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Considerations for using swept-frequency waveforms in marine seismology

Brian H. Maranda

Defence R&D Canada – Atlantic

Technical Memorandum
DRDC Atlantic TM 2012-264
January 2014

Canada

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Abstract

This report has been written as part of a joint research project between Defence Research and Development Canada (DRDC) Atlantic and the Offshore Energy Environmental Research (OEER) Association. One of the research goals is to increase the understanding of alternative geophysical exploration methods for the marine environment, and this report highlights many of the issues that need to be addressed when applying *frequency-modulated* (FM), or *swept-frequency*, waveforms to geophysical prospecting of the ocean seabed. A key component of the signal processing for FM pulses is the matched filter, which is described at an introductory level. Examples are given to illustrate the excellent resolving capability of the linear-FM pulse, which is a consequence of the pulse-compression property of the matched filter. Also discussed are other, more general, kinds of signal modulation. Comments are then made on issues that arise in the specific application of FM waveforms to reflection seismology in the marine environment. Finally, the report also contains an Annex on the physical units relevant to marine seismology.

Résumé

Le présent rapport a été rédigé dans le cadre d'un projet de recherche conjoint entre Recherche et développement pour la défense Canada (RDDC) Atlantique et l'Offshore Energy Environmental Research (OEER) Association (association de recherche environnementale en énergie extracôtière). La recherche visait notamment à mieux comprendre d'autres méthodes d'exploration géophysique de l'environnement marin. Le présent rapport souligne un grand nombre des problèmes à corriger lors de l'application d'ondes *modulées en fréquence* (FM) ou sous forme *d'un balayage de fréquences* à la prospection géophysique du fond marin. Le rapport comporte une introduction au filtre adapté, qui constitue l'un des éléments clés du traitement de signaux d'impulsions FM. On y donne des exemples en vue de montrer l'excellente capacité de résolution d'une impulsion de modulation linéaire en fréquence, capacité qui découle de la propriété de la compression d'impulsions du filtre adapté. Le rapport traite également de types plus généraux de modulation de signaux et comporte des commentaires sur les problèmes survenant lors de l'application particulière d'ondes FM à la sismique réflexion dans l'environnement marin. Enfin, le rapport comprend une annexe sur les unités physiques pertinentes à la sismique marine.

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Executive summary

Considerations for using swept-frequency waveforms in marine seismology

Brian H. Maranda; DRDC Atlantic TM 2012-264; Defence R&D Canada – Atlantic; January 2014.

Background: In reflection seismology, as used for oil prospecting, an acoustic source first transmits a propagating wave into the earth. Any reflected energy is recorded on receivers and then used to construct a mapping of the physical structure below the earth's surface. For marine seismology, the acoustic sources and receivers are usually towed in the water column by a ship. The underwater sources used in current practice, such as air guns, are impulsive in nature. Although impulsive waveforms satisfy the requirements of seismic exploration from a purely technical standpoint, they attain high peak pressures that have raised questions about their environmental impact. An alternative approach is to use *frequency-modulated* (FM), or *swept-frequency*, waveforms.

Results: A key component of the signal processing for FM pulses is the matched filter, which is described in the paper at an introductory level. Examples are given to illustrate the excellent resolving capability of the linear-FM pulse, which is a consequence of the pulse-compression property of the matched filter. In seismology, high spatial resolution is desired in order to map small structural features. Also discussed are other, more general, kinds of signal modulation. Comments are then made on issues that arise in the specific application of FM waveforms to reflection seismology in the marine environment. Finally, the report also contains an Annex on the physical units relevant to marine seismology.

Significance of the work: The work presented herein is not original, but is intended to be an accessible introduction to the concept of pulse compression and its application to reflection seismology. It will be useful to those workers who need to understand the basics of matched filtering and pulse compression without excessive mathematical overhead.

Sommaire

Considerations for using swept-frequency waveforms in marine seismology

Brian H. Maranda ; DRDC Atlantic TM 2012-264 ; R & D pour la défense Canada – Atlantique ; janvier 2014.

Contexte : En sismique réflexion, une source acoustique servant à l'exploration pétrolière émet d'abord une onde de propagation dans la terre. Les récepteurs enregistrent toute énergie réfléchie, puis celle-ci sert à modéliser la structure physique sous la surface de la Terre. En sismique marine, les sources acoustiques et les récepteurs sont habituellement remorqués derrière un navire dans la colonne d'eau. Les sources sous-marines actuelles, comme les canons à air, sont impulsives. Bien que les ondes impulsives respectent les exigences liées à l'exploration sismique d'un point de vue purement technique, la pression acoustique de crête élevée atteinte par celles-ci a soulevé des questions quant aux répercussions sur l'environnement. Une autre méthode consiste à utiliser des ondes *modulées en fréquence* (FM) ou sous forme *d'un balayage de fréquences*.

Résultats : Le rapport comporte une introduction au filtre adapté, qui constitue l'un des éléments clés du traitement de signaux d'impulsions FM. On y donne des exemples en vue de montrer l'excellente capacité de résolution d'une impulsion de modulation linéaire en fréquence, capacité qui découle de la propriété de la compression d'impulsions du filtre adapté. En sismique, une résolution spatiale élevée est souhaitable pour modéliser de petites caractéristiques structurales. Le rapport traite également de types plus généraux de modulation de signaux et comporte des commentaires sur les problèmes survenant lors de l'application particulière d'ondes FM à la sismique réflexion dans l'environnement marin. Enfin, le rapport comprend une annexe sur les unités physiques pertinentes à la sismique marine.

Portée : Le contenu du présent rapport n'est pas original ; il s'agit plutôt d'une introduction au concept de la compression d'impulsions et à l'application de celle-ci à la sismique réflexion. Il sera utile aux travailleurs qui doivent comprendre la base du filtrage adapté et de la compression d'impulsions sans nécessiter de connaissances mathématiques considérables.

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Acknowledgements

This work was partially funded by the Offshore Energy Environmental Research (OEER) Association through a joint Canada – OEER Association Collaborative Agreement, dated March 2010 and entitled "Feasibility of a Marine Vibroseis System to Minimize Potential Impacts of Seismic Surveying on Commercial Marine Invertebrates".

1 Introduction

This report has been written as part of a joint research project between Defence Research and Development Canada (DRDC) Atlantic and the Offshore Energy Environmental Research (OEER) Association. One of the research goals is to increase the understanding of alternative geophysical exploration methods for the marine environment. Given that DRDC Atlantic has considerable expertise in underwater transducer technology, our interest is in the potential of using a coherent swept-frequency source instead of impulsive sources such as air guns.

The purpose of this report is to highlight many of the issues that need to be addressed when applying swept-frequency waveforms to geophysical prospecting in the marine domain. Many of these issues pertain also to land-based applications, since many relationships in signal theory are based on basic mathematical laws and therefore delimit the truly fundamental constraints in system performance, regardless of the application. It is hoped that a discussion of these signal properties will help to pinpoint the signal features that are most important for seismic prospecting, and therefore guide the choice of parameters for the engineering design of a coherent source. The level of exposition is intended to be accessible to the general technical reader, providing an intuitive understanding of what the pertinent issues are. However, the report also identifies areas where further research is needed.

In the rest of this introductory section, an overview is given of the seismic reflection method of geophysical prospecting. The following sections discuss the detectability and resolving capability of waveforms, with an emphasis on *frequency-modulated* (FM), or *swept-frequency*, pulses. A key component of the signal processing for such pulses is the matched filter, which is described at an introductory level. Examples are given to illustrate the excellent resolving capability of the linear-FM pulse, which is a consequence of the pulse-compression property of the matched filter. Also discussed are other, more general, kinds of signal modulation. Comments are then made on issues that arise in the specific application of FM waveforms to reflection seismology in the marine environment. Finally, the report also contains an Annex on the physical units relevant to marine seismology.

1.1 Reflection seismology

The most prevalent method of geophysical prospecting is *reflection seismology* [1]-[5]. For a land-based survey, a propagating seismic wave is first launched from a surface or near-surface source into the earth. In traditional work, the source is an explosive that generates an impulse-like wavelet. As the seismic wave propagates downward it encounters changes in acoustic impedance, such as occur at the interfaces between stratified layers of different materials; these interfaces are usually oriented close to the horizontal. When the propagating wave encounters an interface, part of the wave is reflected and part of it is transmitted through the interface. It is the reflected energy that makes reflection seismology possible: receivers at the surface record the reflections, which can be used to assemble a seismogram, or image of the sub-surface structure. The seismic reflection method clearly works on the same principle as sonar and radar, despite obvious technical differences (e.g., a radar system transmits electromagnetic waves).

In the early days of reflection seismology, a chart recorder was used to trace the outputs from geophones directly onto a paper chart. The drawback of such a simple method is that it is susceptible to the deleterious effects of interference and noise, which act to obscure the events of interest. By the term *interference* we refer to unwanted signal returns, such as multipath arrivals (or *multiples*), which confuse the picture and cause ambiguity in the interpretation of a seismogram. By *noise* we refer to the random background component of the received data. Although unwanted signal returns are most accurately called interference or reverberation, for brevity it is sometimes to convenient to refer to all unwanted components in the data as noise.

Techniques have been developed to suppress the effect of interference and noise. One procedure is to filter the data in order to eliminate unwanted spectral components; with modern equipment, the data can be reprocessed with different filters under the control of the data interpreter, who seeks the filter that best enhances the signal. Another important means for signal enhancement is to collect multiple data records for different source-receiver geometries having a common mid-point (CMP), as illustrated in Fig. 1. To combine the data records, a correction is first applied to each recorded trace to compensate for the normal moveout (NMO), that is, for the expected differences in travel times of the primary reflections for the different geometries. The result of the NMO correction is to time-align the primary reflections from geologic features at the CMP. The individual traces are then combined (*stacked*), in the simplest case by summing them. The primary reflections will coherently add, or be reinforced, while the noise and multiples will not be reinforced; the random noise adds incoherently, and the multiples for the different geometries are not aligned by the NMO correction.

