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Exploration of the dynamics of spiking neural networks

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Defence Research and Development Canada – Valcartier

Technical Memorandum

DRDC Valcartier TM 2011-005

March 2013

Canada

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Abstract

Liquid state machine is a technique that is well-suited for many spatiotemporal pattern recognition tasks found in defence applications. The technique exploits the complex dynamics of spiking neural networks. In general, Liquid state machines use biologically inspired neuron models such as the leaky-integrate-and-fire model. When structured into a network, leaky-integrate-and-fire neurons interact in a complex and a non-linear manner, making it difficult to understand their behavior in response to an input stimulus. In this memorandum, the response of simple neural systems to input stimulus is studied. The goal is to provide a first exploration of the dynamical state of spiking neurons in reaction to their stimulation by continuous inputs. For this purpose, small neural systems made of, respectively, one single neuron, three serial neurons, and a three-by-three network are analyzed.

Résumé

Les machines à état liquide, en exploitant les dynamiques complexes des réseaux de neurones à impulsions, représentent une technique bien appropriée à plusieurs tâches de reconnaissance spatiotemporelle associées à des applications de défense. Les machines à état liquide utilisent généralement des modèles de neurones biologiques tels que le modèle “leaky-integrate-and-fire”. Lorsque structurés en réseau, les neurones “leaky-integrate-and-fire” interagissent de manière complexe et nonlinéaire, ce qui rend difficile la compréhension de leur comportement lorsque soumis à un stimulus d’entrée. Dans ce rapport, la réponse de systèmes neuronaux simples à des signaux d’entrée est étudiée. Le but est de fournir une exploration de base sur le comportement de l’état dynamique des neurones à impulsions lorsqu’ils sont stimulés par un signal. À cette fin, des petits systèmes neuronaux composés d’un seul neurone, de trois neurones en série et d’un réseau de neurones trois-par-trois sont analysés.

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Executive summary

Exploration of the dynamics of spiking neural networks

F. Rhéaume; D. Grenier; DRDC Valcartier TM 2011-005; Defence R&D Canada –Valcartier; March 2013.

The concept of spiking neural networks has recently emerged as a very promising method in artificial intelligence. Specially, spiking neural networks are well suited for pattern recognition and learning problems with a temporal nature, such as video or sound event recognition, for example. As more to being biologically more realistic, they have been proven to be computationally more powerful than other non-spiking neural network models.

The C2 Decision Support Systems (C2DSS) Section at Defence Research & Development Canada (DRDC) - Valcartier conducts research activities on the recognition of spatio-temporal patterns in military applications. The task is to relate streams of inputs to known phenomenons, where the goal is to have a better understanding of a situation or to control an automated system (*e.g.* robotics). A promising technique is called Liquid State Machines. A Liquid State Machine is made of an untrained spiking neural network, whose stimulation by input streams creates complex interactions among the neurons.

The most commonly known spiking neuron model is the leaky-integrate-and-fire, which exhibits biologically plausible properties in an effective manner. This is the model found in most Liquid State Machine achievements. In this memorandum, leaky-integrate-and-fire neurons are studied, where the response of simple neural systems to input stimulus is analyzed. The goal is to provide a first exploration of the dynamical state of spiking neurons in reaction to their stimulation by continuous inputs. For this purpose, small neural systems made of, respectively, one single neuron, three serial neurons, and a three-by-three network are analyzed.

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Exploration of the dynamics of spiking neural networks

F. Rhéaume ; D. Grenier ; RDDC Valcartier TM 2011-005; R&D pour la défense Canada – Valcartier ; mars 2013.

Les réseaux de neurones à impulsions sont récemment devenus des outils très prometteurs dans le domaine de l'intelligence artificielle. Ces réseaux s'avèrent bien disposés pour les problèmes d'apprentissage et de reconnaissance de formes ayant une nature temporelle. En plus de leur réalisme biologique, ceux-ci ont démontré une meilleure capacité de calcul que les réseaux conventionnels qui ne sont pas à base d'impulsions.

La Section des systèmes d'aide à la décision du commandement et contrôle (SADC2) à Recherche et développement pour la défense Canada (RDDC) - Valcartier mène des activités de recherche sur la reconnaissance des formes spatiotemporelles dans des applications militaires. La tâche consiste à relier des flots de données en entrée à des phénomènes connus et d'intérêt, le but étant d'obtenir une meilleur compréhension d'une situation ou de contrôler un système automatisé (en robotique, par exemple). Une technique approfondie à RDDC est celle des machines à état liquide. Une machine à état liquide est constituée d'un réseau de neurones à impulsions pour lequel la stimulation par des signaux d'entrées crée des interactions complexes parmi les neurones.

Le modèle de neurone à impulsion le plus communément cité est le "leaky-integrate-and-fire", qui reproduit de manière efficace et assez juste les propriétés des neurones biologiques. Ce modèle se trouve dans la plupart des machines à état liquide développées. Dans ce memorandum, les neurones "leaky-integrate-and-fire" sont étudiés, où la réponse de systèmes neuronaux simples à des signaux d'entrée est analysée. Le but est d'explorer le comportement de l'état dynamique des neurones à impulsions en réponse à des signaux d'entrée. À cette fin, de petits réseaux formés respectivement d'un seul neurone, de trois neurones en séries et de neuf neurones en disposition récurrente trois-par-trois, sont analysés.

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1 Introduction

This document details an investigation of the behavior of simple spiking neural systems made of Leaky-Integrate-and-Fire (LIF) neurons. It is a first step towards understanding the more complex behavior of larger networks of spiking neurons used in Liquid State Machines. Since LSM are a recently developed technique for spatiotemporal pattern recognition, explorations are needed into the response behavior characterization.

This document aims at two goals. The first one is to understand how simple configurations of LIF neurons react to input stimulus and how their parameters influence their response. The second goal is to familiarize with the C-language-based Simulator (*CSIM*) and *Circuit-Tool* toolboxes [7, 6, 19, 1] for creating and stimulating networks of spiking neurons. As a first step to achieve these goals, the behavior of a single LIF neuron connected to a single analog input neuron is studied in Chapter 2. Then, the simulation and analysis of a series of three non-recurrent LIF neurons connected to an analog input neuron are made in Chapter 3. Finally, a larger network of three-by-three neurons is studied in Chapter 4.

Note that the tests presented in this document are made using analog input neurons, that simply convert input values into input currents. Other methods of input conversion, such as analog-to-spike conversion, are not covered in this document.

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2 Single Leaky-Integrate-and-Fire neuron connected to a single analog input neuron

2.1 Circuit description

The circuit is made of only one spiking neuron connected to an analog input neuron. Precisely, the spiking neuron is a leaky-integrate-and-fire neuron. The analog input neuron is used to transmit the input signal to the leaky-integrate-and-fire neuron through a static analog synapse. This results in a non-recurrent network shown in Figure 1.

