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Granulometries and pattern spectra for radar signals

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Abstract

Granulometries and pattern spectra are morphological tools used to obtain size distributions in images. This paper presents the application of these techniques to radar signals. It uses a new approach which is more appropriate for RF signals than the classical approach. It uses complex openings, closings and power measurements instead of the usual gray-tone operators and the Lebesgue measure. Examples for simulated and real radar signals are given.

Keywords: Mathematical morphology; Pattern spectrum; Complex signals; Radar

1. Introduction

Granulometry and pattern spectrum are two methodologies that are used with goals similar to spectral analysis. They allow to obtain size distributions of objects in pictures by using morphological openings and closings.

The goal of this paper is to introduce a new type of granulometry and a new kind of pattern spectrum that are more appropriate for the analysis of radio frequency (RF) signals than the classical morphological techniques.

The granulometry and pattern spectrum proposed in this paper share the same axiomatic basis as the classical granulometry. The tools were modified by using complex openings and closings instead of the usual openings; the Lebesgue measure was replaced by power measurements. These modifications are necessary because of the nature of RF signals, which are dramatically different from gray-tone pictures.

Mathematical morphology is mainly considered as an image processing methodology. The main reason for this is because it is based on set operators like unions and intersections instead of linear operators such as convolutions and linear combinations. As Serra observed in [8], set operators are better at modeling the way objects interact with each other in a scene; they occlude each other and the information behind an object is irremediably lost. Objects must be transparent if one wishes to use linear models. Moreover, a shape and its background are two distinct ideas that need to be addressed separately. Linear operators do not make that distinction while set operators do.

Morphology was first applied to binary images and then extended to gray-tone images and functions. The extension to gray-tone images was based on the fact that there is a total ordering relationship between all the function points. This order relationship is used to generate the sets upon which morphological operators are applied. Later, there

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have been some extensions of morphology to color, or multi-valued images, Comer and Delp's work [1,2] being typical. The difficulty in extending morphology to color lies in the fact that there is no natural way to order multi-valued objects such as color pixels, as Barnett [3] observed.

Rivest [4] proposed an extension of morphology to complex signals. This extension has some arbitrariness because this is inevitable with multi-valued functions. However, the choices that were made in the selection of the order relationship were dictated by the requirements of RF signals.

Indeed, RF signals are quite different from gray-tone and even color images. RF signals are mostly classified and understood in terms of their frequencies, amplitudes and phases. This comes from the Fourier analysis of such signals. This also means that RF signals are usually two-valued functions. RF signals are also preferably ordered in terms of their amplitudes. The actual phase of a signal sample is not used to compare it with another sample. Even when an RF signals is purely real, the sign of a sample is understood as a phase rotation of π and is not of great importance.

In contrast, gray-tone images rarely feature negative values. If an image has some negative pixels, these are considered smaller than a zero-valued pixel. A negative RF signal sample is clearly larger than a zero-valued sample in the eyes of an RF practitioner because zero means the absence of a signal.

The average component of an image is an important feature. RF circuit designers, on the other hand, strive to remove that component. It is called by electrical engineers the Direct Current (DC) component and is considered parasitic.

The most natural way to combine images is by using set operators. RF signals are best understood with linear operators and frequency-domain representations. Although images are routinely represented in the frequency domain for lossy data compression purposes such as JPEG, there is no known frequency-based model for set operators. Set operators are analyzed in the spatial or, if applied to RF signals, in the time domain. There is usually a hierarchy of shapes in images; objects often include other objects, noise is embedded into textures and textures are bounded by edges. This feature is often used when designing image processing algorithms. The idea of hierarchy of shapes is rarely used in RF signal processing, except when a signal and its harmonics are considered or when one analyses noise embedded in a signal. There is, however, a clear hierarchy in any RF signal. For instance, communication signals are modulated at a fixed rate called the "Baud Rate." Radar signals are also modulated in amplitude, frequency and phase. Trying to understand the hierarchy in terms of Fourier analysis becomes rapidly futile because of the complexity of the process.

Morphological techniques seem more appropriate for images than linear techniques. Linear techniques seem better than morphological techniques on RF signals. On might ask what can be gained by applying morphological or, for that matter, any non-linear technique to RF signals. These tools open up more possibilities for signal processing, especially when the basic assumptions that are currently used start to break down. The main assumption is that RF signals are combined linearly. It is indeed the case when they are propagated in a medium. Yet, there are non-linearities at the transmission and at the reception of such signals. For instance, receivers are frequently saturated and transmitters overdriven. More importantly, non-linearities are inevitable when on wishes to perform pattern recognition on RF signals. A pattern recognition system destroys information; it acquires megabytes of information and outputs an identification. Sometimes, it reduces all that input data to a single bit: pass/fail, present/not present. Linear systems do not destroy information because they are, at least in theory, reversible. Morphological operators perform information destruction in a controlled way.

