \left(\frac{\partial \beta}{\partial n_x} n_z^3 + \frac{\partial \gamma}{\partial n_x} n_z^2 + \frac{\partial \delta}{\partial n_x} n_z + \frac{\partial \epsilon}{\partial n_x} \right) \bigg/ \frac{\partial F(n_z)}{\partial n_z} \tag{43}$$
$$y(z) = y(0) - \int_0^z dz \left(\frac{\partial \beta}{\partial n_y} n_z^3 + \frac{\partial \gamma}{\partial n_y} n_z^2 + \frac{\partial \delta}{\partial n_y} n_z + \frac{\partial \epsilon}{\partial n_y} \right) \bigg/ \frac{\partial F(n_z)}{\partial n_z}, \tag{44}$$

where we have used the fact that $\partial\alpha/\partial n_x = \partial\alpha/\partial n_y = 0$. The problem of determining the ray path $\mathbf{r}(z) = [x(z), y(z), z]$ has been reduced to the problem of computing one-dimensional integrals of known quantities.

3 Effects of ionospheric irregularities on radar signal properties

This section considers the effects of ionospheric plasma irregularities on the propagation of a radar pulse. Irregularities are density structures within the ionospheric plasma that are small compared to the scale size of the overall ionosphere. The problem was first solved for isotropic media [6], and was later applied to the problem of phase scintillation of VHF and microwave signals [7, 8]. Recently, the isotropic theory has found application in the description of the Doppler spreading of HFOTHR signals [9]. In this section, the theory is extended to the general case of propagation in anisotropic media.

3.1 Path integral formulation of wave packet properties

This subsection shows how the propagation described in Section 2 is perturbed by the presence of ionospheric irregularities. Some of the properties associated with the wave packets include amplitude, phase, polarization, skip distance (change in wave packet location), group delay (elapsed time of wave packet propagation), DOA, and Doppler. Of these seven properties, the first three properties of amplitude, phase, and polarization, are observables that comprise point measurements of a wave field. They do not require measuring the variation of a field in space and time. The four remaining properties of skip distance, group delay, DOA, and Doppler, relate to the spatial-temporal variation of the three point-measurement observables.

Let us adopt the wave packet description of Section 2. Following the reasoning of Equation (33), the wave packet properties can be written as path integrals of Equations (29)–(32), namely the skip distance,

$$\Delta \mathbf{r} = -\int dz \, \frac{\partial k_z}{\partial \mathbf{k}}, \tag{45}$$

the group delay,

$$\Delta t = \int dz \, \frac{\partial k_z}{\partial \omega}, \tag{46}$$

the DOA,

$$\Delta \mathbf{k} = \int dz \, \frac{\partial k_z}{\partial \mathbf{r}}, \tag{47}$$

and the Doppler shift,

$$\Delta \omega = -\int dz \, \frac{\partial k_z}{\partial t}. \tag{48}$$

The derivatives are functions of z and can be calculated in a manner similar to Equations (43) and (44), namely:

$$\frac{\partial k_z}{\partial \mathbf{k}} = -\left(\frac{\partial \beta}{\partial \mathbf{k}} n_z^3 + \frac{\partial \gamma}{\partial \mathbf{k}} n_z^2 + \frac{\partial \delta}{\partial \mathbf{k}} n_z + \frac{\partial \epsilon}{\partial \mathbf{k}}\right) \bigg/ \frac{\partial F}{\partial k_z} \tag{49}$$

$$\frac{\partial k_z}{\partial \omega} = -\left(\frac{\partial \alpha}{\partial \omega} n_z^4 + \frac{\partial \beta}{\partial \omega} n_z^3 + \frac{\partial \gamma}{\partial \omega} n_z^2 + \frac{\partial \delta}{\partial \omega} n_z + \frac{\partial \epsilon}{\partial \omega}\right) \bigg/ \frac{\partial F}{\partial k_z} \tag{50}$$

$$\frac{\partial k_z}{\partial \mathbf{r}} = -\left(\frac{\partial \alpha}{\partial \mathbf{r}} n_z^4 + \frac{\partial \beta}{\partial \mathbf{r}} n_z^3 + \frac{\partial \gamma}{\partial \mathbf{r}} n_z^2 + \frac{\partial \delta}{\partial \mathbf{r}} n_z + \frac{\partial \epsilon}{\partial \mathbf{r}}\right) \bigg/ \frac{\partial F}{\partial k_z} \tag{51}$$

$$\frac{\partial k_z}{\partial t} = -\left(\frac{\partial \alpha}{\partial t} n_z^4 + \frac{\partial \beta}{\partial t} n_z^3 + \frac{\partial \gamma}{\partial t} n_z^2 + \frac{\partial \delta}{\partial t} n_z + \frac{\partial \epsilon}{\partial t}\right) \bigg/ \frac{\partial F}{\partial k_z}. \tag{52}$$

