

Vector approach for analyzing survey questions

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Abstract

The vector approach is an alternative method for analyzing surveys. The survey questions are designed as bi-polar constructs or dimensions. Participants rate the dimension on a scale: in this case, “usefulness”, “ease of use”, and “look and feel” scales from 1 to 7 for six tools used during the Multi-national Experiment 3 (MNE 3). The vector method treats each answer as an element in a p -dimensional vector, where p corresponds to the number of participants. The response vectors are then compared to a reference vector. The percentages of “usefulness”, “ease of use”, and “look and feel” are calculated using the projection of the response vectors onto the reference vector.

A clear advantage of the vector approach over statistical methods is that one does not need to invoke the assumptions typically associated with distributions. Thus the vector method can be applied to a greater variety of data sets, and the results are non-ambiguous compared to the statistical techniques. That is, the vector method does not have (nor does it need) an equivalent concept of the spread of data or uncertainty.

Results specific to MNE 3, which examined Effects Based Planning (EBP), were that the Info Workspace tool and the EBP tool suite were given the highest and lowest rating, respectively from the participants. More work is required to optimize the EBP tool suite.

Résumé

La méthode vectorielle constitue un autre moyen d'analyser les sondages. Les questionnaires sont conçus pour qu'à chaque question on donne une réponse correspondant à un point sur un axe bipolaire, ou construit. Au cours de l'Expérimentation multinationale 3 (MNE 3), on a demandé aux participants d'attribuer à six outils une note de 1 à 7 pour l'« utilité », la « facilité d'utilisation », et l'« aspect et la convivialité ». La méthode vectorielle considère que chaque réponse est un élément d'un vecteur de p dimensions, ou p est le nombre de participants. On compare ensuite les vecteurs réponses à un vecteur de référence. On détermine le pourcentage d'« utilité », de « facilité d'utilisation », et d'« aspect et convivialité » à partir de la projection des vecteurs réponses, sur le vecteur de référence.

Un avantage évident de la méthode vectorielle, par rapport aux méthodes statistiques, est qu'elle ne repose pas sur les hypothèses découlant habituellement des distributions. Ainsi, on peut utiliser la méthode vectorielle sur un ensemble plus divers de séries de données et, par rapport aux techniques statistiques, les résultants ne sont pas ambigus. En d'autres termes, la méthode vectorielle n'a pas (et n'a pas besoin) de notions équivalentes à l'étendue des données ou l'incertitude.

L'analyse par la méthode vectorielle des résultats de l'expérience MNE 3 indique que les participants ont respectivement attribué la meilleure et la pire note à l'outil Info Workspace et la suite Effects Based Planning (EBP). Il sera donc nécessaire de travailler davantage à l'optimisation de la suite d'outils EBP.

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Executive summary

Consistent, clean data are difficult to obtain during large scale, complex experimentation for two reasons. First, participants may not answer the same number of questions if the survey is designed with nested questions. Either the entire question or the participant's responses must be discarded if there are missing data when statistical methods are used to analyse the data. Second, there is no guarantee that survey data will fit a normal distribution, and so there is uncertainty in whether to use mean and standard deviation, or median and skewness as appropriate measures. A new analytical technique is required that is impervious to these assumptions of traditional statistical analyses.

A vector approach for analyzing survey questions is proposed in this paper. The concept is to treat each datum as an element in a vector rather than a number in a distribution. Therefore, Euclidean product and vector difference concepts replace central tendency and data spread concepts. The key here is that the vector approach can contend with missing data and data that come from any type of distribution, and so it is likely appropriate for large scale, complex experimentation.

The Multi-National Experiment (MNE 3) was such a large scale, complex experiment. This paper reports on the results of the technology usability, which was one of two MNE 3 analyses that incorporated the vector approach. The usability survey was designed to accommodate the vector approach. That is, participants marked their response between the two opposite constructs of a bi-polar scale, for example, between "extremely useless (1)" and "extremely useful (7)". The scale's numbers are arbitrary, but consistent for all questions. Therefore each question represents a finite dimension, and therefore all the questions form a multi-dimensional space where the analyst can use vector mathematics.

The conclusion for the vector approach is that it is an alternative to traditional statistical methods. The vector approach provides precise (crisp) results because it has no concept of data distribution. Issues that still require investigation are directly comparing vectors that come from different dimensional spaces, and criteria for determining vector similarity.

