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A MATHEMATICAL APPROACH TO COST MINIMIZATION OF SATCOM SYSTEMS

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ABSTRACT

Traditionally, costing analysis of satellite systems tended to be an iterative process of designing to meet the requirements, adding up the costs. If the total is too large, then the design is modified or requirements changed, the entire process is repeated. Unfortunately, the cost analysis has tended to look at component costs, such as payload and terminal costs, in isolation with little concern about how the costs of one affect the costs of the other. In this paper, a mathematical technique is described that formalizes the process of cost minimization by trading off interrelated parameters. In the first step, the total life-cycle cost function is described as the sum of 7 interrelated major terms. In turn, each of these consist of many terms. The second step is the determination of cost functions terms. Then, equations that tie certain of the component items together are determined. Finally, the cost is minimized by varying the component parameters to give minimum total cost.

To illustrate the approach, three examples are given: 1) terminal power and antenna gain, and payload power and antenna gain are tied together with the link equations, 2) terminal reliability and maintainability are related by the availability relation, and 3) spacecraft sparing and reliability are balanced to achieve the specified constellation reliability. The examples show that cost minimums exist and can be determined with potential for considerable savings. The method is straight forward to extend to the entire life-cycle costing.

1. INTRODUCTION

The traditional costing approach to satcom systems tends to be *ad hoc* or "cut and try" wherein a first system design is made, and the costs added up. Then an attempt is made to reduce the costs by trying various measures, often by dropping of capability. Little is done to do a formal minimization. At best, a tradeoff through use of spread sheets is performed. This approach has the disadvantages of being laborious, not necessarily finding a minimum, and often not even indicating if a clear minimum exists. Furthermore, the costing of space and ground segments are often performed separately thereby losing any saving that could be obtained in a trade off between the two.

In this paper, a mathematical method for the cost minimization of satcom systems is described. In the first step of the method, a trial system is designed that meets the requirements. Then, the total system life-cycle cost function is written as a sum of all cost functions. Each of the cost functions can, in turn, be expanded in terms of their constituent components. Some of these components are described, and where available, existing cost models are used or referenced. The terms are often interdependent, and a component that results in a saving in one term can drive up the costs in one or more other terms. Once these equations are described, then standard well known mathematical analysis can be applied to determine minimum cost. If the minimum total cost is found to be too large, then the trial system may need a major design change, or even a decrease of the capability. The minimization is applied to the revised design, and so on.

The method is illustrated by three examples: 1) The power and antenna gain of the satellite are tied to those of the terminals by the link equations. 2) The terminal-reliability cost is traded against terminal-maintainability cost. 3) The cost of payload reliability is traded against the cost of redundant payloads. Extending the approaches in the examples to cost minimization of complete systems is straight forward although perhaps laborious. The techniques described could be a very useful tool for minimizing costs in future Satcom systems.

2. COST FUNCTION COMPONENTS

2.1 Total Cost Function

The first step in building a mathematical cost model is to write the total cost function of the form

$$\$_{tot} = \$_{trm} + \$_{pld} + \$_{bus} + \$_{launch} + \$_{o\&m} + \$_{control} + \$_{disposal} \quad (1)$$

where the terms are the cost of all terminals, all payloads, all buses, launches, operation and maintenance of entire system, control, and disposal, respectively. Each of these terms can be expanded into constituent components. A few examples will be given later. Some of the terms are interdependent and a component or factor that results in a saving in one term can drive up the costs in one or more other terms. Such interrelationships will be illustrated by a number of examples.

Once these components are described mathematically, they can be used to minimize total cost, $\$_{tot}$. A number of such examples are given later.

2.2 Quantity Multiplier, $L(N)$

The unit production cost of an item will decrease as the quantity produced increases. There have been attempts to quantify the effect mathematically with the so-called learning curve. From [Ref. 1, p. 681] the cost of N units is

$$\$_{Nunits} = L(N) \bullet \$_{1unit} = N^B \bullet \$_{1st} \tag{2}$$

where

$$L(N) = N^B \tag{3}$$

is the cumulative cost factor, $\$_{1st}$ is the cost of the first unit,

$$B = 1 + \ln(S/100)/\ln(2), \tag{4}$$

and S is the learning-curve slope in percent. Wong [Ref. 1, p. 681] recommends S=95% (B=0.93) for 1 to 9 units, 90% (B=0.85) for 10 to 50 units, and 85% for more than 50 units. The average unit cost multiplier, $L(N)/N$, would have the values given in Fig. 1. With this model, there are discontinuities between sections of different slope, but is still a useful guide. The first section is more applicable to

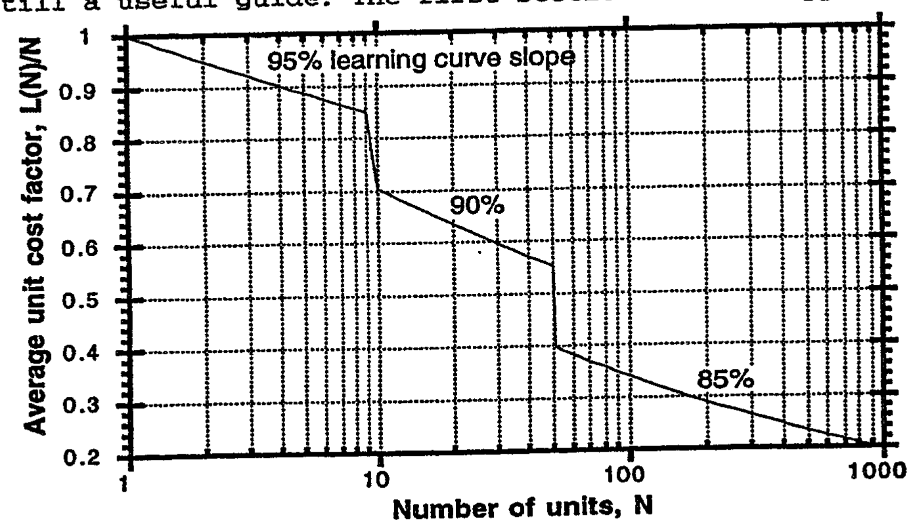

Fig. 1. Normalized unit cost multiplier, $L(N)/N$, as a function of the number of units produced based on a three-segment learning curve.

items such as spacecraft that are produced in low numbers and there is relatively little gained from the learning curve. The middle section is representative of cost reduction obtained for items such as specialized terminals that are produced in limited quantities. In the last section, we see the most cost reduction for large numbers such as would occur for mass produced terminals. For example, the average per-unit cost for 500 units would be about 0.23 of the cost of building only one unit.