Having defined the path integrals, we now embark on a perturbation of k_z to account for the presence of ionospheric plasma density irregularities. We denote the background plasma density as N_0 and the irregularity density as N_1, such that the total plasma density is $N = N_0 + N_1$, and N_1 has zero mean. The perturbed wavenumber is given by

$$k_z(N) = k_z(N_0) + N_1 \left.\frac{\partial k_z}{\partial N}\right|_{N_0} \equiv k_{z0} + k_{z1}, \tag{53}$$

where the numbered subscript denotes the order of perturbation, or in other words the power of N_1 to which the term is proportional. The first-order density perturbation to Equations (45)–(48) can therefore be written

$$\Delta \mathbf{r}_1 = -\int dz \left(N_1 \frac{\partial^2 k_z}{\partial N \partial \mathbf{k}} + \frac{\partial N_1}{\partial \mathbf{k}} \frac{\partial k_z}{\partial N}\right) \tag{54}$$

$$\Delta t_1 = \int dz \left(N_1 \frac{\partial^2 k_z}{\partial N \partial \omega} + \frac{\partial N_1}{\partial \omega} \frac{\partial k_z}{\partial N}\right) \tag{55}$$

$$\Delta \mathbf{k}_1 = \int dz \left(N_1 \frac{\partial^2 k_z}{\partial N \partial \mathbf{r}} + \frac{\partial N_1}{\partial \mathbf{r}} \frac{\partial k_z}{\partial N}\right) \tag{56}$$

$$\Delta \omega_1 = -\int dz \left(N_1 \frac{\partial^2 k_z}{\partial N \partial t} + \frac{\partial N_1}{\partial t} \frac{\partial k_z}{\partial N}\right). \tag{57}$$

Some of the terms can be neglected. In the first and second equations, the second term in the integrand is zero because the density perturbation N_1 is not a function of the incident wave packet properties $\mathbf{k}$ and ω, as long as the incident radio signals are of low enough power to not modify the properties of the plasma medium. In the third equation, we will assume that the scale length of N_1 (the density irregularity) is short compared to the scale length of N_0 (the background ionosphere density, encapsulated in k_z), which allows us to ignore the first term compared to the second. Finally, in the fourth equation, the first term is zero because we assume that the background density N_0 is not a function of time. With these terms ignored, the wave packet properties are given by

$$\Delta \mathbf{r}_1 = -\int dz\, N_1 \frac{\partial^2 k_z}{\partial N \partial \mathbf{k}} \tag{58}$$

$$\Delta t_1 = \int dz\, N_1 \frac{\partial^2 k_z}{\partial N \partial \omega} \tag{59}$$

$$\Delta \mathbf{k}_1 = \int dz\, \frac{\partial N_1}{\partial \mathbf{r}} \frac{\partial k_z}{\partial N} \tag{60}$$

$$\Delta \omega_1 = -\int dz\, \frac{\partial N_1}{\partial t} \frac{\partial k_z}{\partial N}. \tag{61}$$

Thus all properties can be expressed as linear combinations of the quantities N_1, $\partial N_1/\partial \mathbf{r}$, and $\partial N_1/\partial t$. We take these quantities to be zero-mean random variables, and thus the means of the properties $\Delta \mathbf{r}_1$, Δt_1, $\Delta \mathbf{k}_1$, and $\Delta \omega_1$ are also zero. In the next subsection, we examine the second-order statistics of these properties.

3.2 Second-order statistics

Let us write Equations (58)–(61) in the form

$$h(x, y, z, t) = \int_0^z dz'\, g(x, y, z', t) f(z'), \tag{62}$$

where f is deterministic and g is random. Since g and h are random, they have space-time autocorrelations defined as

$$R_g(\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{T}) = E[g(x, y, z, t) g(x + \mathcal{X}, y + \mathcal{Y}, z + \mathcal{Z}, t + \mathcal{T})] \tag{63}$$

$$R_h(\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{T}) = E[h(x, y, z, t) h(x + \mathcal{X}, y + \mathcal{Y}, z + \mathcal{Z}, t + \mathcal{T})], \tag{64}$$

where $E(\cdot)$ is the expectation operator. If we assume that the correlation length of g, associated with the irregularities, is short compared to the length of the wave packet trajectory through the ionosphere, then we can think of R_g and R_h as z-dependent quantities, which we denote as $R_g(\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{T}; z)$ and $R_h(\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{T}; z)$, respectively. From Equation (62), the relationship between R_g and R_h is given by

$$R_h(\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{T}; z) = E\left[\int_0^z dz'\, g(x, y, z', t) f(z')\right. \tag{65}$$

$$\left. \times \int_0^{z+\mathcal{Z}} dz''\, g(x + \mathcal{X}, y + \mathcal{Y}, z'', t + \mathcal{T}) f(z'')\right] \tag{66}$$

$$= \int_0^z \int_0^{z+\mathcal{Z}} dz'\, dz''\, R_g(\mathcal{X}, \mathcal{Y}, z'' - z', \mathcal{T}; z') f(z') f(z''). \tag{67}$$

This expression for R_h is an integration over a rectangular region, bounded by $(0, z)$ and $(0, z + \mathcal{Z})$. However, since the correlation length of g is short compared with the dimensions of the rectangle, the integration will be dominated by the contributions along a narrow strip where $z' \approx z''$. Therefore we can write R_h as

$$R_h(\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{T}; z) \approx \int_0^z dz'\, f^2(z') \int_{-\infty}^{\infty} dz''\, R_g(\mathcal{X}, \mathcal{Y}, z'', \mathcal{T}; z'). \tag{68}$$