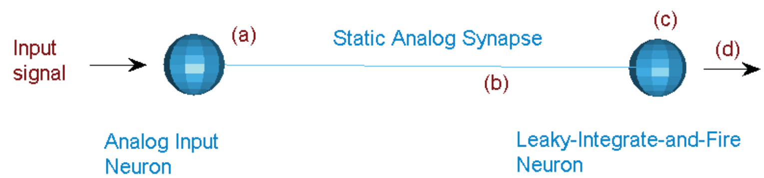


Figure 1: Simple neural circuit. An analog input neuron is connected to a leaky-integrate-and-fire neuron through a static analog synapse.

The circuit follows the third generation of neural network models described in [10].

2.1.1 Analog input neuron

An analog input (AI) neuron plays the role of an interface between the input signal and the neural circuit. The AI neuron used in this work has the following characteristics [7]:

- Resting membrane voltage (Vresting) :	0
- The noise of analog neurons range (Inoise) :	0
- Number of incoming synapses (nIncoming) :	0
- Number of outgoing synapses (nOutgoing) :	1
- Type :	excitatory

Note that the expressions in parentheses correspond to the names of the variables used in *CSIM*. The AI neuron also has a membrane voltage (Vm) which, in this case of the analog input neuron, corresponds to the value that is propagated to the outgoing synapses ($VmOut$). Vm and $VmOut$ are represented by (a) in Figure 1.

2.1.2 Static analog synapse

In this work a very simple synapse model is used, the static analog synapse. Such a model is not representative of biological synapses, but it is useful to transfer analog values into neurons. It mainly consists of a weighting factor that weights the input and of a time delay before transferring the weighted value up to the outgoing neuron. The static analog synapse implemented in the simulated circuit has the following characteristics [7]:

- Noise (Inoise) :	0
- Weigth (W) :	4.9924e-009
- Synaptic transmission delay (delay) :	0.0015

The values were generated according to a default distribution model and some random noise [14]. Precisely, the synapse is related to its input neuron by sending it a postsynaptic response (psr) that is represented by a voltage value. The psr value is the result of weighting the synapse input by a weight W. The psr is represented by (b) in Figure 1. The synapse also has a transmission delay.

2.1.3 Leaky-integrate-and-fire neuron

One of the most widely used spiking neuron models is the leaky-integrate-and-fire (LIF) neuron model [24]. As shown in Figure 2, the neuron is modeled as an electronic circuit consisting of a capacitor in parallel with a resistor, into which a current leaks. A spike is generated when the voltage over the capacitor (Vm) crosses a threshold (υ).

A LIF neuron model is available in *CSIM* and involves different parameters. There are parameters that are constant for all LIF neurons that are created. These include the membrane capacity, the membrane resistance, the spiking voltage threshold, the resting membrane voltage, the absolute refractory period (although it is different whether the neuron is inhibitory or excitatory) and the standard deviation of the noise to be added each integration time constant. Details of these parameters are given in [7, 6]. Other parameters such as the reset voltage (after a spike), the initial membrane voltage and the constant current to be injected into the neuron are determined randomly over some limited range of values and following some specific distribution functions. In this work, the

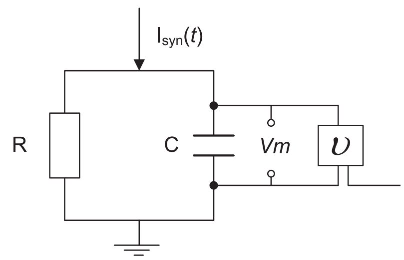


Figure 2: Leaky-integrate-and-fire neuron model. The neuron is modeled as an electronic circuit where a current $I_{syn}(t)$ splits into a capacitor C and a resistor R.

LIF neuron has the following characteristics [7]:

- Membrane capacity (Cm) :	3.0000e-008
- Membrane resistance (Rm) :	1000000
- Spiking voltage threshold (Vthresh) :	0.0150
- Resting membrane voltage (Vresting):	0
- Reset voltage (after a spike) (Vreset):	0.0139
- Initial condition for membrane voltage (at time $t = 0$) (Vinit) :	0.0148
- Lenght of the absolute refractory period (Trefract) :	0.0030
- The standard deviation of the noise to be added each integration time constant (Inoise) :	0
- Constant current to be injected into the neuron (Iinject) :	1.3626e-008
- Number of incoming synapses (nIncoming) :	1
- Number of outgoing synapses (nOutgoing) :	0
- Type :	excitatory

As illustrated in Figure 2, the neuron is also defined by its membrane voltage (Vm), also shown by (c) in Figure 1, and a synaptic input current (Isyn). A spike is emitted when Vm exceeds a threshold υ.

2.2 Circuit stimulation and response

2.2.1 Input

The circuit has at its input a single value which changes at a regular time interval dt_{Input}. Here we have $dt = 0.1s$. The input to the circuit lasts $1s$, so that there are 10 input values. The 10 input values are presented in Table 1 and shown in Figure 3.

Table 1: 10 input values for each time interval dt.

Input values	0.33	0.03	0.04	0.36	0.98	0.97	0.94	0.89	0.72	0.33
Time	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

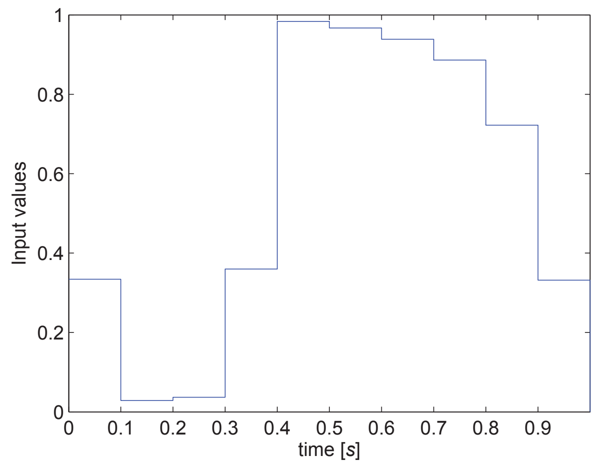
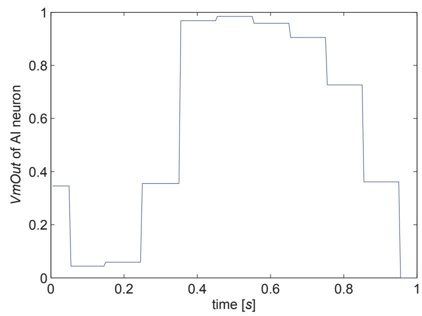


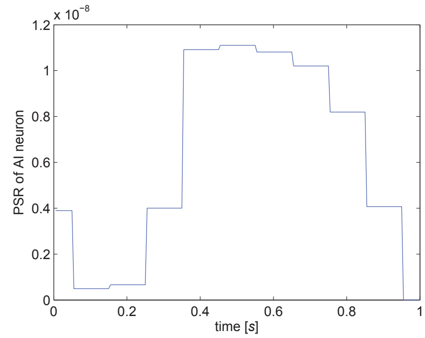
Figure 3: Input values coming out of the analog input neuron through time.