2.3 Terminal Construction Cost Function, $\$_{trm}$

The total cost of the terminals can be written as

$$\$_{trm} = \sum_{i=1}^{\#\,types} \left\{ \$_{devtrm_i} + L(N_{t_i}) \$_{1st\,trm_i} \right\} \quad (5)$$

where

$\$_{devtrm_i}$ = cost of developing terminal of type i
N_{t_i} = total number of terminals of type i
$\$_{1st\,trm_i}$ = cost of building first terminal of type i
$L(N_{t_i})$ = cost multiplier for building N_{t_i} terminals of type i.

The term $\$_{devtrm_i}$ includes non-recurring engineering costs. The term $\$_{1st\,trm_i}$ is a function of many parameters. Rather than trying to incorporate all such factors, only the costs of the terminal power and the antenna size are given explicitly with everything else being lumped together. In this way, the approach is highlighted rather than being all inclusive. Similarly, to simplify the description, only a single terminal type will be considered here so that the subscript "i" will be omitted. Thus, $\$_{1st\,trm_i}$ is expanded as

$$\$_{1st\,trm} = \left(¢_{tp} + X_{tp} P_t \right) + \left(¢_{ta} + X_{ta} G_t \right) + \$_{t\,other} \quad (6)$$

where

P_t = transmit terminal power, Watt
G_t = antenna gain
$\$_{t\ other}$ =cost of all items excluding antennas and RF power
$¢_{tp}$ = a constant cost for terminal power, dollar
X_{tp} = cost multiplier for terminal power, dollar/Watt
$¢_{ta}$ = a constant for antenna, dollar
X_{ta} = cost multiplier for antenna, dollar

The unit of X_{tua} is dollar/(unit gain) but gain is dimensionless so that the units become just dollar. The form (6) assumes a linear relationship between cost and power, and between cost and antenna gain. A linear relationship may not always occur and the form of (6) would have to be modified accordingly. The power and gain cost functions are discussed further below. Associated cost for items such as the cost of achieving the desired reliability and maintainability are assumed to be imbedded in these values.

2.4 Payload Construction Cost Function, $\$_{pld}$

The cost of the payload can be expanded in a similar form to that of the terminal as

$$\$pld = \$_{devpld} + L(N_S)\$_{1st\,pld} \tag{7}$$

where

$\$_{dev\ pld}$ = cost of developing payload
N_S = number of satellites to be built
$\$_{1pld}$ = cost of building first payload

In order to incorporate the effects of payload power and antenna size, $\$_{1pld}$ is expanded as

$$\begin{aligned}\$_{1st\,pld} = &\sum_{k=1}^{N_{hpa}} \left\{ L\left(N_{hpa_k}\right) \bullet \left(¢_{pld\,hpa_k} + X_{pld\,hpa_k} P_{pld_k} \right) \right\} \\ &+ \sum_{j=1}^{N_{ua}} \left(¢_{pld\,ua_j} + X_{pld\,ua_j} G_{pld_{uj}} \right) \\ &+ \sum_{i=1}^{N_{da}} \left(¢_{pld\,da_i} + X_{pld\,da_i} G_{pld_{di}} \right) + \$_{pld\,other}\end{aligned} \tag{8}$$

where

N_{hpa} = number of HPA types
N_{hpa_k} = number of HPAs of type k
N_{da} = number of downlink antennas
N_{ua} = number of uplink antennas
P_{pld_k} = total transmit power of type k HPA
$G_{pld\ uj}$ = gain of jth uplink antenna
$G_{pld\ dj}$ = gain of jth downlink antenna
$\$_{pld\ other}$ = payload costs excluding antennas and HPAs
$¢_{pld\ hpa_k}$ = a constant for the kth HPA type, dollar
$X_{pld\ hpa_k}$ = cost multiplier for kth HPA, dollar/Watt
$¢_{pld\ uaj}$ = a constant for jth uplink antenna, dollar
$X_{pld\ uaj}$ = cost multiplier for jth uplink antenna, dollar
$¢_{pld\ dai}$ = a constant for ith downlink antenna, dollar
$X_{pld\ dai}$ = cost multiplier for ith downlink antenna, dollar.

The number of users, N_{mplx_k}, multiplexed on one type k HPA can be accounted for with

$$P_{pld_k} = N_{mplx_k} \bullet P_{one\,pld_k} \tag{9}$$

where $P_{one\ pld_k}$ = transmit power of one multiplexed user. If operating in broadcast mode, then multiplexing is not needed and the effective value of N_{mplx_k} is 1. The summations in (8) account for the fact that there may be multiple antennas and multiple HPA's. Account must also be taken of limitations on the payload total DC power available and the maximum mass.

2.5 Bus, Launch, O&M and Control Cost Functions

The remaining cost functions will be considered here briefly. The bus cost function would have a form like

$$\$_{bus} = \$_{dev\,bus} + L(N_S) \bullet \$_{1st\,bus} \quad (10)$$

where $\$_{dev\ bus}$ is the cost of developing the bus, and $\$_{1st\ bus}$ is the cost of building the first bus. Some of the *payload* parameters affecting the bus cost are mass, volume, form, and DC power required. These cost effects can be expressed as

$$\$_{1st\ bus} = \$_{bus\ other} + X_{b/p\ pwr} \bullet P_{DC\ pld} + X_{b/p\ mass} \bullet M_{pld} \quad (11)$$

where $\$_{bus\ other}$ is the cost of all other items on the bus, $X_{b/p\ pwr}$ = cost multiplier for bus to supply power to payload, dollar/Watt, $P_{DC\ pld}$ is the power to be supplied to the payload, $X_{b/p\ mass}$ is the bus cost multiplier to support the mass and volume of the payload, and M_{pld} is the mass of the payload. Once again, a linear relationship between cost and power, and mass is assumed. If not linear, then (11) would have to be modified accordingly. Other payload requirements such as station keeping will affect the bus costs through the function $\$_{bus\ other}$.

