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HEMi V.2.02 Engagement Simulation (U)

R. Lestage
DRDC Valcartier

Defence R&D Canada – Valcartier
Technical Memorandum
DRDC Valcartier TM 2006-180
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Abstract

The High Energy Missile Technology Demonstrator (HEMi TD) is a project to demonstrate the lethality of a compact hypervelocity kinetic energy missile to provide light armoured vehicles sufficient lethality to destroy a main battle tank. In support of the development of the HEMi concept, analysis of the missile performance has been performed using modeling and simulation of the engagement. A nominal simulation using constant disturbances composed of boost motor thrust misalignment and dart separation effect permitted the evaluation of the nominal kinematics performance of the missile. Then, a Monte Carlo simulation using randomly generated disturbances was used to evaluate the dispersion and probability of hit of the missile. The rocket motor having been designed for a missile mass of 23kg, the actual mass of 25.8kg leads to a maximum velocity of Mach 5.6 while the objective was Mach 6.5. However top speed is obtained at a range of 325m thus achieving a minimum engagement range better than the 400m objective. The Monte Carlo simulation results demonstrate that the hit probability of 95% required by the customer can be achieved for all target ranges. However, the missile model does not account for guidance sensor induced error. It should be expected that modeling the guidance sensor accuracy would increase the missile dispersion at long ranges. Also, further work is required to estimate the probability of kill achieved by the proposed segmented rod.

Résumé

Le démonstrateur de technologie HEMi est un projet visant à démontrer la létalité d'un missile hypervéloce compact à énergie cinétique pouvant fournir à un véhicule blindé léger la létalité suffisante pour détruire un char d'assaut. Pour soutenir le développement du concept HEMi, l'analyse des performances du système a été effectuée en faisant la modélisation et la simulation de l'engagement. Une simulation nominale utilisant des perturbations de l'alignement du vecteur poussé du moteur et de la séparation du dard a permis l'évaluation de la performance nominale de la cinématique du missile. Puis, une simulation de type Monte Carlo employant des perturbations aléatoires a servi à évaluer la dispersion et la probabilité de frappe du missile. Le moteur-fusée ayant été conçu pour une masse de missile de 23 kg, la masse réelle de 25,8 kg amène une vitesse maximum de Mach 5,6 plutôt que l'objectif de Mach 6,5. Cependant, la vitesse maximale est obtenue à une distance de 325 m réalisant de ce fait une distance minimum d'engagement plus courte que l'objectif de 400 m. Les résultats de simulation Monte Carlo démontrent que la probabilité de frappe de 95 % requise par le client est réalisable pour toutes les distances de cible. Cependant, le modèle de missile ne tient pas compte de l'erreur induite par le capteur de guidage. Modéliser l'imprécision des capteurs de guidage augmenterait la dispersion du missile pour de longues portées. En outre, davantage de travail est requis pour estimer la probabilité de destruction obtenue avec la tige segmentée proposée.

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Executive summary

The High Energy Missile Technology Demonstrator (HEMi TD) is a project to demonstrate the lethality of a compact hypervelocity kinetic energy missile to provide light armoured vehicles sufficient lethality to destroy a main battle tank.

In support of the development of the HEMi concept, analysis of the missile performance has been performed using modeling and simulation of the engagement. For the simulation of system performance, a model was developed. The model allows for parameterization of the system, conduct of simulations using different scenarios, and graphical presentation of the results. The simulation uses six-degrees-of-freedom equations of motion that permits computation of the missile trajectory in three-dimensional space.

A nominal simulation using constant disturbances composed of boost motor thrust misalignment and dart separation effect permitted the evaluation of the nominal kinematics performance of the missile. Then, a Monte Carlo simulation using randomly generated disturbances was used to evaluate the dispersion and probability of hit of the missile.

The rocket motor having been designed for a missile mass of 23kg, the actual mass of 25.8kg leads to a maximum velocity of Mach 5.6 while the objective was Mach 6.5. However top speed is obtained at a range of 325m thus achieving a minimum engagement range shorter than the 400m objective.

The Monte Carlo simulation results demonstrate that the hit probability of 95% required by the customer can be achieved for all target ranges. However, the missile model does not account for guidance sensor induced error. It should be expected that modeling the guidance sensor inaccuracies would increase the missile dispersion at long ranges. Also, further work is required to estimate the probability of kill achieved by the proposed segmented rod.

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Sommaire

Le démonstrateur de technologie HEMi est un projet visant à démontrer la létalité d'un missile hypervéloce compact à énergie cinétique pouvant fournir à un véhicule blindé léger la létalité suffisante pour détruire un char d'assaut.

Pour soutenir le développement du concept HEMi, l'analyse des performances du missile a été exécutée en modélisant et simulant l'engagement. Pour simuler les performances du système, un modèle a été développé. Le modèle permet de paramétrer le système, exécuter les simulations en utilisant différents scénarios, et présenter les résultats sous forme graphique. La simulation emploie des équations 6DOF du mouvement qui permettent le calcul de la trajectoire du missile dans l'espace tridimensionnel.

Une simulation nominale utilisant des perturbations de l'alignement du vecteur poussé du moteur et de la séparation du dard a permis l'évaluation de la performance nominale de la cinématique du missile. Puis, une simulation de type Monte Carlo employant des perturbations aléatoires a servi à évaluer la dispersion et la probabilité de frappe du missile.

Le moteur-fusée ayant été conçu pour une masse de missile de 23 kg, la masse réelle de 25,8 kg amène une vitesse maximum de Mach 5,6 plutôt que l'objectif de Mach 6,5. Cependant, la vitesse maximale est obtenue à une distance de 325 m réalisant de ce fait une distance minimum d'engagement plus courte que l'objectif de 400 m.

Les résultats de simulation Monte Carlo démontrent que la probabilité de frappe de 95 % requise par le client est réalisable pour toutes les distances de cible. Cependant, le modèle de missile ne tient pas compte l'erreur induite par le capteur de guidage. Modéliser l'imprécision des capteurs de guidage augmenterait la dispersion du missile pour de longues portées. En outre, davantage de travail est requis pour estimer la probabilité de destruction obtenue avec la tige segmentée proposée.

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1. Introduction

The High Energy Missile Technology Demonstrator (HEMi TD)[1] is a project to demonstrate the lethality of a compact hypervelocity kinetic energy missile to provide light armoured vehicles sufficient lethality to destroy a main battle tank.