1.2 Wave propagation

Although the basis of reflection seismology is the non-homogeneous nature of the medium, a consideration of wave propagation in a homogeneous medium will illustrate important basic phenomena. In a homogeneous medium, the amplitude of a spherical wave propagating outward from a point source depends on range r according to the equation

$$A(r) = \frac{A_0 r_0}{r} e^{-\alpha(r-r_0)}, \tag{1}$$

where r_0 is a reference range at which the amplitude has value A_0, and α accounts for attenuation due to absorption. The $1/r$ dependence represents spherical spreading, a purely geometrical effect. The corresponding propagation loss in decibels is given by $20\log_{10} r$,

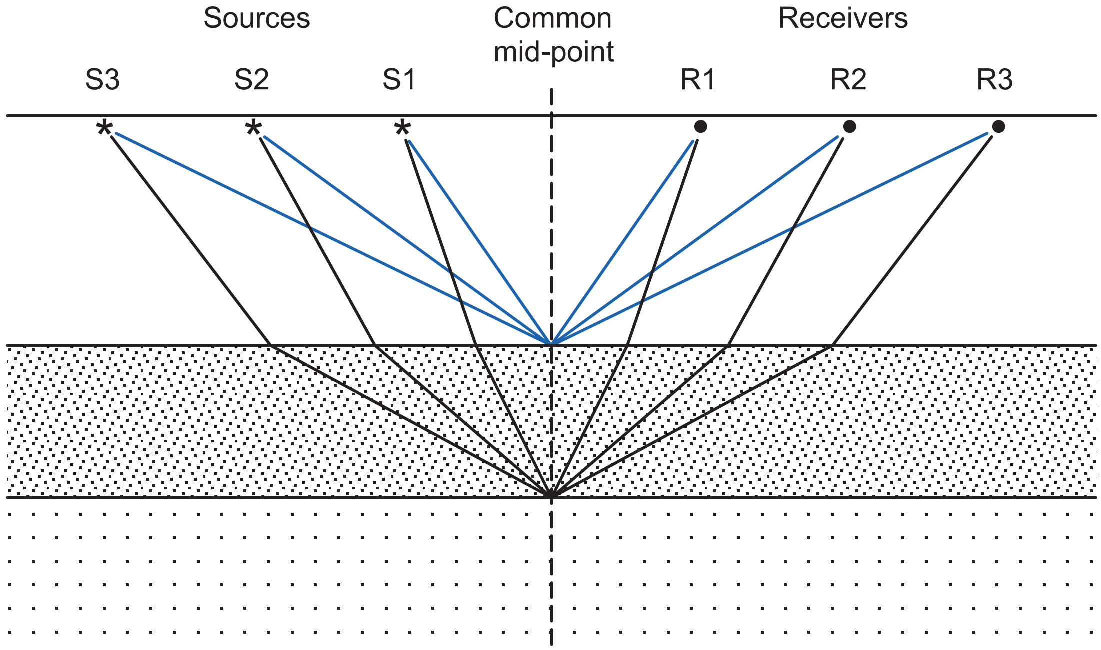

Figure 1: *Ray paths for threefold coverage of a common mid-point.*

where r is measured in meters if the source reference distance is 1 meter. For example, the geometrical spreading loss for one-way propagation to a distance of 1 km is 60 dB, a significant amount. In the marine seismic application, a loss of this magnitude may be incurred before the wavefront even encounters the ocean bottom.

In addition to the spreading loss, there is an attenuation term that depends on the material properties of the medium in which the wave propagates. This component of the loss generally arises through conversion of the wave energy into heat within the medium, and is called absorption. The absorption coefficient α is frequency-dependent, so that Eq. (1) can be considered as applying to each Fourier component of a propagating wave.

At the frequencies used in seismic surveying, the absorption coefficient of seawater is negligible; for example, at 300 Hz the absorption coefficient is a miniscule 10^{-5} dB/m [6, 7]. On the other hand, the solid materials in the earth may exhibit significant absorption at these frequencies. It is important to note that the absorption increases with frequency, so that the high-frequency Fourier components in a seismic wave are continuously reduced relative to the low-frequency components as the wave propagates. The effect is that the signal shape becomes more and more distorted with distance; a sharp impulse-like signal will lose its high definition, continually broadening as it propagates.

Several authors refer to the “seismic band” of frequencies, but there appears to be no consensus on its exact definition. Dobrin and Savit [1, p. 130] refer to the frequency band from 3 to 125 Hz as the “seismic band”; energy above this band is not considered as important for exploration requirements. Sun et al [8] refer to the frequency range from $\sim$5 to 200 Hz as the “seismic frequency band”. Higher-frequency signals may be used for attaining high spatial resolution, but with limited depth penetration.

1.3 Technology

Since this report is concerned primarily with the use of swept-frequency waveforms as an alternative to impulsive signals, it is appropriate to give special attention to the source technology that has seen employment in reflection seismology [1]. We start with land-based operations, as being the oldest historically, and then discuss technology for the marine environment.

The first type of source used for land-based seismic work was a high-energy explosive such as dynamite. Explosives have desirable properties when viewed purely from the standpoint of performance in geophysical prospecting: the energy density is high, and the resulting seismic signal is a compact wavelet that affords good spatial resolution. Despite these desirable characteristics, there are situations where the use of explosives is not suitable — for example, near densely populated areas or in environmentally sensitive regions.

Thus there was motivation for developing alternative source types, and of these we shall only consider the *vibroseis* system. In contrast to the traditional approach of using an impact to generate an impulse-like signal, the vibroseis system generates an FM seismic signal. It should come as no surprise that this technique was developed during the 1950’s and 60’s, since a necessary concept for the time-compression of FM pulses, the matched filter, was also a subject of research in that timeframe. In land-based operations the vibroseis method uses a large truck-mounted source, consisting of a heavy mass driven by a hydraulic vibrator. Multiple trucks are typically used simultaneously while conducting a survey.

For geophysical surveying of the ocean seabeds, the logistical problems are of course more difficult than for land-based work. In a standard configuration, a surface ship tows both an underwater source and an underwater receiver. Since the emphasis here is on source technology, the receivers will be briefly described in a later paragraph. As in land-based operations, explosives were the signal source first used for geophysical prospecting of the seabed. Again, although explosives have desirable properties, these properties are offset by other considerations that discourage their use. Numerous mechanical devices have therefore been developed to replace explosives as underwater sources, but the dominant source technology today is the *air gun* [9, 10].

However, the air gun generates an impulsive waveform with a high peak pressure, and public concern has been voiced about the possible environmental consequences; in particular, the effect of loud underwater sounds on whales and other marine mammals has been a matter of debate. For this reason, alternative technologies have been examined from the standpoint of environmental impact. It is here that the vibroseis technique appears to have an advantage, as the peak pressure from such a system is significantly less than that generated by an air gun [11]. However, employing FM waveforms in marine seismology is a more difficult engineering problem than in land-based operations. Many practical issues are more complex for marine operations: the deployment of large and heavy equipment, the supply of electrical power, maintenance and repair, etc.

Underwater projectors capable of generating FM waveforms for geophysical exploration have been described in the literature. Johnson et al [12] describe a marine vibrator built by Industrial Vehicles International (IVI), and Bird [11] presents results produced with this source using a linear FM sweep from 8 to 96 Hz. Another system is the deep towed acoustics / geophysics system (DTAGS) described in [13]. The underwater projector in DTAGS is a Helmholtz cavity driven by piezoelectric ceramic rings; several versions have been built, the most recent one having a bandwidth capability of 220 – 1000 Hz with a source level of $\sim$ 200 dB re 1 μPa @ 1 m (see Annex A for a review of the units used in underwater acoustics). A notable feature of the Helmholtz projector is that it can operate at great depth, so that spreading loss through the water can be reduced by towing the source close to the seabed. Nevertheless, the relatively high frequency of the source restricts the imaging depth to less than about 1 km into the seabed.

The receiver in marine seismology usually consists of one or more *seismic streamers*, also called *cables* (likely due to analogy with the wire cables that connected together geophones in land-based surveys). These streamers are called *towed arrays* when used by the military for submarine detection. The seismic streamer is constructed as a solid-core or oil-filled hose that is populated along its length with pressure-sensitive hydrophones. The streamer is towed through the water, maintaining a horizontal orientation either by being neutrally buoyant or by active depth-control mechanisms. Data from the hydrophones are telemetered through the tow cable to the tow ship, where the data are recorded for post-processing.

2 Detection of a signal in noise

In many systems, the fundamental limitation in detecting a signal is the background noise against which the detection must be made: as the signal gets weaker, eventually it becomes buried in the noise and cannot be reliably detected. In the seismic application, the effect of noise is to decrease the depth at which a seismogram will provide reliable data. Because the signal is attenuated as it propagates through the water and the sediment, reflections from deeper geologic structures will generally be weaker than those from near-surface structures (depending on the reflection coefficient). In practice, this means that structural detail on the seismogram becomes washed out as the depth increases, because the signals become too weak to be detected against the background noise. In marine seismology the background noise is ambient ocean noise, which has been the subject of extensive study [14].

One goal in system design is to improve the signal-to-noise ratio (SNR) achieved at the output of the signal processor. Often such an increase is achieved by increasing the SNR at the processor input, although this may come at a significant engineering cost: the source level may be increased by building a more powerful acoustic projector, or the noise performance of the sensors may be improved in some fashion (e.g., in a towed array, groups of hydrophones are usually combined at the hardware level in order to average out uncorrelated flow noise). Another approach is to increase the processing gain, defined as the increase in the SNR from the input to the output of the signal processor. The processing gain depends on the type of signal that is used, and the method by which the signal is processed.

The main goal of this section is to introduce the matched filter, which is essential for processing FM waveforms, but for comparison purposes the simpler energy detector will also be discussed. It should be understood that the energy detector is not generally used in seismic processing, as the signal polarity is removed by rectification. Because information is provided by the sign of a reflected signal, techniques that preserve polarity are preferred. Such techniques include least-squares filtering as described in [15]. Nevertheless, there are simple analytical formulas for the detection performance of the energy detector, which the author believes will be indicative of the performance of conventional seismic signal processors that are less amenable to analysis.

2.1 Notation

There are several ways to categorize signals. The type of signal of most interest for this report is a transient signal having a time duration ranging from tens of milliseconds to tens of seconds. The signals will be considered deterministic (that is, not random), although the structure of the signal may not be known in complete detail. For example, the impulsive wavelet produced by an air gun is approximately reproduced in each discharge, but the time series is not reproduced point for point. Such signals are sometimes called *unknown* *deterministic* signals. Much more accurately known are the coherent waveforms produced by underwater projectors, although there may be single parameters, such as the phase, that are unknown from waveform to waveform.

In what follows, the energy of a transient signal $x(t)$ will be defined as

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt, \tag{2}$$

where it is assumed that $x(t)$ is non-zero only on a finite interval. For simplicity it will be assumed that the noise background is additive white Gaussian noise with (one-sided) power spectral density N_0. Although it cannot be expected for the seismic application that the noise will be white across the frequency band occupied by the signal, the assumption of white noise allows us to access a body of theoretical results that provide insight into the issues that affect detection performance. The signal-to-noise ratio will be defined as $\gamma = E_x/N_0$, i.e., as the total signal energy over the noise power spectral density.