Let us take the Fourier transform of R_h:

$$\begin{aligned}
S_h(k_x, k_y, k_z, \omega; z) = & \iiiint d\mathcal{X} d\mathcal{Y} d\mathcal{Z} d\mathcal{T}\, e^{-ik_x\mathcal{X} - ik_y\mathcal{Y} - ik_z\mathcal{Z} + i\omega\mathcal{T}} \int_0^z dz'\, f^2(z') && (69)\\
& \times \int_{-\infty}^{\infty} dz''\, R_g(\mathcal{X}, \mathcal{Y}, z'', \mathcal{T}; z') && (70)\\
= & \, 2\pi\delta(k_z) \int_0^z dz'\, f^2(z') \iiint d\mathcal{X} d\mathcal{Y} d\mathcal{T} e^{-ik_x\mathcal{X} - ik_y\mathcal{Y} + i\omega\mathcal{T}} && (71)\\
& \times \int_{-\infty}^{\infty} dz''\, R_g(\mathcal{X}, \mathcal{Y}, z'', \mathcal{T}; z') && (72)\\
= & \, 2\pi\delta(k_z) \int_0^z dz'\, f^2(z') S_g(k_x, k_y, 0, \omega; z'). && (73)
\end{aligned}$$

Here, the power spectrum S_g is the Fourier transform of the autocorrelation R_g. To find S_g, recall Equations (58)–(61). We are interested in several quantities denoted by g, including N_1, $\partial N_1/\partial x$, $\partial N_1/\partial y$, $\partial N_1/\partial z$, and $\partial N_1/\partial t$. The spectra of the latter four quantities are related to the spectrum of N_1 by the identities

$$\begin{aligned}
S_{\frac{\partial N_1}{\partial x}} &= k_x^2 S_{N_1} && (74)\\
S_{\frac{\partial N_1}{\partial y}} &= k_y^2 S_{N_1} && (75)\\
S_{\frac{\partial N_1}{\partial z}} &= k_z^2 S_{N_1} && (76)\\
S_{\frac{\partial N_1}{\partial t}} &= \omega^2 S_{N_1}. && (77)
\end{aligned}$$

As for S_{N_1} itself, we use a spectrum model for plasma irregularities that follows a 4th-order power law [10], and a dispersion relation for approximately perpendicular-propagating drift wave turbulence [11]. The form is given by

$$S_{N_1}(\mathbf{k}_i, \omega_i; z) = \frac{4\sqrt{2\alpha_i}\pi^2 E[N_1^2(z)] k_0^{-3}}{1 + k_0^{-4}(k_{i\perp}^2 + \alpha_i k_{i\|}^2)^2} \delta(|\omega_i| - k_{i\perp} v_d). \tag{78}$$

Here, v_d is the plasma diamagnetic drift velocity, k_0 is the "outer" scale length parameter, α_i is an anisotropy parameter, $k_{i\perp}$ is the magnitude of the component of the density irregularity wavenumber $\mathbf{k}_i$ that is perpendicular to the earth's magnetic field, $k_{i\|}$ is the magnitude of the component of $\mathbf{k}_i$ along the field, and the variance of the density fluctuations $E[N_1^2(z)]$ is assumed to be a function of altitude z. As in Section 2, we suppose that the magnetic field of the earth follows a unit vector $\hat{\mathbf{l}} = (l_x, l_y, l_z)$. The quantity $k_{i\|}$ is given by the dot product of $\mathbf{k}_i$ and $\hat{\mathbf{l}}$:

$$k_{i\|} = k_{ix}l_x + k_{iy}l_y + k_{iz}l_z, \tag{79}$$

and $k_{i\perp}$ is given by

$$k_{i\perp} = |\mathbf{k}_i - k_{i\|}\hat{\mathbf{l}}|. \tag{80}$$

Although z is constrained to be vertical, at this point we can rotate x and y azimuthally without loss of generality such that the magnetic field vector lies in the y-z plane and $l_x = 0$. In light of Equation (73), we evaluate S_{N_1} for $k_{iz} = 0$:

$$S_{N_1}(k_{ix}, k_{iy}, 0, \omega_i; z) = \frac{4\sqrt{2\alpha_i}\pi^2 E[N_1^2(z)]k_0^{-3}}{1 + k_0^{-4}[k_{ix}^2 + (l_z^2 + \alpha_i l_y^2)k_{iy}^2]^2}\delta\left[|\omega_i| - (k_{ix}^2 + l_z^2 k_{iy}^2)^{1/2} v_d\right]. \tag{81}$$

