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J.-M. Martel
Université Laval

M.Bélanger
A. Guitouni
DRDC Valcartier

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Authors

Jean-Marc Martel, Micheline Bélanger, Adel Guitouni

Approved by

Éloi Bossé

Section Head / Decision Support Systems

Approved for release by

Gilles Bérubé

Chief Scientist

Abstract

This report presents a robustness analysis procedure that was developed for a decision support system based on a multicriterion outranking approach. This work is based on the assumption that any model is a limited representation of reality and that it is impossible to derive exact models of the situation. Therefore, the decision analysis procedure must help identify the best results despite imperfections in the models. In particular, the robustness analysis procedure should produce a ranking of best compromise by considering all plausible values based on the model of the decision-maker's preferences. In this report we propose a definition of the robustness concept within the military decision-making context. We then describe the proposed robustness analysis procedure.

Résumé

Ce rapport présente une procédure d'analyse de la robustesse qui a été développée pour un système d'aide à la décision basé sur une approche de surclassement multicritère. Étant donné l'impossibilité d'obtenir des modèles exacts d'une situation particulière, il est nécessaire que la procédure d'analyse de décision permette l'identification des « bons » résultats en dépit de l'imperfection de ces modèles. Ainsi, une procédure d'analyse de la robustesse devrait produire un rangement des meilleurs compromis en tenant compte de toutes les valeurs plausibles obtenues à partir du modèle des préférences du décideur. Dans ce rapport, nous proposons une définition du concept de la robustesse employé dans le cadre de la prise de décisions. Par la suite, nous décrivons en détail la procédure d'analyse de la robustesse qui est proposée.

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Executive summary

Defence Research and Development Canada – Valcartier (DRDC Valcartier) has developed a prototype decision support system for the command and control of military resources in the context of Canadian airspace protection. This prototype, called CASAP (for Commander's Advisory System for Airspace Protection), was developed on the basis of an analysis of needs identified by knowledge acquisition sessions with the members of an air operations centre (AOC). CASAP consisted of a set of tools used to describe an event in which Canadian airspace is violated, in order to define courses of action (alternatives), then to evaluate these courses of action on the basis of several factors and sub-factors, and arrange the courses of action in order of rank. The procedure followed in this prototype to prioritize and identify the best course of action is PAMSSEM (*Procédure d'Agrégation Multicritère de type Surclassement de Synthèse pour Évaluations Mixtes*).

In recent years, the concepts and algorithms developed in CASAP were migrated into COPlanS (Collaborative Operations Planning System). Several studies have been conducted to improve this type of decision support system. One aspect that was investigated further was the military decision-maker's need for a decision support system employing a multicriterion approach, which would yield valid results despite the difficulty of developing an exact model of the situation under consideration. Such results, described as "robust," would be less affected by issues related to imperfections in the data generated in the evaluation of the courses of action, and to the subjectivity associated with the identification of parameters representing the decision-maker's preferences in the process of modelling a military situation. We then initiated development of a robustness analysis appropriate for a multicriterion decision support system in the context of military decision-making.

This report presents the robustness analysis procedure that was developed for a decision support system based on a multicriterion outranking approach. Since it is impossible to derive exact models of a situation, the decision analysis procedure must help identifying "good" results despite imperfections in the models. In particular, the robustness analysis procedure should produce a ranking of best compromise by considering all plausible values based on the model of the decision-maker's preferences. In this report we propose a definition of the robustness concept within the military decision-making context. We then describe the proposed robustness analysis procedure.

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Sommaire

R&D pour la défense Canada – Valcartier (RDDC – Valcartier) a entrepris des travaux pour développer un prototype de système d'aide à la décision pour le commandement et le contrôle d'interventions dans le cadre de la protection de l'espace aérien canadien. Ce système, appelé CASAP (« Commander's Advisory System for Airspace Protection »), a été mis au point à la suite de sessions d'analyse des besoins et d'acquisition des connaissances qui ont été tenues auprès des membres du centre de commandement et contrôle des forces de l'air canadiennes. Ce système pouvait être utilisé par le personnel militaire pour décrire un événement de violation de l'espace aérien canadien, définir un ensemble de suites d'actions, évaluer et proposer un rangement de ces suites d'actions à partir de plusieurs facteurs et sous-facteurs. La procédure multicritère qui a été utilisée pour proposer ce rangement est PAMSSEM (for "*Procédure d'Agrégation Multicritère de type Surclassement de Synthèse pour Évaluations Mixtes"*).

Au cours des dernières années, les concepts et algorithmes développés dans CASAP ont été repris dans COPlanS (« Collaborative Operations Planning System »). Plusieurs études ont été menées pour améliorer cet outil d'aide à la décision. Un des aspects étudié plus attentivement porte sur les besoins des décideurs militaires à disposer d'un système d'aide à la décision basé sur une approche multicritère et fournissant des résultats valides malgré la difficulté de développer un modèle exact de la situation en cours. Ces résultats, pouvant être qualifiés de robustes, seraient moins affectés par des problèmes liés à l'imperfection des données qui survient dans l'évaluation des suites d'actions et à la subjectivité entourant la fixation des valeurs des paramètres lors de la modélisation des préférences à l'égard d'une situation militaire. On a donc entrepris le développement d'une analyse de robustesse appropriée à un système d'aide à la décision multicritère dans un contexte de décisions militaires.

Ce rapport présente la procédure d'analyse de robustesse qui a été développée pour un système d'aide multicritère à la décision basé sur une approche de surclassement. Étant donné l'impossibilité d'obtenir des modèles exacts de la situation en cours, il est nécessaire que la procédure d'analyse de la décision permette l'identification des « bons » résultats en dépit de l'imperfection de ces modèles. En particulier, une procédure d'analyse de la robustesse devrait produire un rangement des meilleurs compromis, compte tenu de toutes les valeurs plausibles obtenues à partir du modèle de préférence des décideurs. Dans ce rapport, nous proposons une définition du concept de la robustesse employé dans le cadre de la prise de décisions. Par la suite, nous décrivons en détail la procédure d'analyse de la robustesse qui est proposée.

Martel, J.-M., Bélanger, M., Guitouni, A. 2004. Robustness analysis for a multicriterion decision-aid process. DRDC Valcartier TR 2004-379. RDDC Valcartier.

Table of contents

List of figures

List of tables

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1. Introduction

Since 1991, Defence Research and Development Canada – Valcartier (DRDC Valcartier) has been investigating different approaches to the problem of supporting the decision-making process in military command and control situations. One of these investigations concerned a decision-aid system for the command and control of military resources in the context of Canadian airspace protection. In this context, several conflicting and quite incommensurable criteria must be considered and balanced in order to make sound decisions. As our understanding of military command and control processes improved with time, it became clear that most military operational planning processes also consider several conflicting and incommensurable criteria before yielding a decision. Accordingly, a multicriterion decision-aid process was considered appropriate for dealing with most military operational planning activities.

A multicriterion decision-aid process is composed of two steps. The first is the structuration step. Structuration consists in identifying and formalizing the basic elements of a decision-making situation in the form of a model (alternatives, attributes/criteria, evaluations, also represented by *A*, *Λ/C*, *E*). The second step exploits and completes the data supplied by this model with the introduction of some elements of the decision-maker's preference modelling (*M*). Depending on the multicriterion method in use, these elements will consist of coefficients of relative importance for the attributes/criteria, for parameters such as thresholds (of indifference, preference, or veto), or for certain value functions. The values assigned to each attribute/criterion are called "local preferences." These local preferences and some inter-criteria information are then aggregated and exploited on the basis of the chosen decision-making problematics, leading to the formulation of one or more recommendations. The different decision problematics are defined by Roy [1] as the choice of a best alternative (P.α), sorting the alternatives into different categories (P.β), and ranking the alternatives (P.γ) according to their overall performance.