2.2.2 Response

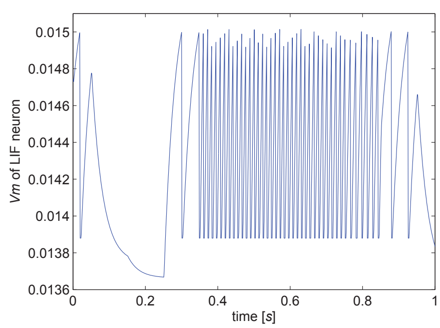
Figure 4 shows the responses of the different circuit components that are illustrated in Figure 1. It can be seen that the membrane voltage transmitted to the outgoing synapse (Figure 4 (a)) has the same value than the input signal (Figure 3). The transmitted voltage is then weighted through the synapse, providing a much lower voltage for the postsynaptic response (Figure 4 (b)). Figure 4 (c) shows the membrane voltage Vm of the LIF neuron. Each time where Vm exceeds 0.015 volts, which corresponds to the voltage threshold (Vthresh), the LIF neuron emits a spike. The spikes emitted during the circuit stimulation are illustrated in Figure 4 (d).



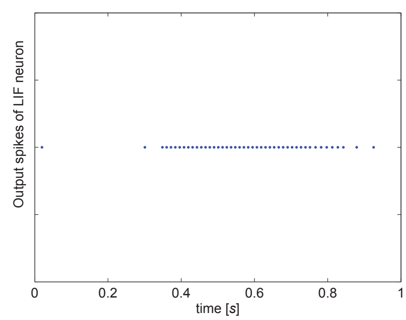
(a) *VmOut* of AI neuron



(b) psr of AI neuron



(c) *Vm* of LIF neuron



(d) Output spikes of LIF neuron

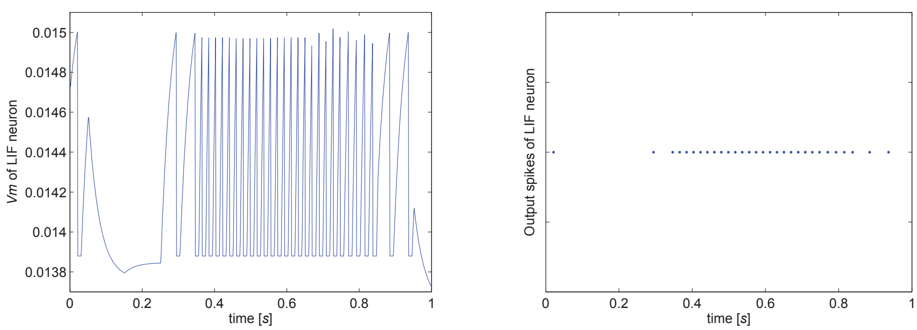
Figure 4: Simulation responses for the circuit of Figure 1. (a) *VmOut* is the voltage which is propagated to the outgoing synapse (b) psr is the postsynaptic response of the static analog synapse (c) *Vm* is the membrane voltage of the leaky-integrate-and-fire (LIF) neuron (d) Firing times of the LIF neuron

2.2.3 Impact of parameters

The impact of different parameters of the neural circuit will be studied in the following paragraphs. The study is intended to show how the LIF neuron reacts under different configurations. Note that there are parameters that have been omitted, such as the spiking voltage threshold, since their influence on the state of the circuit seems easily predictable for this particular single LIF neuron example.

2.2.3.1 Absolute refractory period of the LIF neuron (Trefract)

The absolute refractory period of a spiking neuron is defined as the minimal time distance between two spikes such that when a spike is emitted, another spike cannot be emitted until the refractory period is over [3, 7]. Let us increase the refractory period of the LIF neuron from $0.003s$ to $0.01s$ to generate new results[1]. The LIF neuron should then emit fewer spikes for the same synaptic input. Indeed, Figure 5 cleary shows that the time interval between each spike



(a) Vm of LIF neuron (b) Output spikes of LIF neuron

Figure 5: State of the LIF neuron as a result of increasing the absolute refractory period from 0.003 to 0.01 (a) Vm is the membrane voltage of the leaky-integrate-and-fire (LIF) neuron (b) Firing times of the LIF neuron

is longer and that consequently there are fewer spikes occurring during the $1s$ circuit stimulation, compared to the results in Figure 4.

2.2.4 Synaptic transmission delay (delay)

The synaptic transmission delay is the time it takes for a spike to travel from one neuron to another. For a non-recurrent neural network, such a delay should

[1] Note that in *CSIM* the absolute refractory period of a LIF neuron is randomly set between 0.002 and 0.003.

impact mainly on the phase of the different observed signals. This impact is shown in Figure 6, where the membrane voltage of the LIF neuron is illustrated for two different synaptic transmission delays, $0.0015\,s$ and $0.2\,s$. All other conditions are the same. The membrane voltage signal is nearly similar in both case but delayed by about $0.2\,s$.

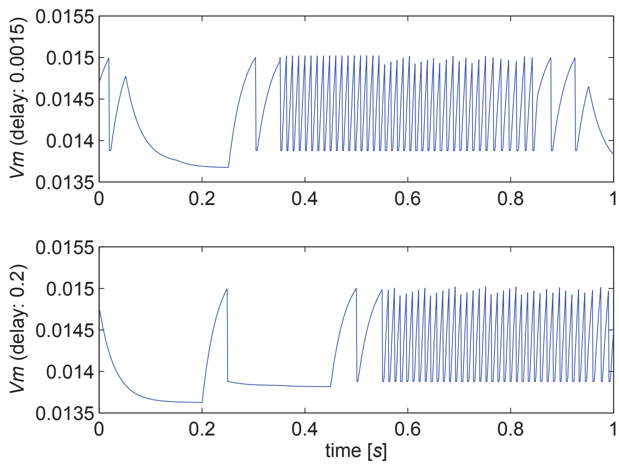


Figure 6: Comparison of the membrane voltage of the LIF neuron for two different synaptic transmission delay values.

2.3 Analysis of the response to the input

2.3.1 Note on time intervals

There are different time intervals playing a role in the circuit simulation. First, there is the discretization interval (dt_{Input}) of the input signal, that can also be referred to as time step of the input. It corresponds to the elapsed time between each value of the discretized signal that is sent to the AI neuron. Out of the AI neuron is a similar discretized signal, except for its different time step (dt_{sim}). What the AI neuron does is that it extends the input signal on time in order to have a value of the signal at each time step dt_{sim} instead of dt_{Input}. An example is shown in Figure 7. Symbol dt_{sim} is referred to as the simulation time step. The time step corresponds to the step at which each calculation in the circuit is advanced. Finally, there is the recording time step (dt_{rec}) that is the time interval at which the analog values are recorded. Normally, the time intervals should be set so that $dt_{Input} \geq dt_{rec} \geq dt_{sim}$. Each of them has an impact on the results gathered from the circuit.