Combining Equations (73), (74)–(77), and (81), we have that the spectra are

$$S_{\Delta r_{x1}} = \frac{a\delta(k_z)\delta[|\omega| - (k_x^2 + l_z^2 k_y^2)^{1/2} v_d]}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2)k_y^2]^2}\int_0^z dz'\, E[N_1^2(z')]\left(\frac{\partial^2 k_z}{\partial N \partial k_x}\right)^2 \tag{82}$$

$$S_{\Delta r_{y1}} = \frac{a\delta(k_z)\delta[|\omega| - (k_x^2 + l_z^2 k_y^2)^{1/2} v_d]}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2)k_y^2]^2}\int_0^z dz'\, E[N_1^2(z')]\left(\frac{\partial^2 k_z}{\partial N \partial k_y}\right)^2 \tag{83}$$

$$S_{\Delta r_{z1}} = \frac{a\delta(k_z)\delta[|\omega| - (k_x^2 + l_z^2 k_y^2)^{1/2} v_d]}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2)k_y^2]^2}\int_0^z dz'\, E[N_1^2(z')]\left(\frac{\partial^2 k_z}{\partial N \partial k_z}\right)^2 \tag{84}$$

$$S_{\Delta t_1} = \frac{a\delta(k_z)\delta[|\omega| - (k_x^2 + l_z^2 k_y^2)^{1/2} v_d]}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2)k_y^2]^2}\int_0^z dz'\, E[N_1^2(z')]\left(\frac{\partial^2 k_z}{\partial N \partial \omega}\right)^2 \tag{85}$$

$$S_{\Delta k_{x1}} = \frac{ak_x^2\delta(k_z)\delta[|\omega| - (k_x^2 + l_z^2 k_y^2)^{1/2} v_d]}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2)k_y^2]^2}\int_0^z dz'\, E[N_1^2(z')]\left(\frac{\partial k_z}{\partial N}\right)^2 \tag{86}$$

$$S_{\Delta k_{y1}} = \frac{ak_y^2\delta(k_z)\delta[|\omega| - (k_x^2 + l_z^2 k_y^2)^{1/2} v_d]}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2)k_y^2]^2}\int_0^z dz'\, E[N_1^2(z')]\left(\frac{\partial k_z}{\partial N}\right)^2 \tag{87}$$

$$S_{\Delta k_{z1}} = \frac{ak_z^2\delta(k_z)\delta[|\omega| - (k_x^2 + l_z^2 k_y^2)^{1/2} v_d]}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2)k_y^2]^2}\int_0^z dz'\, E[N_1^2(z')]\left(\frac{\partial k_z}{\partial N}\right)^2 \tag{88}$$

$$S_{\omega_1} = \frac{a\omega^2\delta(k_z)\delta[|\omega| - (k_x^2 + l_z^2 k_y^2)^{1/2} v_d]}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2)k_y^2]^2}\int_0^z dz'\, E[N_1^2(z')]\left(\frac{\partial k_z}{\partial N}\right)^2, \tag{89}$$

where $a = 8\sqrt{2\alpha_i}\pi^3 k_0^{-3}$. Note that the presence of the $k_z^2\delta(k_z)$ factor in Equation (88) implies that the spectrum $S_{\Delta k_{z1}}$ will be zero. This result arises from the approximations leading to Equation (68). Since in practical HF radars we are not concerned with phase correlations in the vertical direction, there is no need to improve on these approximations. This completes the determination of the second-order statistics.

4 Determination of signal phase

In the previous section, we showed how ionospheric irregularities impacted the radar signal skip distance, group delay, DOA, and Doppler. In this section, we investigate the autocorrelation of the signal phase over space and time to assist in the determination of beamforming and Doppler processing performance, respectively.

4.1 Phase spatial autocorrelation

The first task is to determine the spatial autocorrelation of the phase as observed on the ground. We consider only the first-order phase perturbation due to irregularities in the ionosphere, which is related to the wavenumber perturbations by

$$\Delta k_{x1} = \frac{\partial \phi_1}{\partial x} \tag{90}$$

$$\Delta k_{y1} = \frac{\partial \phi_1}{\partial y}. \tag{91}$$

Hence, the wavenumber spectra are related to the phase spectrum by

$$S_{\Delta k_{x1}} = k_x^2 S_{\phi_1} \tag{92}$$

$$S_{\Delta k_{y1}} = k_y^2 S_{\phi_1}. \tag{93}$$

By inspection of Equations (86) and (87), we see that the phase spectrum is given by

$$S_{\phi_1}(k_x, k_y, k_z, \omega; z) = b(z) \frac{\delta(k_z)\delta[|\omega| - (k_x^2 + l_z^2 k_y^2)^{1/2} v_d]}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2) k_y^2]^2}, \tag{94}$$

where

$$b(z) = a \int_0^z dz' E[N_1^2(z')] \left(\frac{\partial k_z}{\partial N} \right)^2. \tag{95}$$

We now find the autocorrelation $R_{\phi_1}(\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{T})$ at $(\mathcal{Z}, \mathcal{T}) = 0$:

$$R_{\phi_1}(\mathcal{X}, \mathcal{Y}, 0, 0) = \frac{b(z)}{(2\pi)^4} \iiiint dk_x dk_y dk_z d\omega \, e^{ik_x \mathcal{X} + ik_y \mathcal{Y}} \frac{\delta(k_z)\delta[|\omega| - (k_x^2 + l_z^2 k_y^2)^{1/2} v_d]}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2) k_y^2]^2}. \tag{96}$$