The main motivation of this paper is to present a morphological tool called "granulometry." This tool can potentially be used for pattern recognition in radar and RF signals in general. In Electronic Warfare (EW), there is a very strong interest in the interception and analysis of radar signals Each type of radar features its own RF pulse pattern. Radar identification is important because it allows better situational awareness; knowing who is using his radar and where he is, is intelligence data which is of fundamental importance.

Radar signals use all three types of modulations: amplitude, frequency and phase. Most of them transmit pulses of finite duration and variable pulse width and pulse repetition intervals. Some transmit continuously and are frequency or phase-modulated. Modern military radars and imaging radars embed some modulation inside their pulses. Some radars sweep their frequency while transmitting.

These sweeps are called "Chirps." The ground imaging radar cited as an example in this paper features such modulation. Some radars embed binary codes which make them less prone to jamming.

These signals are intercepted by Electronic Support (ES) systems using superheterodyne receivers. As mentioned earlier, there is a hierarchy in these radar signals; this hierarchy is too complex to be analyzed by mere Fourier transforms. There are pulses on a noise background; inside those pulse there is the modulating signal. At the lowest level of the hierarchy there is the RF carrier that pulsates the electromagnetic field a few billion times per second and generates the smallest features of the signal.

2. Granulometry and pattern spectrum

Granulometries are an axiomatization of the sieving process. One can obtain a size distribution of a powder by passing it through progressively finer sieves and by weighting the amount of powder that remains in each sieve. Matheron [5] formalized this process and discovered that a family of openings and closings of size λ was equivalent to a family of sieves. The Lebesgue measure is equivalent to the action of measuring masses. A granulometry of function f with a family of structuring elements B scaled by λ is then defined

$$\mathsf{G}(f, \lambda B) = \mathcal{M}(\gamma_{\lambda B}(f)), \tag{1}$$

where $\mathcal{M}(\gamma_{\lambda B}(f))$ is the Lebesgue measure of the opening of f with structuring element B scaled by a real, positive factor λ. The Lebesgue measure for binary images is the surface of objects. For gray-tone images, it is the volume and for functions, it is

$$\mathcal{M}(f(t)) = \int_{-T/2}^{T/2} f(t)\,\mathrm{d}t. \tag{2}$$

$T \to \infty$ being the length of the integration interval.

Maragos [6] defined the pattern spectrum as

$$\mathsf{PS}(f, \lambda B) = -\frac{\mathrm{d}}{\mathrm{d}\lambda}\,\mathcal{M}(\gamma_{\lambda B}(f)). \tag{3}$$

The pattern spectrum measures what was removed by the openings, that is, the objects that were trapped in the sieves. In contrast, the granulometry measures what passed through the sieves. The pattern spectrum actually measures the difference between successive openings. Maragos also defined the pattern spectrum for negative sizes using closings instead of openings

$$\mathsf{PS}(f, \lambda B) = \frac{\mathrm{d}}{\mathrm{d}\lambda}\,\mathcal{M}(\phi_{-\lambda B}(f)), \quad \lambda < 0. \tag{4}$$

This is the equivalent of performing the pattern spectrum on the complement of an image, because the closing is the dual of the opening. There are granulometries based on closings instead of openings as well. These are also the same as computing a granulometry on the complement of an image.

3. Power granulometry and power pattern spectrum

3.1. Complex morphological operators

RF signals are primarily compared by measuring their relative powers. The usual order relationship, $f(t) \leqslant g(t)$, where $f(t)$ and $g(t)$ are signals, does not reflect this practice. An order relationship has been developed in [4]. It compares two signals samples based first on their relative amplitude or power. If there is an ambiguity because the two signals have the same amplitude the relationship relies on lexicographic ordering [7]. This order relationship $X \preccurlyeq Y$, has been defined on complex signals X and Y; $\Re(X)$ is the real part of X and $\Im(X)$ is the imaginary part and $|X|$ is the modulus of X:

$$X \preccurlyeq Y \text{ If: } \begin{cases} |X| < |Y| \\ \text{or} \\ |X| = |Y| \text{ and } \Re(X) < \Re(Y) \\ \text{or} \\ |X| = |Y| \text{ and } \Re(X) = \Re(Y) \\ \quad \text{and } \Im(X) \leqslant \Im(Y). \end{cases} \tag{5}$$