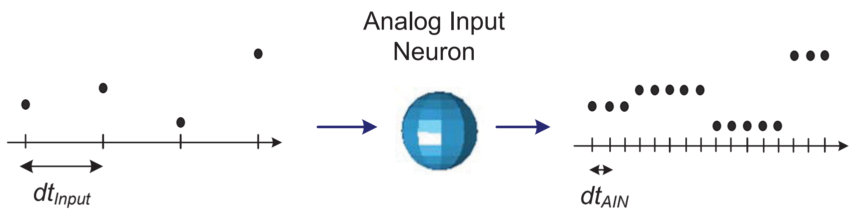


Figure 7: Conversion of time intervals. Each discrete value of the input signal is stretched out on the time line by the AI neuron according to the input time step dt_{Input} and to the AI neuron time step dt_{AIN}.

2.3.2 Results

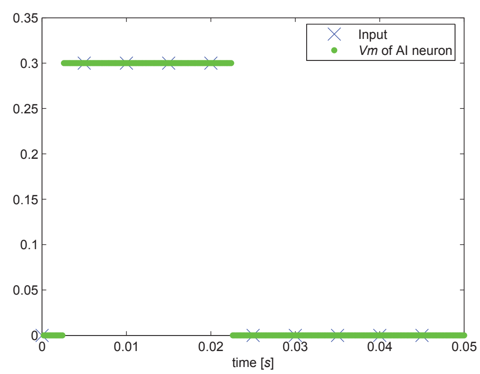
When a signal is presented to the circuit, the LIF neuron emits a sequence of spikes in time which varies depending on the input signal. The following is intended to show broadly how the simple circuit of a single LIF neuron reacts to different input signals (or stimulus). A few questions arise about the circuit response:

1. How does the absolute value of the input affect the response (in terms of spikes and membrane voltage) ?

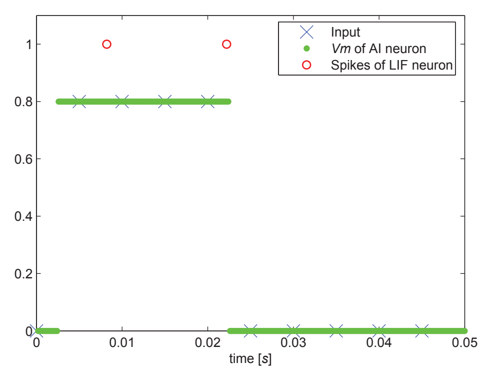
2. How does the frequency of the input signal, *i.e.* its variability over a given interval of time, affect the output ?

3. Is there any memory effect with the single LIF neuron and analog input neuron? In other words, does the response of LIF neuron depend on past inputs, and to what extent?

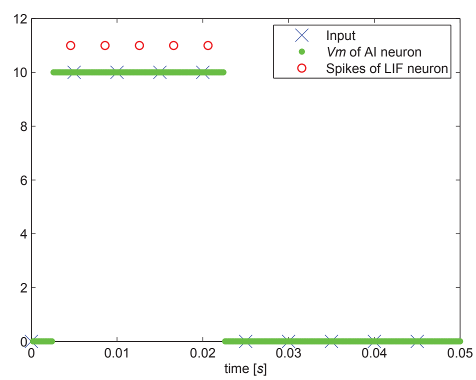
In order to answer these questions, the circuit was presented with different input signals. As a first exercise, three different inputs are created. Each input consists of a signal that is discretized into ten discrete values by a time interval $dt_{Input} = 0.005$. The inputs each have four non-zero, consecutive and equal discrete values. Their magnitudes are 0.3, 0.8 and 10 for input 1, 2 and 3, respectively. The remaining of the signal is zero. The three different signals are shown in Figure 8. Input 1, whose signal has a magnitude of 0.3, does not cause any spike emission by the LIF neuron (Figure 8 (a)). Input 2, which has a higher signal magnitude than Input 1, caused the LIF neuron to emit two spikes (Figure 8 (b)). Finally, raising the signal magnitude from 0.8 to 10 by using Input 3 instead of Input 2, the LIF neuron emits five spikes (Figure 8 (c)). Figures 8 (b) and 8 (c) also show that the first spike is emitted sooner with Input 3 than with Input 2, demonstrating that their is a (non-linear) relation between the magnitude of the signal and the speed at which spikes are emitted ((((linear temporal coding ???? dans Maass.Networks.1997)))). Such a relation depends on the integrate and fire model that is used (see [10, 3, 24, 5] for leaky-integrate-and-fire neuron models). Note that the speed at which spikes are emitted depends on the rate of increase of the membrane voltage of the LIF neuron. Thus, using an analog input neuron connected to LIF neurons can produce some sort of spike time coding (at the output of the LIF neuron) of the input. See [11, 16, 12, 8] for explanations on spike time coding, also known as temporal coding [21] or delay coding [20].



(a) Input 1



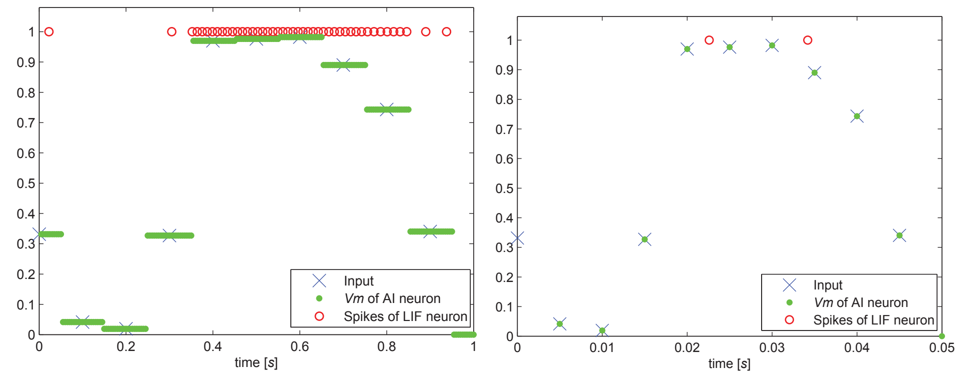
(b) Input 2



(c) Input 3

Figure 8: Circuit responses for similar inputs whose only difference is their absolute values. (a) Input 1 has value 0.3 and yields no spike. (b) Input 2 has value 0.8 and yields two spikes. (c) Input 3 has value 10 and yields 5 spikes.

The relation between the time scale of the input and the different time parameters of the circuit, such as the synaptic transmission delay and the absolute refractory period of the LIF neuron, has a huge impact on the circuit response. Low frequency inputs, *i.e.* whose variations are observable over long periods of time, should prompt setting larger time parameters of the circuit if the latter is deemed to react to low frequency events. Figure 9 shows how the circuit reacts differently to an input that is compressed in time, where the values of the input are not changed, only their time labels are diminished. Precisely, the input features the ten input values of Table 1 where the values are separated by $0.1\,s$ in Figure 9 (a) ($dt_{Input} = 0.1\,s$) and by $0.005\,s$ in Figure 9 (b) ($dt_{Input} = 0.005\,s$).