Missile performance requirements [2] [3] call for a missile that reaches a speed of at least Mach 6.5. The missile should be able to engage a target at a minimum range of 400 m. In order to comply with these specifications, the missile uses a solid rocket motor that propels the missile to Mach 6.5 in 400 m. To control the missile, the rocket motor has thrust vector control (TVC) that provides control of thrust direction in order to guide the boosted missile to the target. The separation of a dart occurs and the dart is guided using control flaps.

In order to direct the development of the technology teams of HEMi, modeling and simulation (M&S) plays an important role. M&S assess the performances of the individual technology in a system context where the performance of the missile is evaluated against the above performance requirements.

During the development of the HEMi TD, more than 150 sets of parameters for the model subsystems have been developed and evaluated [4]. This memorandum presents the simulation results for the final version 2.02 of HEMi [5].

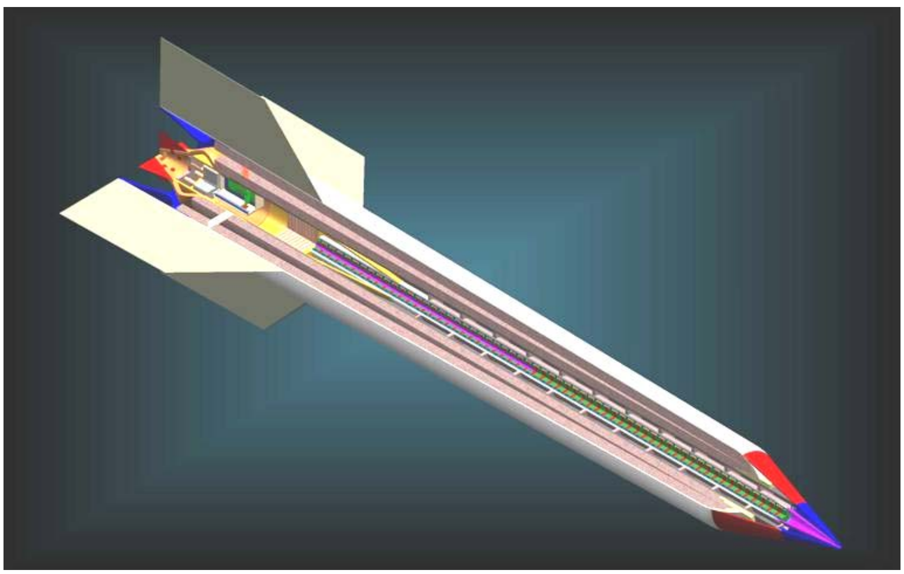

Figure 1. *HEMi V.2.02*

2. Model Description

For the simulation of system performance, a model was developed using methodology described in [6] and [7]. Based on Matlab/Simulink tools[8], the model allows for parameterization of the system, conduct of simulations using different scenarios, and graphical presentation of the results.

The simulation uses six-degrees-of-freedom equations of motion that permits computation of the missile trajectory in three-dimensional space. Annex A presents block diagrams of the Matlab/Simulink models. The top-level block diagram of the engagement comprises three main elements: a fixed target, the booster simulation and the dart simulation blocks. The components of the booster and dart simulation blocks are described in the following sub-sections.

2.1 Dart Airframe Model

The dart airframe is modelled by six-degrees-of-freedom equations of motion. Aerodynamic forces and moments are computed using linear aerodynamic coefficients expressed in function of the Mach number. The coefficients used, derived from the methods presented in [8] are displayed in Table 1.

The mass, moments of inertia and location of centre of gravity are function of time to accommodate possible variations due the propellant combustion. The parameters used are presented in Table 2 [5].

Table 1. *Dart airframe aerodynamic parameters*

Reference diameter: 0.030m. Reference position: 0.889m from nose

Mach	Cx	Cna	Cma	Cmq	Cndelta	Cmdelta	Cl	Clp
6 7 8	0.2831 0.2506 0.2255	10.77 10.60 10.43	-52.31 -49.85 -47.56	-378 -378 -378	0 0 0	0 0 0	0 0 0	-1 -1 -1

Table 2. *Dart airframe mass parameters*

Time [s]	I_{xx}[kg.m²]	I_{yy}[kg.m²]	Mass [kg]	CG Position [m] from nose
-1 2	0.001111 0.001111	1.2525 1.2525	6.877 6.877	0.889 0.889

2.2 Booster Airframe Model

The booster airframe is modelled by six-degrees-of-freedom equations of motions. Aerodynamic forces and moments are computed using linear aerodynamic coefficients expressed in function of the Mach number. The coefficients used, derived from the methods presented in [8] are displayed in Table 3.

The mass, moments of inertia and location of centre of gravity are function of time to accommodate variations due the propellant combustion. The parameters used are presented in Table 4 [5].

Table 3. *Booster airframe aerodynamic parameters*

Reference diameter: 0.050m. Reference position: 0.750m from nose

Mach	Cx	Cna	Cma	Cmq	Cndelta	Cmdelta	Cl	Clp
0.01	2.727	108.44	-459.45	-47544	0	0	0	-847.5
0.5	2.388	109.61	-465.68	-52474	0	0	0	-1010.2
0.7	2.391	108.58	-456.62	-53700	0	0	0	-1062.0
0.75	2.413	108.58	-455.95	-54118	0	0	0	-1076.4
0.77	2.422	108.60	-455.82	-54312	0	0	0	-1082.4
0.8	2.437	108.66	-455.78	-54619	0	0	0	-1091.6
0.85	2.511	108.72	-453.88	-55065	0	0	0	-1098.7
0.9	2.62	108.78	-451.25	-55480	0	0	0	-1105.3
1	4.591	110.87	-439.20	-56660	0	0	0	-1112.9
1.5	3.741	116.36	-509.15	-56881	0	0	0	-1083.1
2	3.056	113.94	-514.26	-62681	0	0	0	-1066.3
2.5	2.596	83.71	-351.43	-55824	0	0	0	-923.3
3	2.219	71.53	-294.97	-54560	0	0	0	-806.7
3.5	1.859	64.00	-247.22	-53752	0	0	0	-714.3
4	1.545	58.95	-204.78	-52756	0	0	0	-644.3
4.5	1.336	55.00	-171.37	-51746	0	0	0	-589.1
5	1.171	51.82	-144.45	-50940	0	0	0	-544.5
6	0.930	46.94	-102.11	-49494	0	0	0	-474.1
7	0.763	43.51	-70.59	-48366	0	0	0	-424.1
8	0.643	43.23	-45.18	-48474	0	0	0	-387.4

Table 4. *Booster airframe mass parameters*

Time [s]	I_{xx}[kg.m²]	I_{yy}[kg.m²]	Mass [kg]	CG Position [m] from nose
0.0	0.06317	2.676	25.77	0.725
0.4	0.01798	1.801	13.04	0.765
3.0	0.01798	1.801	13.04	0.765

2.3 Propulsion and Thrust Vector Control Model

The propulsion and thrust vector control (TVC) model implements a solid rocket motor in the form of a time-thrust curve given by Table 5 and illustrated in Figure 2.