Several special functions will be used in the following discussion. The incomplete Gamma function will be defined by

$$\Gamma(n,x) = \frac{1}{\Gamma(n)} \int_x^{\infty} t^{n-1} e^{-t} dt, \tag{3}$$

where $\Gamma(\cdot)$ is the standard Gamma function. Here it is assumed that $x \geq 0$.

The Marcum Q-function is defined by

$$Q(a,b) = \int_b^{\infty} t \exp\left(\tfrac{1}{2}(t^2 + a^2)\right) I_0(at)\,dt, \tag{4}$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind of order zero. A generalized version of the Marcum Q-function is defined by

$$Q_N(a,b) = \int_b^{\infty} t \left(\frac{t}{a}\right)^{N-1} \exp\left(\tfrac{1}{2}(t^2 + a^2)\right) I_{N-1}(at)\,dt, \tag{5}$$

which reduces to the standard Q-function when $N = 1$.

2.2 Energy detector

The processing performed by the energy detector is quite simple (see Fig. 2): the input signal-plus-noise is fed through a filter that rejects out-of-band noise, is squared, and then averaged. For this detector to perform well, the passband of the front-end filter must be matched to the bandwidth W of the input signal, and the integration time of the back-end integrator must be matched to its duration T. For making binary decisions, the detector output is compared to a threshold, and a signal is declared to be present when the threshold is exceeded. However, the detector output may also be presented visually as a line on a display for subsequent interpretation by a person. When this is done, the human interpreter can extract more information from the detector by observing the height and structure of the peaks, etc.

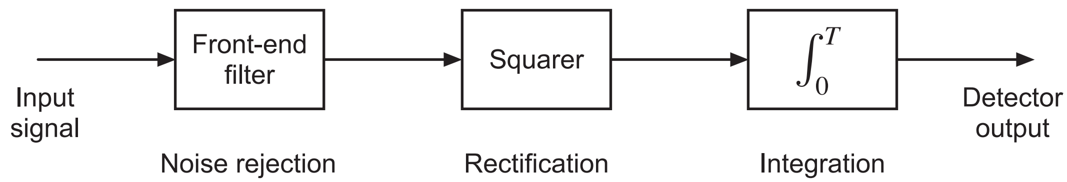

Figure 2: *Block diagram of an energy detector.*

Although the energy detector is simple in its structure, an exact analysis of its performance is complicated by the square-law rectifier that follows the front-end filter. An approximate method of analysis can be found in the paper [16], and essentially the same method can be found in several textbooks [17, 18, 19]. The method is based on approximating a waveform having time-bandwidth product TW by $2TW$ discrete samples. When the background noise has a Gaussian distribution, the sum of squares of these samples at the detector output is distributed as a chi-squared random variable of $2TW$ degrees of freedom, the distribution being either central or non-central depending on whether a signal is present or not. This method has excellent accuracy when the value of TW is large, and analysis presented in [20] suggests that the approximation is suitable even for values as low as $TW = 3$ or 4.

Based on the chi-squared approximation, the probability of detection for the energy detector can be written in terms of the generalized Marcum Q-function in Eq. (5),

$$P_d = Q_{TW}(\sqrt{2\gamma}, \sqrt{2\eta}), \tag{6}$$

where η is a normalized threshold; recall that $\gamma = E_x/N_0$ is the SNR. The probability of false alarm is obtained by taking $\gamma \to 0$ in Eq. (6), and it can be shown that the resulting expression is given in terms of the incomplete Gamma function as

$$P_{fa} = \Gamma(TW, \eta). \tag{7}$$

These equations are employed as follows. First, given the TW of the energy detector and a specified value for P_{fa}, the threshold η is determined from Eq. (7). This value for the threshold is then used in Eq. (6) to compute the value of P_d for a specified value of the SNR γ. Alternatively, Eq. (6) can be used to solve numerically for the SNR required to achieve a desired value of P_d. A numerical example will be presented below.

2.3 The matched filter

The matched filter is a ubiquitous feature of modern signal processing, being used in radar, sonar, data communications, seismology, and no doubt in other applications as well. The matched filter is the linear filter that maximizes the output SNR for a given input signal in additive noise. Also, when the noise is Gaussian, it can be shown by statistical decision theory that the matched filter yields the optimum detection performance. For the reader interested in how the filter is derived mathematically, the author recommends standard textbooks on detection theory [21, 22] or on radar [23, 24, 25].

When the additive Gaussian noise is white (spectrally flat), the matched filter takes on a particularly simple and intuitively attractive form: its impulse response is just a time-reversed replica of the signal. That is, if the signal is given by $x(t)$ on an interval $[0,T]$, then the impulse response of the matched filter is $h(t) = x(T-t)$ on $[0,T]$. The matched filter is often implemented via the correlation receiver, illustrated in Fig. 3(a). Here the incoming waveform (signal plus noise) is multiplied by a replica of the signal $x(t)$ and integrated for the duration T.

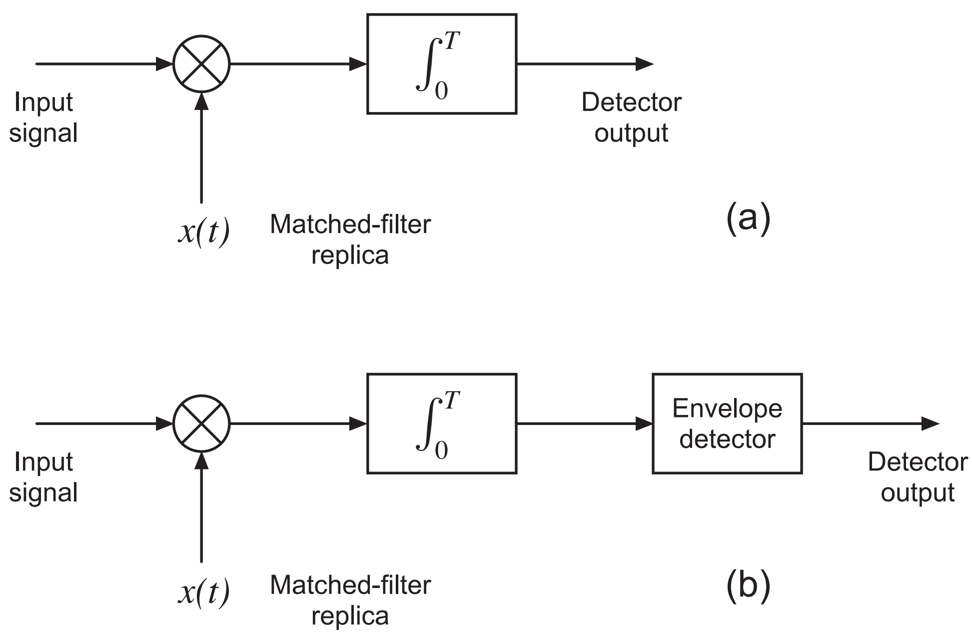

Figure 3: *Block diagram of a matched-filter detector implemented as a correlator. (a) For a completely known signal. (b) For a random-phase signal.*

When implementing the matched-filter detector, it is necessary to be careful in choosing a model for the received signal. (Such precision in the signal model is unnecessary for the energy detector, which makes no assumptions about the signal other than its duration and bandwidth.) The simplest model assumes that the signal is known completely (the *coherent* model), which leads to the detector illustrated in Fig. 3(a). However, for a bandpass signal, it is often the case that the carrier phase of the received signal is unknown or random (a *non-coherent* model). The non-coherent detector, illustrated in Fig. 3(b), places an envelope detector at the output of the correlator. The problem with taking the envelope is that the polarity is removed, as for the square-law energy detector. Much more will be said about this later (Sec. 4.2), and for now it will be assumed that the non-coherent form of the matched filter is being used.

The probability of detection for the non-coherent detector in Fig. 3(b) can be expressed in terms of the Marcum Q-function as

$$P_d = Q(\sqrt{2\gamma}, \sqrt{2\eta'}), \tag{8}$$

where η' is again a normalized threshold; the prime is added in order to distinguish this threshold from the one appearing in the energy detector of Eq. (6). As before, the expression for P_{fa} is found by taking $\gamma \to 0$, and in this case we find simply

$$P_{fa} = e^{-\eta'}. \tag{9}$$

This expression can be inverted to yield $\eta' = -\ln P_{fa}$, and hence Eqs. (8) and (9) can be combined to yield the single equation [24, p. 395]

$$P_d = Q(\sqrt{2\gamma}, \sqrt{-2\ln P_{fa}}). \tag{10}$$

An important point concerning these expressions is that the time-bandwidth product does not appear in them. Indeed, one of the most remarkable properties of the matched-filter detector of a signal in additive white Gaussian noise is that the detection performance is independent of such basic signal parameters as duration and bandwidth [21, p. 175]. Consequently, the signal can be designed to have desirable auxiliary characteristics while leaving its detectability unaffected.

2.4 Detector comparison

The performance of the energy and matched-filter detectors is compared in Fig. 4. The operating point is set at $P_d = 0.5$ and $P_{fa} = 10^{-5}$ for both detectors, and the SNR required to maintain the operating point is computed as the time-bandwidth product TW changes. As noted before, the performance of the matched-filter detector is independent of TW, as indicated in Fig. 4 by the horizontal line at SNR γ =10.4 dB. On the other hand, it is seen from the plot that the performance of the energy detector degrades as TW increases. Although the energy detector is only a few decibels inferior to the matched filter for $TW =$ 10, the SNR differential when $TW = 1000$ is substantial at 11.1 dB.