The launch cost, $\$_{launch}$, is a function of mass of the payload and bus. However, it is nonlinear and exhibits step function increases since, as mass increases beyond a certain point, it becomes necessary to use a different and probably more expensive form of launch. Similarly, form and volume affect $\$_{launch}$ nonlinearly. *Power* requirements of the payload affect launch costs through the mass required for solar panels and batteries.

The operation and maintenance cost function, $\$_{O\&M}$, can be, to a certain extent, traded against terminal costs, $\$_{trm}$. As example, terminal reliability can be balanced against maintenance costs.

The cost function for satellite control is usually only lightly coupled to the other cost functions. Therefore, the control cost function can be optimized largely independently of the other cost functions. However, some balancing of costs can be made between satellite autonomy and control costs. Usually, satellite autonomy is chosen for operational reasons and not for cost balancing reasons.

2.6 Discussion of Some Terminal and Payload Cost Functions

The various cost functions discussed above can be expanded in many ways. For illustration purposes, we will concentrate here on terminal power, and payload power and mass.

2.6.1 Terminal RF Power Cost Functions

The terminal construction costs related to power, $¢_{tp} + X_{tp}P_t$, were assumed in (6) to be linear functions of RF power. Better functions may be available. The two main items in this cost is the cost of the high power amplifier (HPA) and its associated costs, and the cost of the power supply. The cost of the power supply is driven by requirements such as low weight and volume (such as for a manpack), long lifetime etc.

The form of what $¢_{tp} + X_{tp}P_t$ might look like is illustrated in Fig. 2. One curve is for a solid state amplifier (SSA) and the other is for a TWTA. For very low transmit power, the SSA might be less costly than the TWTA but may become more costly than the TWTA after a certain power level. Furthermore, the current technology may limit the maximum power of the SSA, so that beyond this power, the TWTA is the only option. There can be other criteria that lead to the selection of the more expensive amplifier such as weight, mechanical robustness, efficiency, etc.

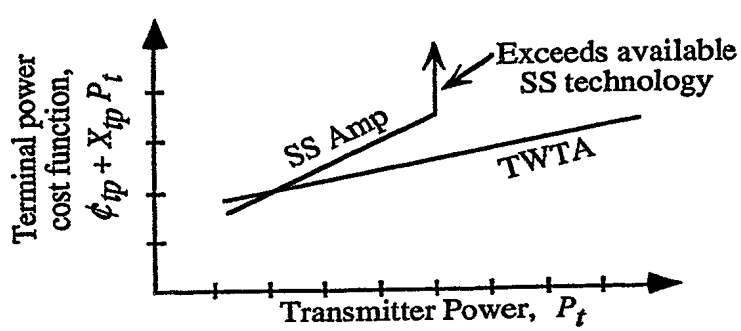

Fig. 2. Cost of providing RF power of P_t for the first terminal as a function of terminal transmit power, P_t.

2.6.2 Payload RF Power Cost Functions

The payload construction costs related to power, $¢_{pld\,hpa_k} + X_{pld\,hpa_k} P_{pld_k}$ from (8), were again assumed to be linear although other forms may be better. The form might look as illustrated in Fig. 3. It is shown as linear up to the point where the DC power required to operate the HPA's exceeds that available from the bus. If it necessary to exceed this power, then there can be a very large cost impact. For example, one could use a larger bus at a very large incremental cost. The slope of the cost can be different after switching to a different bus. Alternatively, one of the other packages or capabilities on the payload could be dropped which might even incur a net cost saving. Of course, this cost saving is at the expense of overall capability.

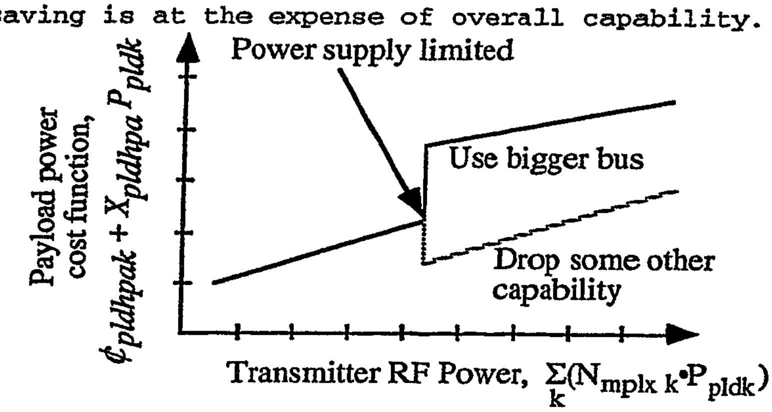

Fig. 3. Cost of providing RF power of P_{pld} for the first payload as a function of total transmit power.

2.6.3 Payload Antenna Cost Functions

Some of the standard cost drivers for onboard antennas are gain, frequency band, type of antenna, surface accuracy, antenna type, antenna construction technology used, method of pointing (steering), pointing accuracy, spatial protection against uplink jamming and interference, and compensation for platform attitude variation.

The cost of construction of payload antennas is given in the second and third terms of (8) and has the form

$$\$_{1st\,ant} = ¢_a + X_a G_a = ¢_a + X_a c_x D_a^2 \tag{12}$$

which is proportional to antenna gain and therefore to area. Here, c_x is a scale factor and D_a is the antenna diameter. In [Ref. 1, p. 667],

The bus cost function, $\$_{bus}$, is a function of mass of the payload M_{pld}, and power, $P_{DC\ pld}$, to be supplied to the payload. But M_{pld} is a function of G_{pld}, and $P_{DC\ pld}$ is a function of P_{pld}. Therefore, $\$_{bus}$ can be expressed functionally as

$$\$_{bus}(M_{pld}(G_{pld}),\ P_{DC\ pld}(P_{pld}))$$

A similar form can be obtained for the launch costs associated with payload transmit power and antenna gain. The multiple-level functionality for the launch cost function would take the form

$$\$_{launch}(M_{pld}(G_{pld}),\ S_{pld}(G_{pld}),\ M_{bus}(P_{pld}))$$

The "shape" function, S_{pld}, of the payload reflects the cost of fitting antennas within the launch vehicle and therefore depends upon the antenna size, and therefore upon the antenna gain. The function $M_{bus}(P_{pld})$ represents a cost function going between three of the major cost elements: the power, P_{pld}, transmitted by the payload requires power, $P_{DC\ pld}$, supplied by the bus; in turn, to generate this power, solar panels are needed which adds mass, which adds to the launch costs. Although this multilayered functionality looks complex, standard mathematical analysis can deal with such functions in a straight forward manner during the cost minimization process.