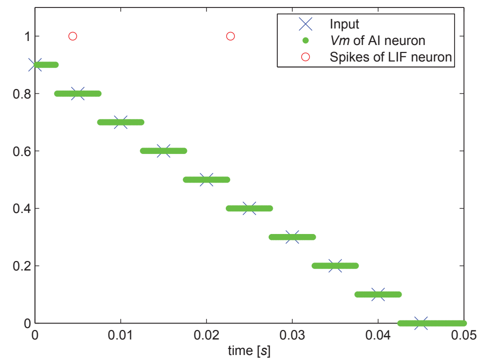


(a) $dt_{Input} = 0.1\,s$ (b) $dt_{Input} = 0.005\,s$

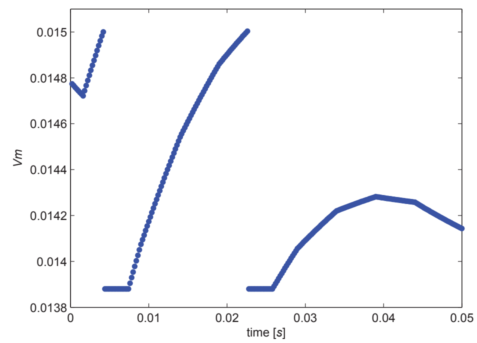
Figure 9: Circuit response for the same input pattern but with two different sampling intervals dt_{Input}.

Applying these two inputs to the same circuit presented in Section 2.1 yields the LIF neuron to behave differently. For the input with $dt_{Input} = 0.1\,s$, the LIF neuron emits a large number a spikes during the most part of the signal (Figure 9 (a)). On the contrary, for the input with $dt_{Input} = 0.005\,s$, the LIF neuron emits only two spikes between $0.02\,s$ and $0.04\,s$, even though the input pattern is the same one than that of Figure 9 (a). Hence, for the same input signal pattern but with different sampling intervals, the response pattern of the LIF neuron is not the same. Although such conclusion is evident, it is very important to understand its significance when developing a neural circuit. If the input sampling interval is too low or too large compared with the neuron reaction time (its time constant RC), the neural circuit will not generate the rich dynamics promulgated with LSMs [23].

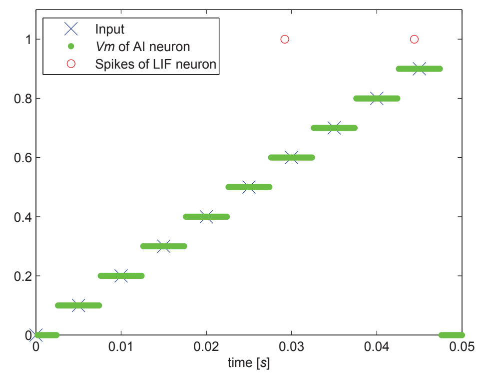
Moreover, the memory effect of the single LIF neuron is clearly visible in Figure 10, where signals of decreasing and increasing magnitude are input to the circuit. The signal of decreasing magnitude is just the result of reversing in time the signal of increasing magnitude. If no memory effect was present, the membrane voltage in Figure 10 (b) would be exactly the opposite in time of the membrane voltage shown in Figure 10 (d). In other words, reversing the axis of time in one of the two figures would yield the same graphic in both figures. This is not the case with the presented results. Therefore, even a single LIF neuron is able to provide a response that is correlated in time, providing some sort of memory. Referring to the LIF neuron model of Figure 2, such a memory is provided by capacitor C's accumulation of charge and discharge abilities, as well as by voltage resetting after a spike and the refractory period of the neuron.



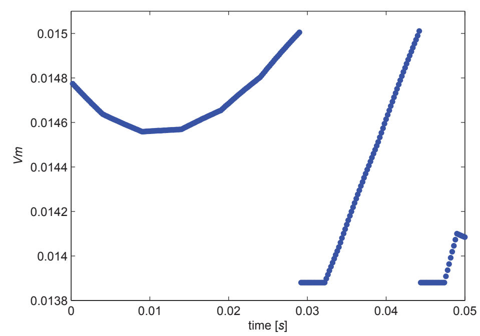
(a) Output spikes of LIF neuron for a monotonically decreasing input



(b) Vm of LIF neuron for monotonically decreasing input



(c) Output spikes of LIF neuron for a monotonically increasing input



(d) Vm of LIF neuron for a monotonically increasing input

Figure 10: Circuit response to: (a),(b) a monotonically decreasing signal (c),(d) a monotonically increasing signal.

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3 Series of 3 non-recurrent Leaky-Integrate-and-Fire neurons connected to an analog input neuron

3.1 Circuit description

The network shown in Figure 11 consists of three Leaky-Integrate-and-Fire (LIF) neurons that are connected serially and where the first of the series is connected to an analog input neuron. The network has no recurrence, so that signals flow in a unique direction. The LIF neurons connect together through static spiking synapses (SSS_1 and SSS_2). In contrast to dynamic synapses, the postsynaptic current of a static synapse has constant amplitude [15, 19, 14]. The analog input neuron just sends the input signal into a static analog synapse that is connected to the first LIF neuron (LIF_1) of the series of three. The static analog synapse is only represented by a weight or scaling factor W_i and a time delay D_i. In this experiment we have $W_i = 1.926 \times 10^{-9}$ and $D_i = 0.0016s$. Note that D_i has no utility in this study. Note that the circuit is free of noise, *i.e.* the neurons have no current noise.

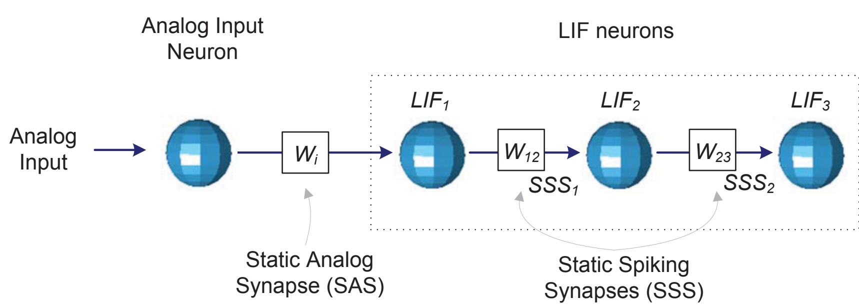


Figure 11: Spiking neural network made of a series of 3 non-recurrent LIF neurons connected through static spiking synapses.