When the signals are real, this becomes

$$X \preccurlyeq Y \text{ If: } \begin{cases} |X| < |Y| \\ \text{or} \\ |X| = |Y| \text{ and } X \leqslant Y. \end{cases} \tag{6}$$

New max $\vee$ and min $\wedge$ operators are created when we apply the new order relationship

$$X \vee Y = \begin{cases} X & \text{if } Y \preccurlyeq X, \\ Y & \text{otherwise.} \end{cases} \tag{7}$$

$$X \wedge Y = \begin{cases} Y & \text{if } Y \preccurlyeq X, \\ X & \text{otherwise.} \end{cases} \tag{8}$$

These two operators are the basic building blocks for all the morphological operators. From them, one builds dilations, erosions, openings and closings.

A new complementation procedure that reverses this order relationship was also introduced in [4]. For a complex signal point $X = Ae^{j\theta}$, the complement of $X \in \mathbb{C}$, $\overline{X}$, is

$$\overline{X} = (M - A)\mathrm{e}^{\mathrm{j}(\theta+\pi)}, \tag{9}$$

where $M \in \mathbb{R}^+$ is the maximum amplitude the signal can take. The amplitude can be limited by a few factors such as the limited dynamic range of the detector or of the signal representation.

Fig. 1 shows the application of a classical opening on a radar pulse used for terrain mapping. The pulse is chirped; signal period varies from 50 samples at the beginning of the pulse to 85 samples at the end of the pulse. The morphological opening is an anti-extensive operator; its output is always smaller than or equal to its input. The amplitude of the signal varies across the pulse, because the period becomes longer; the signal becomes less affected by the structuring element. However, this signal becomes increasingly negative as the structuring element increases in size. Unfortunately, the measured signal power tends to increase with the size of the structuring element. This situation is not satisfactory because RF signals are considered to be smaller when their power is smaller. Anti-extensive operators that actually decrease the output power are needed.

Fig. 2 shows a complex opening applied on the same signal. This opening decreases the power of the signal as the size of the structuring element increases. It is also more symmetrical with respect to the time axis than the classical opening is.

The complex dilation is dual to the complex erosion and so is the complex closing to the complex opening, when the complement operator defined in Eq. (9) is used.

3.2. Power granulometry

Classical granulometries use the Lebesgue measure to assess the amount of signal elements that survived the openings. RF signals are usually symmetrical with respect to the time axis, unless there is an undesirable DC component added to them. Complex morphological operators tend to preserve such symmetry. Consequently, the Lebesgue measure tends to zero for such signals. As λ increases, any deviation of the Lebesgue measure

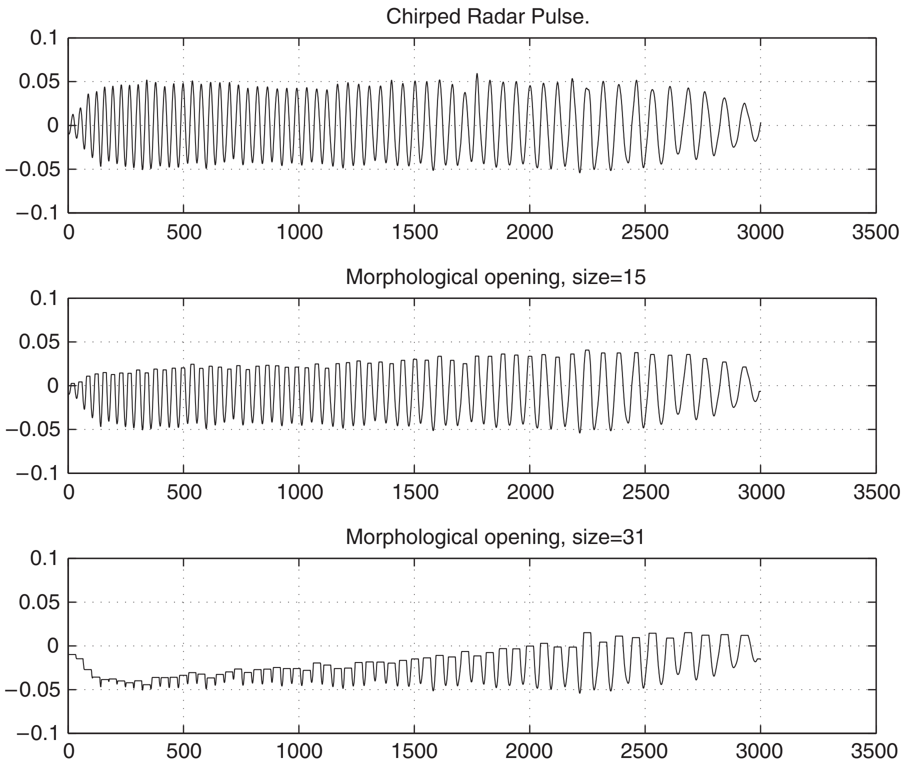

Fig. 1. Morphological opening on a chirped radar pulse.