If we make the substitutions

$$k_y' = k_y (l_z^2 + \alpha_i l_y^2)^{1/2} \tag{97}$$

$$\mathcal{Y}' = \mathcal{Y} (l_z^2 + \alpha_i l_y^2)^{-1/2} \tag{98}$$

$$u^2 = (k_x^2 + k_y'^2)/k_0^2 \tag{99}$$

$$\rho^2 = \mathcal{X}^2 + \mathcal{Y}'^2 \tag{100}$$

$$\psi = \arctan(k_y'/k_x) \tag{101}$$

$$\zeta = \arctan(\mathcal{Y}'/\mathcal{X}), \tag{102}$$

then the autocorrelation can be written

$$R_{\phi_1}(\rho) = \frac{b(z)k_0^2}{8\pi^4(l_z^2+\alpha_i l_y^2)^{1/2}} \int_0^\infty \int_0^{2\pi} du\, d\psi\, \frac{u e^{iuk_0\rho\cos(\psi-\zeta)}}{1+u^4} \tag{103}$$

$$= \frac{b(z)k_0^2}{4\pi^3(l_z^2+\alpha_i l_y^2)^{1/2}} \int_0^\infty du\, \frac{uJ_0(uk_0\rho)}{1+u^4}, \tag{104}$$

where J_0 is the ordinary zero-order Bessel function. It is desirable to extend the integral from the semi-finite u axis to the infinite u axis so that one can use the residue theorem. However, the integrand is an odd function. A useful trick is to rewrite the integrand in zero-order Hankel functions. The required relations are

$$J_0(\xi) = \frac{1}{2}\left[H_0^{(1)}(\xi) + H_0^{(2)}(\xi)\right] \tag{105}$$

$$H_0^{(1)}(-\xi) = -H_0^{(2)}(\xi). \tag{106}$$

Using these relations, we can write Equation (104) as

$$R_{\phi_1}(\rho) = \frac{b(z)k_0^2}{8\pi^3(l_z^2+\alpha_i l_y^2)^{1/2}} \int_{-\infty}^\infty du\, \frac{uH_0^{(1)}(uk_0\rho)}{1+u^4}. \tag{107}$$

The Hankel function of the first kind is bounded in the upper half plane. A closed contour in the upper half plane encloses poles at the locations $e^{i\pi/4}$ and $e^{3i\pi/4}$. Evaluating the residues gives the result

$$R_{\phi_1}(\rho) = \frac{b(z)k_0^2[H_0^{(1)}(e^{i\pi/4}k_0\rho) - H_0^{(1)}(e^{3i\pi/4}k_0\rho)]}{16\pi^2(l_z^2+\alpha_i l_y^2)^{1/2}} \tag{108}$$

$$= E(\phi_1^2)\left[H_0^{(1)}(e^{i\pi/4}k_0\rho) + H_0^{(2)}(e^{-i\pi/4}k_0\rho)\right], \tag{109}$$

where $E(\phi_1^2) = b(z)k_0^2/[16\pi^2(l_z^2+\alpha_i l_y^2)^{1/2}]$, and we recall $\rho^2 = \mathcal{X}^2 + \mathcal{Y}^2/(l_z^2+\alpha_i l_y^2)$. We now turn our attention to the autocorrelation of the complex signal amplitude $A_1 = e^{i\phi_1}$, which we denote as R_{A_1}:

$$R_{A_1}(\rho) = E\left[e^{-i\phi(\rho')}e^{i\phi(\rho'+\rho)}\right]. \tag{110}$$

The central limit theorem implies Gaussian single-point statistics for $\phi(\rho)$. Under this condition, we can use the following identity [12]:

$$E\left[e^{-i\phi_1(\rho')}e^{i\phi_1(\rho'+\rho)}\right] = \exp\left[R_{\phi_1}(\rho) - E(\phi_1^2)\right]. \tag{111}$$

The autocorrelation function is therefore given by

$$R_{A_1}(\rho) = \exp\left\{E(\phi_1^2)\left[H_0^{(1)}\left(e^{i\pi/4}k_0\rho\right) + H_0^{(2)}\left(e^{-i\pi/4}k_0\rho\right) - 1\right]\right\}. \tag{112}$$

In practice, the outer scale length parameter k_0 is about $2\pi/(60\text{ km})$ [13], and thus for spatial separations much less than 60 km it is appropriate to invoke the small-argument expansions for the zero-order Hankel functions:

$$H_0^{(1)}(\xi) = J_0(\xi) + iY_0(\xi) \approx 1 + \frac{2i}{\pi}\left[\log\left(\frac{\xi}{2}\right)\left(1 - \frac{\xi^2}{4}\right) + \Gamma\right] \tag{113}$$

$$H_0^{(2)}(\xi) = J_0(\xi) - iY_0(\xi) \approx 1 - \frac{2i}{\pi}\left[\log\left(\frac{\xi}{2}\right)\left(1 - \frac{\xi^2}{4}\right) + \Gamma\right], \tag{114}$$

where $\Gamma = 0.5772$ is Euler's constant. Inserting these expansions into Equation (112) provides the autocorrelation for spatial distances much less than 60 km:

$$R_{A_1}(\rho) \approx \exp\left[E(\phi_1^2)\frac{k_0^2\rho^2}{\pi}\log\frac{k_0\rho}{2}\right] \tag{115}$$

$$\approx 1 + E(\phi_1^2)\frac{k_0^2\rho^2}{\pi}\log\frac{k_0\rho}{2}. \tag{116}$$

This completes the determination of the signal spatial autocorrelation.