Table 5. *Thrust table*

Time [s]	Thrust [N]
-1.0	0
0.0	0
0.0	78050
0.4	78050
0.4	0
3.0	0

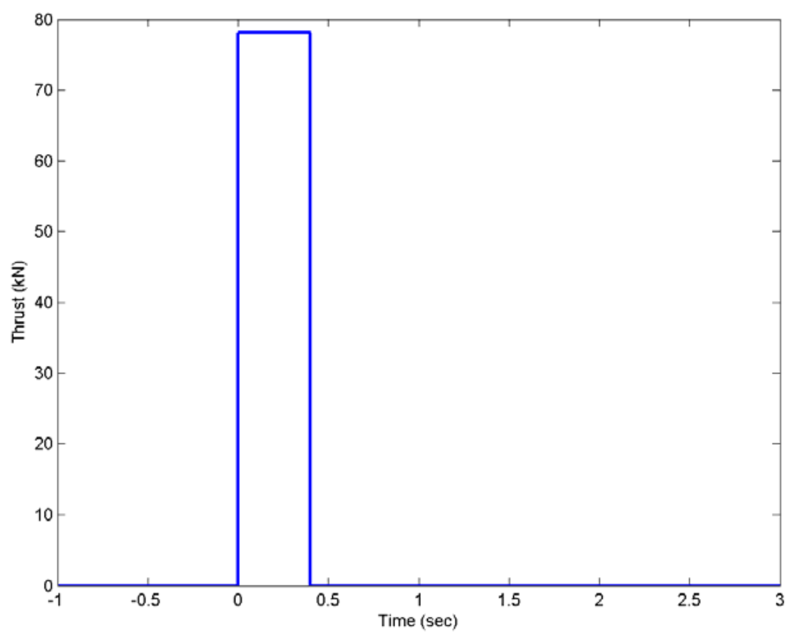

Figure 2. *Thrust curve*

The total impulse of the rocket motor has been determined using the propellant mass of [5] and assuming a specific impulse of 250 lbf.s/lb. Since missile specifications [1] demand a 0.4 sec. boost time, the maximum thrust has been computed accordingly.

The TVC components allow rotating the thrust vector at the nozzle. The location of the nozzle with respect to the missile nose is used to compute the moment. Parameters used are presented in Table 6 and are derived from [5].

Table 6. *Propulsion and TVC parameters*

Nozzle exit area (m²)	0.0178
Nozzle position with respect to nose (m)	1.25
Maximum thrust deflection angle (rad)	0.035

2.4 Booster Guidance Model

The proposed guidance law for the control of the HEMi booster on the guidance beam is based on a linear quadratic regulator (LQR) [10]. This section describes the guidance model based on the LQR regulator and presents trajectory shaping commands use to guide the missile on a super elevated trajectory.

2.4.1 Guidance law

The LQR guidance minimizes the expectancy of objective function J of Equation 1.

$$J = \int_0^{\infty} x^T Q x + \delta^T R \delta \, dt \qquad (1)$$

where

- x is the state vector
- δ is the control command
- Q is the weighting matrix for the state
- R is the weighting matrix for the control command.

The control command δ is given by the following equation:

$$\delta = -K \cdot x \qquad (2)$$

where K is chosen to minimize Equation 1.

Based on a linear model, the beam-rider guidance law can be designed. The guidance objective is to maintain the missile on the beam at Z=0 using available measurements. The state vector of the linear model is:

$$x = \begin{bmatrix} Z \\ \dot{Z} \\ \theta \\ \dot{\theta} \end{bmatrix} \qquad (3)$$

The guidance objective requires minimizing the value of the first state (Z). This leads to a large value for the first value of diagonal of Q. However, we also want the guidance to have minimum oscillations around the beam and we also want to close on the beam with a proper dynamic.

The missile velocity relative to the beam ($\dot{Z}$) and the missile attitude (θ) are not directly measured but can easily be estimated by differentiation and integration of the measurements:

$$\dot{Z} = \frac{d}{dt} Z \qquad (4)$$

$$\theta = \int \dot{\theta}\, dt \qquad (5)$$

Since all states are known, it is possible to implement a state feedback control law.

Chosen weights for matrices Q and R are presented in Table 7. Weights have been adjusted by iteration in order to have the booster reach steady state in 0.4 s with minimum oscillations. This response time of 0.4 s corresponds to the flight time to a range of 400 m, where the missile must have converged on the beam to engage a target.

Table 7. *LQR Weight Matrix Values*

diag(Q)	1000 1 2 4
R	50000

The state feedback gains obtained for each Mach number are presented in Table 8.

Table 8. *LQR Gain Values*

Mach	K_Z	K_{Zdot}	K_θ	$K_{\theta dot}$
1	-0.14142	-0.0078873	0.61109	0.011412
2.5	-0.14142	-0.0076094	0.52818	0.0086236
5	-0.14142	-0.00546	1.2958	0.0078384
7	-0.14142	-0.0041073	3.066	0.0084064

where the guidance law equation is an expansion of Equation 2:

$$\delta = -K_z \cdot Z - K_{zdot} \cdot \dot{Z} - K_\theta \cdot \theta - K_{\theta dot} \cdot \dot{\theta} \tag{6}$$

or using Equations 4 and 5:

$$\delta = -K_z \cdot Z - K_{zdot} \cdot \frac{d}{dt} Z - K_\theta \cdot \int \dot{\theta}\, dt - K_{\theta dot} \cdot \dot{\theta} \tag{7}$$

Each gain computed in Table 8 is based on a linear model developed for constant missile mass and velocity. In actual use, the missile will exhibit continuous velocity variations. It thus requires continuous variations in guidance gains. A gain-scheduling method where the applied gains are obtained by a table look-up function of the actual missile velocity is thus required.