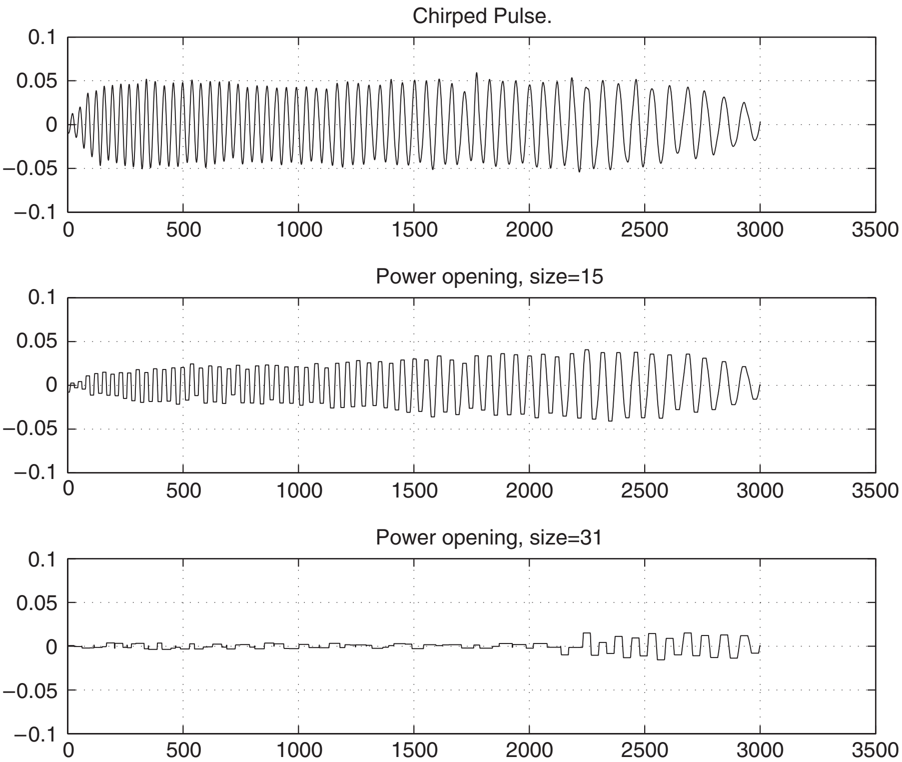

Fig. 2. Complex opening on a chirped radar pulse.

from zero is likely caused by artifacts such as border and sampling effects. This situation is corrected by replacing the Lebesgue measure with the signal power. The power granulometry is then defined

$$\mathsf{G}^P(f,\lambda B) = \frac{1}{T}\int_{-T/2}^{T/2} \gamma^{\mathrm{C}}_{\lambda B}(f(t))\gamma^{\mathrm{C}}_{\lambda B}(f(t))^*\,\mathrm{d}t = \frac{1}{T}\int_{-T/2}^{T/2} |\gamma^{\mathrm{C}}_{\lambda B}(f(t))|^2\,\mathrm{d}t, \tag{10}$$

where $\gamma^{\mathrm{C}}_{\lambda B}(f(t))$ is the complex opening with structuring element B scaled by a factor λ. $\gamma^{\mathrm{C}}_{\lambda B}(f(t))^*$ is the complex conjugate of $\gamma^{\mathrm{C}}_{\lambda B}(f(t))$.

Performing the Lebesgue measure on the difference between successive openings is the same as performing the derivative of the Lebesgue measure over successive openings

$$\lim_{\Delta\to 0}\frac{1}{\Delta}[\mathscr{M}(\gamma_{\lambda B}(f(t)) - \gamma_{(\lambda+\Delta)B}(f(t)))] = -\frac{\mathrm{d}}{\mathrm{d}\lambda}\mathscr{M}(\gamma_{(\lambda)B}(f(t))). \tag{11}$$

It is no longer the case for non-linear measurements such as power. Therefore, it is no longer possible to use the current definition of pattern spectrum in Eq. (3). It is preferable to compute the power of the residues between the successive openings. This yields the following definition for the power pattern spectrum:

$$\mathsf{PS}^P(f,\lambda B) = \lim_{\Delta\to 0}\frac{1}{\Delta T}\int_{-T/2}^{T/2} |\gamma^{\mathrm{C}}_{\lambda B}(f(t)) - \gamma^{\mathrm{C}}_{(\lambda+\Delta)B}(f(t))|^2\,\mathrm{d}t, \quad \lambda \geqslant 0. \tag{12}$$