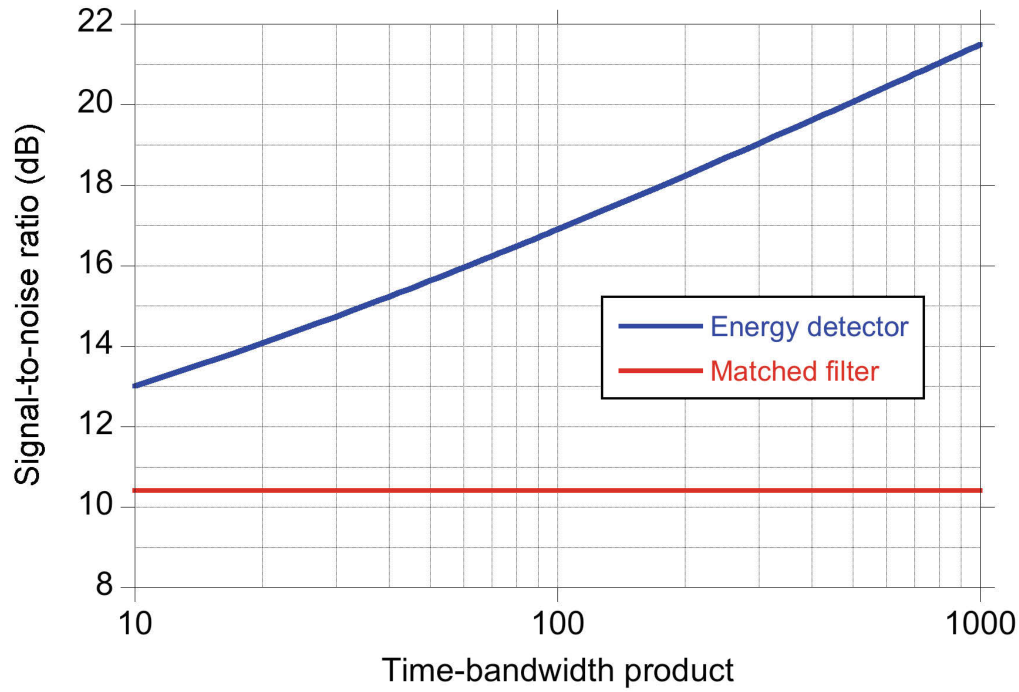

Figure 4: *Detection performance as a function of time-bandwidth product. The operating point is set at* $P_d = 0.5$ *and* $P_{fa} = 10^{-5}$.

One lesson to be drawn from this example is that the energy detector performs best for signals of low complexity. From an historical standpoint, in the period before the development of the matched filter, when the energy detector and even simpler detectors were the "only game in town", there was a good technical reason to use simple waveforms with low TW products. This fact influenced the techniques that were originally established in many fields, including seismology. The flexibility in signal design offered by the matched-filter detector is in stark contrast to the energy detector, whose performance degrades as the complexity of the waveform is increased.

3 Resolving capability and signal design

In this section, we explore the relationship between signal design and resolution. Here the concept of resolution pertains first of all to temporal resolution; that is, how much time separation must there be between two signals arriving at a receiver in order that they can be distinguished from each other. For signals that are propagating as waves, the achievable temporal resolution secondarily implies a certain quality of spatial resolution. For example, for a co-located source and receiver the round-trip travel time from a reflector at range R is given by $\tau = 2R/c$, where c is the propagation speed. Thus to resolve two reflectors that are separated by ΔR in range will require an ability to resolve two reflections (echoes) that are separated by $\Delta\tau = 2\Delta R/c$ in time.

In seismology, as in radar or sonar, it is usually desirable to obtain high spatial resolution, making it possible to distinguish between closely spaced reflectors. Now, the resolving capability depends strongly on the properties of the signal waveform*, and in general improves as the signal bandwidth increases. It is important to note, however, that signals with very different time-domain behaviors can possess wide bandwidths. For example, a short-duration impulsive signal will have a wide bandwidth, while a long-duration FM pulse may also have a wide bandwidth. Here the time-bandwidth product comes to our aid in categorizing signals, as the impulsive signal will have a low TW product and the FM pulse a high TW product.

In the infancy of radar and sonar, the only type of pulse was a gated sinusoid (also called a continuous-wave, or CW, pulse). The range resolution of such pulses is determined by their duration: short-duration CW pulses provide better range resolution than long-duration pulses simply because the echoes can be closer together in time without overlapping. For the same reason, in seismic exploration a transmitted wavelet (pulse) that is more compact in time will afford better spatial resolution of the reflectivity structure. On the one hand, such simple pulses have a low complexity (often with $TW \approx 1$), and hence are well suited for processing by simple detectors, as discussed in the previous Section. On the other hand there is a disadvantage, because by reducing the pulse duration in order to improve resolution, one also reduces the pulse energy and the detection performance in ambient noise.

In the early days of signal processing, it was believed that the only way to improve resolution was to shorten the pulse duration. A big step forward was made in signal-processing theory with the introduction of the matched filter, when it was realized that pulse compression via matched filtering can decouple the achievable time resolution, and hence the range resolution, from the pulse duration. In particular, for an FM pulse the amount of compression is determined by the signal bandwidth, and a long-duration pulse with sufficient swept

*In reflection seismology, the signal properties control the vertical spatial resolution, because the received signals are propagating nearly in the vertical direction. The along-range resolution is determined by other considerations.

bandwidth can be compressed to a short time span at the filter output. In effect there is additional freedom in the signal design, and instead of seeking to use signals with a low TW product the trend in radar was to use FM signals with a large TW product, reaching up even into the 1000's. In the following sub-sections we consider several well-known waveform types that possess desirable properties.

3.1 Linear frequency modulation

The linear frequency modulated (LFM) signal may be considered the prototypical swept-frequency waveform; it appears to have been the first type of swept-frequency waveform implemented in pulse-compression radars, and the radar literature devotes a lot of coverage to it [24, 25]. This type of waveform is often called a chirp waveform, owing to what it sounds like to a human listener when it lies in the aural frequency range. Its analytical form is

$$x(t) = A\cos\left[2\pi(f_0 t + \tfrac{1}{2}\beta t^2)\right], \qquad \text{for } -\tfrac{1}{2}T \leq t \leq \tfrac{1}{2}T. \tag{11}$$

The instantaneous frequency of this waveform is given by the time derivative of the expression in parentheses, and hence is $f_0 + \beta t$. It is seen that the frequency increases linearly with time over the frequency range from f_{lo} to f_{hi}, where

$$\begin{aligned} f_{\text{lo}} &= f_0 - \tfrac{1}{2}\beta T \\ f_{\text{hi}} &= f_0 + \tfrac{1}{2}\beta T. \end{aligned}$$

The parameter β, given by

$$\beta = \frac{f_{\text{hi}} - f_{\text{lo}}}{T},$$

determines the sweep rate and is measured in units of Hz/sec. The frequency spectrum of the LFM pulse cannot be expressed in simple functions, but at high values of TW the spectrum is approximately a rectangle over the range f_{lo} to f_{hi}, as would be intuitively expected [24, 25]; that is, the signal bandwidth is $W \cong f_{\text{hi}} - f_{\text{lo}} = \beta T$.

One of the most useful properties of the LFM pulse is that it is time-compressed at the output of the matched filter. One can think of the matched filter in this case as implementing a frequency-dependent delay, arranged such that the different frequency components arrive at the filter output at the same time; in consequence, the pulse energy is concentrated, or compressed, into a short-duration pulse at the filter output. The resolving capability of the LFM pulse after compression is therefore usually much greater than before compression. It can be shown [24, 25] that the time width of the compressed pulse (after envelope detection) is approximately given by the reciprocal bandwidth $1/W$, regardless of the original duration T. Hence it is the bandwidth of the pulse that is the important factor in determining resolution.

As an illustration using representative numbers for the seismology application, we consider an LFM pulse with a duration $T = 1$ s, sweeping from 20 to 120 Hz. The nominal bandwidth is $W = 100$ Hz and the time-bandwidth product is $TW = 100$. Figure 5 shows the envelope-detected output of the matched filter. The upper plot shows the curve on a linear amplitude scale, normalized so that the maximum is unity, and the lower plot shows the same curve on a decibel scale. Based on the approximate rule stated above, the width of the compressed pulse is expected to be about $(100 \text{ Hz})^{-1} = 10$ ms, while measurement shows that the actual width between the -3 dB points is 8.8 ms. The LFM pulse used in this example was generated and processed with no amplitude shading (a rectangular pulse), and the result of the sharp pulse edges is to generate sidelobes at a high level: the highest sidelobes are only -13 dB below the main lobe.

We next present an example that illustrates the resolving capability of the LFM pulse. Now two pulses are inserted in the same time series, but with a specified time delay between them. The delay is measured between the leading edges of the pulses, so that zero delay would correspond to complete overlap. The pulses have the same duration and bandwidth as before, $T = 1$ s and $W = 100$ Hz, and are given identical amplitudes. Two cases are shown in Fig. 6, with delays of 100 ms (upper plot) and 15 ms (lower plot). As before, the envelope of the matched-filter output is displayed. The significant point here is that the delays are much less than the 1-s pulse duration at the input to the matched filter. For the 100-ms delay, where the pulses are overlapped by 90% at the filter input, they are easily resolved at the filter output. The pulses can even be resolved when they are offset by a delay of only 15 ms.

We make a final comment on pulse compression that is trivial in itself but which may clear up confusion. It is sometimes stated that the compression factor of an LFM pulse is given by its time-bandwidth product TW. This is correct, but because the original pulse duration is T, the width of the compressed pulse is given by $T/(TW) = 1/W$ and hence depends only on the bandwidth. "Improving" the compression factor TW by increasing T in fact has no effect on the absolute width of the compressed pulse.

3.2 Non-linear frequency modulation

Despite the attention often given to the LFM waveform, there is no theoretical reason why the frequency should be swept linearly, and allowing more general behavior opens up further possibilities for signal design. In this sub-section we consider waveforms closely related to the LFM in the sense that the frequency sweep remains continuous. In the following sub-section we consider coded waveforms, in which the frequency or phase is allowed to jump discontinuously.