4.2 Phase temporal autocorrelation

Let us now examine the temporal autocorrelation of the phase as observed on the ground. The phase spectrum was given by Equation (94). We now evaluate the phase autocorrelation function $R_{\phi_1}(\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{T})$ at $(\mathcal{X}, \mathcal{Y}, \mathcal{Z}) = 0$:

$$R_{\phi_1}(0,0,0,\mathcal{T}) = \frac{b(z)}{(2\pi)^4}\iiiint dk_x dk_y dk_z d\omega\, e^{-i\omega\mathcal{T}}\, \frac{\delta(k_z)\delta[|\omega| - (k_x^2 + l_z^2 k_y^2)^{1/2}v_d]}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2)k_y^2]^2} \tag{117}$$

$$= \frac{b(z)}{8\pi^4}\iint dk_x dk_y \frac{\cos[(k_x^2 + l_z^2 k_y^2)^{1/2}v_d\mathcal{T}]}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2)k_y^2]^2}. \tag{118}$$

To proceed, we note the parameter α_i is typically large (≈ 3000) [13]. Thus, the only important contributions to the integral occur when $k_y \ll k_x$, because otherwise the denominator is much greater than the numerator. Therefore, we can approximate the integral as

$$R_{\phi_1}(\mathcal{T}) \approx \frac{b(z)k_0}{8\sqrt{2}\pi^3(l_z^2 + \alpha_i l_y^2)^{1/2}}\iint dk_x dk_y \frac{\cos[(k_x^2 + l_z^2 k_y^2)^{1/2}v_d\mathcal{T}]\delta(k_y)}{1 + k_0^{-4}[k_x^2 + (l_z^2 + \alpha_i l_y^2)k_y^2]^2} \tag{119}$$

$$= \frac{b(z)k_0}{8\sqrt{2}\pi^3(l_z^2 + \alpha_i l_y^2)^{1/2}}\int dk_x \frac{\cos(k_x v_d\mathcal{T})}{1 + k_x^4/k_0^4}. \tag{120}$$

Carrying out the contour integration, we find that

$$R_{\phi_1}(\mathcal{T}) = \frac{b(z)k_0^2}{8\sqrt{2}\pi^2(l_z^2 + \alpha_i l_y^2)^{1/2}}\exp(-k_0 v_d|\mathcal{T}|/\sqrt{2})\cos(k_0 v_d|\mathcal{T}|/\sqrt{2} - \pi/4) \tag{121}$$

$$= \sqrt{2}E(\phi_1^2)\exp(-k_0 v_d|\mathcal{T}|/\sqrt{2})\cos(k_0 v_d|\mathcal{T}|/\sqrt{2} - \pi/4), \tag{122}$$

where, as in Equation (109), $E(\phi_1^2) = b(z)k_0^2/[16\pi^2(l_z^2 + \alpha_i l_y^2)^{1/2}]$.

Finally, we calculate the autocorrelation for the complex amplitude. Inserting R_{ϕ_1} into Equation (111) yields

$$R_{A_1}(\mathcal{T}) = \exp\{E(\phi_1^2)[\sqrt{2}\exp(-k_0 v_d|\mathcal{T}|/\sqrt{2})\cos(k_0 v_d|\mathcal{T}|/\sqrt{2} - \pi/4) - 1]\}. \quad (123)$$

Expanding in small $\mathcal{T}$, we find that

$$\begin{aligned} R_{A_1}(\mathcal{T}) &\approx \exp\left[-E(\phi_1^2)k_0^2 v_d^2 \mathcal{T}^2/2\right] && (124) \\ &\approx 1 - E(\phi_1^2)k_0^2 v_d^2 \mathcal{T}^2/2. && (125) \end{aligned}$$

This completes the determination of the signal temporal autocorrelation.

5 Conclusion

A new theory of radio wave propagation in ionospheric plasma density irregularities has been presented. The theory was motivated by the need to quantify and understand the target-resolving capabilities and clutter characteristics of HF radar systems. The theory assumed an anisotropic ionospheric plasma that is inhomogeneous in the vertical direction only. The formulation is in terms of path integrals of the ray tracing equations. A quiescent space-time signal trajectory was defined by the solution for the ray path in an ionosphere density profile without irregularities. The effect of the irregularities was modelled as small perturbations to the quiescent solution. The plasma density perturbations led to predicted perturbations of the radar signal properties of skip distance, group delay, DOA, and Doppler. The perturbations were evaluated for the case of random plasma density irregularities with a power-law wavenumber spectrum, which led to the predicted power spectra of the signal properties as a function of wavenumber and frequency.

Practical HF radar systems estimate signal properties by analyzing the space-time variation of the radar signal phase. Accordingly, the signal DOA and Doppler power spectra were used to derive the space-time autocorrelation of the signal phase. This autocorrelation was then recast as an autocorrelation of the signal complex amplitude for convenience of use in radar performance predictions. This autocorrelation has a width that depends on the outer scale length of the plasma density irregularities, the phase velocity of the irregularities, and the root mean square phase scintillation incurred during the signal transit of the ionosphere.