Following Maragos, the power pattern spectrum for negative sizes is

$$\mathsf{PS}^P(f,\lambda B) = \lim_{\Delta\to 0}\frac{1}{\Delta T}\int_{-T/2}^{T/2} |\phi^{\mathrm{C}}_{(-\lambda+\Delta)B}(f(t)) - \phi^{\mathrm{C}}_{-\lambda B}(f(t))|^2\,\mathrm{d}t, \quad \lambda \leqslant 0. \tag{13}$$

$\phi^{\mathrm{C}}()$ is the complex closing.

4. Examples

In order to illustrate these concepts, the following signals have been used:

1. A simulated fixed frequency radar pulse. The period of the sinusoid was 50 samples.

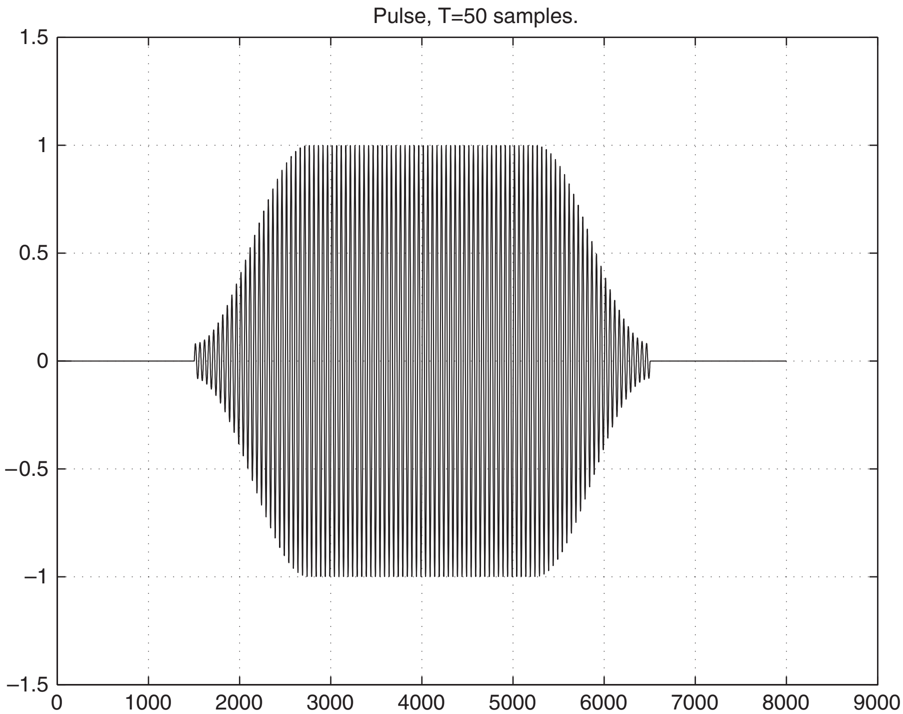

Fig. 3. Simulated radar pulse. Period = 50 samples.