The conclusion for the MNE 3 results using the vector approach is that the Info Workspace tool and the Effects Based Planning (EBP) tool suite were given the highest and lowest rating of usability, respectively. Therefore, more work is required to optimize the EBP tool suite.

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Sommaire

Il est difficile d'obtenir des données cohérentes et claires, d'expériences complexes à grande échelle, et ce, pour deux raisons. Premièrement, si le questionnaire contient des questions imbriquées (c.-à-d. posées en fonction de la réponse donnée à une question précédente), les participants peuvent ne pas avoir tous répondu au même nombre de questions. Si des données sont manquantes, le recours aux méthodes statistiques pour l'analyse oblige à éliminer soit la question soit les réponses données par les participants. Deuxièmement, puisqu'il n'est pas garanti que les données en réponse au questionnaire suivront une distribution normale, il n'est pas sûr que la moyenne et l'écart-type, ou la médiane et l'asymétrie constituent des mesures adéquates. Nous avons besoin d'une nouvelle technique analytique, indépendante de ces hypothèses de l'analyse statistique classique.

Nous proposons dans cet article une méthode vectorielle pour l'analyse des réponses à un sondage. Elle repose sur l'idée de traiter chaque donnée comme un élément d'un vecteur plutôt qu'un nombre dans une distribution. Ainsi, les notions de produit euclidien et de différence vectorielle se substituent aux notions de tendance centrale et d'étendue des données. Le point essentiel est le suivant : la méthode vectorielle tolère les données manquantes et est appropriée à tous les types de distribution, ainsi elle convient probablement mieux aux expériences complexes à grande échelle.

L'Expérimentation multinationale 3 (MNE 3) était l'un de ces expériences complexes à grande échelle. Dans cet article, nous présentons les résultats sur l'« aptitude de la technologie à être utilisée », qui est l'une des deux analyses de données MNE 3 à avoir été effectuées avec la méthode vectorielle. On a conçu le questionnaire sur l'« aptitude à être utilisé » en fonction de la méthode vectorielle. Ainsi, les participants ont indiqué leur réponse entre deux extrémités d'une échelle bipolaire; par exemple entre « extrêmement inutile » (1) et « extrêmement utile » (7). Les nombres qui composent l'échelle sont arbitraires, mais cohérents d'une question à l'autre. Donc, puisque l'univers des réponses à chaque question présente un nombre fini de dimensions, l'ensemble des questions forme un espace multidimensionnel que l'on peut analyser à l'aide de méthodes vectorielles.

Pour conclure, la méthode statistique constitue une solution de remplacement aux méthodes statistiques classiques. L'absence de la notion de distribution de données permet d'obtenir des résultats précis (clairs). Certaines questions doivent encore faire l'objet de recherches, notamment la comparaison directe de vecteurs appartenant à des espaces de différentes dimensions et les critères pour déterminer la similitude des vecteurs.

L'analyse par la méthode vectorielle des résultats de l'expérience MNE 3 indique que les répondants ont jugé que parmi les outils, Info Workspace était le plus utilisable et Effects Based Planning (EBP) le moins utilisable. Donc, il conviendra de travailler à l'optimisation de la suite d'outils EBP.

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This Technical Memorandum was first published in the proceedings of 2004 Command and Control Research and Technology Symposium: The Power of Information Age Concepts and Technologies. June 2004. Also, these results are archived in the CFEC Analysis Report for Multi-National Experiment 3. DRDC Toronto TM-2005-149, Toronto, Canada: Defence R&D Canada. When first reported, the vector approach itself was experimental and was still in its infancy. For a more rigorous examination of the approach please refer to Calculating Effectiveness using Bi-Polar Scales and Vector Algebra. DRDC Toronto TR-2005-148, Toronto, Canada: Defence R&D Canada.

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Introduction

Statistical results of survey questions are often reported without considering the nature of the data set. It is assumed that the mean and standard deviation come from a normally distributed population of the participants' answers. The statistical results for one question can be compared to the results for another question. If the meanings of the questions are shown not to be significantly different, then in certain circumstances the data sets can be combined, but only if the same participants answered each question being compared (i.e., same sample sizes). However, normal distribution and same sample sizes are two assumptions of statistical methods that are not typically realized in large, complex experimentation.