3.2 Circuit stimulation and response

Figure 12 illustrates the response of the network to a constant input signal $I(t) = 10, t \in 0.005 \times \{0, 1, 2, ..., 18\}$. A clearer illustration is presented in Figures 13 and 14, that show the membrane voltage Vm in terms of time for the LIF neurons and the postsynaptic current of the synapses in terms of time, respectively. It is shown that LIF_1 sees its membrane voltage reach its spiking threshold rapidly, so that spikes are emitted regularly at a high frequency. Precisely, Figure 15 shows that LIF_1 emits spikes at an interval of $0.005\,s$. This is close to its absolute refractory period Trefract $= 0.003\,s$. Recall that the absolute refractory period of a spiking neuron is defined as the minimal time distance between two spikes such that when a spike is emitted, another spike cannot be emitted until the refractory period is over [3, 7]. The second LIF neuron of the series, LIF_2, receives as an input the postsynaptic current in synapse SSS_1 and shown on Figure 14 (b). The membrane voltage of LIF_2 increases less rapidly than LIF_1, as a result of the weaker postsynaptic current it receives (see Figures 14 (a) and 14 (b)). Consequently, LIF_2 emits fewer spikes than LIF_1 with an interval of $0.025\,s$ between each spike instead of $0.005\,s$ for LIF_1. The resulting postsynaptic current in synapse SSS_2 is shown in Figure 14 (c) where there are fewer current pulses than with SSS_1. Repeatedly, the membrane voltage of LIF_3 gets slower increases than its precursor and therefore LIF_3 emits fewer spikes than LIF_2. As shown in Figure 15, LIF_3 emits only two spikes compared to three and 17 for LIF_2 and LIF_3, respectively.

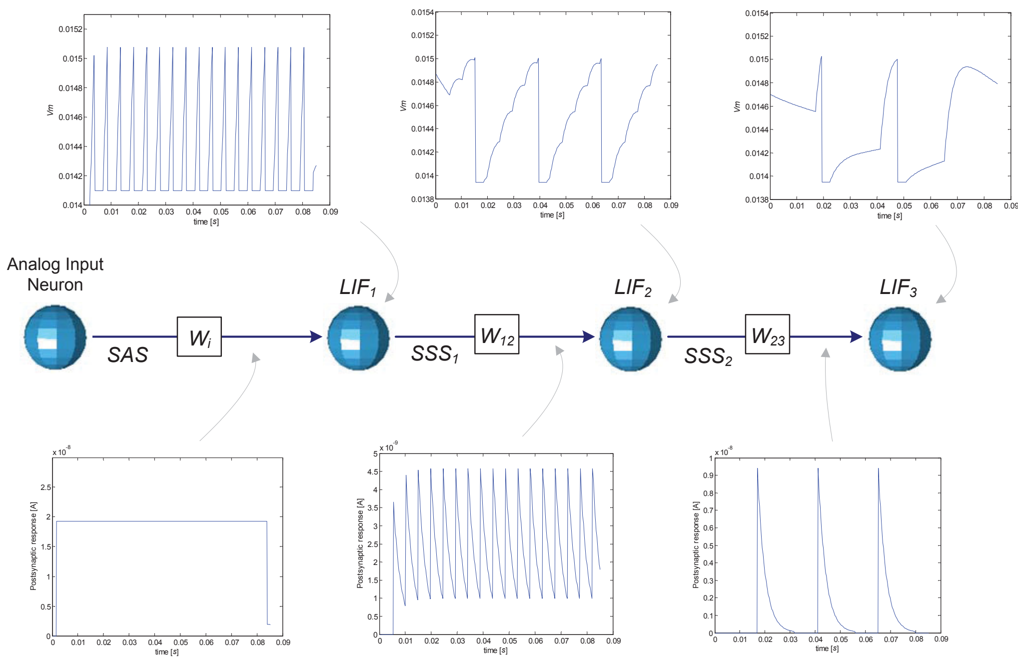


Figure 12: Response of the 3 non-recurrent LIF neuron network to the constant input $I(t) = 10, \forall t$.

The presented results show that the farther a LIF neuron is placed in a non-recurrent series with single synaptic connections, the more its response fades and the more its membrane voltage tends towards the resting stage. However, unless a neuron stays continually at its resting stage, it carries some information about the input. The carried information is different depending on the rank of the neuron in the series. The higher its rank, the higher the integration time.

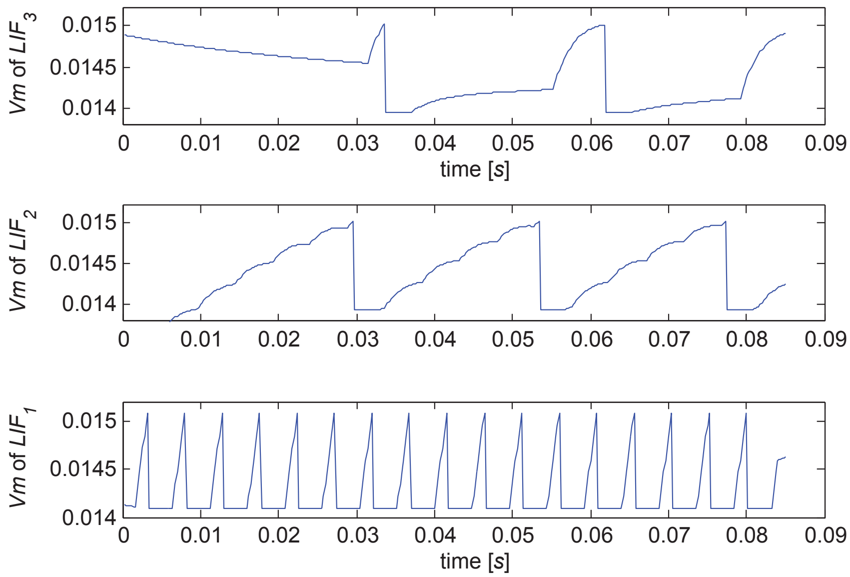
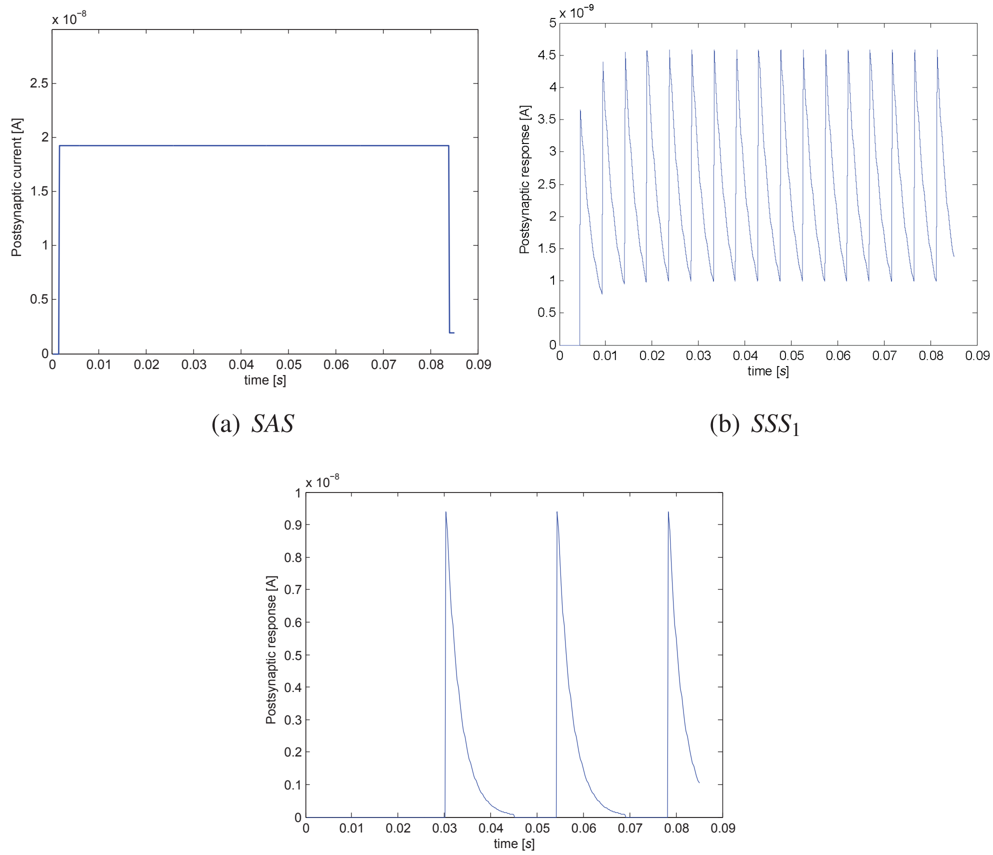