Goupillaud [26] examined different types of swept-frequency waveforms for the seismic application. He noted that the power of an LFM waveform is spread evenly across the signal bandwidth, whereas non-linear FM sweeps have the effect of allocating power unevenly across the spectrum. This latter property provides some flexibility in signal design; in particular, the frequency can be swept more slowly at the higher pulse frequencies in order

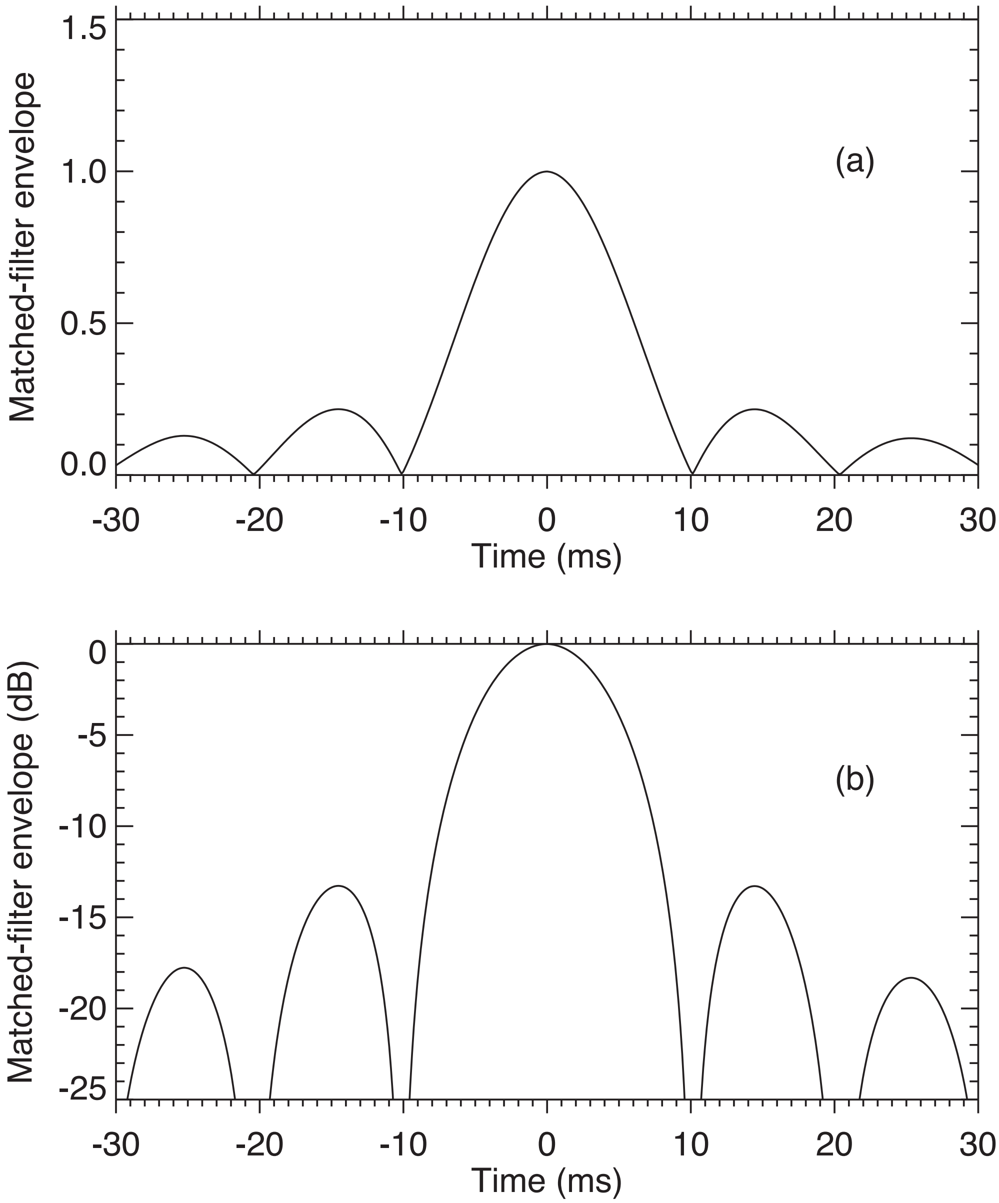

Figure 5: *The envelope-detected output of the matched filter for an LFM pulse having duration 1 s and bandwidth 100 Hz. (a) Linear amplitude scale, (b) decibel scale.*

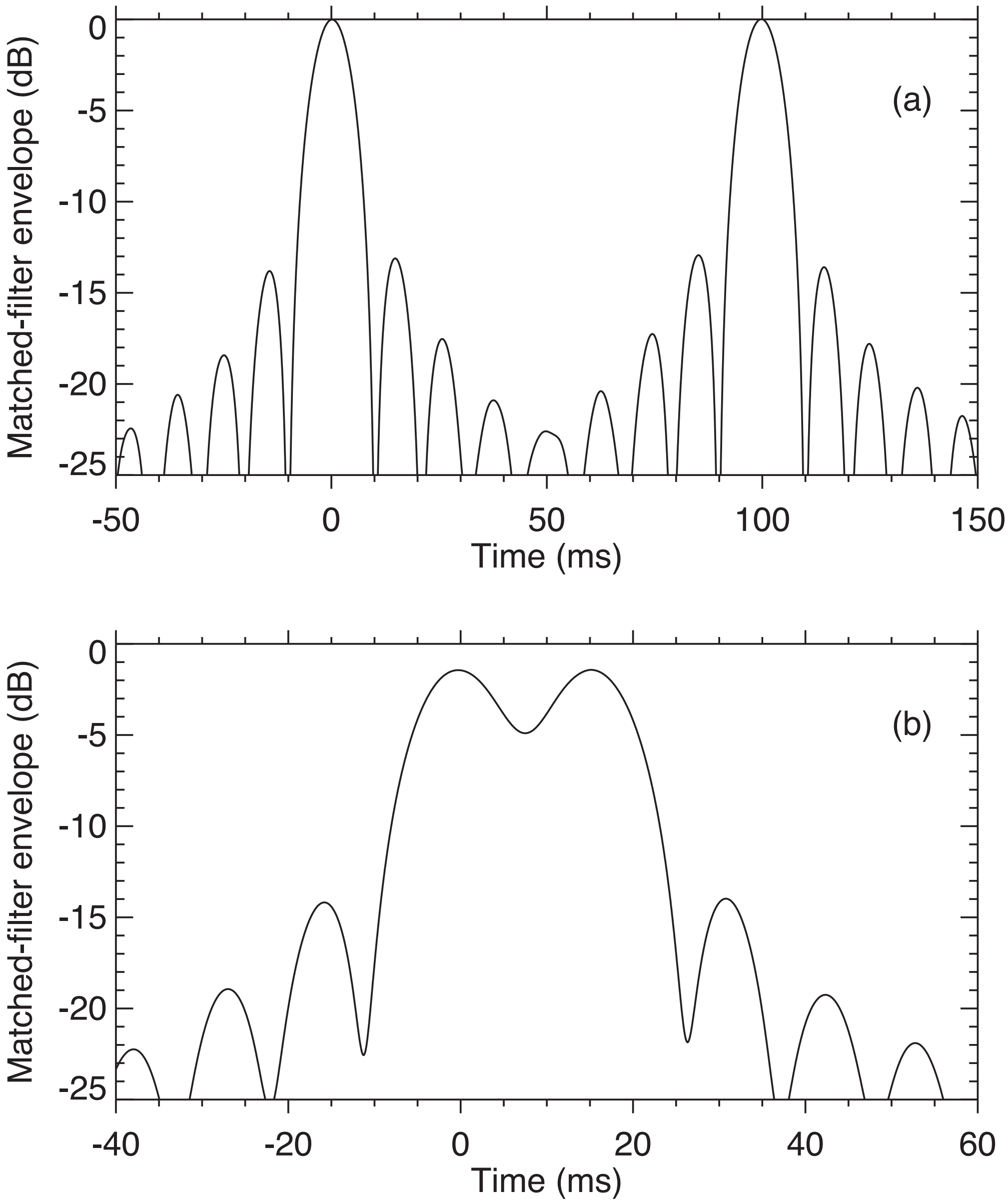

Figure 6: *The envelope-detected output of the matched filter when two identical LFM pulses are received with a time delay between them. (a) 100-ms delay, (b) 15-ms delay.*

to concentrate more energy there. The idea is to counteract the greater absorption loss that is encountered at high frequency, so that more energy at the top end of the frequency band is ultimately received. Specific types of non-linear FM sweep have been proposed for seismic exploration [5].

In sonar applications, a popular type of modulation is hyperbolic frequency modulation (HFM) [27]. Another name for HFM is linear periodic modulation (LPM) because hyperbolic FM results in a linear change in the instantaneous period of the signal (that is, in the reciprocal of its instantaneous frequency). The advantage of the HFM waveform for sonar is that Doppler distortion of the waveform does not greatly reduce the output level of the matched filter; in other words, the HFM waveform is Doppler tolerant [28, 29, 30]. The practical consequence is that the receiver does not have to implement a bank of Doppler-corrected matched filters for the HFM waveform, as is typically required for the LFM waveform. The disadvantage of the HFM signal in a seismic application is that it sweeps more slowly at the low frequencies and concentrates energy there, rather than at the higher frequencies where it is desired.

3.3 Coded waveforms

Another class of waveforms that can be used at high TW products are the coded waveforms. Such waveforms can often be designed to achieve specific goals, such as achieving high resolution in both time and frequency.

There are several approaches to implementing coded waveforms. One common type of waveform employs frequency hopping; that is, a single pulse comprises many CW sub-pulses (called *chips*), each of which has its own specific frequency. Among the frequency-hopped waveforms, the Costas family of waveforms has gained a lot of attention, in both the radar [24] and sonar [27, 31] domains.

A different type of coded waveform employs the same frequency for each chip, but the phase is changed discontinuously at the sub-pulse boundaries. The phase changes are usually specified by means of a pseudo-random sequence, and the waveforms themselves are called pseudo-random noise (PRN) waveforms [27]. It is possible to design a PRN signal set so that the waveforms in the set have low pairwise cross-correlation; such a signal set finds application in vibroseis work when it is desired to have simultaneous or near-simultaneous operation of multiple sources [32].

4 Using FM waveforms in marine seismology

The preceding material has illustrated many of the properties of swept-frequency waveforms. In the vibroseis application of these waveforms, a matched-filter processor precedes much of the standard signal processing that would be used for impulsive waveforms. That is, the matched filter would first be applied to a vibroseis data record, time-compressing the FM pulses encountered. The output data record is then subsequently processed as if the time-compressed pulses were impulsive waveforms, perhaps including deconvolution methods or other advanced techniques [3].

In this Section, some issues arising in the marine application of this technique are discussed. Many of these topics would be suitable for future research, and no attempt is made here to resolve outstanding issues.

4.1 Signal phase

If there is one thread that runs through much of the following material, it is the question of how to model or treat the phase of the received signal, as this will in turn affect how the signal processing must be structured, or place restrictions on parameters (e.g., pulse duration). In marine vibroseis, the effect of source and receiver motion on the received signal phase is a complicating factor that does not appear in land-based surveying.

In some systems, such as radar, the phase of the transmitted signal itself may in fact be random. For example, this situation occurs when the modulation is first induced on a low-frequency carrier that is subsequently heterodyned upward to a high frequency through a mixing process. For the high frequencies used in radar, the mixing would be accomplished using analog technology, and the radio-frequency carrier may be free-running (unsynchronized) relative to the signal-modulation circuitry before the mixer. The result would be a random phase on each transmitted signal.

One would expect to have much greater control over the phase attained in applications at acoustic frequencies, where even a bandpass signal is usually generated directly at the transmit frequency by an digital-to-analog converter. However, the input-output transfer function of an acoustic projector may be quite complicated, with unwanted phase variation across the frequency band where the signal is transmitted. One approach to handling this problem is to equalize the projector response, first measuring the response and then building a compensation (equalization) filter. Another approach is to record each transmitted waveform through an auxiliary sensor and then use the recorded waveforms as the replicas in the signal processing. Finally, a more complicated solution is to control the projector through a feedback loop from an auxiliary sensor in order to track the desired waveform; two phase-locked systems have been described [12, 33] in which the phase error in the transmitted waveform is less than 5° and 3°, respectively.