A new vector approach is considered here for data reduction and interpretation. The vector method treats the responses to each question as a multi-dimensional vector. This response vector is compared to a reference vector. The vector approach does not require any assumptions about distributions or sample sizes, but it expresses the data in terms of a magnitude and direction relative to an ideal data set. This method is similar to the Repertory Grid's principle component analysis technique. This technique is used to elicit and analyse an individual's personal constructs (or perceptions along bi-polar scales) of an event or scenario (Kelly, 1955).

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Vector method

Assume there are p participants and q questions. Each question is bi-polar: that is, the question's response falls between two opposite values (e.g., yes and no, extremely useful and extremely useless). For the current study, each participant responds to the question by selecting an integer value between 1 and 7 inclusively. These numbers are subsequently normalized between -1 and 1 .

A participant's response to a question is an element in a response vector (\mathbf{u}). Thus \mathbf{u} has p elements that lie on p orthogonal axes. \mathbf{u} is a vector sum of answers to a single question, as illustrated in Figure 1a for player 1, player 2, and player 3, and represents a cumulative response to the question. The magnitude and direction fully define \mathbf{u} .

The response vector is compared to a chosen reference vector (\mathbf{v}). In this case, \mathbf{v} is specified as being equivalent to all participants answering 7 (or 100% useful) for a question. After normalizing the scale from $1 \dots 7$ to $-1 \dots 1$, this vector will have the largest possible magnitude, \sqrt{p} , for p participants, and all vectors (including the reference vector itself) are normalized again with respect to this magnitude. Thus \mathbf{v} has a magnitude of one.

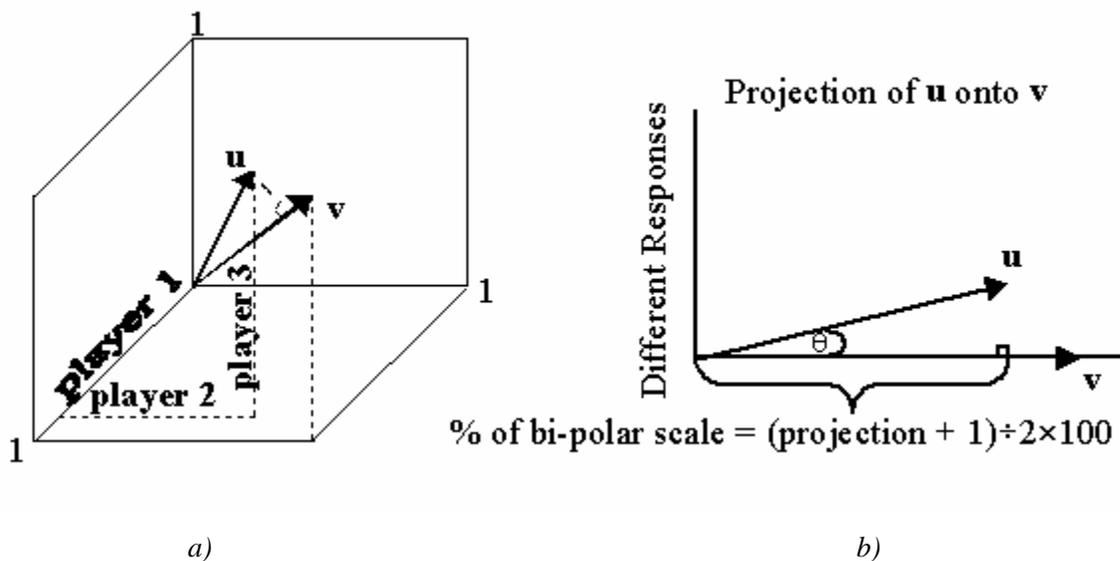


Figure 1: Illustrating the Vector Method

Furthermore, the angle between \mathbf{u} and \mathbf{v} is calculated using the Euclidean inner product or the vector dot product (Anton, 1981). Figure 1b shows the angle between \mathbf{u} and \mathbf{v} . The projection of \mathbf{u} onto \mathbf{v} is related to a percentage of the bi-polar scale. Recall that the scale spans from -1 to 1 . Therefore, if the projection value is 0.5 , for example, then the tool is 75% useful (i.e., $[0.5+1] \div 2 \times 100$). It can be shown that the projection is also related to the participants' responses mean value – assuming that a normal distribution is an appropriate model that describes their responses (Farrell, in review). For the MNE 3 results, the response

distributions were skewed and truncated at both ends, and therefore the median and skew index (Runyon & Haber, 1984) were reported.