3. COST MINIMIZATION EXAMPLES

In this section, three examples of cost minimization will be given. Minimization for real systems would be necessarily much more complex but nonetheless can be done in a straight forward manner.

3.1 Example 1: Payload/terminal capability tradeoff

In this example, only the costs of the antennas and HPAs of the terminal and payload are considered. Furthermore, a single terminal type with a single data rate, and a satellite with a single HPA, receive antenna, and transmit antenna are used. A terminal antenna of diameter D_t is used for both the up and down link. It is assumed that the uplink and downlink frequencies have been previously selected so that wavelength is not varied to minimize the cost. The NRE and other fixed costs are omitted. The number of terminals and satellites is assumed to have been determined from requirements. Under these assumptions, the variable part (non-fixed) of the cost of the terminal plus payload antennas and HPAs is then, using the above equations, symbols and terminology,

$$\left(\$_{trm} + \$_{pld}\right)\Big|_{\text{var}} = L(N_t)\left(X_{tp}P_t + X_a c_x D_t^2\right) + L(N_s)\left(X_{pld\,hpa}N_{mplx} \bullet P_{one\,pld} + X_{pld\,ua}c_x D_u^2 + X_{pld\,ua}c_x D_d^2\right) = c_2 P_t + c_3 D_t^2 + c_4 P_{one\,pld} + D_u^2 c_5 + c_6 D_d^2 \quad (13)$$

where c_2, c_3, c_4, c_5 and c_6 are constants. The cost function, $(\$_{trm}+\$_{pld})\,|_{\text{var}}$, is to be minimized over the 5 variable parameters: P_t, P_{pld}, D_t, D_u, and D_d. The five parameters are tied together by the link equations. An SNR is specified for both the uplink and the downlink that must be met to provide a specified level of performance. These equations are

$$SNR_u = \frac{C_u T_{sym}}{N_{ou}} = \frac{P_t G_{tu} G_{pld\,u} C_u T_{sym} / L_u}{k T_{sys\,u}} \quad (14)$$

and

$$SNR_d = \frac{C_d T_{sym}}{N_{od}} = \frac{P_{one\,pld} G_{td} G_{pld\,d} C_d T_{sym} / L_d}{k T_{sys\,d}} \quad (15)$$

where C_u and C_d are the carrier powers received at the payload and terminal, T_{sym} = symbol period = 1/(symbol rate), L = space loss=$4\pi r/\lambda$, r=path length, λ= wavelength, T_{sys}=noise temperature, K, and k = the Boltzman constant. These equations are valid for both transponding and processing payloads. Gain is calculated from $G = \pi^2 D^2 \eta / \lambda^2$ where η is the efficiency. In one of a variety of approaches, the two link equations are expressed in terms of the 5 variables as

$$P_t = c_u / \left(D_t^2 D_u^2\right) \quad (16)$$

and

$$P_{one\,pld} = c_d / \left(D_t^2 D_d^2\right) \quad (17)$$

where c_u and c_d are constants based upon fixed parameters of the system. The efficiencies, η, of all antennas were set =1 for convenience without affecting the overall approach. These 2 equations are substituted into (13) to eliminate P_t and $P_{one\,pld}$, and the partial differentials of $\left(\$_{trm} + \$_{pld}\right)\Big|_{\text{var}}$ with respect to D_t, D_u, and D_d are found and set to zero. These 3 equations plus (16) and (17) are solved to give the parameters for minimum cost of the variable part as:

$$D_t|_{min} = \left(\left[\sqrt{c_2 c_u c_5} + \sqrt{c_4 c_d c_6}\,\right] / c_3\right)^{1/3} = c_7^{1/3}$$

$$D_{pld\,u}|_{min} = \left(c_2 c_u / \left[c_5 c_7^{2/3}\right]\right)^{1/4} \qquad D_{pld\,d}|_{min} = \left(c_4 c_d / \left[c_6 c_7^{2/3}\right]\right)^{1/4}$$

$$P_t|_{min} = \left(c_5 c_u / \left[c_2 c_7^{2/3}\right]\right)^{1/2} \qquad P_{one\,pld}|_{min} = \left(c_6 c_d / \left[c_4 c_7^{2/3}\right]\right)^{1/2} \qquad (18)$$

These values are substituted into (13) to obtain the minimum total variable part of the cost, $(\$_{trm}+\$_{pld})|_{var}$, for the antennas and HPAs.

As an example, a hypothetical satellite system and cost coefficients are used with the above minimization equations. We chose an SHF system comprised of N_s = 2 satellites, N_t = 100 terminals, and N_{mplx} = 15 simultaneous downlink user channels, with parameters:

$SNR_u|_{dB}$ = 15 dB
$SNR_d|_{dB}$ = 15 dB
$T_{sys\ u}$ = 2000 K
$T_{sys\ d}$ = 1000 K
R_{sym} = 1 Mb/s,
r = 40,000 km
f_u = 8 GHz
f_d = 7 GHz

For the cost coefficients we arbitrarily choose numbers for purposes of illustration and are not based upon any real costing model. The numbers chosen were:

X_{tp} = 15 k\$/Watt
$c_x X_{ta}$ = 20 k\$/meter^2
$X_{pld\ hpa}$ = 500 k\$/Watt
$c_x X_{pld\ ua}$ = 750 k\$/meter^2
$c_x X_{pld\ da}$ = 750 k\$/meter^2

Substitution of these values into the minimization equations and using $P_{pld}|_{min} = N_{mplx} \cdot P_{one\ pld}|_{min}$ gives:

$D_t\|_{min}$ = 1.9 meters $D_u\|_{min}$ = 0.85 meters $D_d\|_{min}$ = 1.53 meters	$P_t\|_{min}$ = 1.2 Watts $P_{pld}\|_{min}$ = 3.5 Watts

Substitution of these values back into (13) gives the minimum costs as

$\$_{trm}|_{var\ min}$ = 5.4 M\$
$\$_{pld}|_{var\ min}$ = 7.7 M\$
$(\$_{trm} + \$_{pld})|_{var\ min}$ = 13.12 M\$

These values become a starting point for the cost minimization process. The next step would be to determine if any of the values exceed technological, operational, etc. requirements. If so, then the numbers would have to be reworked to factor in such considerations. Then one would look for items already developed or off the shelf that have parameters approximately close in value to the ones found for minimum cost. If such items are found, then another costing exercise is done to determine if the savings accrued offset the extra expense caused by the other parameters no longer being at the minimum.