Even when the signal phase is accurately controlled at the transmitter, it may undergo modification as the pulse propagates. In a dispersive medium, the various frequency components of a propagating wave travel at different speeds through the medium [8], and hence the phase structure of the waveform is altered as the waveform propagates. For a CW pulse the dispersion may appear as a simple phase offset or rotation for the entire pulse, which would be correctly handled by a non-coherent receiver processor. For a swept-frequency waveform, however, the effect of dispersion is to progressively alter the internal phase structure of the waveform. As noted below in the Section on Doppler, source and receiver motion may cause effects similar to dispersion.

The effect of dispersion may place an upper bound on the duration that is realistically feasible for a swept waveform in geophysical prospecting in the marine domain. This is an important consideration, since each doubling of the pulse duration increases the energy by 3 dB; one of the least expensive methods of generating additional energy is to increase the pulse duration. [Note, however, that the width of the *compressed* pulse depends on the signal's bandwidth and not on its duration (Sec. 3.1).]

4.2 Envelope detection

As shown in Sec. 2.3, the form of the matched-filter detector is different for coherent and non-coherent signal models. The detector in the non-coherent case implements an envelope detector after the correlator [see Fig. 3(b)], the purpose of which is to remove the dependence on an unknown phase; as a result, the phase information of the signal is lost. In all applications of the matched filter in radar and sonar known to the author, the non-coherent processor is used; i.e., the envelope is taken. However, the bandpass FM signal used in seismic exploration represents an extreme case, because the lower band-edge is often positioned so low that the bandpass signal is more akin to a lowpass signal, spanning well over an octave. For this reason the envelope detector can usually be omitted in seismic processing; see the examples in [34, p. 94]. We now consider this option in more detail.

To illustrate the effect of omitting the envelope detector, two LFM pulses were processed with the coherent matched filter [see Fig. 3(a)]. Both pulses have a 1-s duration and a 100-Hz bandwidth, but the passbands are located at different frequencies. For the upper plot in Fig. 7, the passband is between 20 – 120 Hz, as was used for the examples shown earlier in this report; for the lower plot, it is between 300 – 400 Hz. The solid lines are the compressed pulses without envelope detection, while the dashed lines indicate the positive and negative envelope. Note that the envelope is the same in both plots, because the envelope is determined by the passband width W and not by its position on the frequency axis.

If we think about forming a seismogram directly from the raw compressed pulses, omitting the envelope detector, then there may be advantages in the first example (where the signal spans over two octaves) but much less so in the second example. In the first example, the compressed pulse is fairly well defined, having only one strong peak. In the second

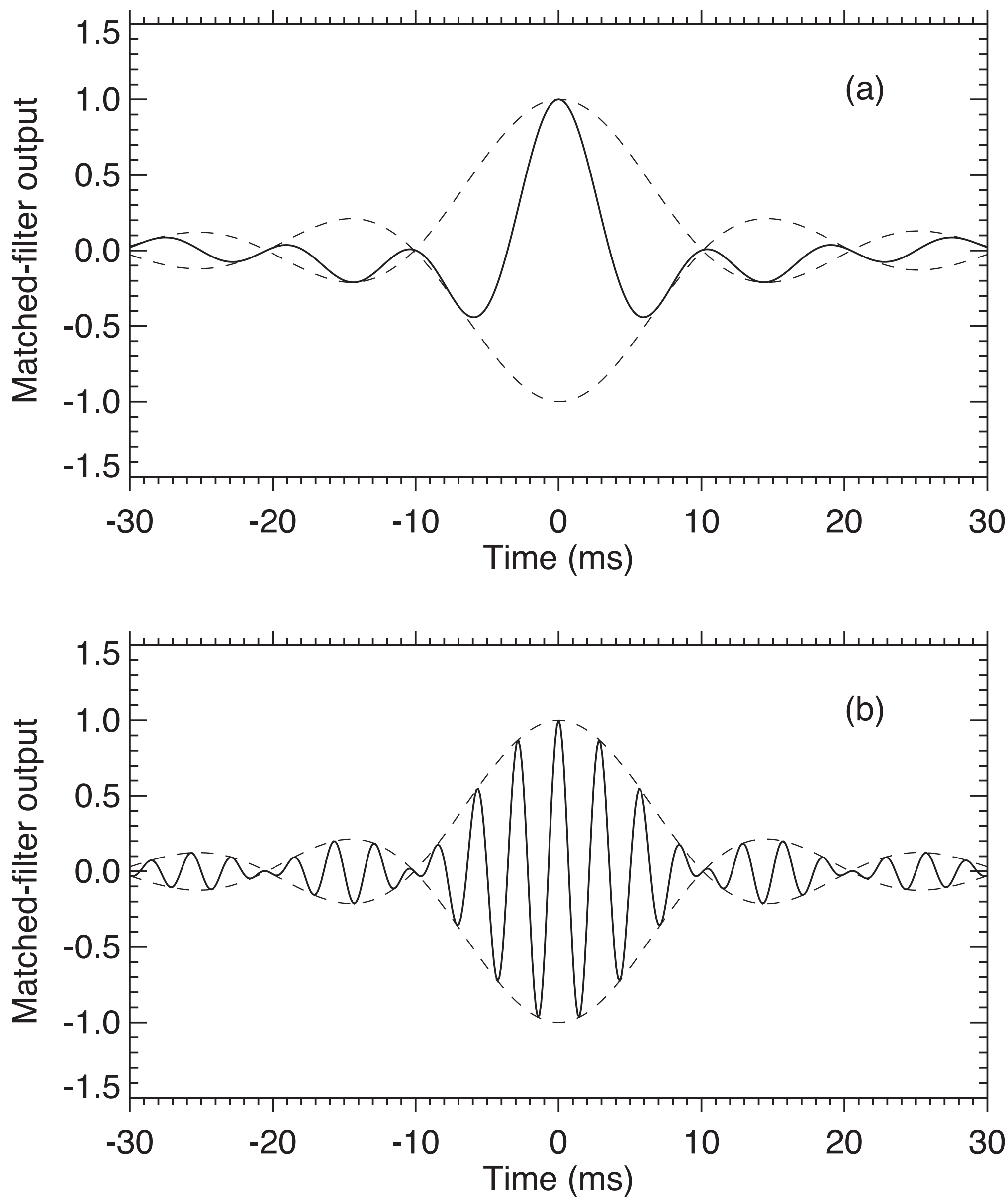

Figure 7: *The matched-filter output for LFM pulses of 1-s duration and 100-Hz bandwidth. The solid line indicates the compressed pulse without taking the envelope, and dashed lines indicate the envelope. (a) Passband from 20 to 120 Hz. (b) Passband from 300 to 400 Hz.*

example, the compressed pulse exhibits a large number of oscillations (at 350 Hz, the center frequency of the LFM pulse) that render the pulse less useful for forming a seismogram. One notable feature is that the processor without envelope detection is polarity-sensitive, as illustrated in Fig. 8. Two pulses have now been inserted into each time series, the second pulse having reversed sign and being delayed by 100 ms. Clearly it is easier to determine that the polarity is reversed in the upper plot, as compared to the lower plot; in a real application, even a slight perturbation of the phase would make it untenable to determine the polarity of the higher-frequency signal.

The plots just presented may make the idea of omitting the envelope detector appear attractive when the FM signal spans more than an octave or, equivalently, has a high fractional bandwidth*. However, so far only an ideal case has been presented, in which all the received signals have zero phase. Figure 9 demonstrates the effect of non-zero phase in the received signal, set at 100° for this example. The envelope is once again unaffected, but the raw compressed pulse is distorted, being almost zero at the correct delay time (as indicated by the peak of the envelope). Certainly one would have to be wary if multiple data records were stacked, as is usual with multifold coverage in a CMP gather. It is clear that sufficiently large phase dispersion in the received signal, or time mis-alignment before the stacking, could lead to cancellation instead of the desired coherent addition.

We recommend the usual approach of placing processing options into the hands of the data interpreter, enabling him or her to try out various options to determine their effect. Of relevance to this discussion is a recent paper [35] in which an attempt is made to "tune" the resolution of the compressed pulses by a heterodyning procedure that allows the user to adjust the position of the passband in the received data. Simulations using a full-wave propagation model would be useful in further investigation.

4.3 The Doppler effect

So far in this report, the Doppler effect has been mentioned only briefly. Given the combination of the very low signal frequencies and the low ship speeds involved in marine seismic exploration, one would surmise that Doppler effects should be negligible. However, there are several papers in the geophysical literature [36, 37, 38] which conclude that Doppler effects are important, particularly when there are dips (i.e., when the geological interfaces are sloped). Dragoset [36] states that the effect of Doppler is to cause phase dispersion in the received waveform. However, Hampson and Jakubowicz [38] suggest that there is no consensus view on the correct way to model the effect of source / receiver motion in marine vibroseis, and that proposed compensation methods are likewise not entirely consistent.

*The fractional bandwidth of a signal is defined as its bandwidth divided by its center frequency.