The y component of \mathbf{u} in Figure 1b indicates that the participants had different answers. If players 1, 2, and 3 all had the same answer of 0.5, for example, \mathbf{u} would lie on the same axis as \mathbf{v} . Conversely, if the answers were 0.5, 0.5, and 0.4, then the \mathbf{u} would not be coincident with \mathbf{v} . One can show that the participants' responses standard deviation is related to the y component, assuming again that the responses fit a normal distribution (Farrell, in review).

In summary, the vector method treats each datum as a single value along a single axis in a multiple dimensional space, computes the response vector, and compares it to a reference vector of similar dimension. The advantage of the vector approach is that it does not invoke any assumptions typically associated with statistical methods, and so the method is applicable to a wider variety of data sets.

Effects Based Planning

Effects Based Operations (EBO) is a transformational concept where command decisions are made based on the desired effects that the nation or coalition would want to see happen within an adverse state or organization. During the planning phase, all four instruments of national power (Diplomatic, Information, Military, Economics) are under consideration in achieving the desired effects. A key activity of EBO is Effects Based Planning (EBP), and key enablers of EBP are collaborative technologies and computer-based planning aids.

Effects Based Planning is a new concept being explored by various nations. EBP aims to generate a single plan that considers desired effects and potential unintended effects during the planning stage. Desired effects are generated from strategic objectives and the Operational Net Assessment (ONA). The ONA is a database that contains political, military, economic, social, information, and infrastructure (PMESII) descriptions of the adverse system. A person, place, or thing in the database is referred to as a node. The ONA contains nodes and links, where the links describe the relationships between nodes. Potential effects are also associated with the nodes and links. Needless to say, the ONA is complex.

In light of the strategic directives and Commander's Intent, the HQ staff assesses the desired effects and the actions required to modify the nodes and links in order to produce the desired effects. The actions may produce unintended or second order effects, and so the staff must reassess the effects and actions until they converge to a reasonable solution. Resources are assigned to the actions, and further iterations may occur if the resources are not available. Once the desired effects, nodes, actions, and resources (ENAR) are identified, the staff prioritizes and sequences the effects and actions, and then produces an Effects Tasking Order (ETO).

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Experiment

Multinational Experiment 3 (MNE 3) examines how an ad hoc coalition would conduct an Effects Based Planning. US Joint Forces Command led the experiment and invited partner nations Australia, Canada, France, Germany, and United Kingdom to participate in the experiment from their own country. They formed the Coalition Task Force Headquarters (CTFHQ), which is based on a structure with Boards, Centers, and Cells (BCC). NATO conducted the same experiment at the same time over the same secure computer network, except that the NATO planning staff was co-located in a single facility. They formed the NATO Response Force (NRF) Headquarters (HQ).

One of the objectives for the Multinational Experiment 3 (MNE 3) was to identify technology requirements to support coalition/NRF Effects Based Planning. The proposition is that technology will augment the human ability to conduct EBP through a suite of tools. Thus, the critical operational issue (COI) for the technology objective is, "What functional requirements are necessary to conduct EBP within a coalition/NRF environment?" Various surveys were designed to answer this question during MNE 3. The following is a sample of the questions asked to the experiment participants.

Was the tool used? YES/NO

If YES

- Rate the usefulness of the tool (1...7)
- The tool was easy to use (1...7)
- Rate the look and feel of the tool (organization/layout, colors, fonts, etc) (1...7)

This nested survey question was asked for six tools:

1. Common Information Environment (CIE) Portal
2. Document Manager
3. Info Work Space (IWS), which is a distributed collaboration tool
4. Operational Net Assessment (ONA) database
5. Effects Based Planning tools
6. WebCOP, which is a Common Operating Picture

The Joint Battle Center's data collection and analysis tool, called JDCAT, was developed in the United States and used to design and distribute these nested questions to the players. However, not everyone used all the tools. It cannot be pre-determined how many nested questions the participants would answer. Thus, the sample size (or p) for each tool will be different.

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Results

The mean and standard deviation were calculated for the 3 questions listed above and for the 6 tools indicated (18 questions in total). The vector methodology was also used to analyze the same data set to highlight the differences with the two methods.