The first implementation of these concepts in an automated decision support system was called CASAP (Commander's Advisory System for Airspace Protection). The method selected for CASAP is called in French PAMSSEM [2, 3, 4] (*Procédure d'Agrégation Multicritère de type Surclassement de Synthèse pour Évaluations Mixtes*), and is based on the *A*, *Λ/C*, *E* model [4]. This method allows mixed evaluations (stochastic, fuzzy, and missing data) for the determination of the multicriterion compromise. PAMSSEM requires that a coefficient of relative importance (π_j) be assigned to each criterion, as well as the value of certain parameters such as discrimination thresholds (q_j representing indifference and p_j representing preference) and veto thresholds ($v_{j,}$) to establish the local preferences:

- π_j represents the "voting power" the decision-maker is willing to assign to each criterion;

- q_j represents the greatest difference between the evaluations of two alternatives according to a criterion *j* where the decision-maker is unable to make a clear choice between these two alternatives, given that everything is the same otherwise;

- p_j represents the smallest difference between the evaluations of two alternatives according to the criterion *j* where the decision-maker is unable to make a clear choice between the two, given that everything is the same otherwise ;

- ν_j represents the smallest difference between the evaluations of two alternatives according to the criterion *j* where the decision-maker declines to choose an alternative a_i over a_k, if the performance of a_k is greater than the performance of a_i and if the difference between a_k and a_i is greater than ν_j .

In brief, the characteristics of our decision-making situation context are:

- $A=(a_1,\ldots,a_i,\ldots,a_m)$

- $\Lambda/C=(g_1,\ldots,g_j,\ldots,g_n)$

- $E=(e_{ij} =g_j(a_i), I=1,...,m; j=1,....,n)$ and

- $M=(\pi_j, v_j(e_{ij}),q_j(e_{ij}),p_j(e_{ij}), I=1,...,m; j=1,...,n)$.

- a multicriterion method, PAMSSEM, within the framework of the ranking problematics.

given *m* alternatives and *n* attributes/criteria.

The determination of these coefficients and parameter values is very specific to the decision-making situation and the decision-maker's value system, beliefs and preferences. At this stage, an analyst should help the decision-maker to determine, as accurately as possible, a coherent set of parameters/values that reflect the decision-maker's "real" preferences. This is essential, considering that the information will include inaccuracies or be incomplete or both.

Due to the complexity of military operations, it is very difficult for military decision-makers to assign very precise values to the different coefficients and parameters when modelling their preferences. Having more than one plausible data set for the parameters to model the preferences of the decision-maker led to the possibility of more than one best compromise in a single decision-making situation. Consequently, military decision-makers needed a decision support system that yields valid results despite the difficulty of developing an exact model of the situation. These valid results, described as "robust," would be less affected by imperfections in the data generated by evaluation of the courses of action and in the instantiation of parameters representing the decision-maker's preferences in the process of modelling a military situation. We then initiated development of a robustness analysis appropriate for a multicriterion decision support system in the context of military decision-making. The objective of this investigation is to find a best compromise that is robust despite imperfections in the information available regarding the values of the coefficients and parameters.

This report is organized as follows: in the next section we present the main characteristics of the robustness analysis procedure that we propose. Sections 3 to 5 describe the creation of the data sets used to model the preferences of the decision-maker, and the algorithm used to extract a synthesis pre-order of plausible decision values (potentially to be qualified as a robust result), and finally we discuss the criterion of robustness itself.

2. Robustness Analysis

The concept of robustness was introduced in operational research several years ago [5, 6, 7, 8]. Robustness refers to the validity of a decision, recommendation, plan or test. Usually it relates to the context of uncertainty. In the fields of operational research (OR) and decision-aid (DA), the concept of robustness is often based on different techniques, notably the optimization process, which is used in sensibility analysis, for instance. Also, the concept of nearness is usually used to make the concept of robustness operational.

According to Kouvelis and Yu [9], uncertainty is a basic structural constituent of several decision environments. The best way to make a decision when faced with uncertainties is to recognize that they exist, understand and model them, and consider them an integral part of the decision support system. According to French [10], uncertainty and, more generally, the imperfect nature of the information input to a decision support process can take various forms and derive from several sources. Roy [11] states that the resolution of a concrete problem often raises many questions regarding the range of numerical values used: what is usually called the "data" represents, in fact, the values used by the analyst to build a specific model. These data can be the result of assumptions aimed at approximating the context of the problem, of estimates based on inadequate knowledge of the real values, of unpredictable choices, or of a forecast of future events. The resulting data set emerges as one plausible representation of the situation. The development of a procedure to identify a robust result is based on these premises.

As Roy [12] indicated, use of the term "robust" in decision-aid is increasingly frequent. Although this descriptor generally means "which considers near-values (*à peu près*)," it may sometimes have quite different meanings. Roy [12] added that this term, in its formalization, is highly subjective and contingent upon the decision-making context under consideration. The multicriterion decision-aid method implemented in the CASAP prototype takes into account the imperfect nature of the values assigned to actions, based on all criteria, in an explicit way (e.g., stochastic, fuzzy, incomplete), and ranks them according to the decision-maker's preferences. In this work, we sought to obtain a robust result that allows for imperfect information in the decision-maker's preference model (i.e., relative importance coefficients, veto thresholds and discrimination thresholds). This kind of imperfection is generally fuzzy and incomplete. Since we are not concerned here with a sequence of decisions, our analysis does not consider flexibility for future choices [13].

The work described in this report is partly inspired by the following definitions proposed by Kouvelis and Yu [9]:

- The solution of a mathematical program is robust, from the point of view of the optimality, if it remains nearby the optimum whatever is the plausible set of data used in the model (however, in this report it is not a question of optimality but of best compromise).

- A specific set of data instantiates a potential realization of the decision model represented by a set of parameters. Since the robustness approach is crucially based on the process of generating these data sets, it requires considerable knowledge of the environment in which the decision is to be made.

Thus, a result will be considered robust if it is not too far away from or not too contradictory [14] of different results obtained for all the data sets representing plausible values in the decision-making model. It is obviously necessary to define a robustness criterion in order to better define "not too far away."

Therefore, from this perspective, three critical elements were identified to compute a robust ranking:

- a method of modelling all the data sets that instantiate the decision-maker's preferences, which are not so well known;
- a method of aggregating the pre-orders generated from each data set;
- a robustness criterion suited to the decision-making situation.

The first step of our robustness analysis is to determine which data sets should be considered good enough to instantiate the decision-maker's preferences. The multicriterion aggregation procedure PAMSSEM, which is implemented in the CASAP prototype, is a procedure based on a synthesis outranking approach. It introduced coefficients of relative importance (c.r.i.) of the criteria (π_j), thresholds of veto ($v_j(e_{ij})$), thresholds of indifference ($q_j(e_{ij})$) and thresholds of preference ($p_j(e_{ij})$). Since the robustness analysis under study here involves modelling the decision-maker's preference parameters, the data sets presented in the next section focus on these parameters.

In the second step, we want to establish a synthesis pre-order $\overline{P}$ of the pre-orders $P^{(t)}$ obtained for each data subset. To do this, we will use the algorithm developed by Xu, Martel and Lamond [15], as discussed in section 4.

Once the synthesis pre-order is established, we must determine whether or not it can be considered robust. According to Vincke [14], a result can be described as robust if it is not too far away from the best compromises obtained for each of the sets of parameters (or "scenarios"), even if it is not the best compromise for all the sets of plausible parameter values. The criterion of robustness as defined in section 5 is based on the maximum distance between the synthesis pre-order $\overline{P}$ and each of the pre-orders $P^{(t)}$ obtained from these data sets. If that maximum value does not exceed a certain threshold (to be determined), then the synthesis pre-order $\overline{P}$ can be qualified as robust. The value of that threshold can be determined on the basis of simulations with various pre-orders.