Figure 13: Membrane voltage (Vm) in terms of time for the three LIF neurons.

Hence, each neuron encodes the input information with a different spiking frequency.



(a) *SAS* (b) SSS_1

(c) SSS_2

Figure 14: Postsynaptic current in terms of time for the static analog synapse and the static spiking synapses.

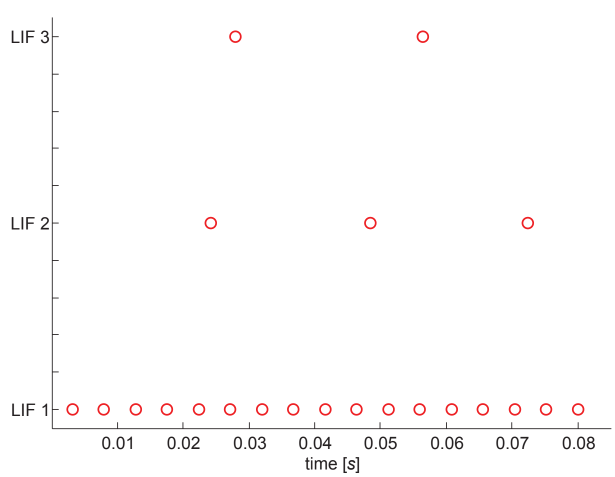


Figure 15: Spike emissions of the three LIF neurons.

4 Single analog input neuron connected to a three-by-three array of LIF neurons

4.1 Circuit description

A circuit is made using $Circuit - Tool$, where a single analog input (AI) neuron feeds a recurrent network of nine LIF neurons arranged in 3 by 3 array. The circuit is shown in Figure 16, where $n_{i,j}$ designate the neuron lying at position i, j on the array. The LIF neuron model is described in [24].

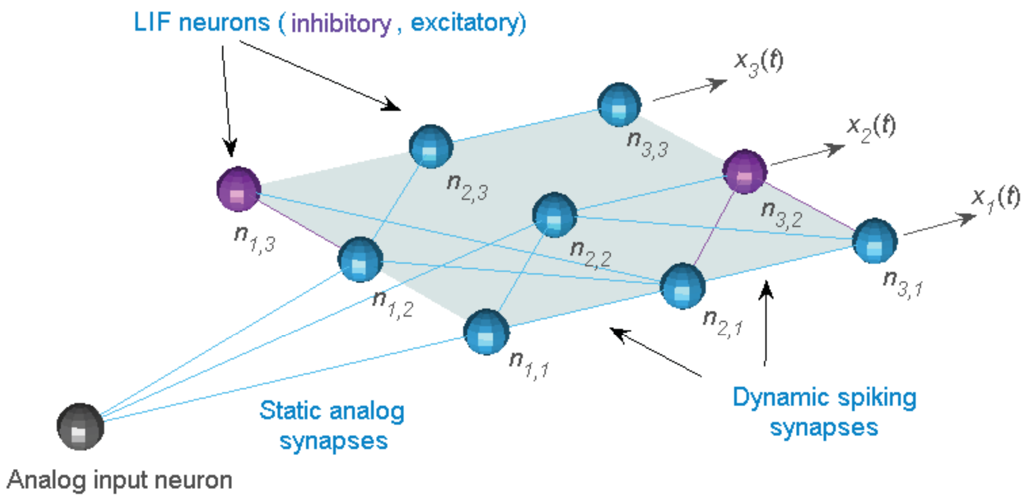


Figure 16: An analog input neuron is connected to a three-by-three array of LIF neurons through static analog synapses. Connections between LIF neurons are made with dynamic spiking synapses. A line may represent a incoming or outgoing neuron synaptic connection.

The connectivity structure of the LIF array is determined randomly by the synaptic connection probability $Ce^{-(D(a,b)/\lambda)^2}$ between neurons a and b, where λ controls both the average number of connections and the average distance between neurons that are synaptically connected [9], C a scaling factor and $D(a,b)$ the Euclidean distance between neurons a and b [14]. The value of C variates depending whether a and b are excitatory (E) or inhibitory (I) (the value of C are 0.1 (EE), 0.2 (EI), 0.4 (IE) and 0.3 (II), where EE, EI, IE and II denote connections between the two types of neurons). See [10, 3] for details about excitatory and inhibitory neurons. Broadly, the resulting configuration generates mostly local connections with a few longer ones. Furthermore, the synaptic dynamics follows the biological model suggested in [17] with a synaptic strength (Wscale

Neuron label	Type	Nb. of incoming connections	Nb. of outgoing connections
$n_{3,1}$	excitatory	3	0
$n_{3,2}$	inhibitory	1	3
$n_{3,3}$	inhibitory	0	1

Table 2: Connection details of neurons $n_{3,1}$, $n_{3,2}$ and $n_{3,3}$.

in CSIM) of 2. Note that this synaptic strength represents the scaling of the A-parameter for the synapse model defined in [17].

The AI neuron is connected to the array of LIF neurons through static synapses and using the same connection probability model as for the array of LIF neurons and a synaptic strength (Wscale) of 2. A static synapse is a synapse model in which the amplitude of each postsynaptic response is equal [7].

The states of neurons $n_{3,1}$, $n_{3,2}$ and $n_{3,3}$ are observed. Suppose the array of LIF neurons represents the 'liquid' of a LSM. The observed states then represent the liquid state discussed in [13, 18, 23, 22, 2, 4]. The states are stored in a vector $\boldsymbol{x}(t) = [x_1(t), x_2(t), x_3(t)]$ for each time t.