Figure 2 is a histogram for the usefulness of the IWS tool question (raw data). This histogram is typical of most questions. Table 1 contains the normalized statistical data for all 18 questions. For example, the mean, standard deviation, median, and skew index for the usefulness of IWS question is 0.63, 0.35, 0.67, and -1.4 respectively.

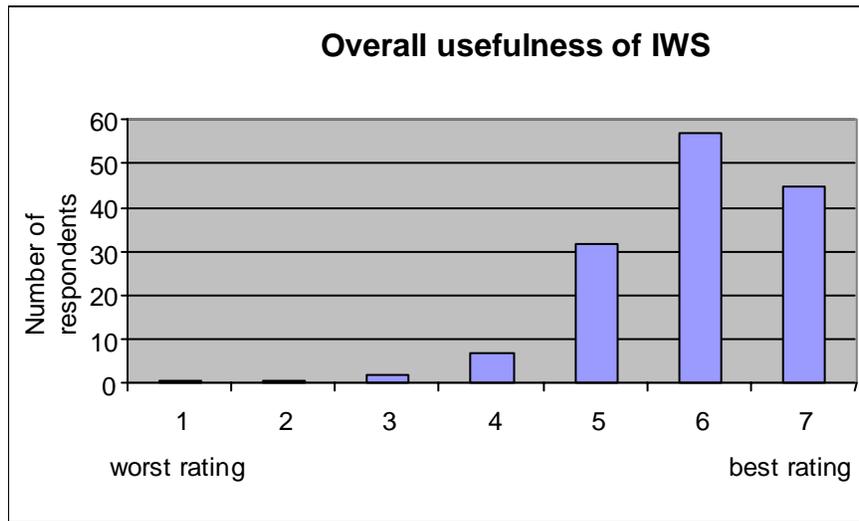


Figure 2: Histogram results of "Usefulness" rating for IWS Tool

Table 1: Statistical Values for Usefulness, Ease of Use, and Look and Feel of Tools

Tool	Sample size	Usefulness				Ease of Use				Look and Feel			
		mean	s.d.	med.	skew	mean	s.d.	med.	skew	mean	s.d.	med.	skew
CIE Portal	145	.28	.43	.33	-.35	.22	.47	.33	-.67	.10	.43	.33	-.40
Document Manager	55	.38	.38	.33	.10	.33	.42	.33	-.20	.32	.40	.33	.07
IWS	145	.63	.35	.67	-1.4	.58	.30	.67	-.50	.55	.31	.67	-.50
ONA Database	114	.30	.45	.33	-.15	.19	.47	.33	-.25	.23	.42	.33	-.30
EBP tools	62	-.09	.52	0	.18	-.44	.52	-.67	.84	-.22	.49	-.33	.17
WebCOP	28	.06	.59	0	-.12	.07	.59	0	-.18	.13	.52	0	-.16