Therefore, the procedure proposed for the determination of a robust result consists of four steps:

- establish the data sets (section 3);
- develop the pre-orders corresponding to each data set (section 3);

- develop a synthesis pre-order from the pre-orders (section 4);
- verify whether or not this synthesis pre-order can be qualified as robust (section 5).

3. Estimating Model Parameter Values

In outranking methods, thresholds are often used for modelling the imperfection of information related to the action evaluations and the decision-maker's preferences [16]. Since mixed evaluations (noted *E*) are allowed in PAMSSEM, the current imperfections (stochastic, fuzzy, missing data) related to *E* are thus modelled and taken into account for the determination of the multicriterion compromise. So, we agree that the data sets for the determination of a robust result could consist of values for π_j, $v_j(e_{ij})$, $q_j(e_{ij})$, and $p_j(e_{ij})$, $i=1,2,\ldots,m$; $j=1,2,\ldots,n$. This reduces the variety and the quantity of sets of plausible values; but still this quantity can be very large.

The data sets are developed based on the available information. If the decision-maker, in consultation with the analyst, is able to supply a lot of information, all this information must be used to develop the data sets. It is essential that these sets cover all the possibilities for modelling the decision-maker's preferences. Where possible, our report considers the use of intervals to express the degree of imprecision of the values used to model the decision-maker's preferences in a robustness analysis.

3.1 Coefficients of relative importance of the criteria

The coefficients of relative importance (c.r.i.) of the criteria $g_{(j)}$ range from 0 to 1 and their sum is 1:

$$0 < \pi_j < 1, \quad j = 1,\ldots,n \text{ with } \sum_{j=1}^{n} \pi_j = 1 .$$

If the decision-maker can determine a precise value for the importance of each criterion, these values can be used as a reference to establish other data sets. If he can go as far as expressing intervals of plausible values, we can then divide these intervals into small increments to make a detailed analysis. To limit the number of possible combinations that may develop in the course of a robustness analysis, it is suggested that only three increments be considered for each interval (two extreme values and one central value):

$$\pi^1_{(j)}, \frac{\pi^1_{(j)} + \pi^2_{(j)}}{2}, \pi^2_{(j)} \quad \forall j = 2,\ldots,n-1$$

where each set of coefficients must be normalized.

3.1.1 Intervals based on decision-maker's explicit values

If the decision-maker provides explicit values for the coefficient of importance of the criteria, it is proposed to construct an interval centred on each explicit coefficient value. The interval for the coefficients of each criterion would be between 0 and 1 and defined as:

$$\left[\pi^1_{(1)},\pi^2_{(1)}\right],\ldots,\left[\pi^1_{(j)},\pi^2_{(j)}\right],\ldots,\left[\pi^1_{(n)},\pi^2_{(n)}\right]$$

$$\text{with} \quad \pi^1_{(n)}\rangle 0 \quad \text{and} \quad \pi^2_{(1)}\langle 1 \quad \text{and} \quad \pi^1_{(j)} \geq \pi^2_{(j+1)} \quad \text{and} \quad \pi^2_{(j)} \leq \pi^1_{(j-1)} \quad \forall j = 2,\ldots,n-1 .$$

The limits of these intervals for the c.r.i. could be, as shown in Table 1, 10% to 20% greater or smaller than the given value; in respecting the constraints at the limits, $\pi^1_{(j)} \geq \pi^2_{(j+1)}$.

Table 1. Intervals for the coefficients of relative importance of the criteria

g_j	π^0_j	$\pi^0_j \pm 10\%$	$\pi^0_j \pm 20\%$
$g_{(1)}$	0.30	[0.27,0.33]	[0.24,0.36]*
$g_{(2)}$	0.24	[0.216,0.264]	[0.192,0.288]*
$\overline{g_{(3)}},\overline{\overline{g_{(3)}}}$	0.18,0.18	[0.162,0.198], [0.162,0.198]	[0.144,0.216], [0.144,0.216]*
$g_{(4)}$	0.10	[0.09,0.11]	[0.08,0.12]

* indicates particular cases where the constraint $\pi^1_{(j)} \geq \pi^2_{(j+1)}$ is not respected

$\overline{g_{(j)}},\overline{\overline{g_{(j)}}}$ are two different criteria having the same c.r.i. value

Considering that three values are used for each interval (two extreme values and one central value), and that each set of coefficients must be normalized, this leads to the identification of $3^n - 2$ sets of coefficients (-2 since the combination of extreme values $\pi^1_{(j)}, \quad j = 1,2,\ldots,n$ and $\pi^2_{(j)}, \quad j = 1,\ldots,n$ are normalized; this in turn leads to the same set of parameters as the normalized central values $\overline{\pi_{(j)}} = \dfrac{\pi^1_{(j)} + \pi^2_{(j)}}{2}$).

In the particular cases where the constraints are not respected (such as those shown in Table 1 with an *), we could use the central value between the limits, i.e., if $\pi^1_{(j)} \geq \pi^2_{(j+1)}$ then, by considering $\overline{\pi^1_{(j)}} = \overline{\pi^2_{(j+1)}} = \dfrac{\pi^1_{(j)} + \pi^2_{(j+1)}}{2}$ we get $\overline{\pi^1_{(j)}} \leq \pi^0_{(j+1)}$ and $\overline{\pi^2_{(j)}} \geq \pi^0_{(j+1)}$. For our example, since $\pi^1_{(1)} = 0.24 < \pi^2_{(2)} = 0.288$, we can use $\overline{\pi^1_{(1)}} = \overline{\pi^2_{(2)}} = \dfrac{0.24 + 0.288}{2} = 0.264$ (<0.30 and >0.24).

In this case, the combination of normalized extreme values does not lead to the same set of parameters as the combination of normalized central values, and we will get 3^n sets of parameters.

3.1.2 Intervals based on decision-maker's intervals

If the decision-maker is able to provide an interval for the coefficients of each criteria, such as:

$$\left[\pi^1_{(1)},\pi^2_{(1)}\right],\ldots,\left[\pi^1_{(j)},\pi^2_{(j)}\right],\ldots,\left[\pi^1_{(n)},\pi^2_{(n)}\right] \text{ with } \quad \pi^1_{(n)}\rangle 0 \quad \text{and} \quad \pi^2_{(1)}\langle 1$$

and if the relative importance of criterion $g_{(1)}$ is greater than the relative importance of criterion $g_{(2)}$ and so on, where $g_{(1)}=\left[\pi^1_{(1)},\pi^2_{(1)}\right]$, $g_{(j)}=\left[\pi^1_{(j)},\pi^2_{(j)}\right]$ and $g_{(n)}=\left[\pi^1_{(n)},\pi^2_{(n)}\right]$, we would use these intervals directly.

If no two criteria have equal importance, it is possible to identify a limited number of values for each interval. Considering that three values are used for each interval (two extremes and one central), and that each set of coefficients must be normalized, this leads to the identification of 3^n-2 sets of coefficients (for the same reasons as mentioned in 3.1.1).

It is possible that two or more criteria have equal importance, and this may appear in more than one place in the pre-order. These equally important criteria must be processed as one (i.e., one block). For example, if there are only two criteria that have the same relative importance, we will have, in order:

$$g_{(1)},\ldots,\left(\overline{g_{(j)}},\overline{\overline{g_{(j)}}}\right),\ldots,g_{\left(n^1=n-1\right)}.$$

The same process will apply where there are two criteria of equal importance or where there are other groups of equivalent criteria.