Future work will involve employing the theory presented in this memorandum to make quantitative predictions of the performance of HFOTHR and HFSWR systems. Of particular interest are the target-resolving capabilities of HFOTHR, and the physical limitations on useful radar waveform bandwidth, antenna aperture, and integration time. These results also carry over to hybrid HFOTHR-HFSWR systems that use an ionospheric transmit path and a surface wave receive path. Furthermore, the theory will be adapted to quantify the performance of HFSWR ionospheric clutter cancellation technologies, such as elevation angle-resolving planar arrays and Space-Time Adaptive Processing (STAP) algorithms. However, it should be kept in mind that the theory cannot be easily adapted to a three-dimensionally inhomogeneous background ionosphere, so modifications to the theory would be necessary in regions of known horizontal plasma density gradients, such as the equatorial anomaly and the auroral cusp.

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Annex A: List of symbols

a	$8\sqrt{2\alpha_i}\pi^3 k_0^{-3}$
A	coefficient of n^4 in plasma dispersion relation
A_1	complex amplitude $e^{i\phi_1}$
b	$a\int_0^z dz' E[N_1^2(z')] \left(\frac{\partial k_z}{\partial N}\right)^2$
B	coefficient of n^2 in plasma dispersion relation
B_0	magnitude of static component of magnetic field
$\mathbf{B}$	magnetic field
$\mathbf{B}_0$	static component of magnetic field
c	speed of light
C	constant in plasma dispersion relation
D	element of plasma conductivity tensor
$E(\cdot)$	expectation operator
E_{1x}	magnitude of component of $\mathbf{E}_1$ in x direction
E_{1y}	magnitude of component of $\mathbf{E}_1$ in y direction
E_{1z}	magnitude of component of $\mathbf{E}_1$ in z direction
$\mathbf{E}$	electric field
$\mathbf{E}_1$	linearized electric field
f	$\in \left\{\frac{\partial^2 k_z}{\partial N \partial \mathbf{k}}, \frac{\partial^2 k_z}{\partial N \partial \omega}, \frac{\partial k_z}{\partial N}\right\}$
$F(\cdot)$	Booker quartic
g	$\in \left\{N_1, \frac{\partial N_1}{\partial \mathbf{r}}, \frac{\partial N_1}{\partial t}\right\}$
$G(\cdot)$	implicit form of plasma dispersion relation
h	$\in \{\Delta\mathbf{r}_1, \Delta t_1, \Delta\mathbf{k}_1, \Delta\omega_1\}$
$H_0^{(1)}(\cdot)$	zero-order Hankel function of first kind
$H_0^{(2)}(\cdot)$	zero-order Hankel function of second kind
i	imaginary unit
$\mathbf{I}$	identity tensor
$J_0(\cdot)$	zero-order ordinary Bessel function
$\mathbf{J}$	internal self-consistent current density
$\mathbf{J}_e$	externally applied current density
$\mathbf{J}_1$	linearized internal self-consistent current density
k	magnitude of $\mathbf{k}$
k_0	outer scale length parameter
k_x	magnitude of component of $\mathbf{k}$ in x direction
k_y	magnitude of component of $\mathbf{k}$ in y direction
k_z	magnitude of component of $\mathbf{k}$ in z direction
k_{z0}	quiescent k_z
k_{z1}	perturbation of k_z
$k_{i\parallel}$	magnitude of component of $\mathbf{k}_i$ parallel to magnetic field
$k_{i\perp}$	magnitude of component of $\mathbf{k}_i$ perpendicular to magnetic field

k_{ix}	magnitude of component of $\mathbf{k}_i$ in x direction
k_{iy}	magnitude of component of $\mathbf{k}_i$ in y direction
k_{iz}	magnitude of component of $\mathbf{k}_i$ in z direction
k'_y	$k_y(l_z^2 + \alpha_i l_y)^{1/2}$
Δk_{x1}	magnitude of component of $\Delta\mathbf{k}_1$ in x direction
Δk_{y1}	magnitude of component of $\Delta\mathbf{k}_1$ in y direction
Δk_{z1}	magnitude of component of $\Delta\mathbf{k}_1$ in z direction
$\mathbf{k}$	wavenumber
$\mathbf{k}_i$	plasma density irregularity wavenumber
$\Delta\mathbf{k}$	change in wavenumber
$\Delta\mathbf{k}_1$	linearized change in wavenumber
l_x	magnitude of component of $\hat{\mathbf{l}}$ in x direction
l_y	magnitude of component of $\hat{\mathbf{l}}$ in y direction
l_z	magnitude of component of $\hat{\mathbf{l}}$ in z direction
$\hat{\mathbf{l}}$	unit vector in direction of magnetic field
L	element of plasma conductivity tensor
m_s	particle mass of species s
n	magnitude of index of refraction
n_x	magnitude of component of $\mathbf{n}$ in x direction
n_y	magnitude of component of $\mathbf{n}$ in y direction
n_z	magnitude of component of $\mathbf{n}$ in z direction
$\mathbf{n}$	index of refraction
N_0	quiescent plasma denstiy
N_1	perturbation of plasma density
N_s	density of species s
P	element of plasma conductivity tensor
q_s	electrical charge of species s
Δr_{x1}	magnitude of component of $\Delta\mathbf{r}_1$ in x direction
Δr_{y1}	magnitude of component of $\Delta\mathbf{r}_1$ in y direction
Δr_{z1}	magnitude of component of $\Delta\mathbf{r}_1$ in z direction
$\mathbf{r}$	spatial position
$\Delta\mathbf{r}$	change in spatial position
$\Delta\mathbf{r}_1$	linearized change in spatial position
R	element of plasma conductivity tensor
R_g	space-time autocorrelation of g
R_h	space-time autocorrelation of h
R_{A_1}	space-time autocorrelation of A_1
R_{ϕ_1}	space-time autocorrelation of ϕ_1
s	plasma species (electrons, ions)
S	element of plasma conducitivity tensor
S_g	wavenumber-frequency power spectrum of g
S_h	wavenumber-frequency power spectrum of h