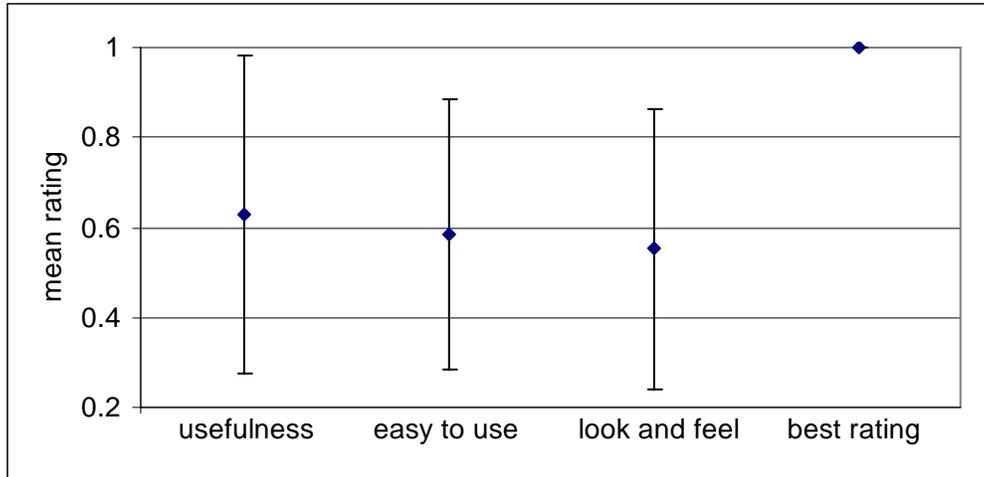


Figure 3: Mean and Standard Deviation for IWS

Figure 3 illustrates that the questions themselves are not significantly different (i.e., standard deviations overlap). Participants seemed to interpret the usefulness, ease of use, and look and feel questions in the same manner. Also for the usefulness of the IWS tool question (mean = .63 and s.d. = .35), the normal cumulative distribution up to and including one is .85. That is, if this were a normal distribution, 15% of the participants would give an answer greater than 100% useful, which would not make sense. This is the primary difficulty for reporting mean and standard deviation with data sets that do not come from normal distributions.

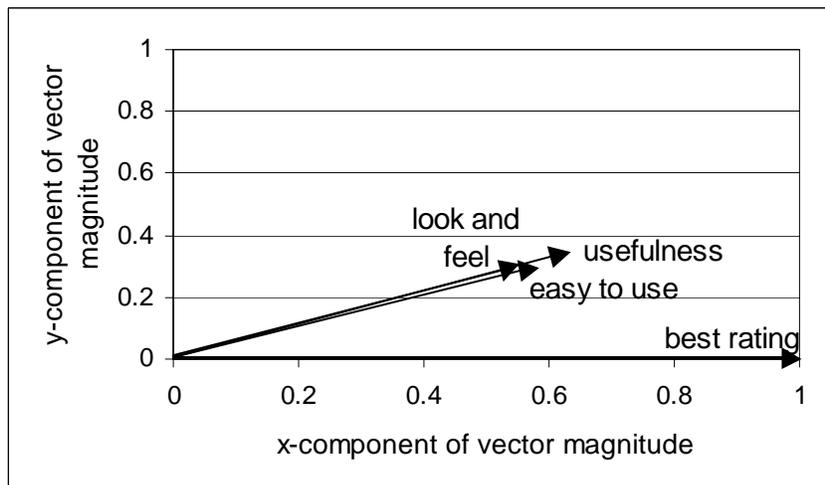


Figure 4: Vector Methodology Applied to the Responses to the IWS Questions

The vector approach was applied to all 18 questions. Figure 4 shows an example of the three response vectors for the IWS questions: that is, the magnitude of the three multi-dimensional vectors, and their angles relative to a reference vector (shown along the x-axis). Therefore, the axes are dimension At a glance, one might say that the three questions are similar since all three vectors have similar angles with respect to the reference vector (29.2° for “usefulness”, 27.1° for “ease of use”, and 29.3° for “look and feel”). However, in a multi-dimensional

space, it is possible have similar angles relative to a reference vector, but with the vectors actually pointing in different directions.

In order to resolve this issue, the relative angles are calculated between the vectors: 21.2° between “ease of use” and “look and feel”, 29.3° between “usefulness” and “ease of use”, and 27.8° between “look and feel” and “usefulness”. Figure 5 is a 3-dimensional conceptualization of what this might look like. Thus, vectors could be near the reference vector while at the same time far from each other. The degree of closeness requires cluster analyses of both the relative angle and relative distance between vector pairs. For further discussion, see (Farrell, in review).

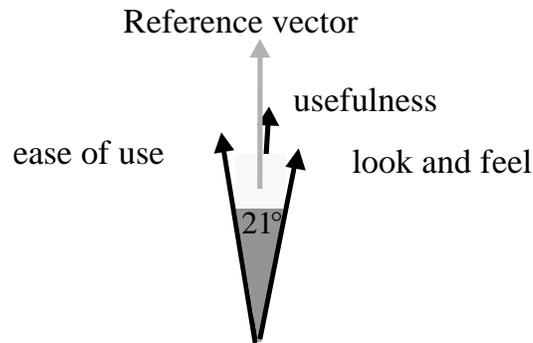


Figure 5: Conceptualization of Three Vectors having Similar Angles Relative to the Reference Vector, but Pointing in Different Directions

Also, the projection of \mathbf{u} onto \mathbf{v} is not ambiguous but it is a single number. This number will always fall within the limits of the bi-polar scale, unlike normal distributions where their tails theoretically extend to infinity. The projection is used to calculate the percentage of the bi-polar scale. Table 2 lists these percentages for all six tools and the three questions. The highest rating from the participants was IWS and the lowest was the EBP tools.