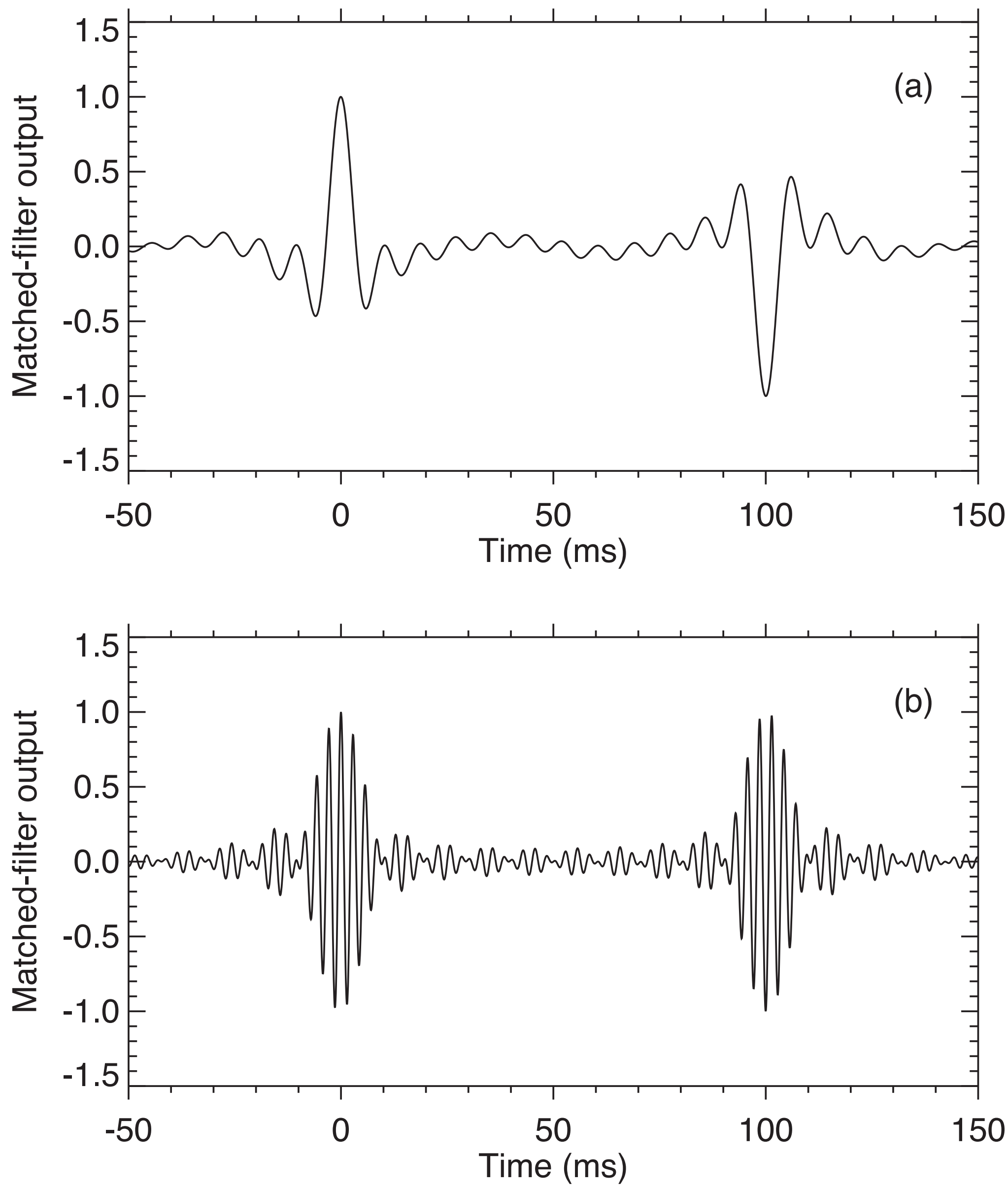

Figure 8: *The output of a matched filter for opposite-polarity LFM pulses having duration 1 s and bandwidth 100 Hz. (a) Passband from 20 to 120 Hz, (b) Passband from 300 to 400 Hz.*

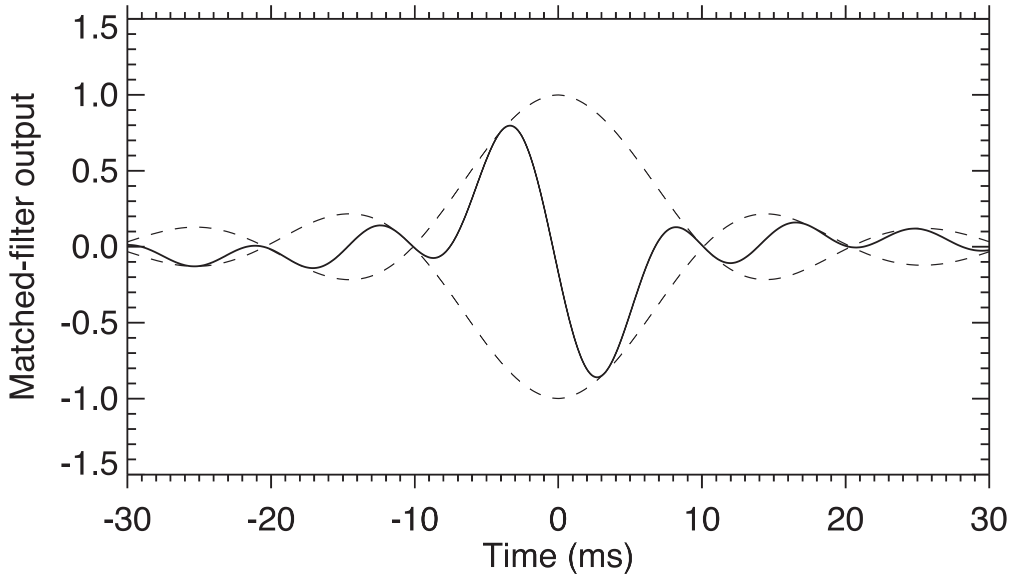

Figure 9: *The output of a matched filter for an LFM pulse having duration 1 s and bandwidth 100 Hz. The received signal no longer has zero phase.*

It was noted earlier (Sec. 3.2) that the HFM pulse is Doppler tolerant. An important point is that it remains Doppler tolerant even at the large fractional bandwidths that would be encountered in seismic exploration. The author is unaware if any research has appeared in the geophysical literature on the applicability of HFM waveforms to marine vibroseis, although it would appear to be a matter of interest.

In summary, the impact of the Doppler effect on the marine application of swept-frequency waveforms, and appropriate methods of compensation, are relevant topics for future research.

4.4 Sidelobe control

In Sec. 3.1, an example was given showing the resolution capability of the LFM waveform. In that example, the two signals had the same magnitude, and the sidelobes in the matched-filter output were no cause for concern. In a more general situation, however, the sidelobes of a strong pulse may obscure the presence of a weaker pulse. The standard rectangular pulse, with its discontinuous transitions at the leading and trailing edges, generates sidelobe levels that are only -13 dB below the mainlobe. The standard way of reducing the sidelobes is to shape the pulse envelope to make it smoother, a procedure called *shading*, *tapering*, etc.

For a completely matched system, both the transmitted pulse and the matched-filter replica would be shaded in the same way. However, the downside of this approach is that the energy of the transmitted pulse is reduced, at the same time that the system designer may be struggling to increase the pulse energy. Indeed, increasing the pulse energy by 1 dB by building a more powerful acoustic projector may be very expensive, and so the idea of reducing the energy by shading goes against the grain. Thus one finds in the radar literature [24, 25] the idea of deliberately introducing mismatch into the processing chain: an unshaded (rectangular) pulse is transmitted in order to maximize pulse energy, and sidelobe control is performed by shading only the processor replica. It should be noted that in some systems a rectangular pulse must be sent because the power amplifier in the transmitter is a switching amplifier that can only go from "rail to rail".

More investigation would be warranted to examine and assess the methods of sidelobe control that have been tried in vibroseis. One approach would be to transmit an unshaded pulse, and subsequently to enable the data interpreter to re-process the data with different shading options on the replica. This approach would provide the flexibility to look at different parts of the seismogram in different ways, instead of imposing a compromise solution across the entire image.

5 Conclusion

There are two basic approaches to achieving spatial resolution in seismic exploration: (1) use an impulsive signal with a relatively low time-bandwidth product, or (2) use a swept-frequency signal with a large time-bandwidth product, followed by a pulse-compression filter. This report examined the detection and resolution capability of the swept-frequency waveform, illustrating the pulse-compression concept through examples. Issues concerning the application of swept-frequency pulses to geophysical prospecting in the marine domain were discussed, and potential areas for future investigation were identified.

From a purely technical standpoint, impulsive waveforms as produced by air guns satisfy the requirements of seismic exploration. However, these impulsive waveforms attain high peak pressures, and there is concern about their environmental impact. For the swept-frequency pulse, the peak pressures are generally much lower, more through necessity than through design. That is, the swept waveform must be produced through a well-controlled transduction process; in underwater work, the required projector will typically use a piezo-electric ceramic to convert an electrical signal into an acoustic wave. This kind of source technology cannot attain the peak power that can be generated by an impulsive source. The use of a matched filter to perform pulse compression of a swept-frequency pulse is then essential, as the pulse duration can be increased in order to yield more pulse energy without a concomitant loss in resolution. Ultimately, however, the application of the vibroseis technique in marine seismic exploration hinges on the engineering of a transducer capable of producing sufficient power over the desired frequency band.

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Annex A: Units

In this section we consider the units for several acoustical quantities of interest. The issue of units, although at first glance a simple one, has been a topic of debate because units are sometimes treated in an inconsistent manner or with different conventions by different researchers. Carey, in an editoral [39] that appeared in the IEEE Journal of Oceanic Engineering in 1995, discussed the standard definitions for sound levels in underwater acoustics. This editorial prompted additional articles [40, 41], and Carey again took up the topic in 2006 with a much longer article [42]. Another article specifically addresses the issue of marine seismic energy sources [43].

The above articles have certainly helped to clarify what the issues are, and the following discussion is meant to provide general coverage of the area. Some general remarks are made in the next sub-section. The following sub-sections consider the spectral analysis of both transient and random-noise signals. The discussion wraps up with comments on a few topics that can lead to confusion, such as the difference between one-sided and two-sided spectrum levels.

A.1 General remarks

Before embarking on a detailed discussion, a few points will be clarified. First of all, it is common in underwater acoustics to omit the acoustic impedance ρc from the dimensional analysis. For example, the square pressure (having units Pa^2) and integrated square pressure (units $\mathrm{Pa}^2 \cdot \mathrm{s}$) are conventionally referred to as "power" and "energy", when technically speaking they should be divided by the acoustic impedance (having dimensions of *rayls*) to obtain intensity* in $\mathrm{W/m}^2$ and energy flux in $\mathrm{J/m}^2$. Although this convention is open to objection, it is nevertheless almost universally adopted in the underwater acoustics community and will be used without further comment.

Also, the reader is assumed to be conversant with the use of decibels (see [42], or standard textbooks [6, 44]). A quantity stated in decibels is called a *level*. For example, the power spectral density of random noise, when stated or plotted on a decibel scale, is called the *noise power spectrum level*. However, there is little consistency in the nomenclature used by different researchers, and one encounters almost all possible variations; in the example just given, the terms *noise spectrum level* or simply *spectrum level* are often used when the context is understood.

*The relation $I = \langle p^2 \rangle / \rho c$ holds exactly only for plane and spherical waves [6], but is approximately true in the far-field of an arbitrary source when the wavefront is almost planar.