For example, with eight criteria respecting the following pre-order:

$$g_{(1)},\left(\overline{g_{(2)}},\overline{\overline{g_{(2)}}}\right),g_{(3)},\left(\overline{g_{(4)}},\overline{\overline{g_{(4)}}},\overline{\overline{\overline{g_{(4)}}}}\right),g_{(5)}$$

(where $g_{(1)}$ is the most important criterion and $g_{(5)}$ the least important criterion),

we have:

$$
\begin{array}{l||l}
g_{(1)} & \left[\pi^1_{(1)}, \pi^2_{(1)}\right] \\
(\overline{g}_{(2)}, \overline{\overline{g}}_{(2)}) & \left[\pi^1_{(2)}, \pi^2_{(2)}\right], \left[\pi^1_{(2)}, \pi^2_{(2)}\right] \\
g_{(3)} & \left[\pi^1_{(3)}, \pi^2_{(3)}\right] \\
\left(\overline{g}_{(4)}, \overline{\overline{g}}_{(4)}, \overline{\overline{\overline{g}}}_{(4)}\right) & \left[\pi^1_{(4)}, \pi^2_{(4)}\right], \left[\pi^1_{(4)}, \pi^2_{(4)}\right], \left[\pi^1_{(4)}, \pi^2_{(4)}\right] \\
g_{(5)} & \left[\pi^1_{(5)}, \pi^2_{(5)}\right]
\end{array}
$$

Each of the three criteria in the fourth rank has the same interval of values for its coefficient of relative importance (c.r.i.); we can then adopt the assumption that the coefficient importance value for criteria of equal importance is the same. This assumption allows us to accept $3^{n'} - 2$ (in our example: $3^5 - 2$) sets of coefficients, if we use three values per interval.

3.1.3 Intervals based on a pre-order of importance

The decision-maker may only be able to provide a pre-order on these coefficients:

$$\pi_{(1)} \geq \pi_{(2)} \geq \ldots \geq \pi_{(n)} \text{ with } \pi_j > 0 \text{ and } \sum_{j=1}^{n} \pi_j = 1,$$

and potentially the existence of equivalents. In that case, we could accept six value (data) sets $\underline{\pi}^1$, $\underline{\pi}^2$, ..., $\underline{\pi}^6$ where $\underline{\pi}^1$ corresponds to the case where the criteria are equally balanced and $\underline{\pi}^6$ corresponds to the case where the c.r.i. are decreasing from 1 to 1/*n* (before normalization of the c.r.i. of this vector). For the four data sets between $\underline{\pi}^1$ and $\underline{\pi}^6$, we progressively reduce the values of the c.r.i. in stages of *(n-2)/4*, while respecting the pre-order. If (*n*-2)/4 is not an integer, then fractional values must be considered. For example, if we have 15 criteria to consider, then *(n-2)/4* = 3.25, and the four less important criteria will be modified, i.e., $\underline{\pi}^2$: *(n-1/n, n-2/n, n-3/n, n-4/n)* and progressively in stages of three criteria for the three other data sets $\underline{\pi}^3$, $\underline{\pi}^4$, $\underline{\pi}^5$. These data sets are illustrated in Table 2.

Table 2. *Six data sets*

$\underline{\pi}^1$	$\underline{\pi}^2$	$\underline{\pi}^3$	$\underline{\pi}^4$	$\underline{\pi}^5$	$\underline{\pi}^6$
1	1	1	1	1	1
1	1	1	1	1	$\frac{n-1}{n}$
1	1	1	1	$\frac{n-1}{n}$	$\frac{n-2}{n}$
1	1	1	1	$\frac{n-2}{n}$	$\frac{n-3}{n}$
1	1	1	1	$\frac{n-3}{n}$	$\frac{n-4}{n}$
1	1	1	$\frac{n-1}{n}$	$\frac{n-4}{n}$	$\frac{n-5}{n}$
1	1	1	$\frac{n-2}{n}$	$\frac{n-5}{n}$	$\frac{n-6}{n}$
1	1	1	$\frac{n-3}{n}$	$\frac{n-6}{n}$	$\frac{n-7}{n}$
1	1	$\frac{n-1}{n}$	$\frac{n-4}{n}$	$\frac{n-7}{n}$	$\frac{n-8}{n}$
1	1	$\frac{n-2}{n}$	$\frac{n-5}{n}$	$\frac{n-8}{n}$	$\frac{n-9}{n}$
1	1	$\frac{n-3}{n}$	$\frac{n-6}{n}$	$\frac{n-9}{n}$	$\frac{n-10}{n}$
1	$\frac{n-1}{n}$	$\frac{n-4}{n}$	$\frac{n-7}{n}$	$\frac{n-10}{n}$	$\frac{n-11}{n}$
1	$\frac{n-2}{n}$	$\frac{n-5}{n}$	$\frac{n-8}{n}$	$\frac{n-11}{n}$	$\frac{n-12}{n}$
1	$\frac{n-3}{n}$	$\frac{n-6}{n}$	$\frac{n-9}{n}$	$\frac{n-12}{n}$	$\frac{n-13}{n}$
1	$\frac{n-4}{n}$	$\frac{n-7}{n}$	$\frac{n-10}{n}$	$\frac{n-13}{n}$	$\frac{1}{n}$

3.2 Threshold values

Our study considered the use of intervals to express the imprecision of the values used to model decision-maker preferences in a robustness analysis. The three types of thresholds used in the decision-maker preference model in PAMSSEM are indifference threshold, preference threshold and veto threshold. In this investigation we considered criteria that have cardinal evaluations.

3.2.1 Indifference threshold

We need to establish an interval for the indifference threshold, such as: $\left[q_j^1, q_j^2\right] \quad \forall j, \quad \text{with} \quad q_j^1 \geq 0$. Intervals for the indifference threshold (q_j) can vary between 0 and E_j, which corresponds to the range of the scale associated with the criteria g_j.

If the decision-maker provides an interval for the indifference threshold, then we will use it directly.

If the decision-maker provides an exact value for this threshold, we propose to define the interval by using two other values: 80% and 60% of $q_j^{'}$, for example $q_j^2 = 0.8q_j^1$ and $q_j^3 = 0.6q_j^1$ are two lower values, since the reference value is probably overestimated. These values (80% and 60%) were obtained from simulations, and therefore are not absolute values.

If the decision-maker is not able to provide anything for the indifference threshold, we [4] suggest calculating a default value using:

$$q_j^{'} = 0.15X\,0.25E_j$$

then an interval can be defined using 80% and 60% of $q_j^{'}$.

3.2.2 Preference threshold

One needs to establish an interval for the preference threshold, such as:

$$\left[p_j^1, p_j^2\right] \quad \forall j \text{ with } p_j^1 \geq q_j^2 \quad \text{and} \quad p_j^2 \leq E_j\,.$$

Intervals for the preference threshold (p_j) can vary between q_j and E_j, which corresponds to the range of the scale associated with the criteria g_j.

If the decision-maker provides an interval for the preference threshold, we will use it directly.

If the decision-maker provides an exact value for this threshold, we propose to define the interval by using two other values: 80% and 60% of $p_j^{'}$; for example $p_j^2 = 0.8p_j^1$ and $p_j^3 = 0.6p_j^1$ are two lower values, since the reference value is probably overestimated with the constraint: $p_j^1 \geq q_j^2 \quad \forall j$. These values (80% and 60%) were obtained from simulations, and therefore are not absolute values.

If the decision-maker is not able to provide anything for this preference threshold, we [4] suggest calculating a default value using:

$$p_j^{'} = 0.25E_j + 0.05\left(v_j - q_j\right),$$

where v_j is the veto threshold described in subsection 3.2.3

then defining an interval using 80% and 60% of $p_j^{'}$.

3.2.3 Veto threshold

If the decision-maker provides an interval for the veto threshold, such as

$$v_j \Rightarrow \left[v_j^1, v_j^2\right]$$

then we will use it directly.

If the decision-maker provides an exact value for this threshold, we propose to determine the interval:

$$v_j \Rightarrow \left[v_j^1, v_j^2\right] \quad \text{with} \quad v_j^1 > p_j^2$$

by using two other values: 80% and 60% of $v_j^{'}$. For example, $v_j^2 = 0.8v_j^1$ and $v_j^3 = 0.6v_j^1$ are two lower values, since the reference value is probably overestimated with the constraint:

$$v_j \geq p_j \quad \forall j$$

These values (80% and 60%) were obtained from simulations, and therefore are not absolute values.