Table 2: Percentage of Usefulness, Ease of Use, and Look and Feel

Tool	Usefulness (%)	Ease of Use (%)	Look and Feel (%)
CIE Portal	63.8	60.9	55.2
Document Manager	69.1	66.4	66.1
IWS	81.5	79.2	77.6
ONA Database	64.9	59.6	61.7
EBP tools	45.2	28.0	39.2
WebCOP	53.0	53.6	56.5

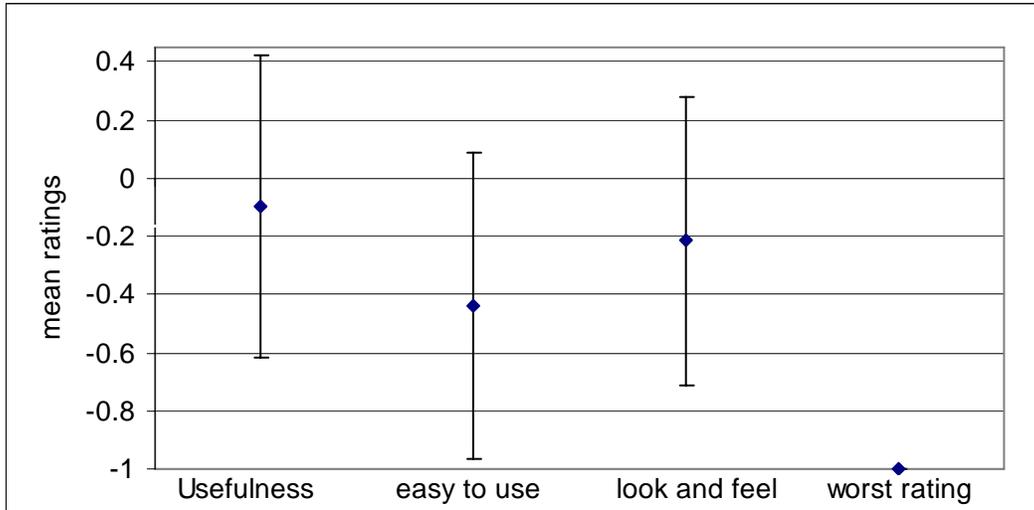


Figure 6: Mean and Standard Deviation for the EBP Tools

Figures 6 and 7 show the results for the EBP tools. The statistical results may be interpreted from 70% useful to extremely difficult to use because of the spread of the data. In contrast the vector approach shows 45% useful, 28% ease of use, and 39% look and feel. It is clear that participants rated this tool as being poor.

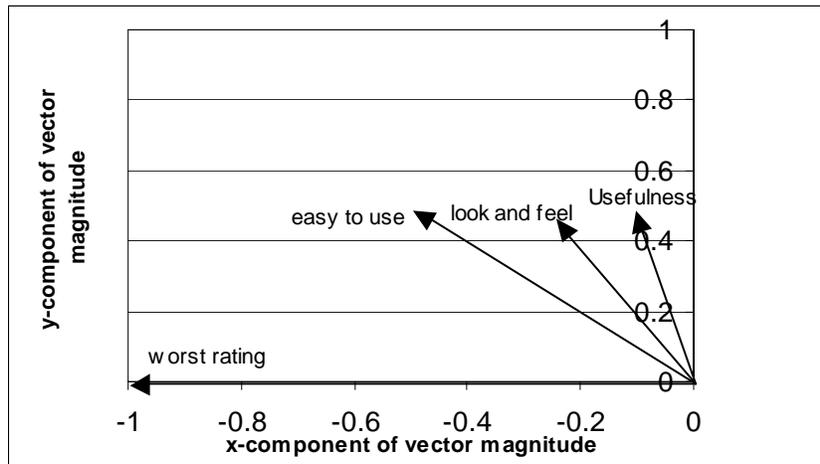


Figure 7: Vector Methodology Applied to the EBP Tools Data

Consider the three tools: CIE Portal, Document Manager and IWS. Fifty-three participants rated these three tools across the three dimensions for a total of 477 ratings. If we assume that the questions are not statistically different then the data can be combined to produce a single statistic, since all 53 participants used all three tools and answered all three questions. Figure 8 shows the statistical result.