The real benefit to the minimization process is to study the cost tradeoffs among the 5 variables. To illustrate that these values indeed give the minimum, it would be necessary to use a six dimensional plot. A more realistic 2-dimensional plot, and a 3-dimensional plot will be used. In the 2-dimensional illustration, the parameter P_t was allowed to vary about the value, $P_t|_{min}$. Because of the link equations, a change in P_t must compensated by a change in one or more of the other 4 parameters. For the example, the payload uplink antenna diameter, D_u, was allowed to vary and the remaining three parameters were fixed at the minimum value and the results plotted in Fig. 5.

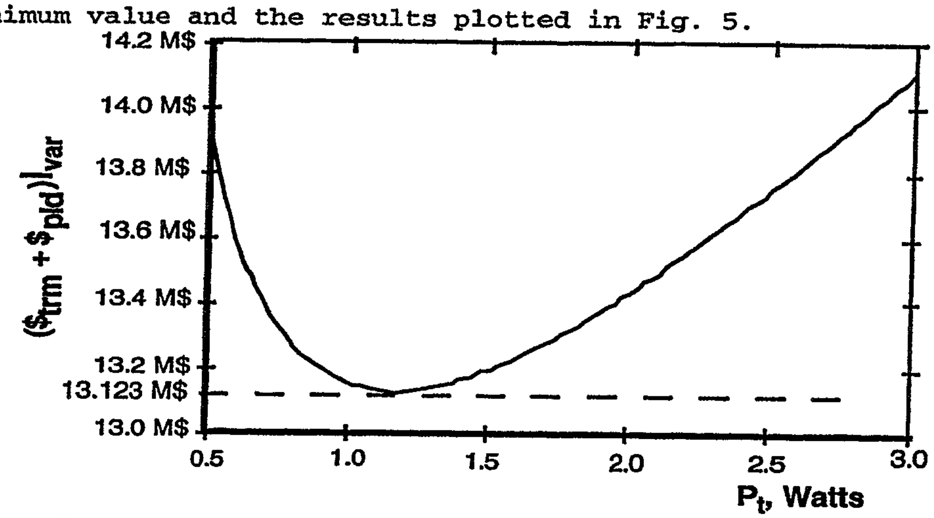

Fig. 5. A plot of the variable part of $\$_{trm}$ + $\$_{pld}$ as P_t is varied about its minimum value with D_u varying, and the other 3 parameters fixed.

The minimum does indeed occur at the level of 1.2 W. Keeping the power between 0.8 and 1.5 W would keep the cost close to the minimum. As P_t gets small, the cost rises rapidly since D_u must become very large to compensate.

In a 3-dimensional illustration the parameters P_t and P_{pld} were allowed to vary about the values, $P_t|_{min}$ and $P_{pld}|_{min}$, while D_u and D_d were allowed to vary and D_t was fixed at $D_t|_{min}$. A resulting plot is shown in Fig. 6. Because the surface is concave, it was difficult to chose a view that best displayed the minimum. It appears that the minimum occurs at the values calculated above. Again, the total cost rises rapidly as P_t and P_{pld} become small because D_u and D_d must be correspondingly large.

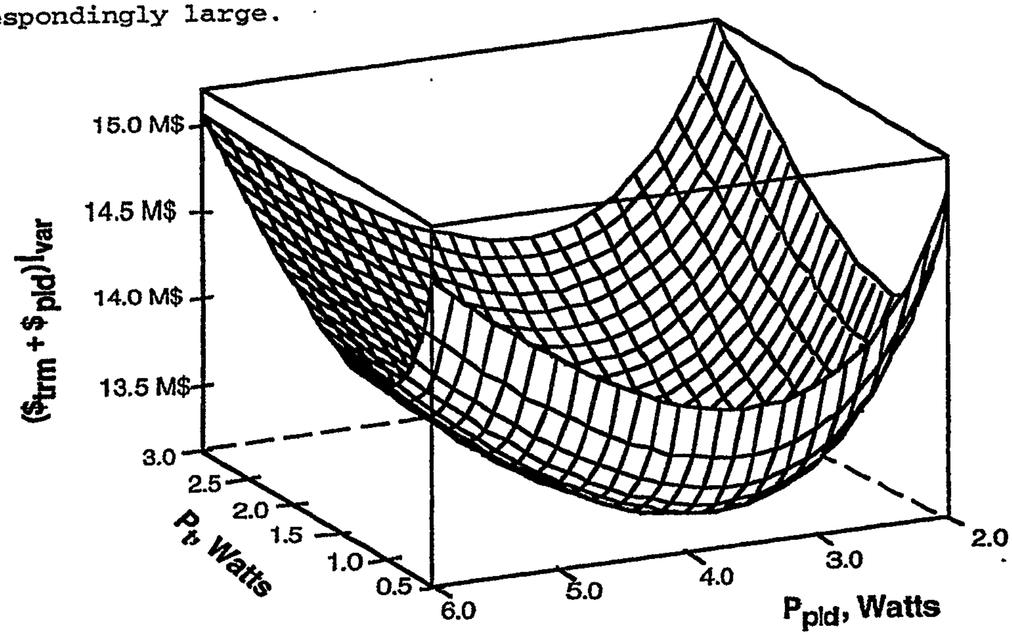

Fig. 6. A plot of the variable part of $\$_{trm} + \$_{pld}$ as P_t and P_{pld} are varied about their minimum values.