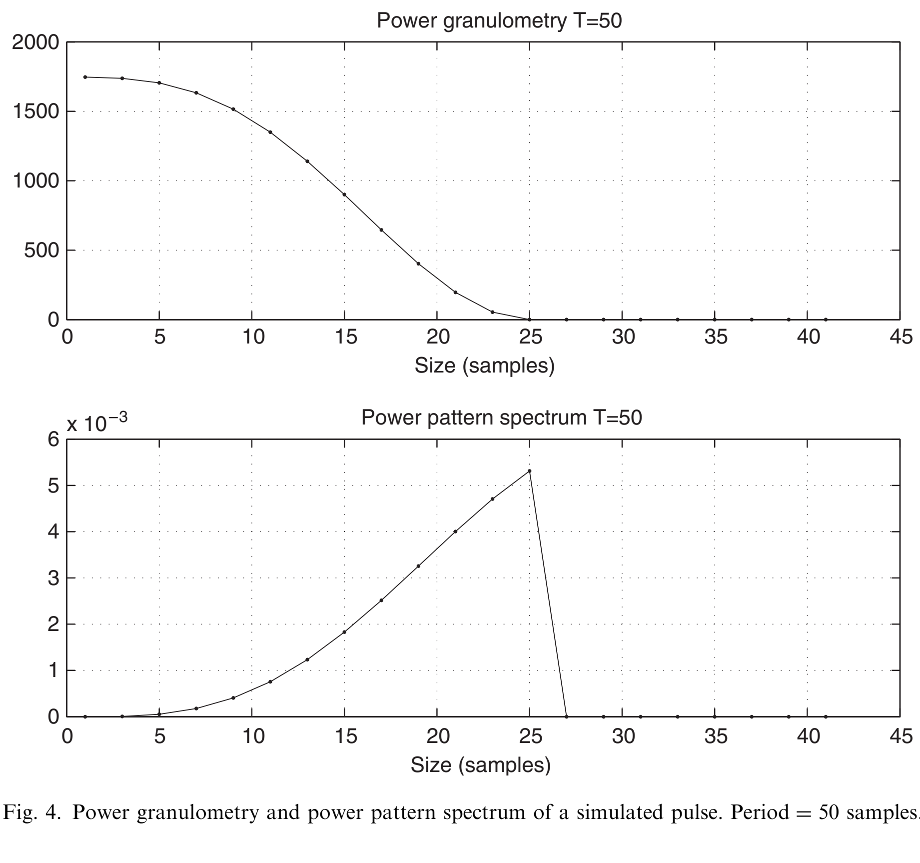

Fig. 4. Power granulometry and power pattern spectrum of a simulated pulse. Period = 50 samples.

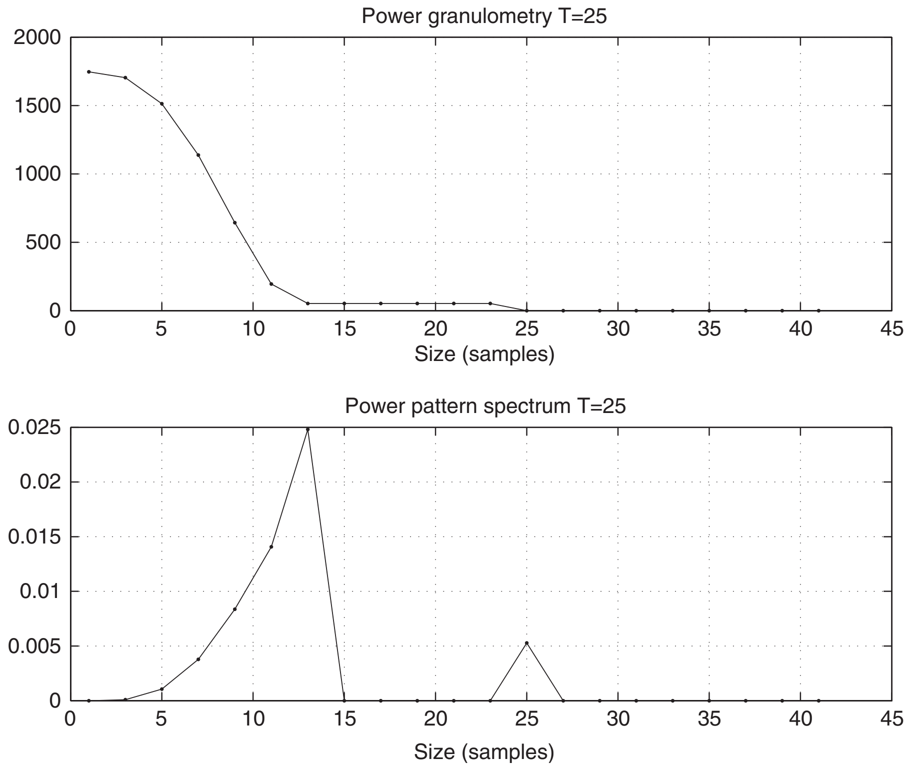

Fig. 5. Power granulometry and power pattern spectrum of a simulated pulse. Period = 25 samples.