If the decision-maker is not able to provide an exact value for this threshold, we [4] suggest calculating a default value such as:

$$v_j = \frac{0.25E_j}{\sqrt{\pi_j}}$$ where E_j is the range of the scale associated with the criterion g_j.

However, we must make sure that $v_j > p_j^2 \quad \forall j$.

Since this value varies depending on the coefficients of relative importance of the criteria, no interval will be defined.

3.3 Identification of plausible data sets

Considering all plausible parameter values will determine the number of data sets to be considered in the robustness analysis. As mentioned previously, this work considers the use of intervals to express the imprecision of the values used to model the decision-maker's preferences in a robustness analysis. Accordingly, the number of data sets to be considered in a robustness analysis can be rather large. Even if the number of values is limited to three per interval, the number of data sets can be quite impressive. Although this is not unexpected (for example, Roy and Bouyssou [17] processed 136 data sets in their analysis of robustness), we need to find a way to process these combinations in an acceptable time frame. The method we propose is to treat the different parameters as groups or blocks.

For this example, we will consider the indifference and preference thresholds only. If there are *m* cardinal criteria, we can form 9^m sets of thresholds. We can reduce this number to 9 by grouping (blocking) the extreme and central values of all the criteria. See the example in Table 3:

Table 3. *Treatment of parameters in blocks*

	q_j			p_j		
g_j	q_j^1	$\overline{q_j}$	q_j^2	p_j^1	$\overline{p_j}$	p_j^2
g_1	q_1^1	$\overline{q_1}$	q_1^2	p_1^1	$\overline{p_1}$	p_1^2
g_2	q_2^1	$\overline{q_2}$	q_2^2	p_2^1	$\overline{p_2}$	p_2^2
g_3	q_3^1	$\overline{q_3}$	q_3^2	p_3^1	$\overline{p_3}$	p_3^2

By computing the values in blocks, we will have 9 combinations instead of 9^3. Therefore, we suggest that this method be used to reduce the number of combinations in order to complete the robustness analysis within an acceptable period of time.

Formally, we designate by $J_t=(\pi_t, v_t, q_t, p_t)$, a selected plausible data set for the development of a robust result. The reader should note that J_t is not part of the problem data, but the result of an interactive process with the decision-maker [18]. Our robustness analysis plan will consider s data sets ($J_1,\ldots,J_s;\ t=1,\ldots,s$), forming a representative subset of all the sets of plausible values. Using the multicriterion performance table and a multicriteria aggregation procedure (such as PAMSSEM [4] in our case), a pre-order ($P^{(t)},t=1,\ldots,s$) is determined for each of the data sets established.

4. Algorithm to Obtain a Synthesis Ranking

A synthesis pre-order $\overline{P}$ of the *s* pre-orders $P^{(t)}$ is obtained by the use of an algorithm (developed by Xu et al. [15]) that determines two complete pre-orders reflecting the dominant and dominated characters of each alternative a_i, $i=1, 2,\ldots, m$. The algorithm then uses the intersection of these pre-orders as the synthesis pre-order (as in the ELECTRE III procedure [19]). Generally, this synthesis pre-order is partial. We consider that all the data sets have the same importance, but the algorithm can be applied with unequal importance.

4.1 Notations

The algorithm is implemented in three steps:

1. compute a dominant pre-order $P_{\succ}$;
2. compute a dominated pre-order $P_{\prec}$; and
3. find the intersection pre-order $\overline{P}$ of the dominant and dominated pre-orders.

The pre-orders obtained for each data set can be represented by square matrices expressing the binary relations R between every pair of alternatives. These relations are:

$\succ$	: preference
$\prec$	: inverse preference, i.e., not preferred
$\approx$	: indifference
?	: incomparability.

We designate by:

A	: set of alternatives a_i with $i=1, 2, \ldots, m$
a,b	: $a, b \in A$ where $a \neq b$
P	: pre-order
s	: number of data sets, and
t	: one of the data sets; $t=1,\ldots,s$.

We measure the distances between binary relations as proposed by Jabeur, Martel and Ben Khelifa [20], illustrated in Table 4.

Table 4. *Distances between binary relations of alternatives a and b*

	$a \approx b$	$a \succ b$	$a ? b$	$a \prec b$
$a \approx b$	0	1	$4/3$	1
$a \succ b$	1	0	$4/3$	$5/3$
$a ? b$	$4/3$	$4/3$	0	$4/3$
$a \prec b$	1	$5/3$	$4/3$	0

4.2 Dominant pre-order $P_{\succ}$

We propose an iterative algorithm to determine the classes of equivalencies in decreasing order of importance of the alternatives. So, class $A_1 \subset A$ contains the alternative or alternatives occupying the first rank, class A_2 contains the alternatives occupying the second rank, and so on. To do this, we define for every alternative $a \in A$ an index $\Phi_1^{\succ}(a)$ which measures the divergence of the relation $a \succ b$ (alternative b $\in$ A) with respect to the other binary relations $R^{(t)}$ which bind alternatives a and b in the pre-orders $P^{(t)}$, $t=1, 2,\ldots,s$. So, to determine class A_1, one calculates for every alternative $a \in A$ the index $\Phi_1^{\succ}(a)$ as follows:

$$\Phi_1^{\succ}(a) = \sum_{b \neq a} \sum_{t=1}^{s} \Delta\left(\succ, R^{(t)}(a,b)\right)$$

$$\text{where } R^{(t)}(a,b) = \begin{cases} \approx & \text{if} \quad a \approx^{(t)} b \\ \succ & \text{if} \quad a \succ^{(t)} b \\ ? & \text{if} \quad a\,?^{(t)}\, b \\ \prec & \text{if} \quad a \prec^{(t)} b \end{cases}$$

and where Δ is the distance between the pair of binary relations $\left(\succ, R^{(t)}(a,b)\right)$ (see Table 4).

Given that $\Phi_1^{\succ}(a)$ measures the distance of alternative a from the first rank, class A_1 is determined in the following way:

$$A_1 = \left\{a \in A / \Phi_1^{\succ}(a) \text{ is minimal}\right\}.$$

To determine class A_2, it is necessary to calculate for every alternative $a \in A\text{-}A_1$ the index $\Phi_2^{\succ}(a)$:

$$\Phi_2^{\succ}(a) = \Phi_1^{\succ}(a) - \sum_{b \in A_1} \sum_{t=1}^{s} \Delta\left(\succ, R^{(t)}(a,b)\right).$$

The alternatives minimizing the difference $\Phi_2^{\succ}(a)$ for $a \in A\text{-}A_1$ constitute class A_2, or:

$$A_2 = \left\{a \in A - A_1 / \Phi_2^{\succ}(a) \text{ is minimal}\right\}.$$

Generally speaking, to determine class of equivalence A_k, one determines for each alternative $a \in A - \bigcup_{i=1}^{k-1} A_i$ the index $\Phi_k^{\succ}(a)$ as follows:

$$\Phi_k^{\succ}(a) = \Phi_1^{\succ}(a) - \sum_{i=1}^{k-1} \sum_{b \in A_i} \sum_{t=1}^{s} \Delta\left(\succ, R^{(t)}(a,b)\right).$$

The alternatives minimizing the difference $\Phi_k^{\succ}(a)$ for any alternative $a \in A - \bigcup_{i=1}^{k-1} A_i$ constitute class A_k. So the algorithm to determine the complete pre-order $P_{\succ}$ appears as follows:

Initialization

$Y = A$

$k = 1$

As long as $Y \neq \phi$, do:

$$\Phi_k^{\succ}(a) = \Phi_1^{\succ}(a) - \sum_{i=1}^{k-1} \sum_{b \in A_i} \sum_{t=1}^{s} \Delta\left(\succ, R^{(t)}(a,b)\right)$$

$$A_k = \left\{a \in Y / \Phi_k^{\succ}(a) \text{ is minimal}\right\}$$

$$k = k + 1$$

$$Y = A - \bigcup_{i=1}^{k-1} A_i$$

end do

Illustration. To illustrate this algorithm, we limit ourselves to four data sets. Let us start with some pre-orders obtained by applying a synthesis outranking multicriterion aggregation procedure, so we can determine partial pre-orders. These pre-orders are given in Figure 1.