3.2 Example 2: Terminal Reliability/Maintainability tradeoff

Availability relates to the ability of an item to perform its required function at a stated instant in time [Ref. 2]. There are a variety of forms of availability. In this example, the availability of a terminal as governed by the reliability and maintainability is considered. This availability is the fraction of time that the equipment is serviceable. Therefore, it is related to the mean time between failures (MTBF) and the mean time to repair (MTTR) by [Ref. 2]

$$A_{R\&M} = MTBF/(MTBF + MTTR) \tag{19}$$

A simple cost model is that the cost of building a terminal is proportional to the MTBF and inversely proportional to the MTTR so that the R&M cost function would have the form

$$\$_{R\&M} = X_R MTBF + X_M (1/MTTR) \tag{20}$$

where $1/MTTR$ can be considered the measure of maintainability. Then (19) is solved for MTBF and MTTR which are substituted into (20), to obtain

$$\$_{R\&M} = \frac{X_R MTTR}{(1/A_{R\&M})-1} + \frac{X_M}{MTTR} = X_R MTBF + \frac{X_M}{[(1/A_{R\&M})-1]MTBF} \qquad (21)$$

By taking the derivative of \$R&M with respect to MTTR and MTBF, and equating to zero, we find

$$(1/MTTR)|_{min} = \sqrt{\frac{A_{R\&M}X_R}{(1-A_{R\&M})X_M}} \qquad (22)$$

and

$$MTBF|_{min} = \sqrt{\frac{A_{R\&M}X_M}{(1-A_{R\&M})X_R}} \qquad (23)$$

As an example, we have chosen arbitrarily the cost multipliers of X_R=1.0 \$/hour, and X_M=800 \$-hour. These values are used in (21) with two values of $A_{R\&M}$, 0.990 and 0.995, and the results plotted in Fig. 7.

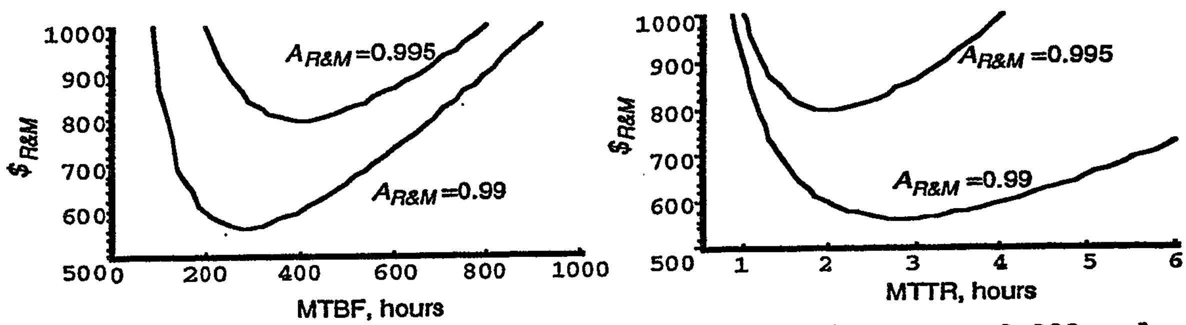

Fig. 7. Plots of $\$_{R\&M}$ vs MTBF MTTR for availabilities $A_{R\&M}$ = 0.990 and 0.995.

The values of MTTB and MTTR for minimum $\$_{R\&M}$, as determined from (22) and (23), are also given in Table 1 and can be seen to agree with the minima in Fig. 7. The graphs gives rough guidelines about where to put the emphasis between reliability and maintainability. One could similarly take the next step of including the cost of maintenance, both in spare parts and personnel.

Table 1. Values of MTBF and MTTR to achieve the lowest incremental cost of a single terminal, and for two values of $A_{R\&M}$.

$A_{R\&M}$	MTBF|min	MTTR|min	$\$_{R\&M}$|min
0.990	281 hours	2.8 hours	\$563/terminal
0.995	399 hours	2.0 hours	\$798/terminal

3.3 Example 3: Payload reliability/number tradeoff

3.3.1 Introduction

The subject of reliability and the related subject of maintainability (R&M) are well covered in the literature including a variety of standard texts [Ref. 4 and 5] and an entire Transaction of the IEEE. For payloads, maintainability will not be considered here. The main methods of improving reliability is through more reliable units and through redundancy. The "unit" can be from the smallest component up to complete constellation of satellites.

One way to look at the cost of spacecraft reliability is to use Hecht's [Ref. 3, p. 641] approach of using the "expected cost of failure" which is the cost of replacement multiplied by the probability of spacecraft failure. Unfortunately, the term "expected" is in the probabilistic sense and in fact will not likely correspond to any actual final cost regardless of the outcome. This term would cause considerable misunderstanding at funding agencies. An alternative approach is to determine the cost to provide the specified level of reliability. The following attempts to show how this can be done, and for the minimum cost.

3.3.2 Reliability concepts

The study of reliability involves use of probability theory. The reliability function, $R(t)$, is the probability that a unit is still functioning at time t. Other functions of use are:

$$
\begin{aligned}
MTBF &= \int_0^\infty R(t)dt \\
f(t) &= \text{failure probability density} = \mathrm{d}F(t)/\mathrm{d}t = -\mathrm{d}R(t)/\mathrm{d}t \\
F(t) &= \text{cummulative probability of failure at time t,} \\
&= \int_0^\infty f(t)dt = 1 - R(t) \\
h(t) &= \text{hazard rate} = f(t)/R(t)
\end{aligned}
\tag{24}
$$

There are many forms for the reliability function [Ref. 4] such as the commonly used exponential distribution. For the exponential form, $R(t)=e^{-\lambda t}$, where λ is a constant, $f(t)=\lambda e^{-\lambda t}$, $h(t)=\lambda$, i.e. a constant hazard rate, and MTBF = $1/\lambda$.

There are 2 main methods of improving the reliability. One is by increasing the basic reliability of a unit by through improved design, technology, etc. The other is by redundancy. The cost of each approach will be considered separately and then combined.

Hecht [Ref. 3, p. 643] gives a cost function for estimating the incremental cost of improving the reliability of a unit. Let $\$_0$ be the cost of building a unit with a basic reliability of $R_0(t_{life})$ (to be abbreviated to R_0 hereafter) at the design lifetime, t_{life}, and $\$_{\Delta rel}$ be the incremental cost of improving the reliability to be $R_{improved}(t_{life})$. Then from [Ref. 3] the cost function is found to be

$$\$_{\Delta rel} = \left[\frac{\ln(1-R_{improved})}{\ln(1-R_0)} - 1\right]\$_0 \tag{25}$$

To demonstrate, consider a spacecraft that costs 200 M$ for a basic reliability of 0.85. The incremental cost for improving the reliability is then plotted in Fig. 8. It starts at zero for the basic reliability and rises very rapidly above 0.95 reliability.