S_{N_1}	wavenumber-frequency power spectrum of N_1
$S_{\frac{\partial N_1}{\partial x}}$	wavenumber-frequency power spectrum of $\frac{\partial N_1}{\partial x}$
$S_{\frac{\partial N_1}{\partial y}}$	wavenumber-frequency power spectrum of $\frac{\partial N_1}{\partial y}$
$S_{\frac{\partial N_1}{\partial z}}$	wavenumber-frequency power spectrum of $\frac{\partial N_1}{\partial z}$
$S_{\frac{\partial N_1}{\partial t}}$	wavenumber-frequency power spectrum of $\frac{\partial N_1}{\partial t}$
$S_{\Delta r_{x1}}$	wavenumber-frequency power spectrum of Δr_{x1}
$S_{\Delta r_{y1}}$	wavenumber-frequency power spectrum of Δr_{y1}
$S_{\Delta r_{z1}}$	wavenumber-frequency power spectrum of Δr_{z1}
$S_{\Delta t_1}$	wavenumber-frequency power spectrum of Δt_1
$S_{\Delta k_{x1}}$	wavenumber-frequency power spectrum of Δk_{x1}
$S_{\Delta k_{y1}}$	wavenumber-frequency power spectrum of Δk_{y1}
$S_{\Delta k_{z1}}$	wavenumber-frequency power spectrum of Δk_{z1}
$S_{\Delta\omega_1}$	wavenumber-frequency power spectrum of $\Delta\omega_1$
S_{ϕ_1}	wavenumber-frequency power spectrum of ϕ_1
t	time
Δt	change in time
Δt_1	linearized change in time
$\mathcal{T}$	correlation lag in time direction
u	$(k_x^2 + k_y'^2)^{1/2}/k_0$
v_d	plasma diamagnetic drift velocity
v_{s1x}	magnitude of component of $\mathbf{v}_{s1}$ in x direction
v_{s1y}	magnitude of component of $\mathbf{v}_{s1}$ in y direction
v_{s1z}	magnitude of component of $\mathbf{v}_{s1}$ in z direction
$\mathbf{v}_s$	fluid velocity of species s
$\mathbf{v}_{s1}$	linearized fluid velocity of species s
x	spatial position on x axis
$\hat{\mathbf{x}}$	unit vector along x axis
X	ω_{pe}^2/ω^2
$\mathcal{X}$	correlation lag in x direction
y	spatial position on y axis
$\hat{\mathbf{y}}$	unit vector along y axis
Y	ω_{ce}/ω
$Y_0(\cdot)$	zero-order modified Bessel function
$\mathcal{Y}$	correlation lag in y direction
$\mathcal{Y}'$	$\mathcal{Y}(l_z^2 + \alpha_i l_y^2)^{-1/2}$
z	spatial position on z axis
z'	variable of integration
z''	variable of integration
$\hat{\mathbf{z}}$	unit vector along z axis
$\mathcal{Z}$	correlation lag in z direction
α	coefficient of n_z^4 in Booker quartic

α_i	plasma density irregularity anisotropy parameter
β	coefficient of n_z^3 in Booker quartic
γ	coefficient of n_z^2 in Booker quartic
Γ	Euler constant (0.5772...)
δ	coefficient of n_z in Booker quartic
$\delta(\cdot)$	Dirac delta function
ϵ	constant in Booker quartic
ϵ_0	permittivity of free space
$\boldsymbol{\epsilon}$	permittivity tensor
ζ	$\arctan(\mathcal{Y}'/\mathcal{X})$
θ	angle of **k** or **n** with respect to direction of magnetic field
μ_0	permeability of free space
$\boldsymbol{\sigma}$	conductivity tensor
ξ	Bessel or Hankel function argument
ρ	$\sqrt{\mathcal{X}^2 + \mathcal{Y}'^2}$
ρ'	variable of integration
τ	parameter for ray trajectory
ϕ_1	linearized signal phase
ψ	$\arctan(k'_y/k_x)$
ω	wave frequency
ω_i	plasma density irregularity frequency
ω_{cs}	cyclotron frequency of species s
ω_{ps}	plasma frequency of species s
$\Delta\omega$	change in wave frequency
$\Delta\omega_1$	linearized change in wave frequency

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A new theory of radio wave propagation in ionospheric plasma density irregularities is presented. The theory is motivated by the need to quantify and understand the target-resolving capabilities and clutter characteristics of high-frequency radar systems. The theory is formulated in terms of path integrals of the ray tracing equations, which lead to expressions for the effects of the irregularities on the radar signal properties of skip distance, group delay, direction of arrival, and Doppler. The expressions are evaluated for the case of random density irregularities with a power-law wavenumber spectrum, which leads to predicted power spectra for the signal properties. The signal phase spatial-temporal autocorrelation function is also derived.

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