Figure 8: Statistical Results for 3 Tools and 3 Questions, and the Same Participants

The vector method allows us to consider all participants' ratings. Some participants did not answer all the questions, however these data can still be used in the analysis. This is possible because each individual answer corresponds to a value on a single axis that forms a multi-dimensional vector. In this case there are 963 dimensions (or ratings). The percent usefulness is 65.2% (in contrast to 71.2% usefulness calculated from the mean of Figure 8).

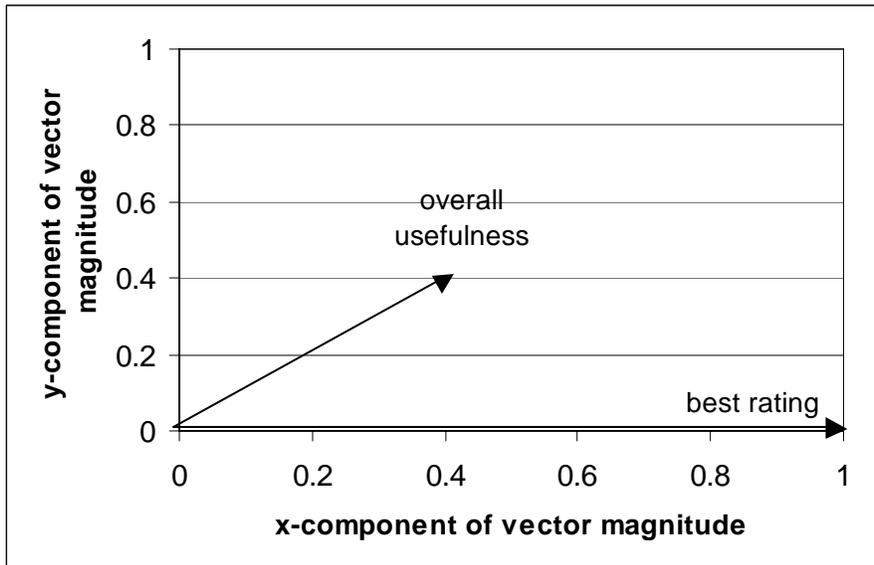


Figure 9: Vector Results for 3 Tools and 3 Questions, and All Participants

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Conclusions

A vector approach was used to reduce and interpret data from MNE3, a large and complex experiment. This method was compared and contrasted with typical statistical methods. A advantage of the vector approach is that it does not invoke assumptions typically associated with those statistical methods. Thus the vector method can be applied to a wider variety of data sets. As well, the results produced are non-ambiguous compared to the traditional techniques.

The statistical method indicates that the participants seemed to have the same interpretation of the three questions on usefulness. However, the vector method indicates that these three questions are not coincident, and further analysis is required to determine whether the questions are similar.

Results specific to MNE 3 were that the Info Workspace tool and the EBP tool suite were given the highest and lowest rating, respectively from the participants. More work is required to optimize the EBP tool.

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List of symbols/abbreviations/acronyms/initialisms

DRDC	Defence R&D Canada
EBP	Effects Based Planning
ENAR	Effects, Nodes, Actions, Resources
ETO	Effects Tasking Order
HQ	Headquarters
MNE	Multinational Experiment
ONA	Operational Net Assessment
p	Number of players
PMESII	Political, Military, Economic, Social, Information, Infrastructure
q	Number of questions
u	response vector
v	reference vector
WebCOP	Web Common Operating Picture

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Glossary

Technical term	Explanation of term
Effectiveness	The degree to which a system reaches its aim, objectives, and/or goals.
Measure of Performance	Measurement of the system's full capabilities under controlled conditions
Efficiency	A measure of energy loss
Vector Method	Application of Bi-Polar scales and Vector Algebra to calculate effectiveness.
Bi-Polar Scales	Describe key dimensions of the question(s) under investigation. The scales have minimum and maximum values that represent the exact opposite construct (i.e., cold and not cold).
Nominal Scale	Categorical scale where "Relative distance" between the two points on this scale does not make sense (e.g., on and off).
Ordinal Scale	"Relative distance" may make sense, but it is subjective (e.g., not useful – useful).
Interval Scale	A number scale where only addition and subtraction operations make sense (e.g., temperature scale in degrees Celcius).
Ratio Scale	A number scale where there exists a natural zero and multiplication and division operations make sense (e.g., temperature scale in degrees Kelvin).
Euclidean Vector Product	Commonly known as the Dot Product, it calculates the magnitude of the projection of one vector onto another.