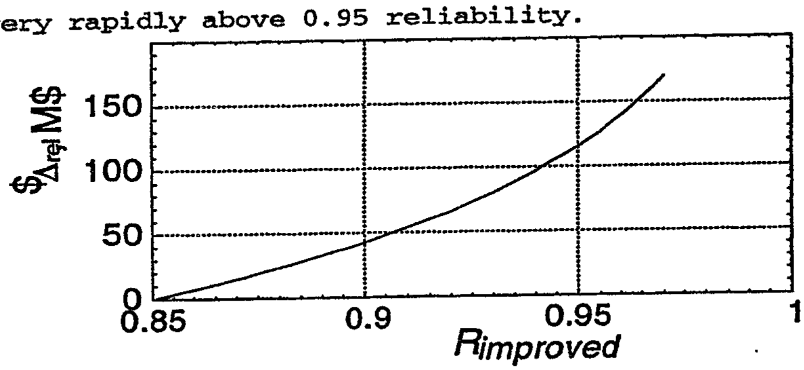

Fig. 8. Example of incremental cost of improving reliability without using redundancy.

The spacecraft reliability can also be improved by the use of redundancy in the form of spares. We consider here the spares to be stored on the ground and that these "standby" spares have perfect reliability (=1) until the time they are launched. This is distinguished from in-orbit spares which can be put into service much more rapidly, but could fail while still as a spare. However, it may have a reliability function that is better than that of the operational payloads. The analysis below can be easily modified for in-orbit spares.

Consider a single-satellite system. The total system reliability at the end of the specified life will be derived for one on-the-ground standby and two standbys. The relevant probability functions for one standby are:

$$f_0(t) = \text{failure prob. density of basic satellite}$$
$$f_1(t_1) = P_r[\text{sat \# 1 fails in interval } t_1 \text{ to } t_1 + dt_1] = f_0(t_1)$$
$$f_2(t_2|t_1) = P_r\begin{bmatrix}\text{sat \# 2 fails in interval } t_2 \text{ to } t_2 + dt_2 | \\ \text{sat \# 1 failed in interval } t_1 \text{ to } t_1 + dt_1\end{bmatrix} = f_0(t_2 - t_1)$$
$$f_{1,2}(t_2) = P_r[\text{sat \# 1 failed and \# 2 fails in interval } t_2 \text{ to } t_2 + dt_2]$$
$$= \int_0^{t_2} f_1(t_1) f_2(t_2|t_1) dt_1$$
$$F_{1,2}(t_{life}) = P_r[\text{sat \# 1 failed and \# 2 failed in interval } t_2 = 0 \text{ to } t_{life}] \qquad (26)$$
$$= \int_0^{t_{life}} f_{1,2}(t_2) dt_2$$
$$R_{1,2}(t_{life}) = P_r[\text{1 sat is still functioning at } t_{life}] = 1 - F_{1,2}(t_{life})$$

If 2 standby units are used, then the additional functions necessary are:

$$f_3(t_3|t_2) = P_r[\text{sat \# 3 fails in interval } t_3 \text{ to } t_3 + dt_3 |$$
$$\text{\# 1 failed and \# 2 failed in interval } t_2 \text{ to } t_2 + dt_2]$$
$$= f_0(t_3 - t_2)$$
$$f_{1,2,3}(t_3) = P_r[\text{sat \# 1 and \# 2 failed and \# 3 fails in interval } t_3 \text{ to } t_3 + dt_3]$$
$$= \int_0^{t_3} f_{1,2}(t_2) f_3(t_3|t_2) dt_2$$
$$F_{1,2,3}(t_{life}) = P_r[\text{all 3 sats fail in interval 0 to } t_{life}] \qquad (27)$$
$$= \int_0^{t_{life}} f_{1,2,3}(t_3) dt_3$$
$$R_{1,2,3}(t_{life}) = P_r[\text{1 sat is still functioning at } t_{life}] = 1 - F_{1,2,3}(t_{life})$$

The launch reliability can be included as illustrated by the modification to $f_1(t_1)$ as

$$f_1(t_1) = P_r[\text{sat \# 1 launch failure or fails in interval } t_1 \text{ to } t_1 + dt_1]$$
$$= (1 - R_{launch}) \bullet \delta(t_1) + R_{launch} \bullet f_0(t_1) \qquad (28)$$

where the probability density of a launch failure at time t_1 is expressed by a delta function. The other probability functions would be modified in a similar manner to account for launch reliability.

In the approach given here, the system reliability $R_{sys}(t_{life}) = R_{1,2}(t_{life})$ for one standby or $= R_{1,2,3}(t_{life})$ for 2 standbys, is specified and it is necessary to determine the baseline single unit reliability, $R_0(t_0)$, by working backwards through the above equations! Working backwards through (26), (27), and (28) may appear formidable even for simple reliability distributions. However, modern symbolic mathematical software such as Mathematica or Maple can be used systematically to solve these equations very easily, rapidly, and accurately. The results below used Maple.

If the number of simultaneously operating satellites is $N_{oper} > 1$, then system reliability, $R_{sys}(t_{life})$, that N_{oper} satellites are still functioning at t_{life} can be derived from the basic reliability, $R_0(t_0)$, of one satellite by the use of combinatorics and extensions of the above methods. However, these techniques can get quite messy because of all the combinations of satellite failures possible. Fortunately, some reasonably good approximations can be applied that simplify the analysis, and will be illustrated in one of the examples below.

Because of the perception of complexity of the above analysis, reliability analysis has usually been done by Monte Carlo simulation. Unfortunately, as a "cut and try" method, it will not normally indicate if minimum cost is achieved, and can well be replaced with the following approach.

3.3.3 General spacecraft reliability cost minimization

The total cost of launch and spacecraft as driven by reliability is

$$\$_{total} = N_s \$_{1launch} + L(N_s)(\$_0 + \$_{\Delta rel}) \tag{29}$$

where N_s is the total number of satellites required to enable N_{oper} satellites to be operating over the t_{life} so that $N_s \geq N_{oper}$. It is assumed that no learning curve is applied to launch costs, and that the launch cost is incurred regardless of whether all satellites are launched (in retrospect, the "non launches" should have not been added).