A.2 Transient signal

The spectral analysis of a transient or impulsive waveform $x(t)$, assumed to be deterministic, begins with the Fourier transform

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-i2\pi ft} dt. \tag{A.1}$$

The *energy spectral density* of the signal $x(t)$ is given by $S_{xx}(f) = |X(f)|^2$. When $x(t)$ is a pressure with units μPa, it follows directly from the dimensional analysis of integral (A.1) that the units of $X(f)$ are $\mu\text{Pa}\cdot\text{s}$. Hence the units of $S_{xx}(f)$ are $\mu\text{Pa}^2\cdot\text{s}^2$, which can be written more meaningfully in the form of an energy spectral density, $\mu\text{Pa}^2\cdot\text{s/Hz}$. The physical significance of the energy spectral density follows from Parseval's theorem, which states that the integral of $S_{xx}(f)$ will yield the total signal energy in $\mu\text{Pa}^2\cdot\text{s}$,

$$E_x = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-\infty}^{\infty} S_{xx}(f)\, df.$$

The theory can also be carried out in terms of the correlation function defined by

$$R_{xx}(t) = \int_{-\infty}^{\infty} x(u)x(u+t)\, du, \tag{A.2}$$

as a simple derivation based on Fourier theory shows that

$$S_{xx}(f) = |X(f)|^2 = \int_{-\infty}^{\infty} R_{xx}(t) e^{-i2\pi ft} dt.$$

That is, the spectral density is the Fourier transform of the correlation function.

In decibels, the energy spectral density is termed the *energy spectrum level*; it indicates the frequency region(s) where the signal energy is concentrated. The total received energy in decibels is called the *sound energy level* or *sound exposure level* (SEL) in dB re 1 $\mu\text{Pa}^2\cdot\text{s}$. When the SEL is measured at a reference point 1 m from the source (usually by making a far-field measurement and applying a range correction), it is termed the *energy source level* (ESL), stated in dB re 1 $\mu\text{Pa}^2\cdot\text{s}$ @ 1 m. Finally, for a flat-topped pulse of duration T, the *source level* (SL) can be computed using the equation

$$\text{SL} = \text{ESL} - 10\log T.$$

The source level is a measure of radiated acoustic power, stated either in dB re 1 μPa^2 @ 1 m or in dB re 1 μPa @ 1 m (see Sec. A.5). More generally, SL can be computed for a signal of any kind, in which case it represents the mean power over the time T.

A.3 Random noise

The background ambient noise in the ocean is most appropriately modeled as a stochastic process, which for analytical simplicity is often considered stationary. The assumption of stationarity can often be justified when the duration of the analysis is on the order of the pulse duration. Historically, one of the impediments to developing methods for the spectral analysis of random processes was the recognition that the Fourier integral (A.1) is not directly applicable: a stationary noise signal $n(t)$ persists for all time, and hence the integral will not converge. It would take us too far afield to give a full development of the spectral density in this case, and the reader may consult standard references [21, 45]. The basic result can be summarized by the Wiener-Khinchine theorem, which is now discussed.

First, we define the correlation function of the noise process $n(t)$ (assumed here to have zero mean) as

$$\mathcal{R}_{nn}(\tau) = \mathcal{E}\{n(t)n(t+\tau)\}, \tag{A.3}$$

where $\mathcal{E}$ denotes statistical expectation. The stationarity of the noise process is reflected in the fact that the correlation function depends only on the time separation τ and not on the time t. A different symbol has been used for the correlation function in (A.3) to distinguish it from the correlation function (A.2) for deterministic signals. The Wiener-Khinchine theorem then states that the correlation function $\mathcal{R}_{nn}(\tau)$ and the power spectral density $\mathcal{S}_{nn}(f)$ of the random noise $n(t)$ are Fourier transform pairs:

$$\begin{aligned} \mathcal{S}_{nn}(f) &= \int_{-\infty}^{\infty} \mathcal{R}_{nn}(\tau) e^{-i2\pi f\tau} d\tau \\ \mathcal{R}_{nn}(\tau) &= \int_{-\infty}^{\infty} \mathcal{S}_{nn}(f) e^{i2\pi f\tau} df. \end{aligned} \tag{A.4}$$

It can be shown that $\mathcal{S}_{nn}(f) \geq 0$ at all frequencies.

The units here are different from those in the deterministic case. For a noise pressure $n(t)$ having units of μPa, the correlation function (A.3) has units μPa^2 and $\mathcal{S}_{nn}(f)$ as given by (A.4) has units $\mu\text{Pa}^2/\text{Hz}$.

A.4 One- and two-sided representations

The development above has been carried out in terms of two-sided spectra; that is, the spectra are defined for both positive and negative frequencies. For real-valued signals, as are considered here, these two-sided spectra will exhibit symmetry about the origin (zero frequency); hence it is possible to work entirely with spectra defined only on the positive part of the frequency axis, that is, with one-sided spectra. The choice of working with either one-sided or two-sided spectra is a matter of convention, but it is important when dealing with physical units to maintain the proper bookkeeping. The basic normalization for a density is that it must integrate to the correct total value. Thus the two-sided power spectral density of a noise signal, when integrated over the entire line, must equal the total noise power:

$$P_n = \int_{-\infty}^{\infty} \mathcal{S}_{nn}(f)\, df.$$

It can be shown that $\mathcal{S}_{nn}(-f) = \mathcal{S}_{nn}(f)$, and so the previous result can be obtained by integrating over positive frequencies only,

$$P_n = 2\int_{0}^{\infty} \mathcal{S}_{nn}(f)\, df.$$

This last equation leads to the definition of the one-sided power spectral density as

$$\mathcal{G}_{nn}(f) = \begin{cases} 2\,\mathcal{S}_{nn}(f) & \text{for } f \geq 0 \\ 0 & \text{for } f < 0 \end{cases}$$

for which

$$P_n = \int_{0}^{\infty} \mathcal{G}_{nn}(f)\, df.$$

Thus the one-sided power or energy densities are greater by a factor of two compared to the two-sided values.

It is conventional to state (and plot) spectrum levels using the one-sided representation. For example, commercial spectrum analyzers display power spectra that are calibrated according to the one-sided convention. The advantage of this convention is that it is not necessary to insert a factor of two when performing band analysis. For example, given an ambient-noise spectrum level of 60 dB re 1 $\mu\text{Pa}^2/\text{Hz}$, assumed to be flat over a frequency band having a width of 100 Hz, the total noise power entering through that band (the so-called *band level*) can be directly computed as 60+10 log(100) = 80 dB re 1 μPa^2. If the two-sided convention were used, the spectrum level would be stated as 57 dB re 1 $\mu\text{Pa}^2/\text{Hz}$ and the total power entering through a 100-Hz bandwidth would have to be calculated as 57+10 log(200) = 80 dB re 1 μPa^2. The band level must of course work out to the same value using either representation, but the first (one-sided) representation is more commonly used.

The one-sided version of the Wiener-Khinchine relationship can be stated explicitly as [46]

$$\mathcal{G}_{nn}(f) = 4\int_{0}^{\infty} \mathcal{R}_{nn}(\tau)\cos(2\pi f\tau)\, d\tau$$

and

$$\mathcal{R}_{nn}(\tau) = \int_{0}^{\infty} \mathcal{G}_{nn}(f)\cos(2\pi f\tau)\, df.$$

In modern work these relations are not much used, as the two-sided Fourier formulas are the more natural and convenient. However, when spectral values have initially been computed on a two-sided basis, it must be remembered to insert a factor of two when computing the spectrum level.

A.5 Ambiguities in the handling of units

The use of decibels can lead to ambiguity in the reference units. The rule is that $10\log(\cdot)$ is used to compute decibels for squared quantities, such as energy or power, and $20\log(\cdot)$ for linear quantities, such as signal amplitudes. However, the simplicity of converting between the two forms can lead to the use of apparently different units when referring to the same quantity. For example, the units of power spectral density that follow from the definitions above are $\mu\mathrm{Pa}^2/\mathrm{Hz}$, but owing to the equivalence

$$10\log\left(\frac{\mathcal{G}_{nn}(f)}{\mu\mathrm{Pa}^2/\mathrm{Hz}}\right) = 20\log\left(\frac{\sqrt{\mathcal{G}_{nn}(f)}}{\mu\mathrm{Pa}/\sqrt{\mathrm{Hz}}}\right)$$

one may encounter the units of power spectral density stated in the apparently non-physical form $\mu\mathrm{Pa}/\sqrt{\mathrm{Hz}}$. Note that the numerical value in decibels will be the same; for example, an ambient-noise spectrum level of 60 dB re 1 $\mu\mathrm{Pa}^2/\mathrm{Hz}$ and 60 dB re 1 $\mu\mathrm{Pa}/\sqrt{\mathrm{Hz}}$ in fact mean the same thing. However, on a linear (non-decibel) scale, the power spectral density of the noise would be stated as either $10^6\,\mu\mathrm{Pa}^2/\mathrm{Hz}$ or $10^3\,\mu\mathrm{Pa}/\sqrt{\mathrm{Hz}}$. Many researchers today would frown upon the usage in the classic text by Urick [44], in which a noise spectrum level is stated in dB re 1 μPa with an implicit understanding that the noise is measured in a 1-Hz band.

A similar manipulation can occur for the energy spectral density:

$$10\log\left(\frac{G_{xx}(f)}{\mu\mathrm{Pa}^2\cdot\mathrm{s}/\mathrm{Hz}}\right) = 20\log\left(\frac{\sqrt{G_{xx}(f)}}{\mu\mathrm{Pa}/\mathrm{Hz}}\right) = 20\log\left(\frac{\sqrt{2}\,|X(f)|}{\mu\mathrm{Pa}/\mathrm{Hz}}\right).$$

Hence one may cite values of the energy spectrum level in either dB re 1 $\mu\mathrm{Pa}^2\cdot\mathrm{s}/\mathrm{Hz}$ or dB re 1 $\mu\mathrm{Pa}/\mathrm{Hz}$. The rightmost expression illustrates why the energy spectrum level may also be called the amplitude spectrum, in which case the units dB re 1 $\mu\mathrm{Pa}/\mathrm{Hz}$ would be typical.

The author's view is that decibels are inherently a measure of energy or power, and hence the units should be quoted in that fashion.

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This report has been written as part of a joint research project between Defence Research and Development Canada (DRDC) – Atlantic and the Offshore Energy Environmental Research (OEER) Association. One of the research goals is to increase the understanding of alternative geophysical exploration methods for the marine environment, and this report highlights many of the issues that need to be addressed when applying frequency-modulated (FM), or swept-frequency waveforms to geophysical prospecting of the ocean seabed. A key component of the signal processing for FM pulses is the matched filter, which is described at an introductory level. Examples are given to illustrate the excellent resolving capability of the linear-FM pulse, which is a consequence of the pulse-compression property of the matched filter. Also discussed are other, more general, kinds of signal modulation. Comments are then made on issues that arise in the specific application of FM waveforms to reflection seismology in the marine environment. Finally, the report also contains an Annex on the physical units relevant to marine seismology.

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