2.4.2 Trajectory shaping

Trajectory shaping commands are introduced in the guidance law to control the missile path during the boost phase[1][10].

The commands Z_{CMD} on the position and $\dot{Z}_{CMD}$ on the vertical velocity are added to the guidance law:

$$\delta = -K_z \cdot \left(Z - Z_{CMD}(t)\right) - K_{zdot} \cdot \left(\frac{d}{dt} Z - \dot{Z}_{CMD}(t)\right) - K_\theta \cdot \int \dot{\theta}\, dt - K_{\theta dot} \cdot \dot{\theta} \tag{8}$$

The commands are varied as a function of time to progressively guide the missile onto the beam by the end of the boost phase. Figure 3 presents the trajectory shaping profile.

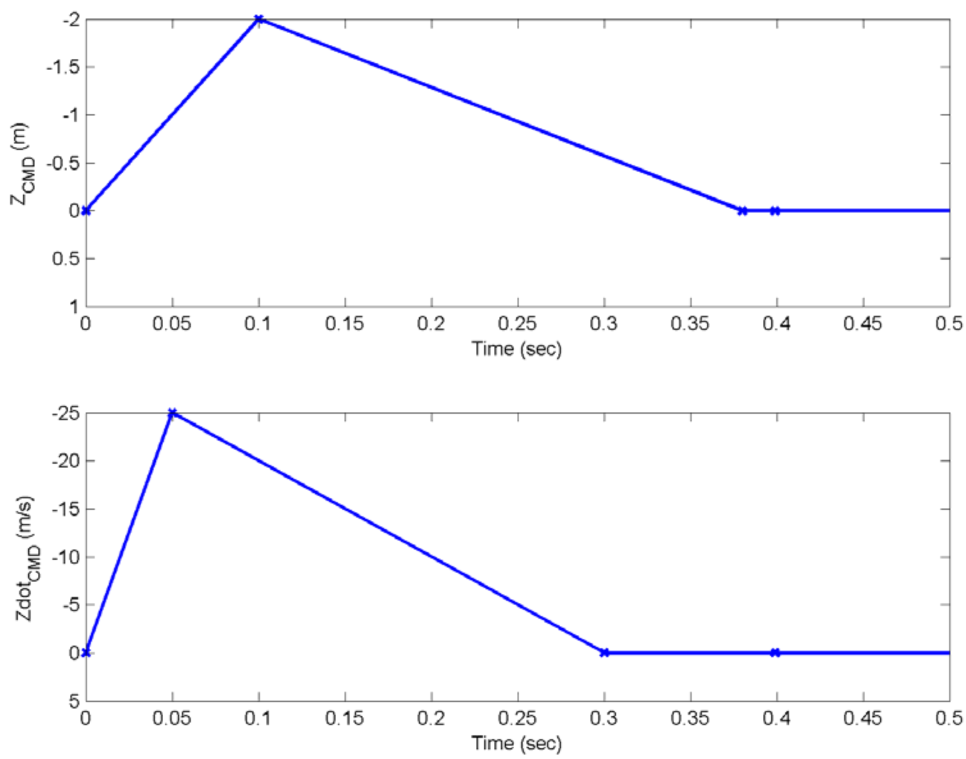

Figure 3. *Trajectory shaping commands*

2.5 Predictive Dart Guidance Model

The guidance for the HEMi dart is composed of two elements, first a continuous time feedback gain command δ_{AP} and a discrete-time guidance command $\delta_{GUIDANCE}$ as illustrated in Figure 4.

The continuous time feedback gain provides basic damping to the airframe. The guidance is based on a discrete-time prediction of the state of the missile at impact [11] and is thus a discrete-time command.

Since the control effectors is not modeled, the commands are control forces applied directly to the tail of the dart.

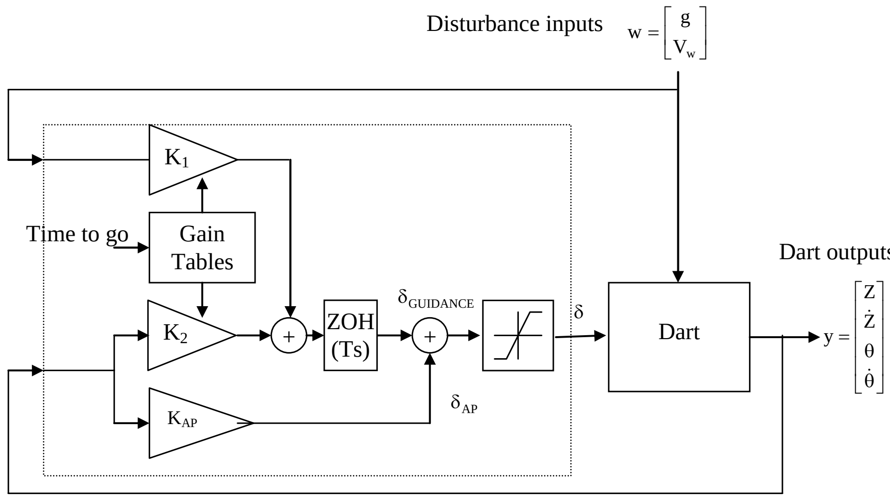

Figure 4. *Dart guidance block diagram*

The continuous time feedback command applied is:

$$\delta_{AP} = K_{AP} \cdot y \tag{9}$$

Where

$$y = \begin{bmatrix} Z \\ \dot{Z} \\ \theta \\ \dot{\theta} \end{bmatrix} \tag{10}$$

with the following missile measurements:

$\dot{\theta} = q$	Missile angular pitch rate (rad/s)
θ	Missile angular position (rad)
$\dot{Z}$	Missile velocity relative to beam (positive down) (m/s)
Z	Missile position relative to beam (positive down) (m)

The discrete-time guidance command uses a gain that is function of the number n of time-step estimated before impact [11]. The gains apply compensations for the missile outputs and the wind and gravity disturbances:

$$\delta_{GUIDANCE} = K_1(n) \cdot y + K_2(n) \cdot w \tag{11}$$

where

$$w(k) = \begin{bmatrix} g(k) \\ V_w(k) \end{bmatrix} \quad (12)$$

with the following anticipated disturbances:

g	Gravity
V_w	Wind velocity

The sum of the commands is truncated to a maximum value δ_{max}. Parameter values used in the simulation are presented in Table 9. Gains in function the number of time step left are presented in Table 10.