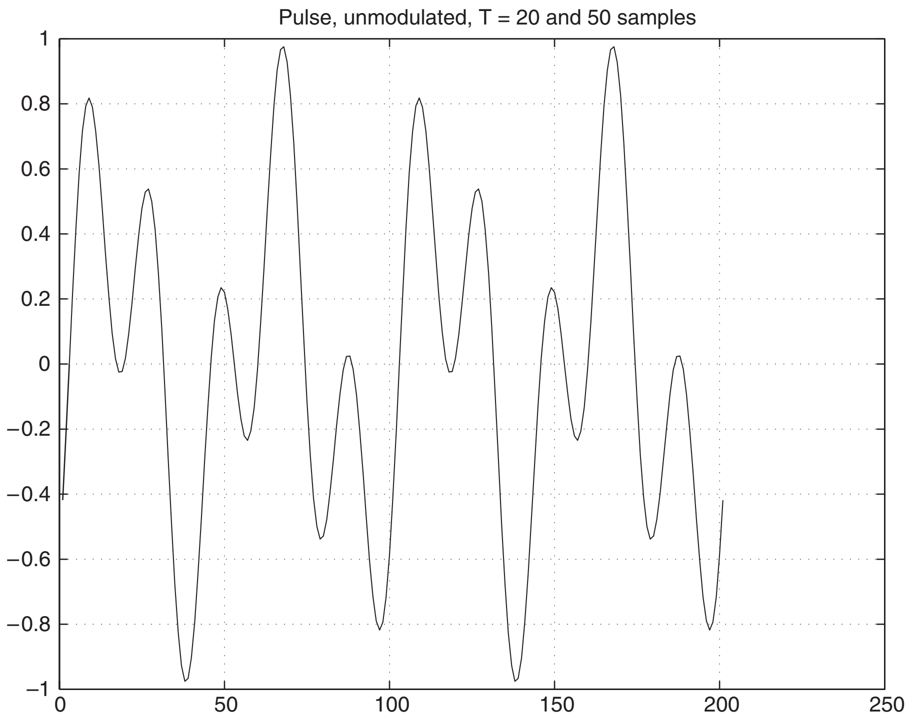

Fig. 6. Pulse fragment, superposition of two tones (period: 20 and 50 samples).

2. Another simulated fixed frequency radar pulse. The period of the sinusoid was 25 samples.
3. A simulated signal composed of two tones. The period of the first tone was 50 samples and it was 25 for the second tone.
4. A real chirped radar signal, with the period of the carrier starting at 50 samples and ending at 85 samples.

These signals were analyzed with a power granulometry and a power pattern spectrum with a flat structuring element of diameter $\lambda = \{3, 5, 7, \ldots, 41\}$.

Fig. 3 shows the simulated pulse with a period of 50 samples. The signal instantaneous power is periodic with a 25-sample period. Fig. 4 shows the resulting power granulometry and power pattern spectrum. The main feature of the power pattern spectrum is its peak at 25 samples. This corresponds well with the period of the power fluctuations present in the signal. The rate of growth of this peak increases with λ. This reflects the shape of the sinusoid; the structuring element progressively brings the signal amplitude down to zero, where the power fluctuations are the largest.

Fig. 5 shows the power granulometry and pattern spectrum of the simulated pulse featuring a carrier period of 25 samples. The granulometry and the pattern spectrum scaled by a factor 2 along the λ axis. The peak in the pattern spectrum has moved to a period of 13 samples. The remaining peak, at $\lambda = 25$ samples, was caused by sampling artifacts.

Fig. 7 shows the measurements of a two-tone signal. It should be noted that even though this signal is the result of the linear combination of two signals, this is not visible in the pattern spectrum. This is to be expected, because the pattern spectrum shows the shapes in the time domain instead of the frequency domain. The successive peaks can be observed in Fig. 6, which details a section of the pulse. These peaks and valleys do indeed exhibit the various sizes shown on the pattern spectrum.

Fig. 8 shows the measurements of a chirped radar signal. The difference with the other signals is striking. The chirped radar features wider peaks because the size of the instantaneous power varia-