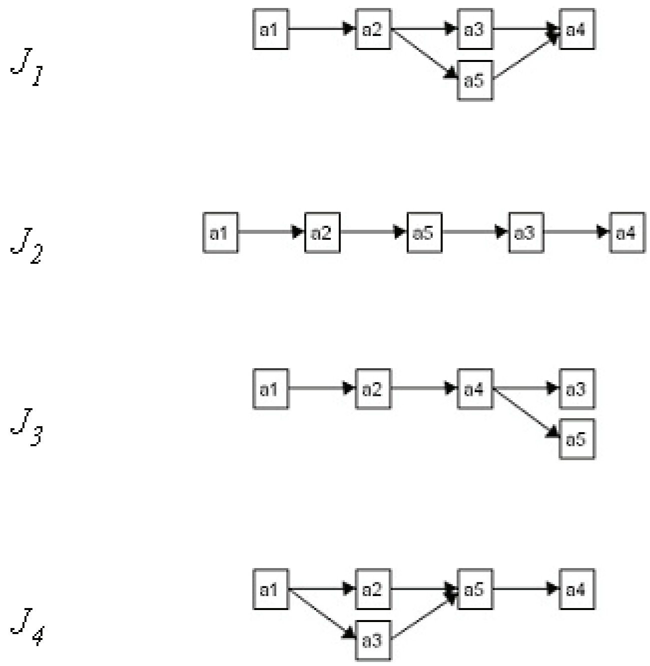

Figure 1. *Pre-orders obtained with 4 data sets*

First, to determine pre-order $P_{\succ}$, one calculates the divergences $\Phi_k^{\succ}(a)$ of the relation $a \succ b$ for every alternative $a \in A$ (see Table 5, where * indicates the value of A_k). For instance, following is the development of $\Phi_1^{\succ}(a_2)$:

$$\Phi_1^{\succ}(a_2) = \sum_{b \neq a_2} \sum_{t=1}^{4} \Delta\left(\succ, R^{(t)}(a_2, b)\right)$$

$$\Phi_1^{\succ}(a_2) = \sum_{b \neq a_2} \Delta\left(\succ, R^1(a_2, b)\right) + \sum_{b \neq a_2} \Delta\left(\succ, R^2(a_2, b)\right) + \sum_{b \neq a_2} \Delta\left(\succ, R^3(a_2, b)\right) + \sum_{b \neq a_2} \Delta\left(\succ, R^4(a_2, b)\right)$$

$$\Phi_1^{\succ}(a_2) = \left(\frac{5}{3} + 0 + 0 + 0\right) + \left(\frac{5}{3} + 0 + 0 + 0\right) + \left(\frac{5}{3} + 0 + 0 + 0\right) + \left(\frac{5}{3} + \frac{4}{3} + 0 + 0\right)$$

$$\Phi_1^{\succ}(a_2) = \frac{24}{3}.$$

Table 5. Divergences of the relation $a \succ b$

	$\Phi_1^{\succ}(a_i)$	$\Phi_2^{\succ}(a_i)$	$\Phi_3^{\succ}(a_i)$	$\Phi_4^{\succ}(a_i)$
a_1	$\frac{0}{3}$ *			
a_2	$\frac{24}{3}$	$\frac{4}{3}$ *		
a_3	$\frac{57}{3}$	$\frac{37}{3}$	$\frac{18}{3}$ *	
a_4	$\frac{70}{3}$	$\frac{50}{3}$	$\frac{30}{3}$	
a_5	$\frac{58}{3}$	$\frac{38}{3}$	$\frac{18}{3}$ *	

* indicates the value of A_k

The resulting pre-order $P_{\succ}$ is (Figure 2):

$A_1 = a_1$; $A_2 = a_2$; $A_3 = a_3, a_5$; $A_4 = a_4$.

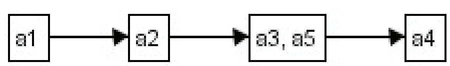

Figure 2. *Resulting dominant pre-order* $P_{\succ}$

4.3 Dominated pre-order $P_{\prec}$

Here again, an iterative algorithm is used to determine the classes of equivalent alternatives A_k, where $1 \le k \le m$. However, these classes are determined on the basis of the index $\Phi_1^{\prec}(a)$, which measures the divergence of the relation $a \prec b$ with respect to the relations that bind a and b in the pre-orders $P^{(t)}$, $t=1, 2, \ldots, s$. So, to determine class A_1, one defines for every $a \in A$ the index $\Phi_1^{\prec}(a)$:

$$\Phi_1^{\prec}(a) = \sum_{b \ne a} \sum_{t=1}^{s} \Delta\left(\prec, R^{(t)}(a,b)\right).$$

Given that $\Phi_1^{\prec}(a)$ measures the distance of *a* from the last rank, class A_1 is determined in the following way:

$$A_1 = \{a \in A / \Phi_1^{\prec}(a) \text{ is maximal}\}.$$

To determine A_2, it is necessary to calculate for every action $a \in A\text{-}A_1$ the index $\Phi_2^{\prec}(a)$:

$$\Phi_2^{\prec}(a) = \Phi_1^{\prec}(a) - \sum_{b \in A_1} \sum_{t=1}^{s} \Delta\left(\prec, R^{(t)}(a,b)\right), \text{ and}$$

$$A_2 = \{a \in A - A_1 / \Phi_2^{\prec}(a) \text{ is maximal}\}.$$

Generally speaking, to determine a class of equivalence A_k, we calculate for each action $a \in Y = A - \bigcup_{i=1}^{k-1} A_i$ the index $\Phi_k^{\prec}(a)$:

$$\Phi_k^{\prec}(a) = \Phi_1^{\prec}(a) - \sum_{i=1}^{k-1} \sum_{b \in A_i} \sum_{t=1}^{s} \Delta\left(\prec, R^{(t)}(a,b)\right) \text{ and}$$

$$A_k = \left\{a \in A - \bigcup_{i=1}^{k-1} A_i / \Phi_k^{\prec}(a) \text{ is maximal}\right\}.$$

The algorithm determining the complete pre-order $P_{\prec}$ appears as follows:

Initialization

$Y = A$

$k = 1$

As long as $Y \neq \phi$, do:

$$\Phi_k^{\prec}(a) = \Phi_1^{\prec}(a) - \sum_{i=1}^{k-1} \sum_{b \in A_i} \sum_{t=1}^{s} \Delta\left(\prec, R^{(t)}(a,b)\right)$$

$$A_k = \left\{a \in A - \bigcup_{i=1}^{k-1} A_i / \Phi_k^{\prec}(a) \text{ is maximal}\right\}$$

$$k = k + 1$$

$$Y = A - \bigcup_{i=1}^{k-1} A_i$$

end do

Illustration. To determine the pre-order $P_{\prec}$ of the four pre-orders presented in Figure 1, one must compute the divergence $\Phi_1^{\prec}(a)$ of the relation $a \prec b$ for every alternative $a \in A$ (Table 6).