Table 9. *Dart predictive guidance parameters*

Gravity (m/s^2)	9.81
Wind velocity (m/s)	0
δ_{max}(N)	250
K_{AP}	-44.721 -13.984 299.06 57.473
Sample time(s)	0.05

Table 10. *Dart predictive guidance parameters as a function of step to go.*

Step to go n	$K_1(n)$		$K_2(n)$			
1	-0.40771	-2.8792	-44.721	-14.259	2237.7	-0.97876
2	2.8412	1.0406	590.54	47.852	-1504.7	-54.994
3	2.8161	0.1986	303.13	31.718	-946.28	-34.904
4	2.7188	0.27249	158.97	20.821	-601.61	-22.183
5	2.6139	0.22496	94.693	14.621	-407.04	-15.941
6	2.4188	0.091085	59.758	10.468	-276.96	-11.33
7	2.2583	0.12769	36.906	7.4032	-196.31	-8.1115
8	2.0419	0.055246	23.644	5.3066	-135.76	-5.8312
9	1.8293	0.05625	14.453	3.6883	-93.631	-4.0584
10	1.5999	0.029784	8.7158	2.5362	-62.936	-2.8069
11	1.3691	0.018471	4.9552	1.6859	-41.065	-1.8667
12	1.1419	0.0077948	2.6191	1.0834	-25.903	-1.2058
13	0.92418	-0.001201	1.2269	0.66664	-15.531	-0.74379
14	0.72283	-0.006463	0.44911	0.38792	-8.8125	-0.4349
15	0.54197	-0.010924	0.06526	0.21121	-4.6176	-0.23802
16	0.38502	-0.013085	-0.088217	0.10498	-2.185	-0.11912
17	0.25327	-0.014261	-0.11648	0.046228	-0.88782	-0.053015
18	0.14653	-0.014301	-0.088218	0.017087	-0.2855	-0.019897
19	0.063305	-0.013643	-0.043156	0.0051673	-0.07086	-0.0061363
20	0.0011724	-0.0125	-0.0018309	0.0021726	-0.045134	-0.0024649

3. Simulation of Nominal Engagement

In order to evaluate the kinematics performance of the HEMi missile against its performance specifications [1], a nominal simulation scenario of the missile using deterministic disturbances is performed.

3.1 Simulation parameters

The simulation parameters include the initial conditions of the booster at the exit of the launch tube, the environment parameters, motor thrust misalignment and the effects of the separation of the dart from the booster. In this scenario, deterministic values are used.

3.1.1 Missile Initial Condition

This element of the model defines the initial conditions of the missile at the beginning of the simulation.

The position is in meters. The X,Y and Z axis correspond respectively to the north, east and down directions. Velocities UVW and angular velocities PQR are given in the missile body system of reference. A slight elevation of the pitch Euler angle θ is used.

The parameters used are presented in Table 11.

Table 11. *Missile Initial Conditions*

Position vector [X,Y,Z] (m)	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
Velocity vector [U,V,W] (m/s)	$\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$
Euler angles $[\psi, \theta, \phi]$ (rad)	$\begin{bmatrix} 0 \\ 0.1745 \\ 0 \end{bmatrix}$
Angular velocity vector [P,Q,R]	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

3.1.2 Simulation Environment Model Description

The simulation environment model is used to define environment parameters common to the complete simulation. It includes the wind and gravity.

The parameters used are presented in Table 12.

Table 12. *Simulation environment parameters*

Gravity (m/s²)	9.81
Wind vector [U,V,W] (m/s)	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

3.1.3 Motor Thrust Misalignment

The motor thrust misalignment is a representation of the motor-to-motor variation in the orientation of the motor thrust vector. The thrust misalignment induces pitching moment on the airframe. The moment should be compensated by the thrust vector control system and the guidance.

The parameters used are presented in Table 13.

Table 13. *Motor thrust misalignment*

Yaw thrust misalignement angle (rad)	0
Pitch thrust misalignement angle (rad)	0.01

3.1.4 Separation Effect Model Description

The separation effect model is used to describe disturbances occurring at the separation of the dart from the booster.

Given the condition of the booster/dart assembly before separation, additive offsets are added to obtain the dart initial condition after separation.

The parameters used are presented in Table 14 and are detailed in [12] and [13].

Table 14. Separation effect parameters

Separation induced pitch rate (rad/s)	2.35
Separation induced yaw rate (rad/s)	0

3.2 Nominal Engagement Results

The nominal engagement is simulated with the parameter values of the previous section. The two types of disturbances used here are a thrust misalignment on the booster motor and a dart separation tip-off. Figure 5 presents the trajectories for target ranges of 400m and 5000m. These two target ranges have been selected because they represent the minimum and maximum range requirements. Since all simulated disturbances are in the vertical plan, only results for this plan are presented since no manoeuvre occurs in the horizontal plane.

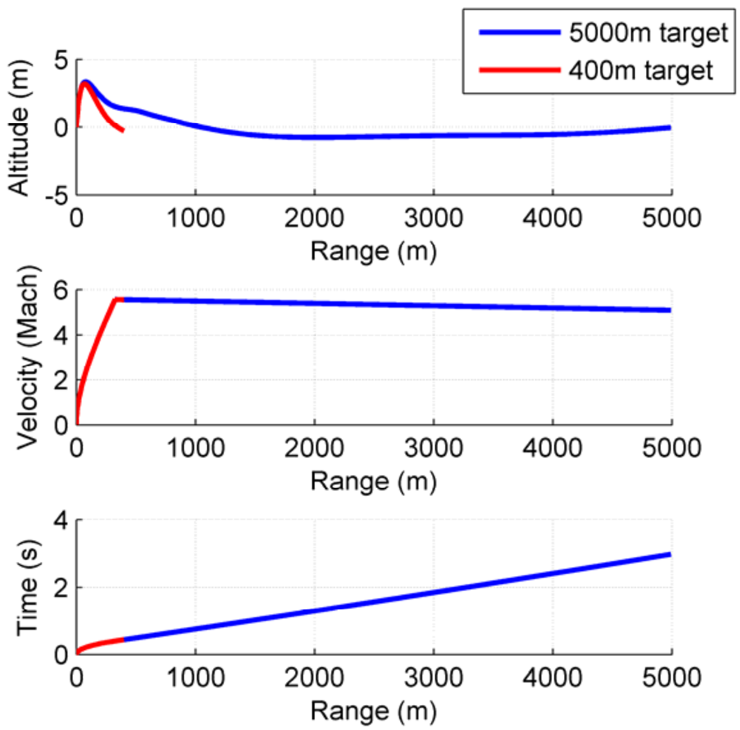

Figure 5. *Nominal engagement results*

Figure 6 presents the angular information for the same simulations. Results show that maximum velocity is obtained at a range of 325m. Design objective was to attain

400m range in 0.4sec. at Mach 6.5. However, because of its excess weight, the missile reaches a maximum velocity of only Mach 5.6 at 325m in 0.4sec.