The objective is to make $\$_{total}$ in (29) a minimum subject to meeting the specified $R_{sys}(t_{life})$. Cost function (29) reflects the two reliability drivers with $\$_{\Delta rel}$, given by (25), governing the cost of improving individual spacecraft reliability, and N_s governing the cost of spares. The factor $\$_{\Delta rel}$ is a continuous function of reliability whereas the factor N_s takes only integer values. Therefore, the minimization is done in steps of N_s to see which N_s gives the minimum cost.

3.3.4 Application to one- and four-satellite constellations with on-ground standbys

Some examples will now be used to illustrate the approach to cost minimization with respect to reliability. For these examples, the following parameters were chosen:

N_{oper} = 1 and 4 $R(t) = e^{-\lambda t}$ $R_0 = R_0(8 \text{ years}) = 0.85$ $R_{launch} = 0.85$	$R_{sys}(20)=0.9$ t_{life} = 20 years $\$_0$ = 200 M\$ $\$_{launch}$ = 100 M\$

N_{oper}= 1

For a single satellite constellation, equations (25) through (28) are applied to get the following results.

N_S	N_{stdby}	$R_{improved}(8)$	$\$_{total}$
1	0	>1 (impossible)	∞
2	1	0.903	667 M$
3	2	$0.761<R_0$	852 M$

The launch reliability causes it to be impossible to get the specified $R_{sys}(20)$ with zero standbys. With 1 standby, the single-spacecraft reliability must be increased from 0.85 to 0.903 which ends up giving the minimum cost. For 2 standbys, the single spacecraft reliability needed is actually less than the basic value which one would not change. The result would be a $R_{sys}(20)$ slightly higher than the one specified but at substantially higher cost. Clearly, 1 standby gives the minimum cost.

N_{oper}= 4

For a 4-satellite constellation equations (25) through (28) are used along with the following concepts. We know from the previous example that no standbys for a single satellite does not give the specified $R_{sys}(20)$. Therefore, for the 4-satellite system we will use one standby for each slot as the starting point, i.e. we start with N_S = 4+4, and do the costing. Then we will repeat for N_S = 9, and 10. The first simplification is to assume that for N_S = 8, there will be 1 original satellite plus one standby assigned to each of the 4 slots. This is a good approximation as it has a much higher probability than any other events such as one slot failing twice. If we define

$$p_{x,1stdby} = P_r\left[\text{slot x fails in interval } t_2 = 0 \text{ to } t_{life}\right] = 1 - R_{x,1,2}\left(t_{life}\right) \qquad (30)$$

and then the probability that n slots fail (probability that both the original and the standby replacement fail) is

$$P_r[\text{exactly } n \text{ slots fail}] = \binom{4}{n} p_{x,1stdby}^{n} \left(1 - p_{x,1stdby}\right)^{4-n} \tag{31}$$

Applying all the techniques used for N_{oper}=1, plus (30) and (31), plus common techniques in combinatorics, the following costing summary was obtained.

N_s	N_{stdby}	$R_{improved}$(8)	$\$_{total}$
8	4	0.995	4398 M$
9	5	0.899	2666 M$
10	6	0.849<R_0	2600 M$
11	7	<R_0	2860 M$

With 4 standbys, the single spacecraft reliability must be increased from 0.85 to 0.995 which drives the cost up considerably. The minimum cost is obtained for 6 standbys, and, since $R_{improved}$ is less than the basic value which one would not change, R_{sys}(20) would be slightly higher than the one specified. Clearly N_s = 10 is a "best buy" since it not only costs less than for N_s=9, but gives slightly higher system reliability. These results are very important because it shows that, to achieve the specified reliability, it is not necessary to have more than 2 second-standby spares for all slots which results in very substantial savings.

3.3.5 Spacecraft component reliability cost minimization

The techniques applied above to entire spacecraft can also be applied to reliability cost analysis for components on board the satellite. The combined series reliability is

$$R_{pld} = R_{antUL} \bullet R_A \bullet R_B \bullet R_C \bullet R_{antDL}, \tag{32}$$

for the illustrative payload of Fig. 9 which results in a lower reliability than for the individual components.

The unit reliability improvement cost is still governed by (25). The reliability cost for redundancy is still just the product of the unit cost times the total number of units needed. The main difference in analysis arises in from the use of different forms or redundancy. Two types are illustrated in Fig. 9, and are given by

$$\begin{aligned} R_A &= 1 - (1 - R_{LNA1}) \bullet (1 - R_{LNA2}) \\ R_C &= R_{HPA}(1 - \ln R_{HPA}) \end{aligned} \tag{33}$$

for active and standby redundancy, respectively. Analysis proceeds as before but using (33) for the subsystem reliability.

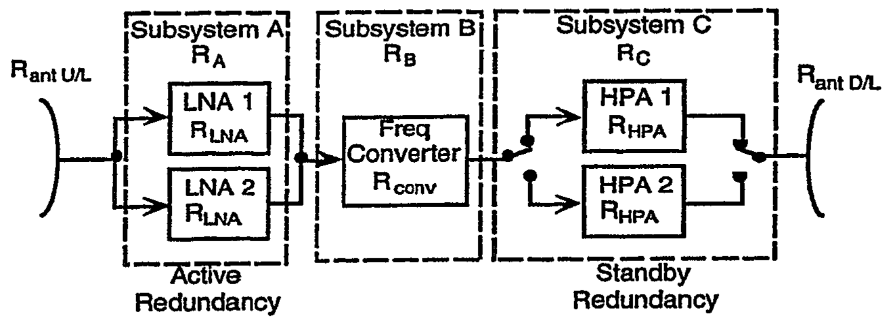

Fig. 9. A block diagram of a simple payload to illustrate the computation of system reliability.

4. CONCLUSION

A methodology is described that has potential for analytically estimating parameters that lead to minimum cost. The methodology was further explained by means of some examples of various tradeoffs of interrelated parameters. Application to any satcom system can give good guidance as to how to achieve lowest cost. Unlike previous "spread-sheet" approaches, this method will indicate if a minimum even exists, as well as giving the parameter values for the minimum. Much work remains to be done such as better component cost functions, and developing how to account for step changes in cost.

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SESSION III

OPERATIONAL ASPECTS

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