The connections between neurons are all illustrated in Figure 16, although it does not specify the direction of the connection. For instances, the neurons chosen for representing the liquid state have their connections detailed in Table 2.

One particularity of the network is that one of its neurons, neuron $n_{3,3}$ has no incoming connection. It is isolated from the network such that it cannot react to any stimulus. It is therefore useless since it generates no meaningful information about the inputs.

4.2 Circuit stimulation and response

4.2.1 Input

Each input to be introduced in the circuit consists of a signal that is discretized in time according to a time interval dt_{Input}. Such resulting signal is fed to the AI neuron. Accordingly, the network of LIF neurons has at its input a single value which changes at a regular time interval dt_{Input}. The input to the circuit lasts for a duration T_{sim}. Figure 17 shows an example of a 1s signal with $dt_{Input} = 0.1$.

4.2.2 Response

The circuit response will be analyzed through the liquid state vector that was defined in Section 4.1 and that consists of the state of neurons $n_{3,1}$, $n_{3,2}$ and $n_{3,3}$. Recall that the liquid state is represented by a vector $\boldsymbol{x}(t) = [x_1(t), x_2(t), x_3(t)]$ for each time t. As a first study, the observed state to will be the membrane voltage of the LIF neurons. Note that instead of membrane voltage, firing rates could also be a relevant representation of a neuron's state.

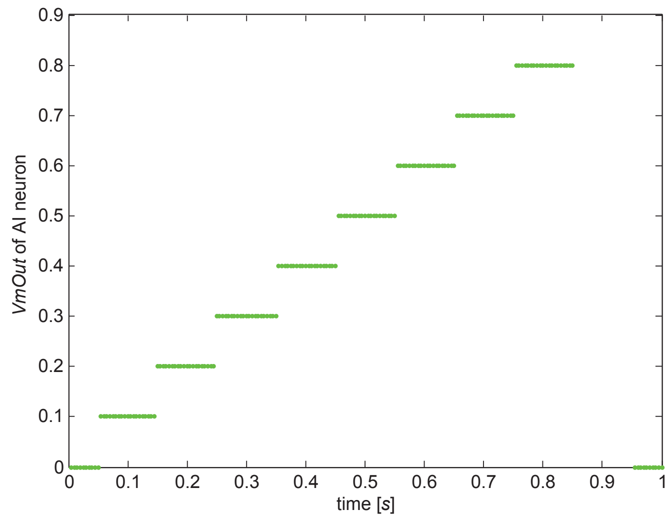


Figure 17: Input to the network of LIF neurons in terms of time. The input corresponds to the membrane voltage of the AI neuron.

5 Conclusion

The presented experimentations aimed at understanding the basics of simple spiking neural network. It underlined the relation between the input and the neurons' output according to different parameters. The results presented in this memorandum are only meant to preset a complete study of LSM and its application to temporal learning applications. They will help to understand the dynamics of larger networks. Among the temporal learning applications that may support situation awareness in military operations, we find object and event recognition from many sources, such as video, sound, infrared sensors, seismic sensors, magnetic sensors, radar (target profiles), as well as any source that report a large an continuous amount of information. The applications may also include surveillance of information systems, such as detection of anomalies or suspicious behaviors.

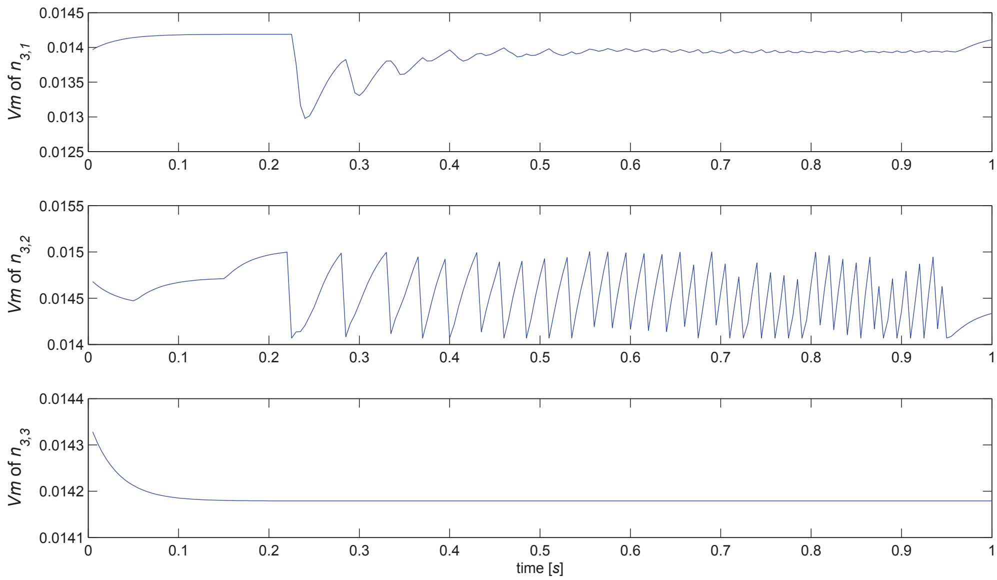


Figure 18: Membrane voltage (Vm) in terms of time for neurons $n_{3,1}$, $n_{3,2}$ and $n_{3,3}$.

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Liquid state machine is a technique that is well-suited for many spatiotemporal pattern recognition tasks found in defence applications. The technique exploits the complex dynamics of spiking neural networks. In general, Liquid state machines use biologically inspired neuron models such as the leaky-integrate-andfire model. When structured into a network, leaky-integrate-and-fire neurons interact in a complex and a non-linear manner, making it difficult to understand their behavior in response to an input stimulus.

In this memorandum, the response of simple neural systems to input stimulus is studied. The goal is to provide a first exploration of the dynamical state of spiking neurons in reaction to their stimulation by continuous inputs. For this purpose, small neural systems made of, respectively, one single neuron, three serial neurons, and a three-by-three network are analyzed.

Les machines à état liquide, en exploitant les dynamiques complexes des réseaux de neurones à impulsions, représentent une technique bien appropriée à plusieurs tâches de reconnaissance spatiotemporelle associées à des applications de défense. Les machines à état liquide utilisent généralement des modèles de neurones biologiques tels que le modèle “leaky-integrate-and-fire”. Lorsque structurés en réseau, les neurones “leaky-integrate-and-fire” interagissent de manière complexe et nonlinéaire, ce qui rend difficile la compréhension de leur comportement lorsque soumis à un stimulus d’entrée. Dans ce rapport, la réponse de systèmes neuronaux simples à des signaux d’entrée est étudiée. Le but est de fournir une exploration de base sur le comportement de l’état dynamique des neurones à impulsions lorsqu’ils sont stimulés par un signal. À cette fin, des petits systèmes neuronaux composés d’un seul neurone, de trois neurones en série et d’un réseau de neurones trois-par-trois sont analysés.

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