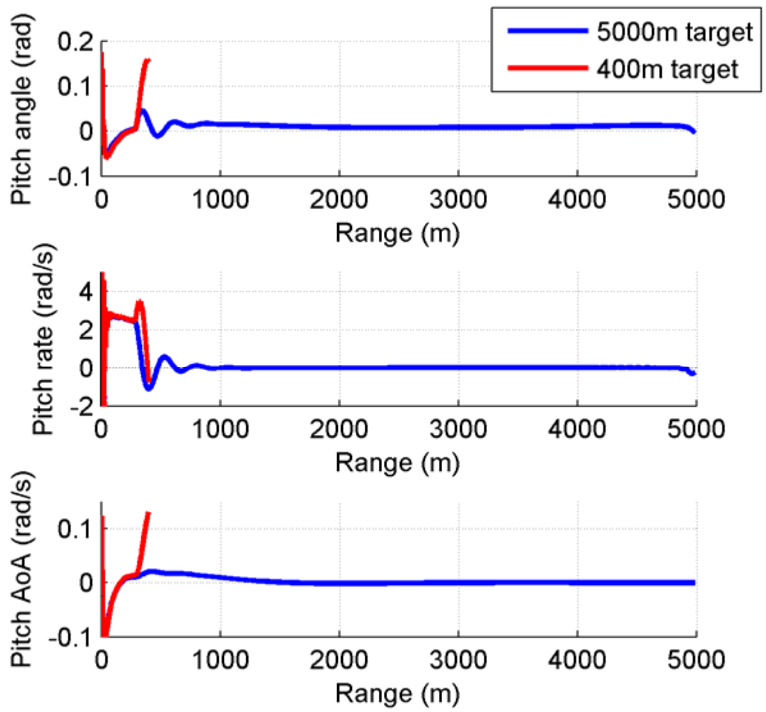

Figure 6. *Nominal engagement results(2)*

At the end of boost at 325m, the booster is stabilized at a small angle of attack.

For a long range target at 5000m, the dart coasts with minimum corrections. At 5000m, the dart has decelerated to Mach 5.1.

For the simulated cases, the next table present maximum and minimum values for different variables.

4. Monte-Carlo Simulation

In order to compute the dispersion and accuracy performance of HEMi, this section presents the parameters and results of a Monte-Carlo simulation. In a Monte-Carlo simulation, the simulation is run repetitively with parameters selected randomly according to their estimated distribution.

4.1 Simulation Parameters

Simulation parameters used for the simulation are either constant or random. Random parameters are expressed by $N(\mu, \sigma^2)$ which is a random number drawn from a normal distribution with mean μ and variance σ^2. The Marsaglia's Ziggurat algorithm build in Matlab[8] has been used to generate random numbers.

4.1.1 Missile Initial Conditions

This element of the model defines the initial conditions of the missile at the beginning of the simulation.

The position is in meters. The X,Y and Z axes correspond respectively to the north, east and down directions. Velocities UVW and angular velocities PQR are given in the missile body system of reference. A slight elevation of the pitch Euler angle θ is used.

The parameters used are presented in Table 15.

Table 15. *Missile Initial Conditions*

Position vector [X,Y,Z] (m)	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
Velocity vector [U,V,W] (m/s)	$\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$
Euler angles $[\psi, \theta, \phi]$ (rad)	$\begin{bmatrix} 0 \\ 0.1745 \\ 0 \end{bmatrix}$

Angular velocity vector [P,Q,R]	$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

4.1.2 Simulation Environment Model Description

The simulation environment model is used to define environment parameters common to the complete simulation. It includes the wind and gravity.

The parameters used are presented in Table 16.

Table 16. *Simulation environment parameters*

Gravity (m/s²)	9.81
Wind vector [U,V,W] (m/s)	$\begin{bmatrix} N(0,25) \\ N(0,25) \\ N(0,25) \end{bmatrix}$

4.1.3 Motor Thrust Misalignement

The motor thrust misalignment is a representation of the motor-to-motor variation in the orientation of the motor thrust vector. The thrust misalignment induces pitching moment on the airframe. The moment should be compensated by the thrust vector control system and the guidance.

The parameters used are presented in Table 17.

Table 17. *Motor thrust misalignement*

Yaw thrust misalignement angle (rad)	$N(0,0.000025)$
Pitch thrust misalignement angle (rad)	$N(0,0.000025)$

4.1.4 Separation Effect Model Description

The separation effect model is used to describe disturbances occurring at the separation of the dart from the booster.

Given the condition of the booster/dart assembly before separation, additive offsets are added to obtain the dart initial condition after separation.

The parameters used are presented in Table 18 and are detailed in [12] and [13].

Table 18. *Separation effect parameters*

Separation induced pitch rate (rad/s)	$N(0,1)$
Separation induced yaw rate (rad/s)	$N(0,1)$

4.2 Monte-Carlo Simulation Results

The simulation has been performed for target range varying from 100m to 5000m. For each range, 100 missile simulations have been performed with the parameters defined in the previous section.

Compilation of the simulation results is presented in Figure 7. The first plot presents the 50% and 95% circular error probable (CEP) dispersions. They represent the radius of circular targets that contain respectively 50% and 95% of all missile impacts.

The second plot of Figure 7 presents the probability of hit for a square target of 1m x 1m and a standard NATO target of 2.3m x 2.3m.

Results show that at any range, the missile meets the 95% probability of hit requirements for a standard 2.3m x 2.3m NATO target.

Transient disturbances created by motor thrust misalignments and dart separation effect disturbances slightly decrease the missile accuracy for ranges less than 800m. For long range targets, the dart guidance fully compensates the disturbance and achieves greater accuracy. However, the missile model does not account for guidance sensor induced error [14]. It should be expected that modeling the guidance sensor accuracy would increase the missile dispersion at long range.