Table 6. *Divergences of the relation $a \prec b$*

	$\Phi_1^{\prec}(a_i)$	$\Phi_2^{\prec}(a_i)$	$\Phi_3^{\prec}(a_i)$	$\Phi_4^{\prec}(a_i)$
a_1	$\frac{80}{3}$ *			
a_2	$\frac{59}{3}$	$\frac{59}{3}$ *		
a_3	$\frac{32}{3}$	$\frac{32}{3}$	$\frac{28}{3}$	$\frac{15}{3}$ *
a_4	$\frac{10}{3}$	$\frac{10}{3}$	$\frac{10}{3}$	$\frac{5}{3}$
a_5	$\frac{33}{3}$	$\frac{33}{3}$	$\frac{33}{3}$ *	

* indicates the value of A_k

$$\Phi_1^{\prec}(a_1) = \sum_{b \neq a_1} \sum_{t=1}^{4} \Delta\left(\prec, R^{(t)}(a_1, b)\right)$$

$$\Phi_1^{\prec}(a_1) = \sum_{b \neq a_1} \Delta\left(\prec, R^1(a_1, b)\right) + \sum_{b \neq a_1} \Delta\left(\prec, R^2(a_1, b)\right) + \sum_{b \neq a_1} \Delta\left(\prec, R^3(a_1, b)\right) + \sum_{b \neq a_1} \Delta\left(\prec, R^4(a_1, b)\right)$$

$$\Phi_1^{\prec}(a_1) = \left(\frac{5}{3}+\frac{5}{3}+\frac{5}{3}+\frac{5}{3}\right) + \left(\frac{5}{3}+\frac{5}{3}+\frac{5}{3}+\frac{5}{3}\right) + \left(\frac{5}{3}+\frac{5}{3}+\frac{5}{3}+\frac{5}{3}\right) + \left(\frac{5}{3}+\frac{5}{3}+\frac{5}{3}+\frac{5}{3}\right)$$

$$\Phi_1^{\prec}(a_1) = \frac{80}{3}.$$

The resulting pre-order $P_{\prec}$ is (as in Figure 3):

$A_1 = a_1$; $A_2 = a_2$; $A_3 = a_5$; $A_4 = a_3$; $A_5 = a_4$.

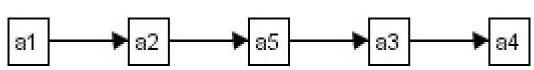

Figure 3. *Resulting dominated pre-order $P_{\prec}$*

4.4 Synthesis pre-order $\overline{P} = P_{\succ} \cap P_{\prec}$

The intersection $\overline{P} = P_{\succ} \cap P_{\prec}$ is obtained according to the following rules (as they are in ELECTRE III):

1. $a \succ b$ in $\overline{P}$ $\Leftrightarrow$ $a \succ b$ in $P_{\succ}$ and $P_{\prec}$ where $a \succ b$ in one of the two pre-orders and $a \approx b$ in the other one;

2. $a \approx b$ in $\overline{P}$ $\Leftrightarrow$ $a \approx b$ in $P_{\succ}$ and $P_{\prec}$;

3. $a ? b$ in $\overline{P}$ $\Leftrightarrow$ $a \succ b$ in one pre-order and $b \succ a$ in the other one.

N.B. The pre-orders $P_{\succ}$ and $P_{\prec}$ are complete, but $\overline{P}$ may be partial.

Illustration. The synthesis pre-order $\overline{P}$ in our example is (as in Figure 4):

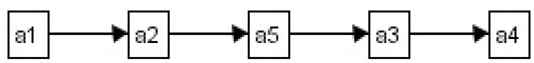

Figure 4. *Synthesis pre-order* $\overline{P}$

5. Robustness Criterion

The next step is to verify whether or not the synthesis pre-order $\overline{P}$ obtained can be qualified as robust, i.e., if it satisfies the criterion of robustness. The criterion of robustness is based on the concept of "not too far away," i.e., the synthesis pre-order must not be too far away from the pre-orders $P^{(t)}$ obtained from each set of plausible data values of the parameters used to model the decision-maker's preferences. The concept of "not too far away" is made operational by using a measure of distance between two complete or partial pre-orders of alternatives. As for the coefficients of rank correlation in usual statistical measures, most of the available measures apply only to complete pre-orders [21, 22, 23].

Roy and Slowinski [24] developed a measure (of distance) for the treatment of partial pre-orders. We suggest using a modified form of this measure for building a criterion of robustness. The element modified is the distance between the binary relations that bind two pairs of alternatives (see Table 4), considering the importance of an alternative in a pre-order. From Spearman's correlation index [25], Wang and Shen [23] derived a correlation index between two vectors of ranks that is based on the following assumption: a permutation of the alternatives occupying the best ranks should be more penalized in the measure of the difference between these two vectors than a permutation of the alternatives occupying the last ranks.

Inspired by the Wang and Shen paper, Jabeur [26] proposed an index of correlation between two complete or partial pre-orders. This index, representing the importance of an alternative a_i in a pre-order $P^{(t)}$, is the ratio obtained by counting the number of times this alternative is preferred to the other alternatives, plus one, over the total score of all the alternatives plus one for each alternative of the same pre-order. This importance for an alternative a_i, noted $w_{a_i}^{(t)}$, is formalized in the following way:

$$w_{a_i}^{(t)} = \frac{score\ a_i + 1}{\sum_{a_i} (score\ a_i + 1)} \ .$$

From this index of correlation, the measure of distance $D^{(t)}$ between a pre-order $P^{(t)}$, $t=1,2,\ldots,s$ and the pre-order synthesis $\overline{P}$ was extracted:

$$D^{(t)} = \left(\frac{3}{2(m-1)\Delta(\succ, \succ^{-1})} \right) \sum_{i=1}^{m-1} \sum_{k>i} \max(w_{a_i}^{(t)}, w_{a_k}^{(t)}) \Delta(R_{ik}^{(t)}, \overline{R_{ik}})$$

where $w_{a_i}^{(t)}$ designates the importance of the alternative a_i in the pre-order $P^{(t)}$ and where Δ is the distance between the binary relations $R_{ik}^{(t)}$ (in the pre-order $P^{(t)}$) and $\overline{R_{ik}}$ (in the synthesis pre-order $\overline{P}$).

This measure varies in a monotonous way between 0 and 1: its value is 0 if the two pre-orders are identical and 1 if they are totally inversed.

Illustration. We can compute the importance index of each alternative within the four pre-orders described in the Figure 1. First, the scores of each alternative are presented in Table 7.

Table 7. *Scores of the alternatives within pre-orders*

Pre-order	a_1	a_2	a_3	a_4	a_5
J_1	4	3	1	0	1
J_2	4	3	1	0	2
J_3	4	3	0	2	0
J_4	4	2	2	0	1

Second, the importance indices of the alternatives within the four pre-orders J are presented in Table 8. For example, for the pre-order $P^{(t)}=J_1$, one obtains:

$$w_{a_1}^{(t)} = \frac{4+1}{14} = \frac{5}{14}; \; w_{a_2}^{(t)} = \frac{4}{14}; \; w_{a_3}^{(t)} = \frac{2}{14}; \; w_{a_4}^{(t)} = \frac{1}{14}; \text{ and } w_{a_5}^{(t)} = \frac{2}{14}.$$

Table 8. *Importance indices of the alternatives within pre-orders*

Pre-orders	a_1	a_2	a_3	a_4	a_5
J_1	$\frac{5}{14}$	$\frac{4}{14}$	$\frac{2}{14}$	$\frac{1}{14}$	$\frac{2}{14}$
J_2	$\frac{5}{15}$	$\frac{4}{15}$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{3}{15}$
J_3	$\frac{5}{14}$	$\frac{4}{14}$	$\frac{1}{14}$	$\frac{3}{14}$	$\frac{1}{14}$
J_4	$\frac{5}{14}$	$\frac{3}{14}$	$\frac{3}{14}$	$\frac{1}{14}$	$\frac{2}{14}$