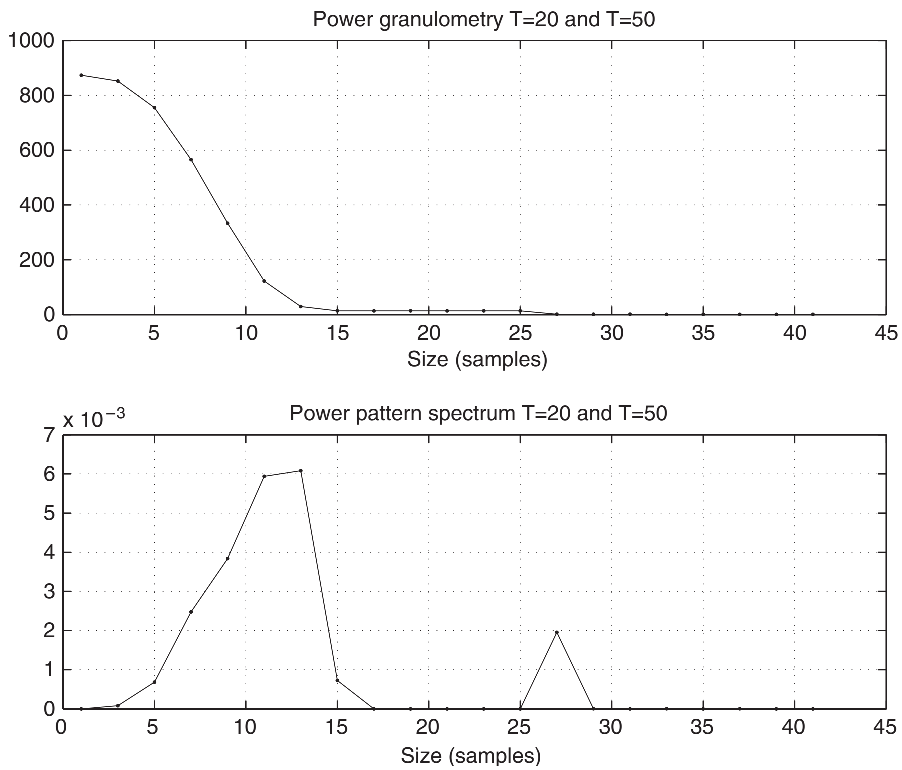

Fig. 7. Power granulometry and power pattern spectrum of a simulated pulse. Superposition of two tones (period: 20 and 50 samples).

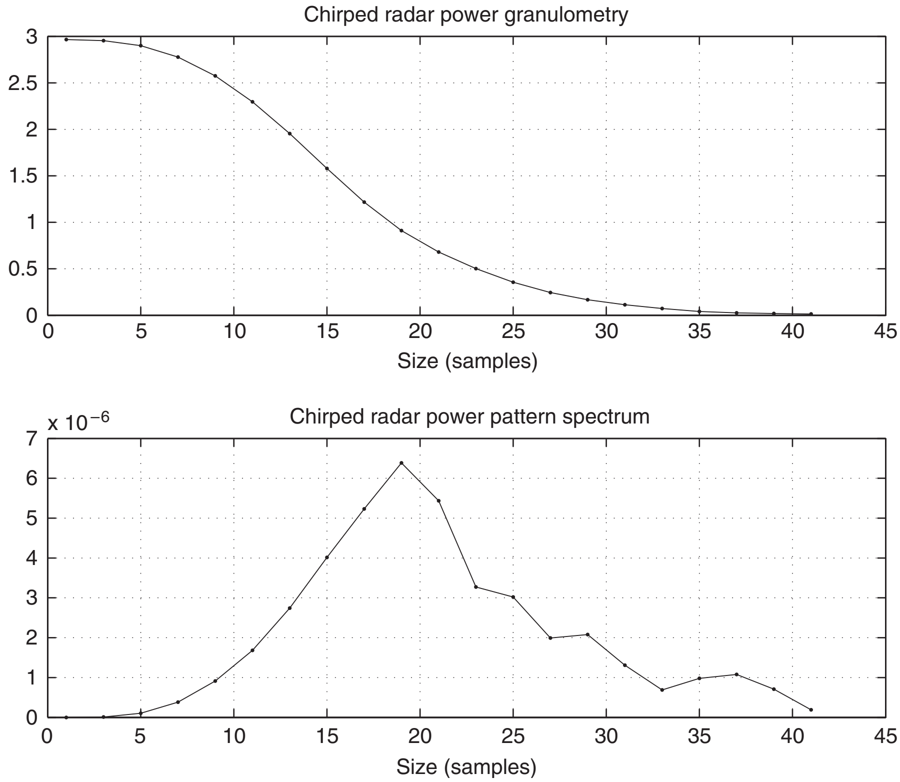

Fig. 8. Power granulometry and power pattern spectrum of a chirped radar pulse.

tions vary more widely than in the case of the simulated signals.

5. Conclusion

This paper presents a new type of granulometry and pattern spectrum. These are more appropriate to the context of RF signal processing than the classical granulometry and pattern spectrum for the following reasons: first, RF signals are often represented using complex functions and second, these signals are compared using their amplitude and power instead of the value of the signals themselves.

The granulometry was modified in two ways. The Lebesgue integral was substituted with the power measurement. The morphological opening was also replaced with a complex opening. It turned out that this complex opening was more appropriate for purely real RF signals than the classical opening.

The definition of pattern spectrum was also modified. Instead of computing the first derivative of the granulometric curve relative to the structuring element size λ, the power measurement on the difference between adjacent openings was computed.

Granulometries and pattern spectra perform time–domain measurements of objects embedded in signals. The transformations used to obtain these measurements are non-linear; therefore, the concept of linear superposition is not valid in this context. Two sinusoids cannot be separated in this fashion; the superposition of two pattern spectra does not directly correspond to the pattern spectrum of the two signals that featured these spectra. Granulometries and pattern spectra have similar goals to spectral analysis; yet they are complementary to this methodology.

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