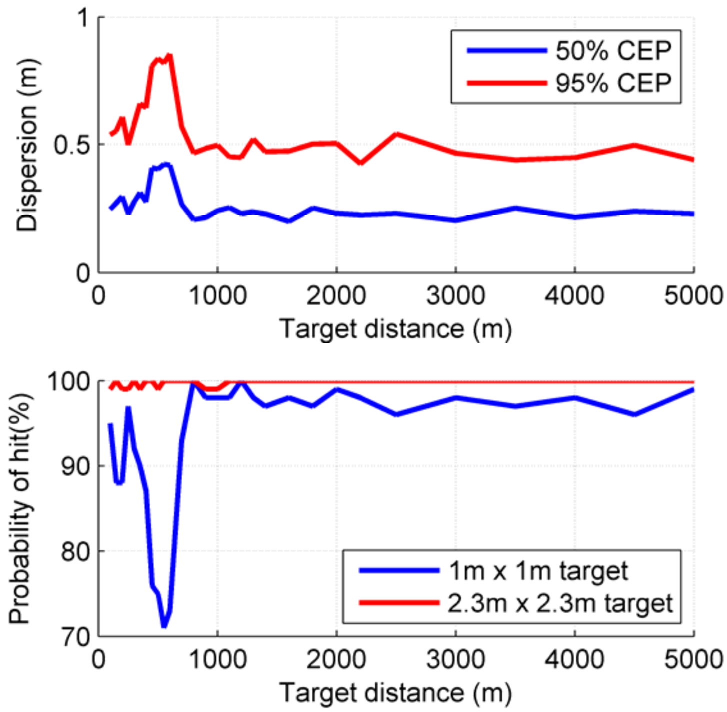

Figure 7. *Dispersion and probability of hit as a function of target range.*

5. Conclusion

In support of the development of the HEMi concept, analysis of the missile performance has been performed using modeling and simulation of the engagement.

A nominal simulation using constant disturbances composed of boost motor thrust misalignments and dart separation effects permitted the evaluation of the nominal kinematics performance of the missile.

Then, Monte Carlo simulations using randomly generated disturbances were used to evaluate the dispersion and probability of hit of the missile.

Table 19 presents a summary of the achieved HEMi missile performance by comparison with the design objective from the missile system level specifications [2][3].

Table 19. *Summary of HEMi performances*

Characteristic	***Objective***	***Achieved***
Mass (kg)	23	25.8
Length (m)	1.25	1.264
Minimum range(m)	400	325
Maximum range(m)	5000	5000+
Top speed (Mach)	6.5	5.6
Flight time to 5000m (sec)	2.4	3.0
Hit probability (400-5000m, 2.3x2.3m stationary target)	95%	95+%

The rocket motor having been design for a missile mass of 23kg, the actual mass of 25.8kg leads to a maximum velocity of Mach 5.6 while the objective was Mach 6.5. However top speed is obtained at a range of 325m thus achieving a minimum engagement range shorter than the 400m objective.

The Monte Carlo simulations result demonstrate that a hit probability of at least 95% can be achieved for all target range. However, the missile model does not account for guidance sensor induced error [14]. It should be expected that modeling the guidance sensor inaccuracies would increase the missile dispersion at long range. Also, further work is required to estimate the probability of kill of the proposed segmented rod [15].

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Annex A

Block diagrams of the Matlab/Simulink models.