The distances $D^{(t)}$ between each of the four pre-orders $P^{(t)}$ and the synthesis pre-order $\overline{P}$ are given in Table 9. For example, for the pre-order $P^{(t)}=J_1$, one obtains:

$$D^{(1)} = \frac{9}{40}\left\{ \max\left(\frac{5}{14},\frac{4}{14}\right)^{(1,2)} \Delta(P,P) + \max\left(\frac{5}{14},\frac{2}{14}\right)^{(1,3)} \Delta(P,P) + \max\left(\frac{5}{14},\frac{1}{14}\right)^{(1,4)} \Delta(P,P) + \max\left(\frac{5}{14},\frac{2}{14}\right)^{(1,5)} \Delta(P,P) + \right.$$

$$\max\left(\frac{4}{14},\frac{2}{14}\right)^{(2,3)} \Delta(P,P) + \max\left(\frac{4}{14},\frac{1}{14}\right)^{(2,4)} \Delta(P\ \ ,P\ \ ) + \max\left(\frac{4}{14},\frac{2}{14}\right)^{(2,5)} \Delta(P,P) +$$

$$\max\left(\frac{2}{14},\frac{1}{14}\right)^{(3,4)} \Delta(P\ \ ,P\ \ ) + \max\left(\frac{2}{14},\frac{2}{14}\right)^{(3,5)} \Delta(?,P^{-1}) +$$

$$\left. \max\left(\frac{1}{14},\frac{2}{14}\right)^{(4,5)} \Delta(P^{-1},P^{-1}) \right\}$$

$$D^{(1)} = \frac{9}{40}\left\{\left(\frac{5}{14}\times 0\right) + 0 + 0 + 0 + 0 + 0 + 0 + 0 + (\frac{2}{14}\times\frac{4}{3}) + 0\right\}$$

$$D^{(1)} = 0.043$$

Table 9. *Distances $D^{(t)}$ between $P^{(t)}$ and $\overline{P}$*

	Distances
Pre-order: *J1*	0.043
Pre-order: *J2*	0.000
Pre-order: *J3*	0.182
Pre-order: *J4*	0.145

When we calculate the distance $D^{(t)}$ for every data set (a kind of robustness score), we determine the maximum distance obtained. If this maximum is lower than a threshold to be determined, we can conclude that the synthesis pre-order is a robust result. Our criterion (test) of robustness is thus:

$$\underset{t}{Max}\, D^t \leq \alpha \text{ (a threshold).}$$

To determine an acceptable value for threshold α, we calculated the distance D between several pre-orders that were simulated so as to cover extreme cases (similar and totally opposite) and intermediate cases. As we indicated, this distance varies between 0 and 1. We concluded that a value of roughly 0.25 to 0.30 would be appropriate for the threshold. As is the case for the significance level (also a kind of threshold) in statistics, it is difficult to fix this threshold absolutely.

Illustration. Taking the last example, one has

$$\underset{t}{Max}\, D^{(t)} = \max(J1, J2, J3, J4)$$

$$\underset{t}{Max}\, D^{(t)} = \max(0.043, 0.000, 0.182, 0.145)$$

$$\underset{t}{Max}\, D^{(t)} = 0.182$$

If one fixes threshold α at 0.25, then one can conclude that the synthesis pre-order $\overline{P}$ (see Figure 4) is robust, since it passes the robustness test. In fact,

$$\underset{t}{Max}\, D^{(t)} = 0.182 \leq 0.25.$$

We can conclude that the synthesis pre-order $\overline{P}$ is "not too far away" from the four pre-orders $P^{(t)}$ and can qualify it as robust.

6. Conclusion

The various studies in the operational research and decision-aid domains are almost unanimous in their justification of the concept of robustness. It is generally presented as a palliative to inaccurate or incomplete knowledge of decision-maker preferences, to the inherent limitations of modelling, or to the fact that the values used in a model are not readily available. The robustness concept, however, is examined and defined in different ways in different domains. Moreover, it is sometimes interpreted differently within the same domain.

In this report, the robustness analysis is presented to deal with the plausible values that some preference modelling variables may take. The multicriterion method is used to obtain a best compromise result despite conflicting criteria. The proposed procedure for identifying a robust result in the ranking of alternatives yields the best compromise while considering all plausible values of the preference modelling variables. This procedure is designed to be used in a multicriterion decision-aid process. As mentioned earlier, the characteristics of our decision-making context are:

- $A=(a_1,\ldots,a_i,\ldots,a_m)$
- $\Lambda/C=(g_1,\ldots,g_j,\ldots,g_n)$
- $E=(e_{ij}=g_j(a_i),\ i=1,...,m;\ j=1,....,n)$ *and*
- $M=(\pi_j,\ v_j(e_{ij}),\ q_j(e_{ij}),\ p_j(e_{ij}),\ i=1,...,m;\ j=1,...,n).$
- A multicriterion method, PAMSSEM, within the framework of the ranking problematics.

In this investigation we considered the imperfect nature of the information contained in the elements of the set *M*. The proposed robustness analysis targets the values of those elements. This study considered the use of intervals to express the imprecision of the values used to model decision-maker preferences in a robustness analysis. Consequently, the procedure proposed in this report for determining a robust result consists of four steps:

- establishing the data sets;
- developing the pre-orders corresponding to the data sets;
- developing a synthesis pre-order from the pre-orders;
- verifying whether or not the synthesis pre-order can be qualified as robust.

This approach is different from that proposed by Roy and Bouyssou [17] in that it uses an algorithm to determine a synthesis pre-order, thus a robustness criterion. Although this procedure is proposed here for use with PAMSSEM, it is equally appropriate for any multicriterion aggregation procedure based on synthesis outranking.

The proposed procedure is highly context-dependent, as generally the case in robustness analysis. Besides, one may well ask whether it is sufficient to consider only the imperfect nature of the information used to model the decision-maker's preferences. Although the multicriterion aggregation procedure used in the CASAP prototype treats distributional evaluations, it is possible that more than one distributional evaluation of an alternative according to a criterion would exist. We agree with Roy's assertion [27] that we could use the fuzzy subsets language to represent the as-yet undetermined values of the parameters required to model preferences. More simulations should also be done to determine the value of the threshold for ascribing the quality of robustness to a given alternative.

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List of symbols/abbreviations/acronyms/initialisms

A, Λ/C, E	Alternatives, Attributes/Criteria, Evaluations
AOC	Air Operations Centre
CASAP	Commander's Advisory System for Airspace Protection
c.r.i.	coefficients of relative importance
DA	Decision-Aid
DND	Department of National Defence
DRDC	Defence Research and Development Canada
OR	Operational Research
PAMSSEM	"Procédure d'Agrégation Multicritère de type Surclassement de Synthèse pour Évaluations Mixtes"
RDDC	R&D pour la défense Canada

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This report presents a robustness analysis procedure that was developed for a decision support system based on a multicriterion outranking approach. This work is based on the assumption that any model is a limited representation of reality and that it is impossible to derive exact models of the situation. Therefore, the decision analysis procedure must help identify the best results despite imperfections in the models. In particular, the robustness analysis procedure should produce a ranking of best compromise by considering all plausible values based on the model of the decision-maker's preferences. In this report we propose a definition of the robustness concept within the military decision-making context. We then describe the proposed robustness analysis procedure.

Ce rapport présente une procédure d'analyse de la robustesse qui a été développée pour un système d'aide à la décision basé sur une approche de surclassement multicritère. Étant donné l'impossibilité d'obtenir des modèles exacts d'une situation particulière, il est nécessaire que la procédure d'analyse de décision permette l'identification des « bons » résultats en dépit de l'imperfection de ces modèles. Ainsi, une procédure d'analyse de la robustesse devrait produire un rangement des meilleurs compromis en tenant compte de toutes les valeurs plausibles obtenues à partir du modèle des préférences du décideur. Dans ce rapport, nous proposons une définition du concept de la robustesse employé dans le cadre de la prise de décisions. Par la suite, nous décrivons en détail la procédure d'analyse de la robustesse qui est proposée.

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