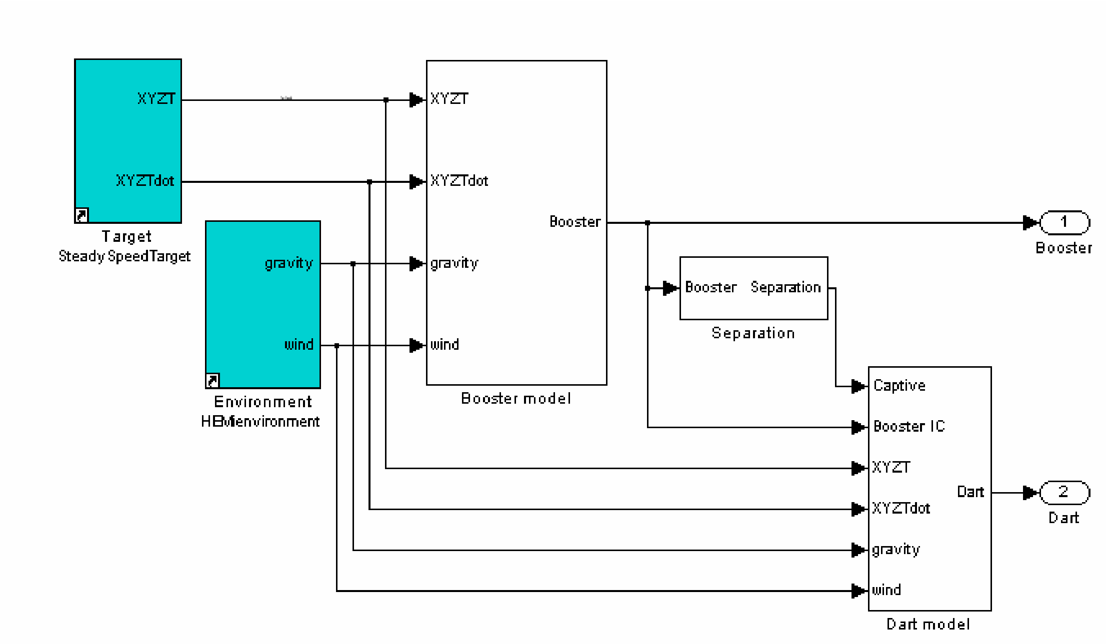

Figure 8. *Top-level of the engagement model*

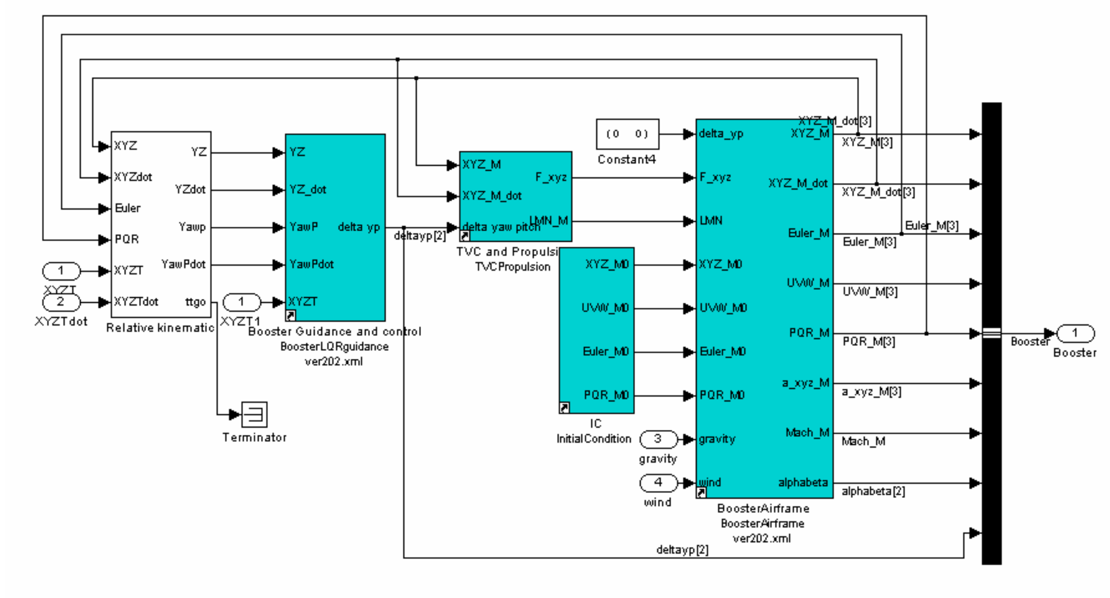

__Figure 9.__ Booster subsystem of the engagement model

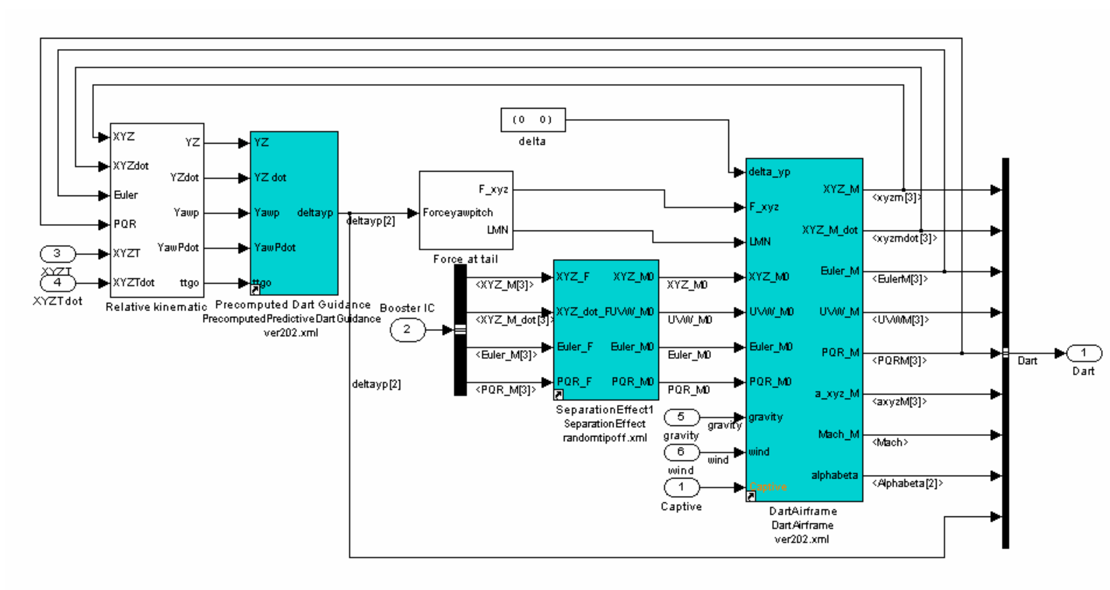

__Figure 10.__ Dart subsystem of the engagement model

List of symbols, abbreviations and acronyms

δ	Control command
$[\psi, \theta, \phi]$	Euler angles
AoA	Angle of attack
CEP	Circular error probable
CG	Centre of gravity
Cl	Roll moment coefficient
Clp	Roll damping coefficient
Cma	Pitching moment coefficient
Cmdelta	Yaw moment control effectors coefficient
Cmq	Pitch damping coefficient
Cna	Lift coefficient
Cndelta	Yaw moment control effectors coefficient
Cx	Axial drag coefficient
DND	Department of National Defence
g	Gravity
HEMi TD	High Energy Missile Technology Demonstrator
I_{xx}	Polar moment of inertia for x-axis
I_{yy}	Polar moment of inertia for y-axis
J	Optimization objective function
K	Guidance gain

$N(\mu, \sigma^2)$	Random number draw from a normal distribution with mean μ and variance σ^2
NATO	North Atlantic Treaty Organisation
[P,Q,R]	Angular velocity vector
Q	Weighting matrix for the state
R	Weighting matrix for the control command.
[U,V,W]	Velocity vector
TVC	Thrust vector control
V_w	Wind velocity
[X,Y,Z]	Position vector
x	State vector

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The High Energy Missile Technology Demonstrator (HEMi TD) is a project to demonstrate the lethality of a compact hypervelocity kinetic energy missile to provide light armoured vehicles sufficient lethality to destroy a main battle tank. In support of the development of the HEMi concept, analysis of the missile performance has been performed using modeling and simulation of the engagement. A nominal simulation using constant disturbances composed of boost motor thrust misalignment and dart separation effect permitted the evaluation of the nominal kinematics performance of the missile. Then, a Monte Carlo simulation using randomly generated disturbances was used to evaluate the dispersion and probability of hit of the missile. The rocket motor having been designed for a missile mass of 23kg, the actual mass of 25.8kg leads to a maximum velocity of Mach 5.6 while the objective was Mach 6.5. However top speed is obtained at a range of 325m thus achieving a minimum engagement range better than the 400m objective. The Monte Carlo simulation results demonstrate that the hit probability of 95% required by the customer can be achieved for all target ranges. However, the missile model does not account for guidance sensor induced error. It should be expected that modeling the guidance sensor accuracy would increase the missile dispersion at long ranges. Also, further work is required to estimate the probability of kill achieved by the proposed segmented rod.

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